

Why are the borders of Palestine/Israel and Wallonia/Flanders so different?: Entropic Analysis of a Schelling model with hierarchically structured initial conditions.

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Argentina



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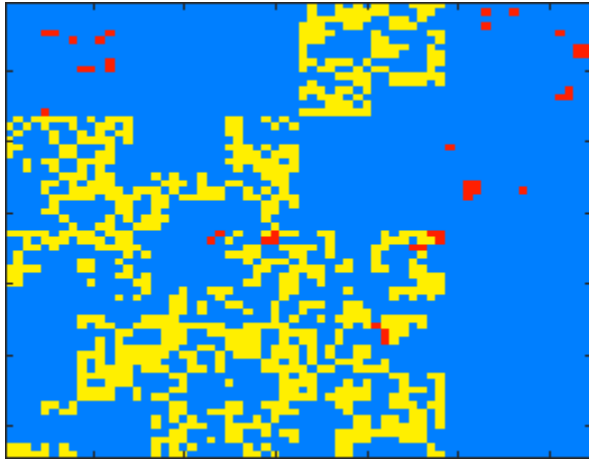
Flanders/Wallonia (Belgium)



[2] A. Rutherford, D. Harmon, J. Werfel, A.S. Gard-Murray, S. Bar-Yam, A. Gros, R. Xulvi-Brunet and Y. Bar-Yam (2014). Good Fences: The Importance of Setting Boundaries for Peaceful Coexistence. PLoS ONE 9(5): e95660. <https://doi.org/10.1371/journal.pone.0095660>

[3] Y. Bar-Yam, *Making Things Work: Solving Complex Problems in a Complex World* (NECSI Knowledge Press, Boston, 2004).

T.C. Schelling (1971). Dynamic Models of Segregation. Journal of Mathematical Sociology 1.2: 143-186.



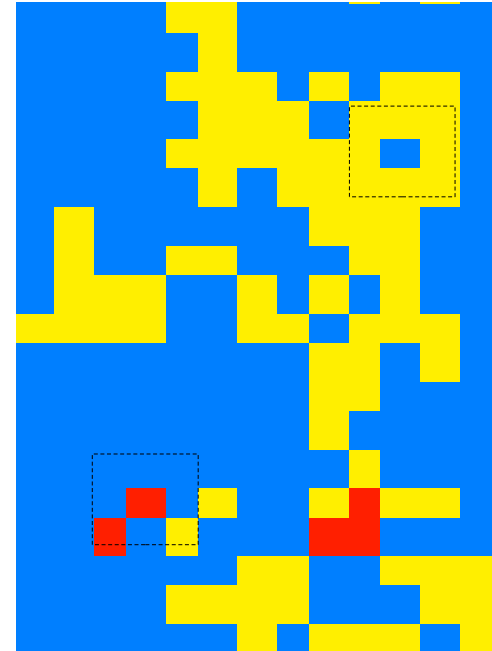
yellow: the
minority
(5%-40%)

blue: the
majority

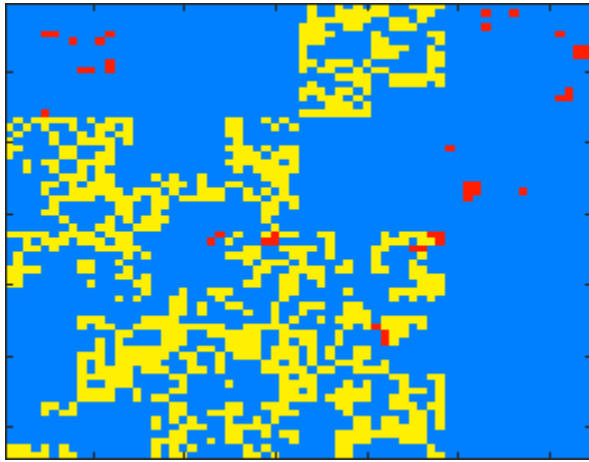
red: empty
sites (1%)

$t \rightarrow t + dt$

“if an agent is surrounded by more individuals belonging to the opposite social group, he will move to an empty place in which he becomes surrounded by comparatively more individuals of his own social group”



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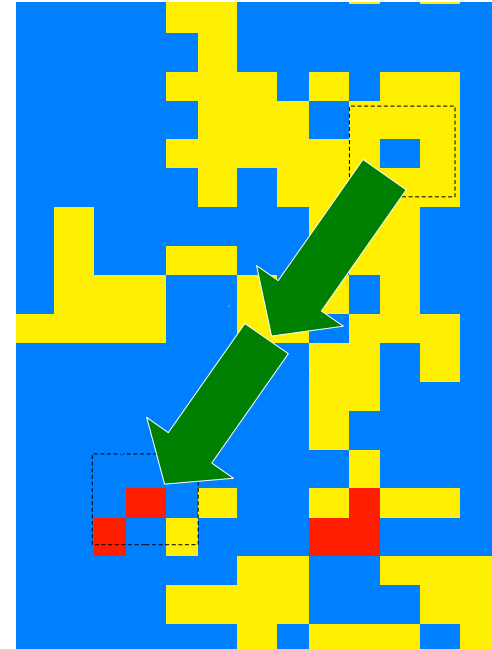
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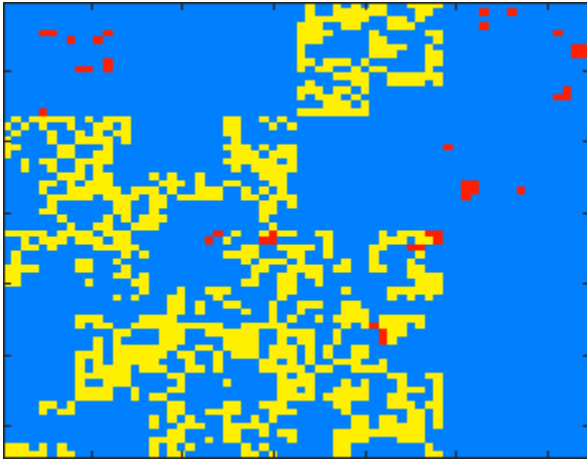
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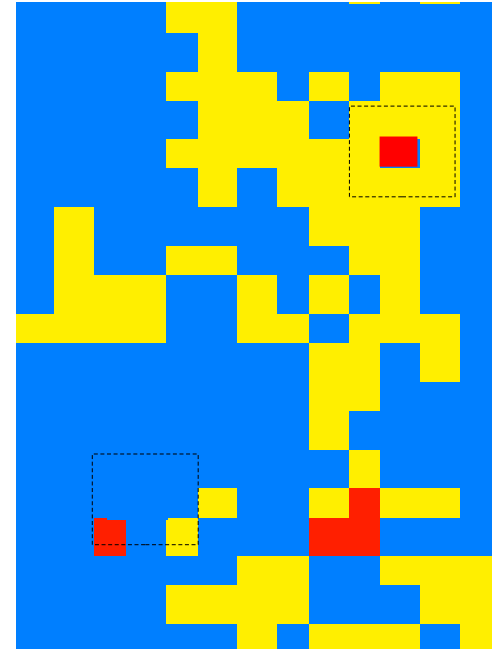
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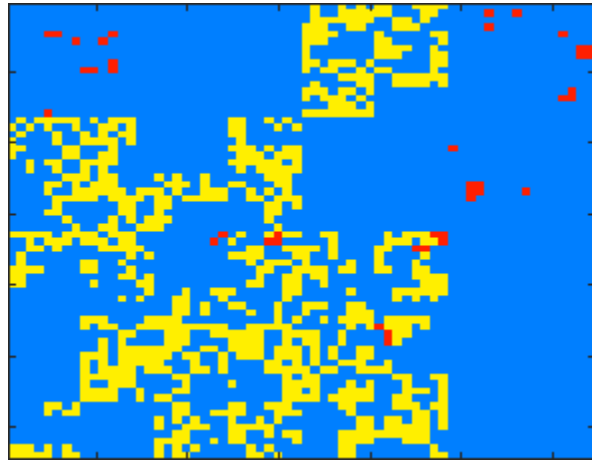
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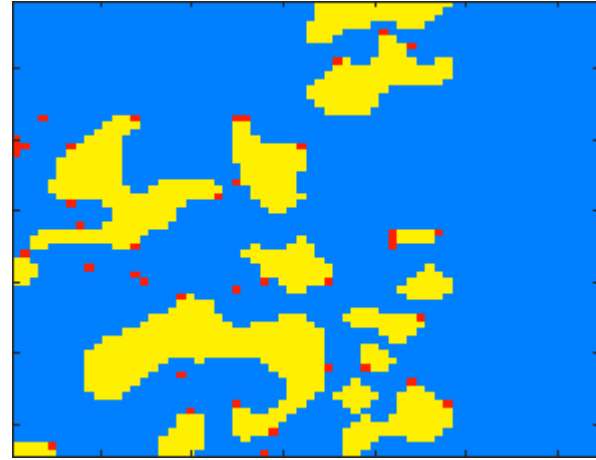
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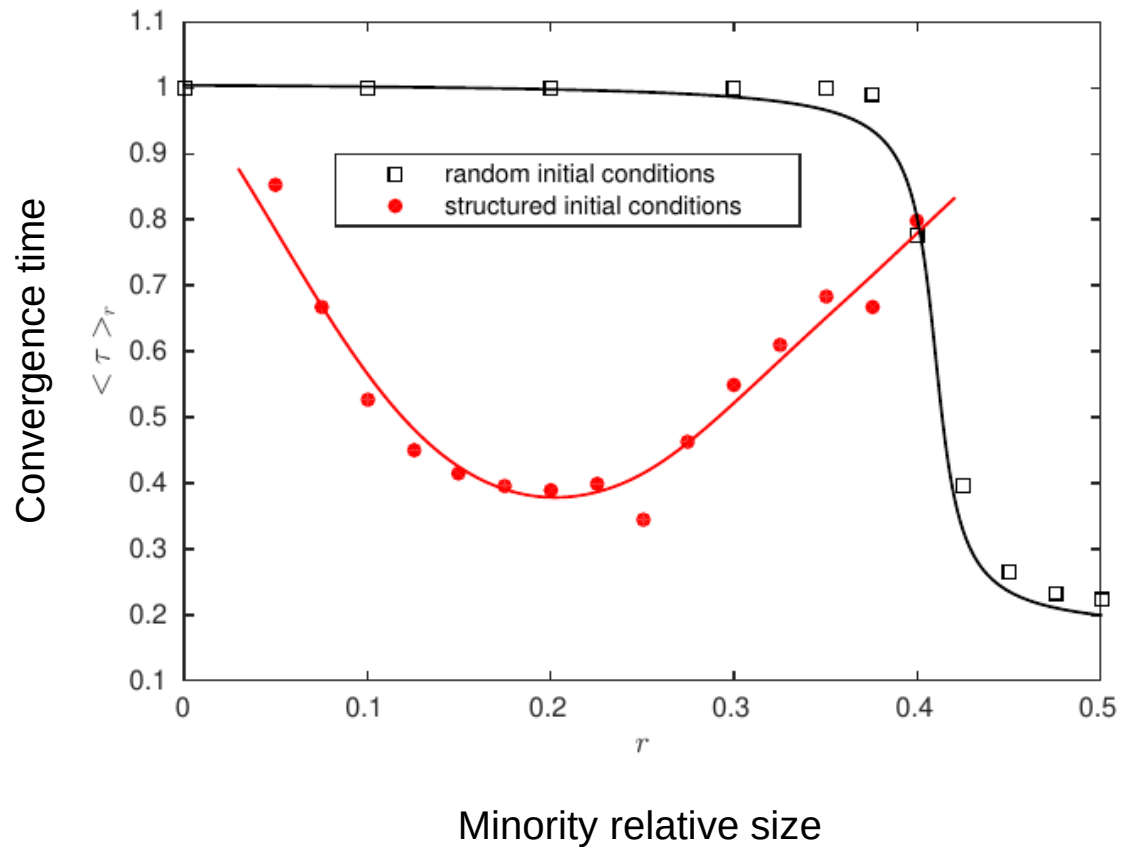
$t = 0$



Schelling

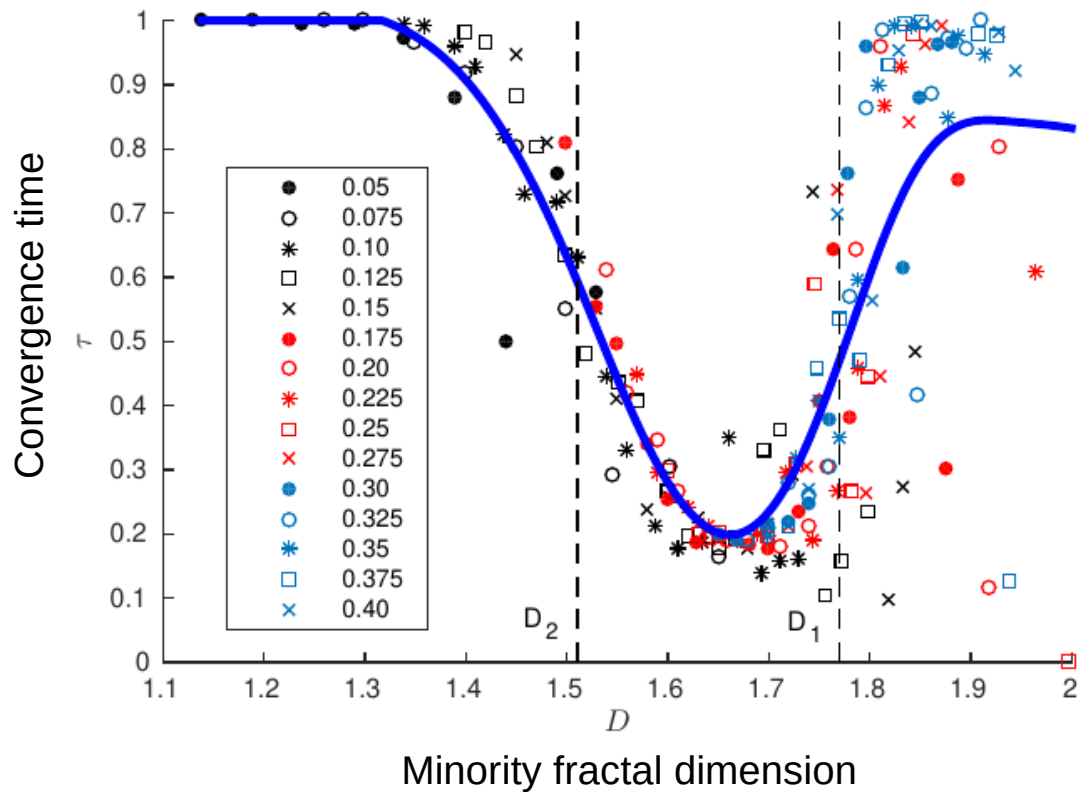


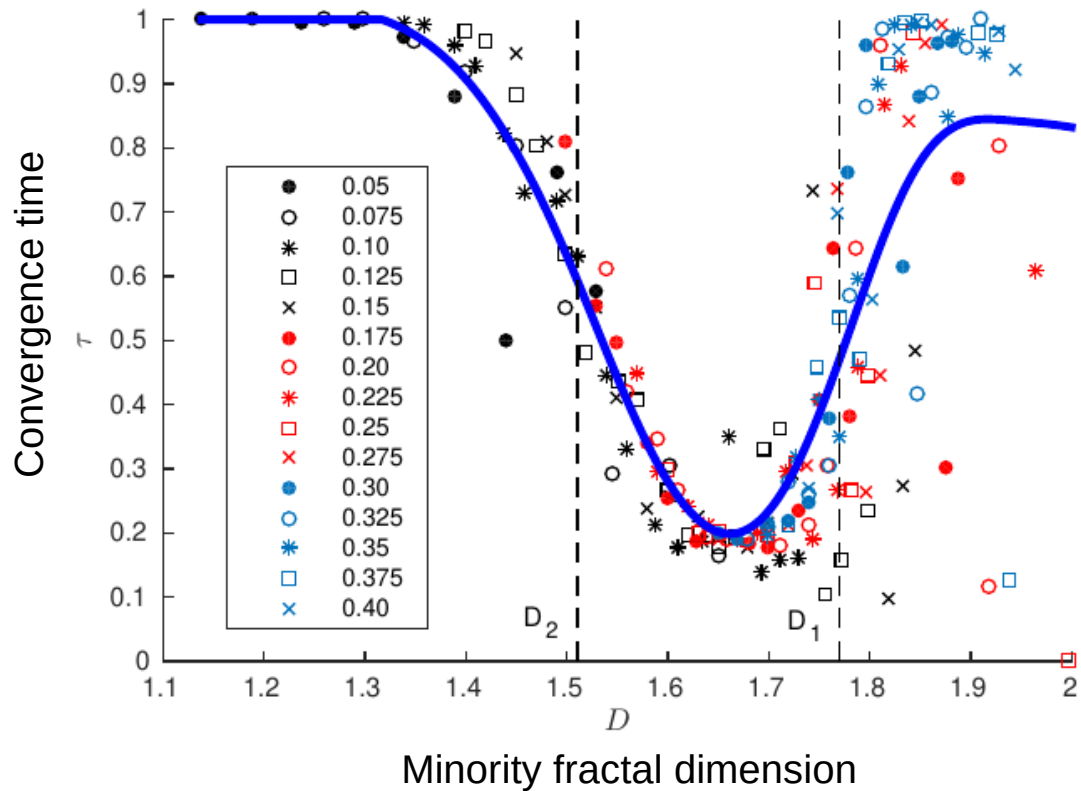
$t \rightarrow \infty$



$$\tau = \frac{2}{\pi} \tan^{-1} \frac{T}{T_0}$$

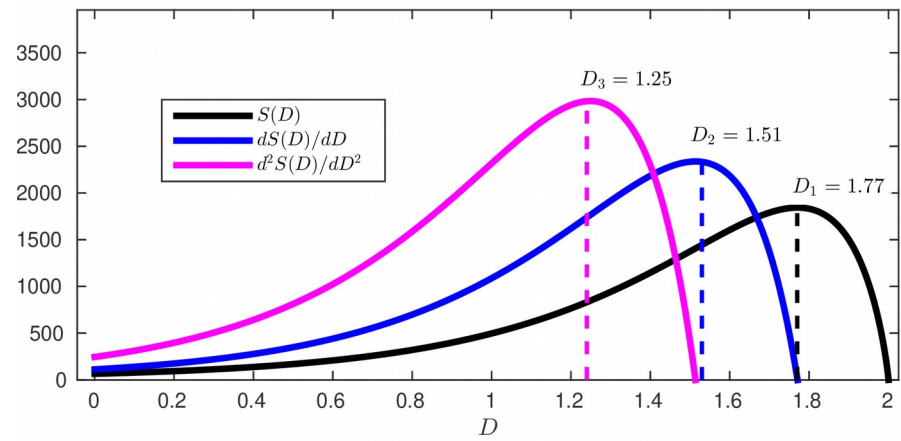
$\tau \rightarrow 1$ means $T \rightarrow \infty$

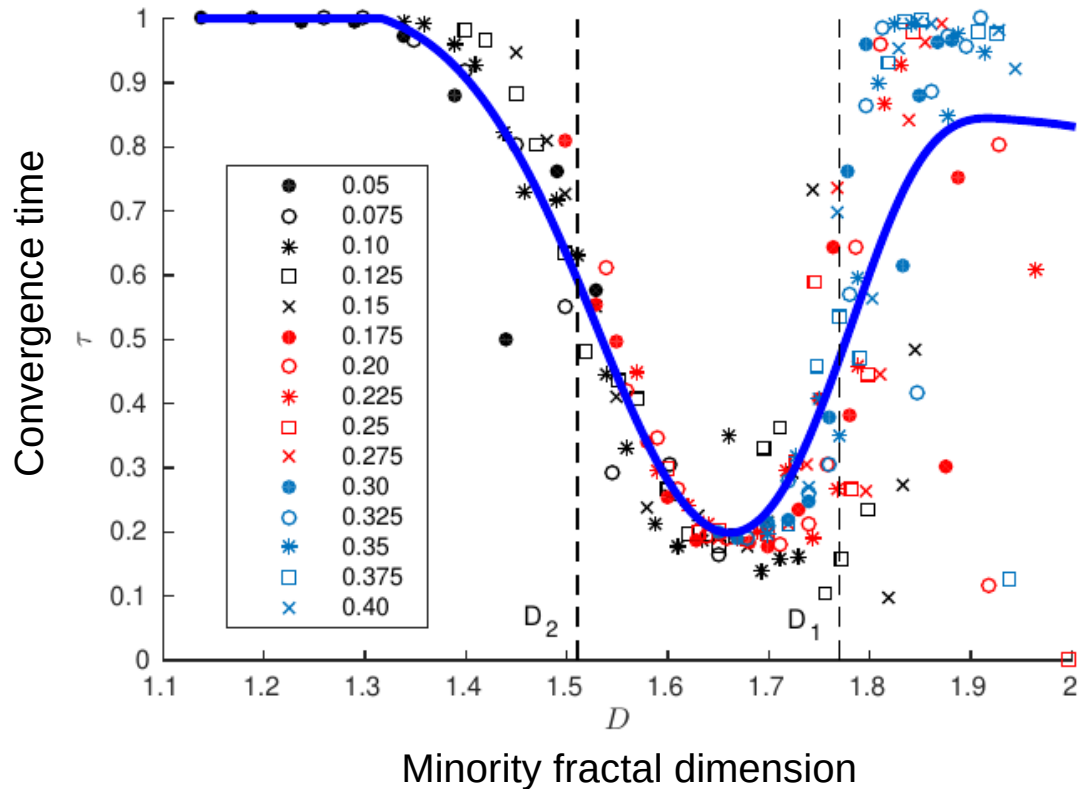




M.E. Gaudio (2015). An Entropical Characterization for Complex Systems Becoming out of Control. *Physica A* **440**, 185.

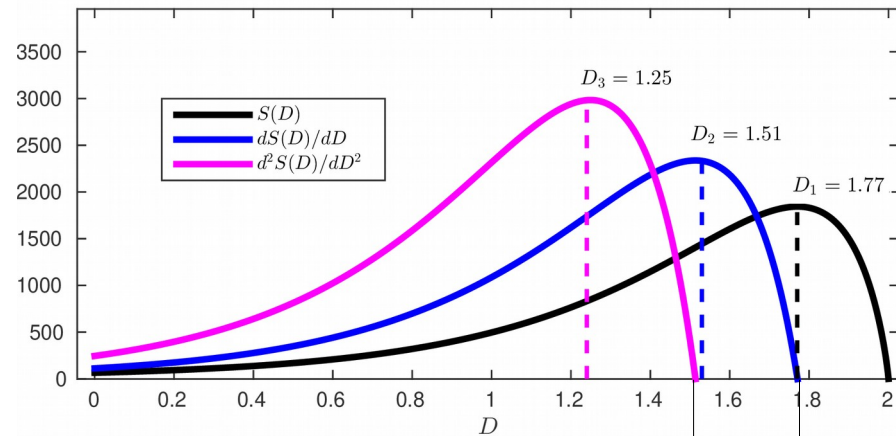
$$S(d) = \log \Omega(d) \quad \text{where } \Omega(d) \text{ is the total number of possible patterns having fractal dimension } d \text{ in a } \lambda \times \lambda \text{ array of pixels}$$





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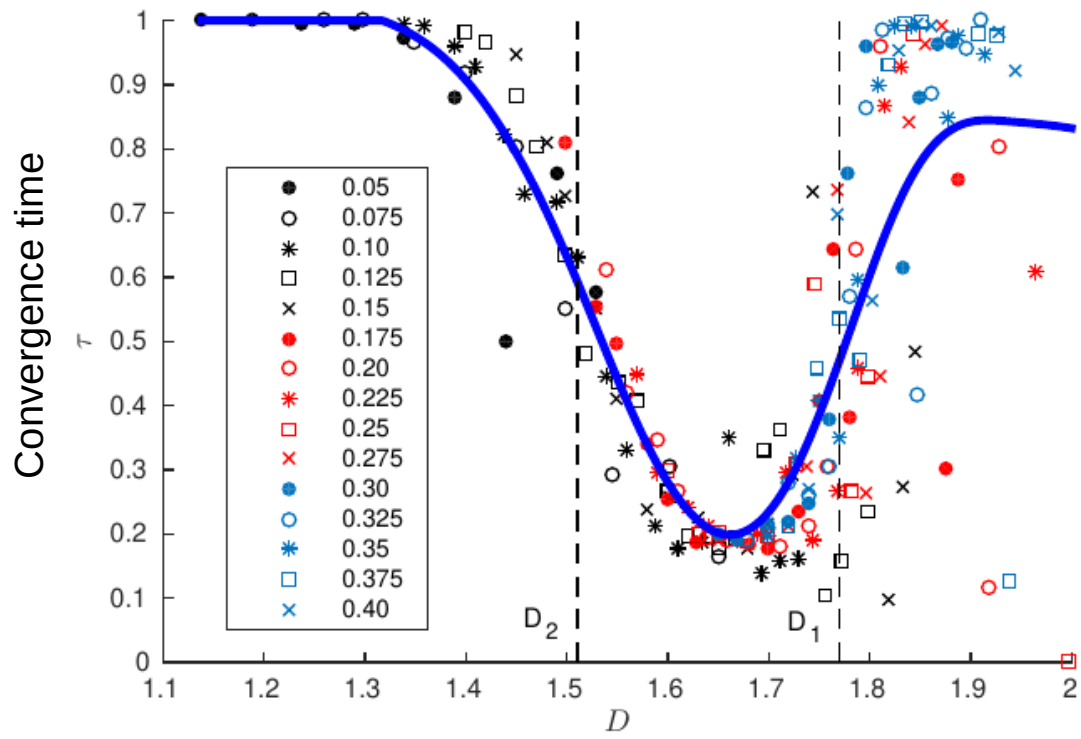
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$D_2 < D < D_1$
the most uncontrollable regime

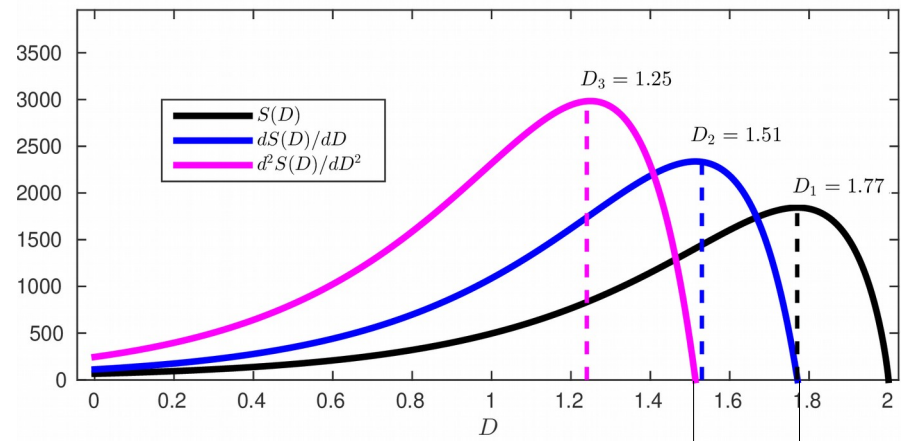
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Minority fractal dimension

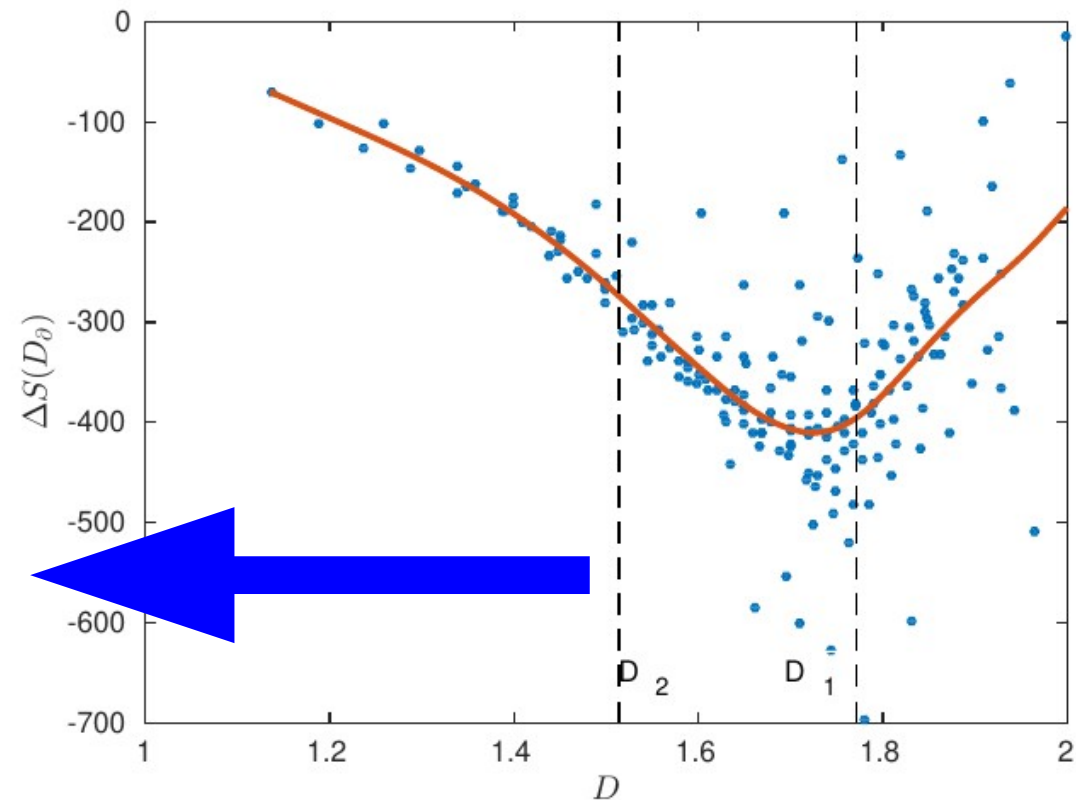
$1.51=D_2 < D < D_1=1.77$
The most segregationist regime



$D_2 < D < D_1$
the most uncontrollable regime

M.E. Gaudiano (2015). An Entropical Characterization for Complex Systems Becoming out of Control. *Physica A* **440**, 185.

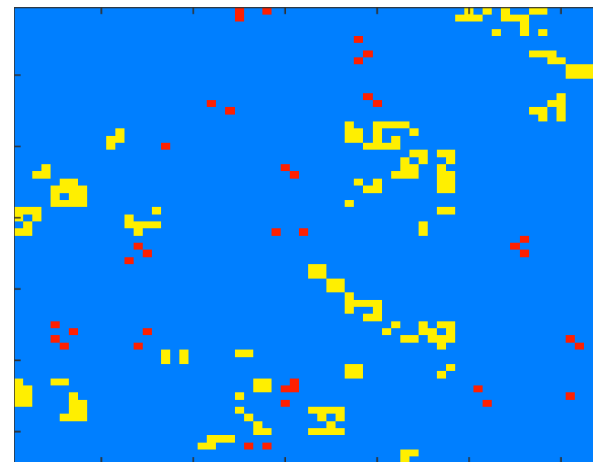
$$S(d) = \log \Omega(d) \quad \text{where } \Omega(d) \text{ is the total number of possible patterns having fractal dimension } d \text{ in a } \lambda \times \lambda \text{ array of pixels (see$$



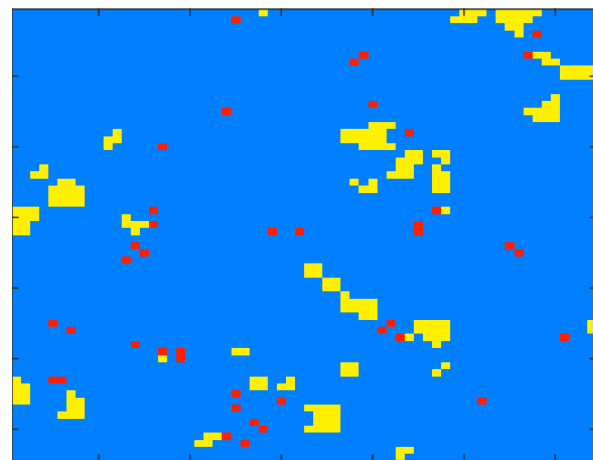
low entropy variations
=
less temporal evolution.

“ *status quo tends to prevail* ”

$t = 0$



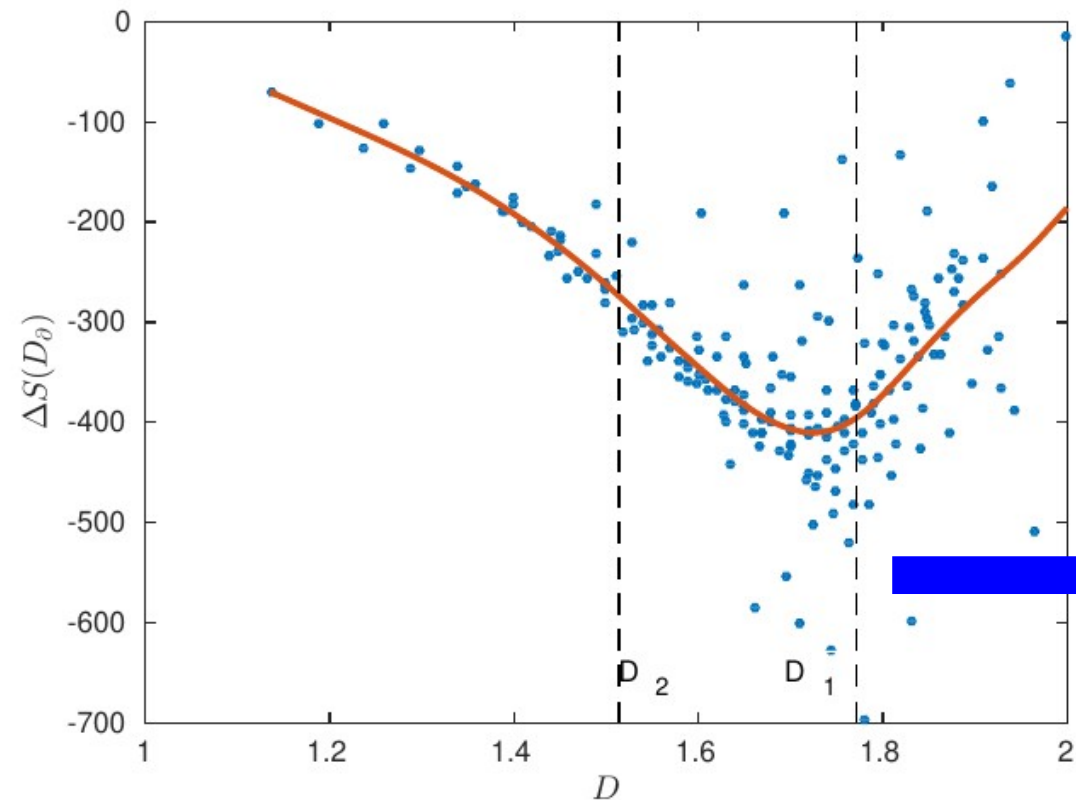
$t = \infty$



$D=1.14, r=0.05$

M.E. Gaudiano (2015). An Entropical Characterization for Complex Systems Becoming out of Control. Physica A **440**, 185.

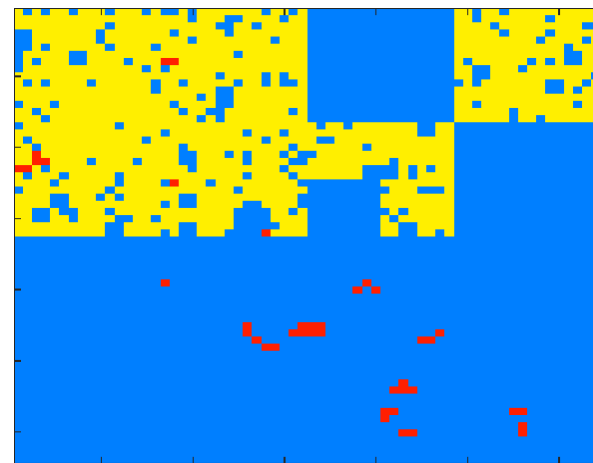
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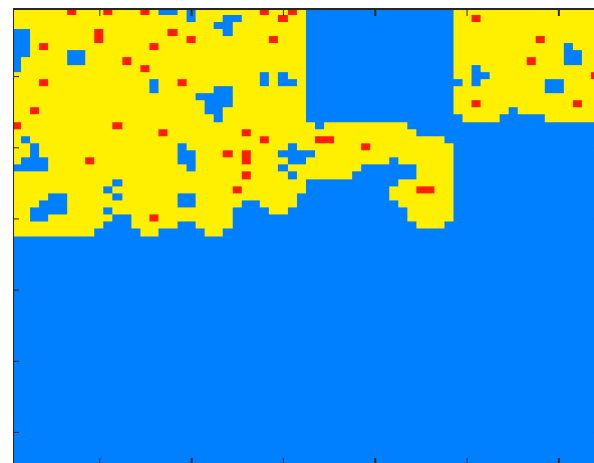
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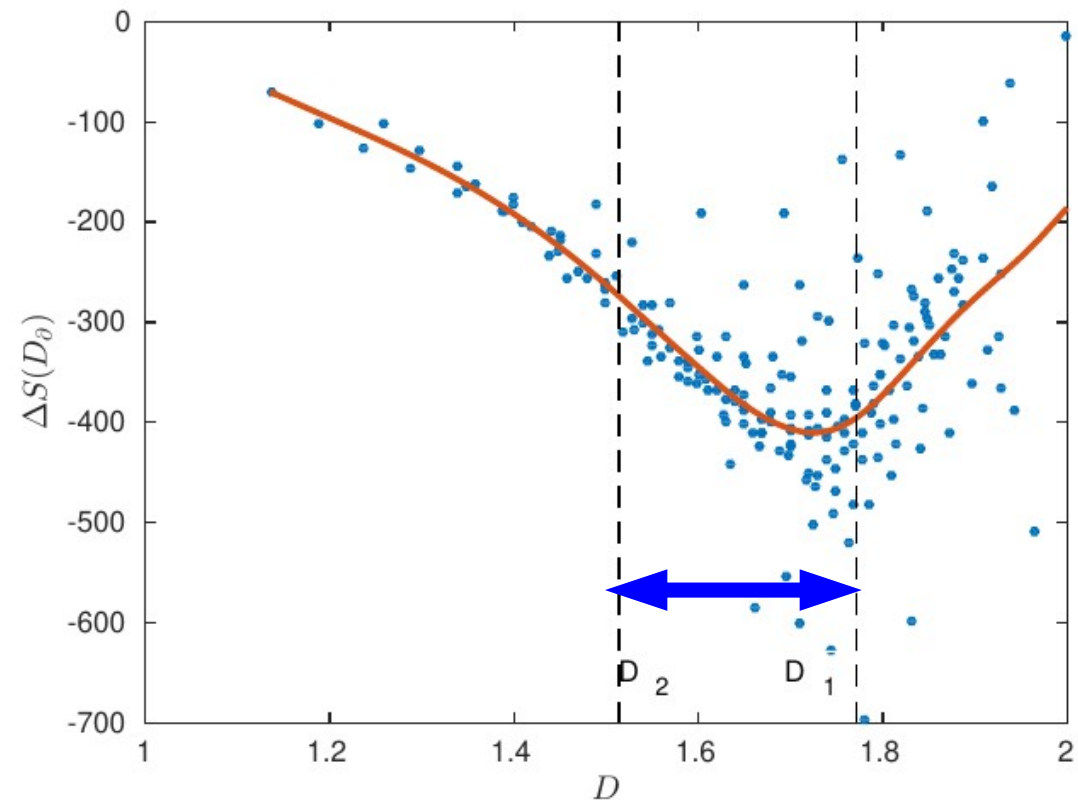
$t = \infty$



$D=1.89, r=0.30$

M.E. Gaudiano (2015). An Entropical Characterization for Complex Systems Becoming out of Control. *Physica A* **440**, 185.

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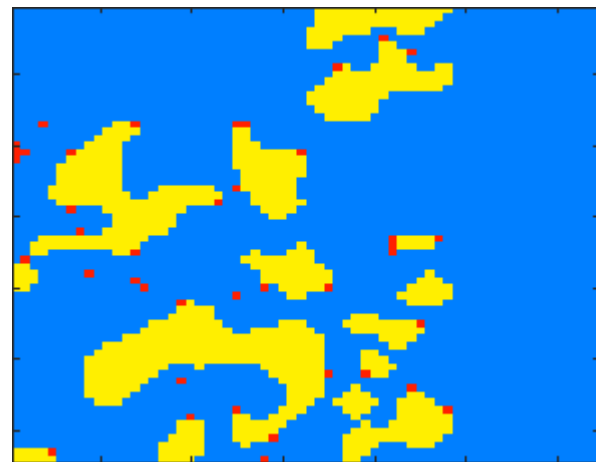


$D_2 < D < D_1$
the most segregationist regime

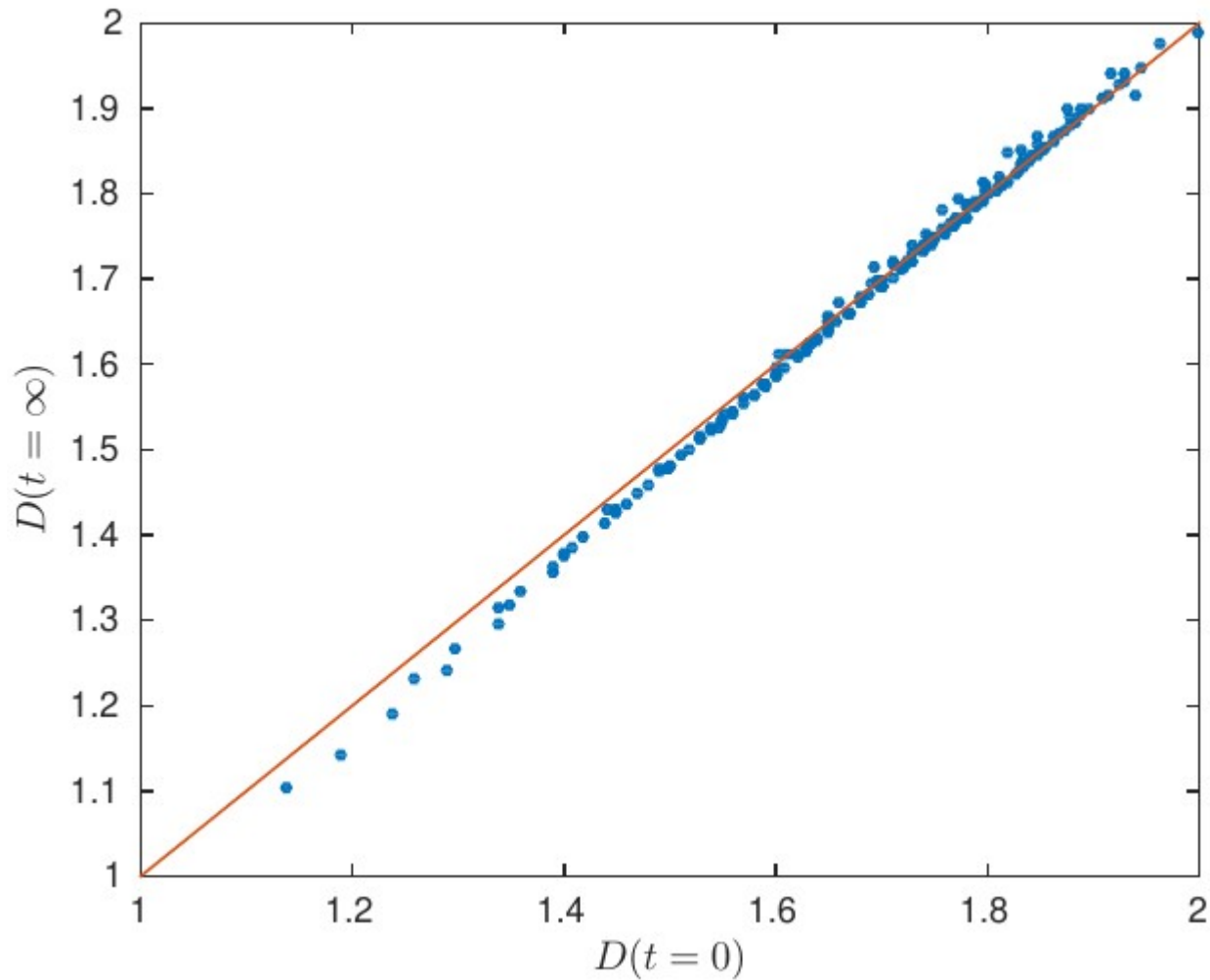
$t = 0$



$t = \infty$



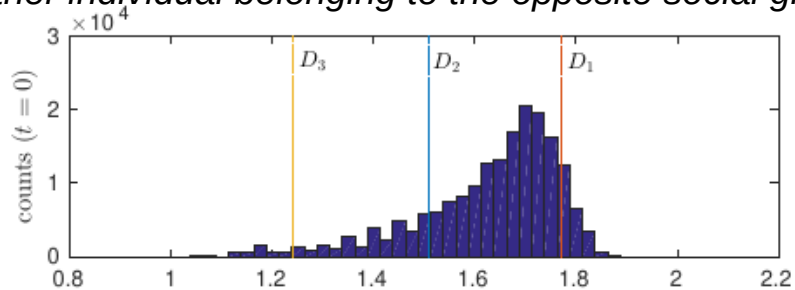
$D=1.59, r=0.20$



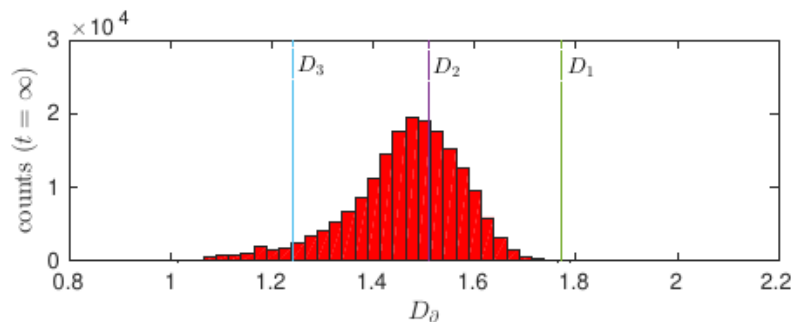
Minority fractal dimension is basically time-invariant

“System’s dynamics basically occurs at the borders”

“An agent belongs to the border of his social group when he is next to another individual belonging to the opposite social group”



$t = 0$



$t = \infty$

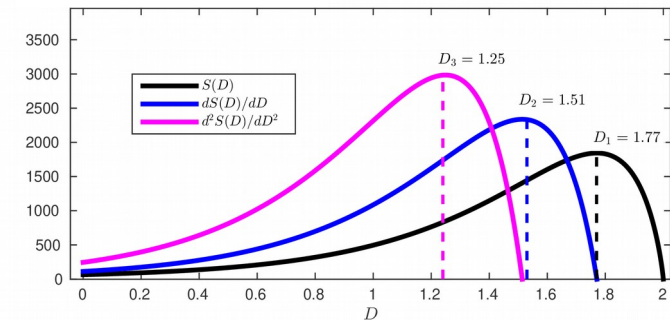
Minority border dimension

$$D_\partial \lesssim D_1$$

(Borders have a basically volatile nature)

$$S(d) = \log \Omega(d)$$

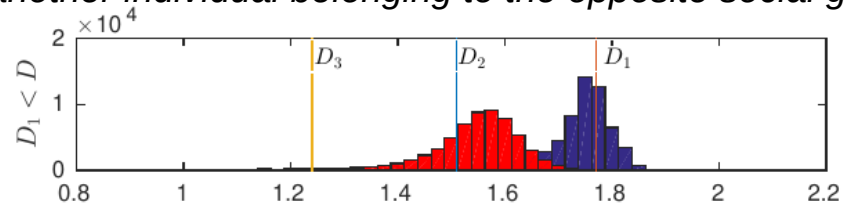
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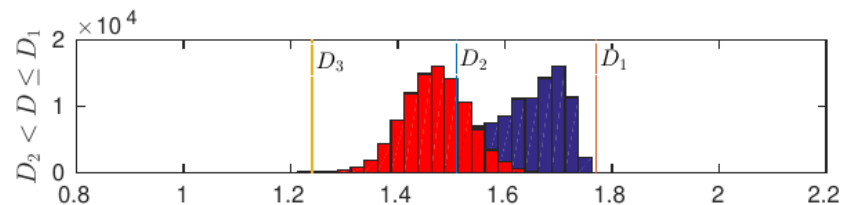
$D_2 < D < D_1$
The most uncontrollable regime

$$D_2 < D_\partial < D_1$$

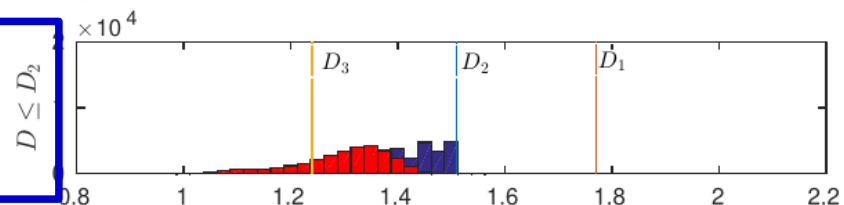
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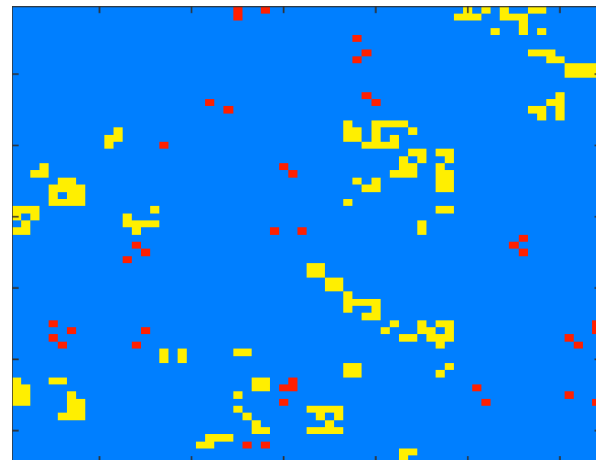
$t = 0$



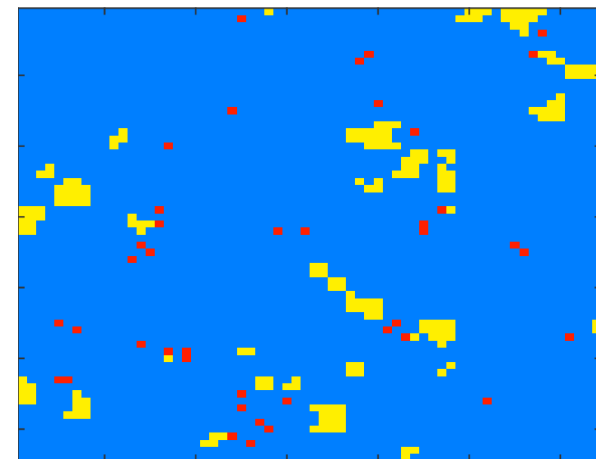
$t = \infty$



Minority border dimension



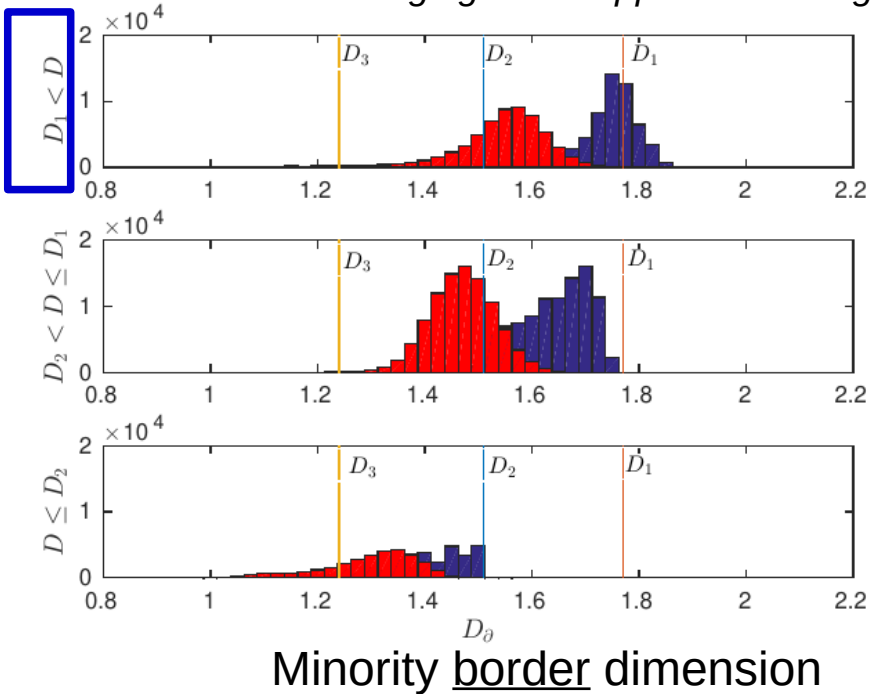
$t = 0$



$t = \infty$

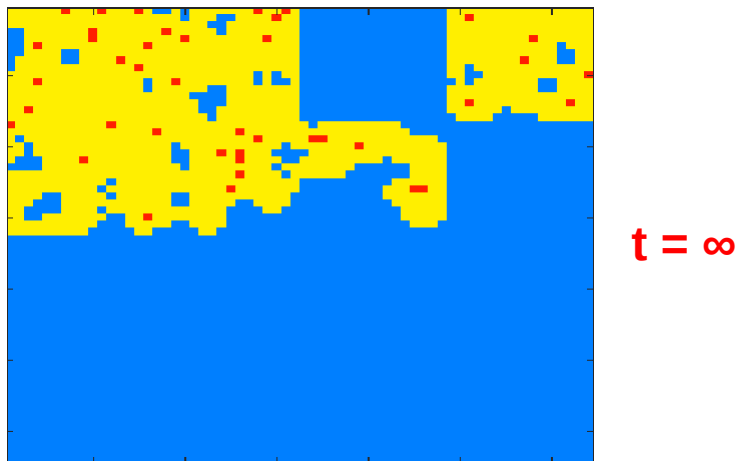
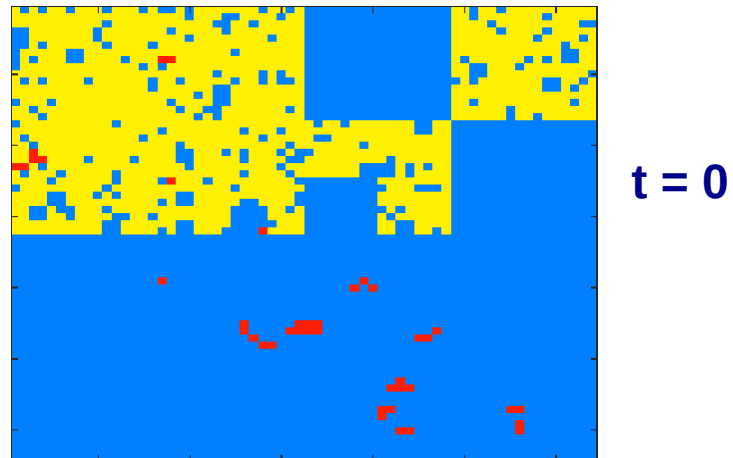
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“An agent belongs to the border of his social group when he is next to another individual belonging to the opposite social group”



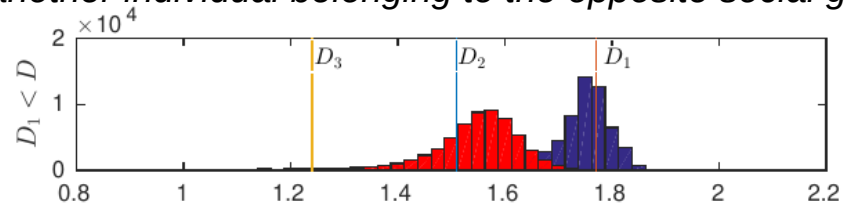
$t = 0$

$t = \infty$

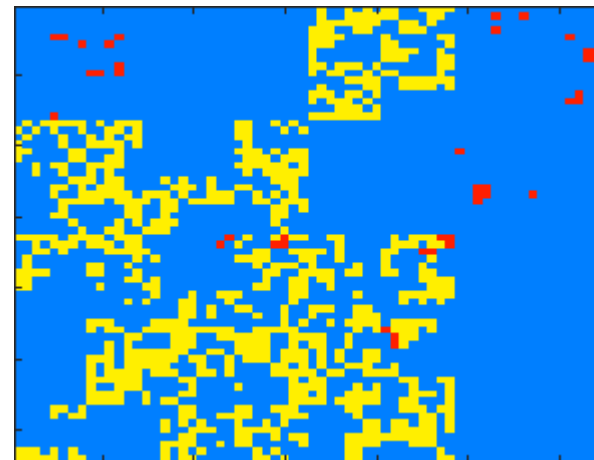


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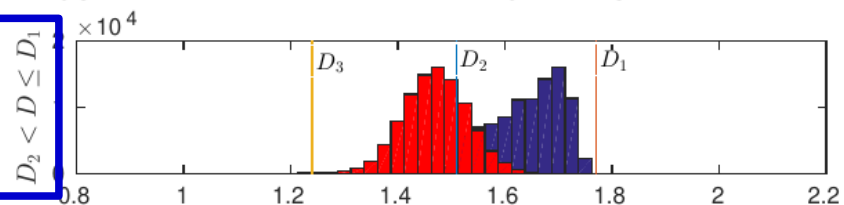
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$t = 0$



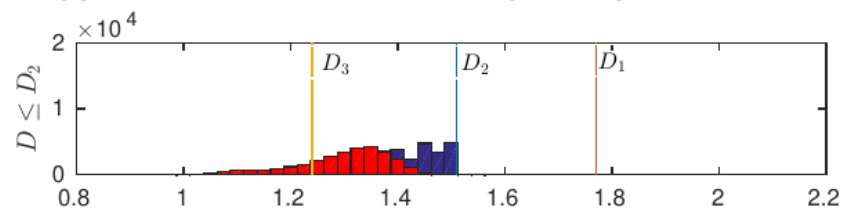
$t = 0$



$t = \infty$



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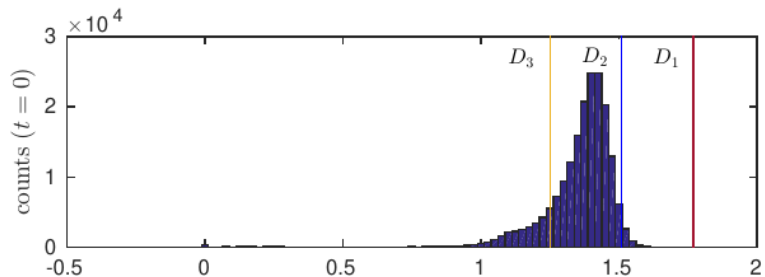


Minority border dimension

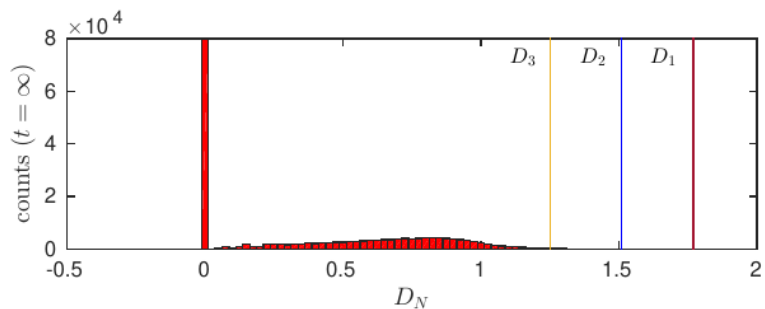
$$D_2 < D < D_1$$

The border substantially changes its shape:
There occurs an entropic regime change

“An agent is unsatisfied when he is surrounded by more individuals belonging to the opposite social group”



$t = 0$



$t = \infty$

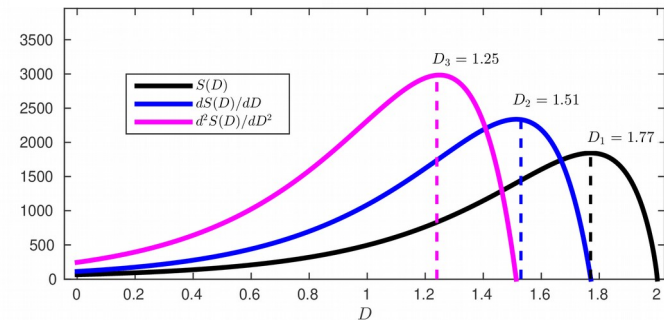
fractal dimension of the unsatisfied agents

The set of unsatisfied agents (usually undetectable) is even more volatile than the borders.

$$D_N \lesssim D_2$$

$$S(d) = \log \Omega(d)$$

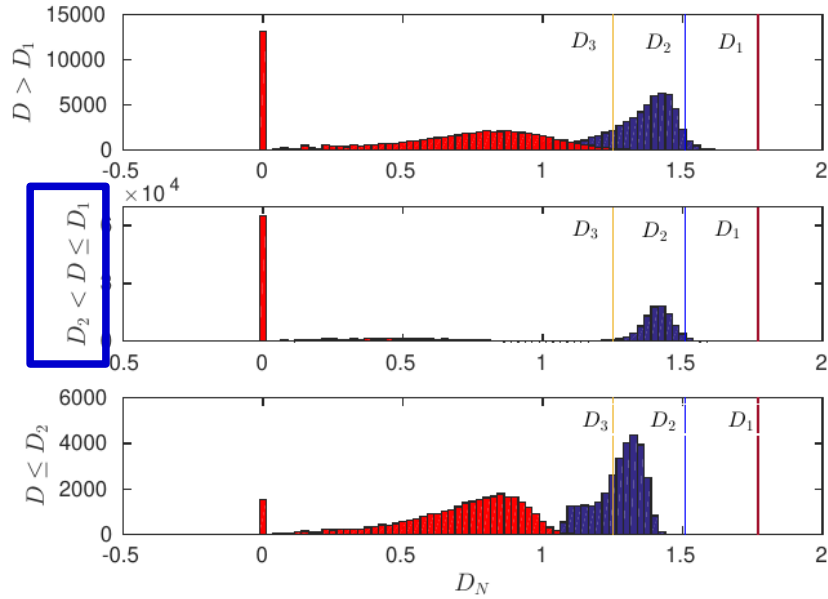
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$D_2 < D < D_1$
The most uncontrollable regime

$D_3 < D_N < D_2$
The most uncontrollable regime of a space of dimensionality D_1

“An agent is unsatisfied when he is surrounded by more individuals belonging to the opposite social group”



fractal dimension of the unsatisfied agents

$$D_2 < D < D_1$$

The set of unsatisfied agents substantially changes its shape: there occurs an entropic regime change

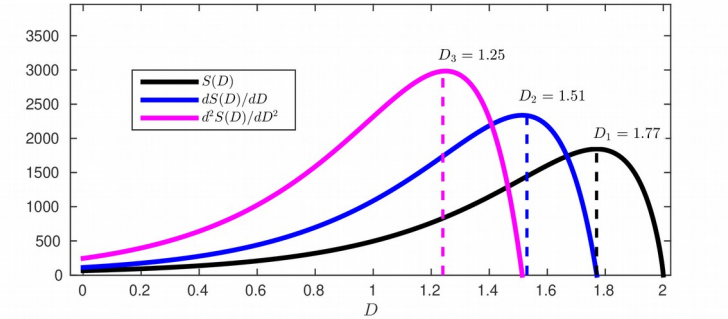
$t = 0$

$t = \infty$

M.E. Gaudiano (2015). An Entropical Characterization for Complex Systems Becoming out of Control. *Physica A* **440**, 185.

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$D_2 < D < D_1$
The most uncontrollable regime

CONCLUSIONS:

1- Basic Schelling model + structured initial conditions = quite different temporal evolution in comparison with random initial conditions (widely found in the Literature). Many social aspects are naturally reproduced without introducing artificial parameters into the model.

“On the role of structured initial conditions in the Schelling model”.

M. Gaudiano, J. Revelli. Physica A, october 2021,
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- 3- For D outside of $D_2 < D < D_1$: Tolerance and status quo prevail. Time just passes by with a relatively non-substantial system evolution.

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3- For D outside of $D_2 < D < D_1$: Tolerance and status quo prevail. Time just passes by with a relatively non-substantial system evolution.

4- $D_2 < D < D_1$: Segregation easily blows up. Existence of recurrent segregation processes [include into the model a kind of regeneration mechanism (like e.g percolation)]. It corresponds to the out-of-control regime predicted in the general complex system formalism of [Gaudiano, 2015, Physica A 440, 185].

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**THANKS FOR
YOUR ATTENTION!!!**

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