Majority Vote and BCS model on Complex Networks

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18 de outubro de 2021

Introdution

- Majority-vote model (MVM) on square lattice (2D)- *M.J. Oliveira, J. Stat. Phys.* **66**, *273 (1992)*;
- MVM on Erdös-Rényi random graphs (ER)- L.F.C. Pereira, F.G. Brady Moreira, Phys. Rev. E **71**, 016123 (2005);
- MVM on directed ER- F.W.S. Lima, A.O. Sousa, M.A. Sumuor, Physica A **387**, 3503 (2008);
- On infinite dimensions- Biswas-Chatterjee-Sen (BCS model), *Biswas, S.; Chatterjee, A.; Sen, P. Physica A* **391**, *3257 (2012)*;

Introdution

- BCS model on 2D and 3D- Sudip Mukherjee and Arnab Chatterjee Phys. Rev. E **94**, 062317 (2016);
- BCS model on directed Barabási-Albert networks (BANs)- F. Welington S. Lima and J. A. Plascak, Entropy **21**, 942 (2019).
- BCS model on BANs- G.A. Alves, T.F.A. Alves, F.W.S. Lima, A. Macedo-Filho, Physica A **561**, 125267 (2021);
- BCS model on Apollonian networks (ANs)- F.W.S. Lima, Muneer A. Sumour, André A. Moreira, Ascânio D. Araújo, Physica A **571**, 125834 (2021).

Introdution- BCS model variants

- Nuno Crokidakis and Celia Anteneodo, Phys. Rev. E **86**, 061127 (2012).
- Nuno Crokidakis, Physica A **414**, 321 (2014).
- Allan R. Vieira and Nuno Crokidakis, Physics Letters A **380**, 2632 (2016).
- Marcelo A Pires, André L Oestereich and Nuno Crokidakis, J. Stat. Mech., 053407 (2018).
- A.L. Oestereich, M.A. Pires, S.M. Duarte Queirós, N. Crokidakis, Chaos, Solitons and Fractals bf137, 109893 (2020).

MVM: dynamics evolution

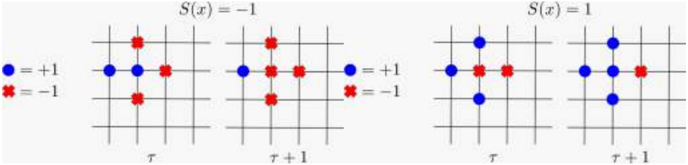
The MVM dynamics evolution is as follows.

- Initially, we have a spin variable σ with values ± 1 at each node of the network.
- At each time step, we try to spin-flip a node.
- The flip is accepted with probability

$$w_i(\sigma) = \frac{1}{2}\left[1-(1-2q)\sigma_i S\left(\sum_{\delta=1}^{k_i}\sigma_{i+\delta}\right)\right], \quad S(x) = \begin{cases} 1, & \text{if } x>0; \\ 0, & \text{if } x=0; \\ \text{-1}, & \text{if } x<0; \end{cases}$$

- The i site can follow the neighboring majority opinion with probability q ("Social Temperature") and the minority sign with probability $p = 1-q$
- $S(x)$ is a signal function, associated to neighborhood majority opinion and sum runs over the number k_i of neighbors of i-th spin.

MVM - signal function $S(x)$



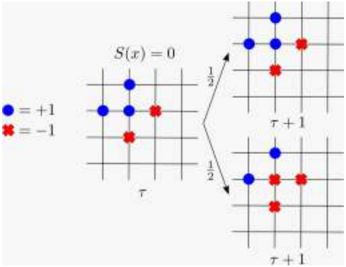


Figura: Signal function $S(x)$.

BCS model-discreet version: dynamics evolution-Pairwise interactions

The Kinetic Monte Carlo rules of the BCS model are written as follows:

- (i) For each network site, one agent or individual is assigned with discrete opinion variables $o_i(t)$ between only three values -1, 0, and $+1$.
- (ii) For each time step, we randomly select a network site to be updated;
- (iii) Next, we randomly select only one of its bonds and choose the affinity μ_{ij} of the bond for every pass of the dynamics. Here, the affinity parameter is a discrete variable and assumes a value $+1$, which can be turned negative with a probability q. The parameter q acts as external noise, modeling local discordances;

BCS - Biswas-Chatterjee-Sen model

- (iv) The two sites i and j that share the selected bond are updated according to the following:

$$o_i(t+1) = o_i(t) + \mu_{ij} o_j(t) \,, \tag{2}$$

$$o_j(t+1) = o_j(t) + \mu_{ij} o_i(t) \,, \tag{3}$$

  where the variables $o_i(t)$ and $o_j(t)$ are the previous opinion states while the $o_i(t+1)$ and $o_j(t+1)$ stand for the updated opinion states of the two sites i and j, respectively.

- (v) If any updated opinion state is larger (lower) than $+1$ (-1), then it is made equal to $+1$ (-1), to preserve the opinion states in the interval $[-1, 1]$.

Complex Networks: Barabási-Albert networks, Erdös-Rényi random graphs, Apollonian networks

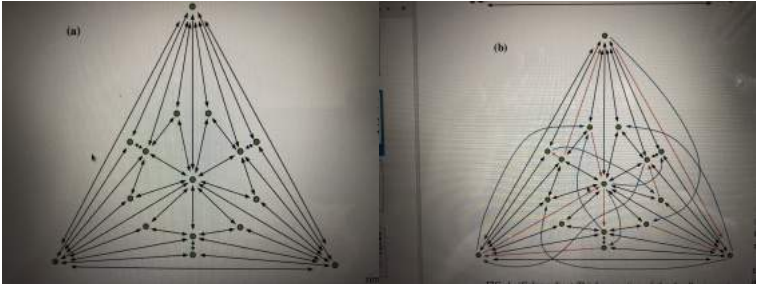


Figura: Third generation of the Apollonian network before (a) and after (b) redirecting the links with probability $p = 0.1$. In (a) the black arrows represent all (incoming and outgoing) links, whereas in (b) the red arrows indicate the preserved incoming links, and the blue arrows indicate the redirected outgoing links. The black arrows indicate links not affected by redirecting.

Observables

Order Parameter or average opinion in the time step

$$O = |\sum_{i}^{N} o_i|/N. \tag{4}$$

Order Parameter in function of noise q

$$O(q) = \left[\langle O \rangle_t\right]_{av}, \tag{5}$$

Susceptibility

$$OF(q) = N\left[\langle O^2 \rangle_t - \langle O \rangle_t^2\right]_{av}, \tag{6}$$

Binder cumulant

$$O_4(q) = 1 - \left[\frac{\langle O^4 \rangle_t}{3\langle O^2 \rangle_t^2}\right]_{av}, \tag{7}$$

Finite-size scaling relations

$$O(q) = N^{-\beta/\nu} f_O((q - q_c)N^{1/\nu}), \tag{8}$$

$$OF(q) = N^{\gamma/\nu} f_{OF}((q - q_c)N^{1/\nu}), \tag{9}$$

where ν, β, and γ are the usual critical exponents of the correlation length, order parameter and susceptibility, respectively.

Results: directed Barabási-Albert networks

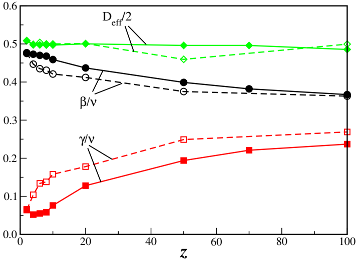


Figura: Critical exponents ratio β/ν and γ/ν, and half value of the effective dimension D_{eff} as a function of the connectivity z. Full symbols correspond to the present BCS model, and open symbols to the MVM , both on the same DBAN. Full and dashed lines are only guide to the eyes.

Results: directed Small-World-Apollonian networks

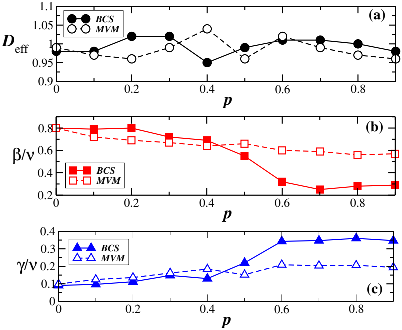


Figura: The effective dimension D_{eff} (a), critical exponents ratio β/ν (b) and critical exponents ratio γ/ν (c) as a function of the probability p.

Results: Erdös-Rènyi random graphs

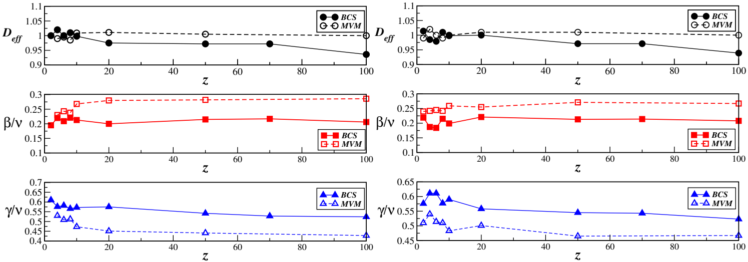


Figura: D_{eff}, β/ν, γ/ν as a function of the connectivity z. DER(left) and ER(right).

Results: Erdös-Rènyi random graphs

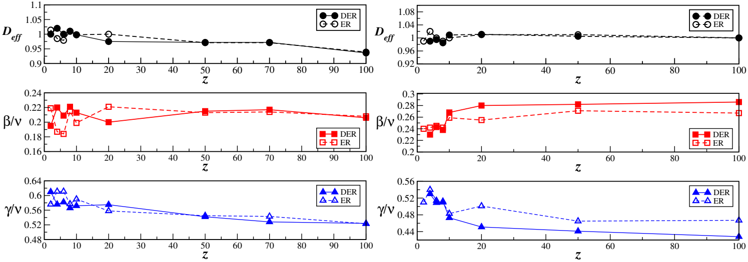


Figura: D_{eff}, β/ν, γ/ν as a function of the connectivity z. BCS (left) and MVM (right) model.

Conclusion

Finaly, we can see that for both models, the ratio exponents can be in different universality class:

- Ising exponents': BCS and MVM on square lattice
- Mean-Field: BCS on Barabási-Albert newtorks (continuos opinions)
- Other: BCS and MVM on directed Small-World-Apollonian networks, Erdös-Rènyi random graphs, and directed Barabási-Albert newtorks.

Big master and friend

Figura: Dietrich Stauffer- 2005 until 2019.