

Few-Body Physics of Emitter(s) Coupled to Bath

Doerte Blume

Center for Quantum Research and Technology (CQRT) Department of Physics and Astronomy The University of Oklahoma, Norman.

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All Four Lectures At A Glance



Emitter(s) Coupled to Cavity Array

Lecture 4: Emitter(s) coupled to cavity array \rightarrow Radiation dynamics.

Want to consider situation where the cavity array exhibits universal physics. More specifically, want universal two-body bound state.

Very rough idea:

Emitter = two-level system (atom, quantum dot,...): States $|e\rangle$ and $|g\rangle$.

Initialize in $|e\rangle$.

How does probability P_e to be in state $|e\rangle$ change?

Depends on

- how the emitter is coupled to cavity array and
- the characteristics of the cavity array.

Broad Motivation



Very broad motivation of our work:

What can we learn by having two probes as opposed to one probe? We gain access to spatial correlations (not accessible with one probe).

How much more information can we extract if we use two probes that are entangled?

More Specifically



Perspective 1: Use the mode structure of the environment to modify the radiance of the system (e.g., super- and subradiance).

Perspective 2: Deduce the mode structure from the dynamics of the probe.

How Does Cavity Array Look Like?

Want to consider situation where the cavity array exhibits universal physics. More specifically, want universal two-body bound state.

If we want to have a bound state, then we need to have photon-photon interactions: assume presence of Kerr-like non-linearity ($U \neq 0$). This is an effective or induced interaction (photons are massless and non-interacting).

Tight-binding + non-linearity:

$$\hat{H}_{b} = \hbar\omega_{c}\sum_{n=1}^{N} \hat{a}_{n}^{\dagger}\hat{a}_{n} - J\sum_{n=1}^{N} \left(\hat{a}_{n}^{\dagger}\hat{a}_{n+1} + \hat{a}_{n+1}^{\dagger}\hat{a}_{n}\right)$$
$$+\frac{U}{2}\sum_{n=1}^{N} \hat{a}_{n}^{\dagger}\hat{a}_{n}^{\dagger}\hat{a}_{n}\hat{a}_{n}.$$
$$Discrete space:$$
$$n = 1, 2, \cdots, N!$$
$$\hat{n}_{n}(\hat{n}_{n} - 1)$$



Photon-Photon Bound State: When Is It Universal?

Images of 1D and 2D photonic crystals



Question: If we have a regular crystal structure with lattice spacing *a*, what is the condition on the size of the two-body bound state for the state to be universal?

Figure from A. Piper, D. V. Timotijevic, and D. M. Jovic, Phys. Scr. T157, 014023 (2013).



Tight-binding + non-linearity: $\hat{H}_{b} = \boxed{\hbar\omega_{c}\sum_{n=1}^{N}\hat{a}_{n}^{\dagger}\hat{a}_{n} - J\sum_{n=1}^{N}\left(\hat{a}_{n}^{\dagger}\hat{a}_{n+1} + \hat{a}_{n+1}^{\dagger}\hat{a}_{n}\right)} + \frac{U}{2}\sum_{n=1}^{N}\hat{a}_{n}^{\dagger}\hat{a}_{n}\hat{a}_{n}\hat{a}_{n}.$

Photon number operator: $\widehat{N} = \sum_{n=1}^{N} \widehat{a}_n \widehat{a}_n^{\dagger}.$

Since $[\widehat{N}, \widehat{H}_b] = 0$, the Hamiltonian \widehat{H}_b can be diagonalized separately for $\langle \widehat{N} \rangle = 1, 2, \cdots$. Single-photon energy: $E_k = \hbar \omega_c - 2J \cos(ka).$

Single-photon wave number k is a good quantum number: $ka \in [-\pi, \pi]$. Energy band of width 4*J*.

Approximately quadratic around k = 0 but not near band edge: Non-trivial single-particle dispersion.

Group velocity: $v_g = \hbar^{-1} \frac{\partial E_k}{\partial k}$ $\Rightarrow v_g = \frac{2Ja}{\hbar} \sin(ka).$

Tight-binding + non-linearity:

$$\hat{H}_{b} = \hbar\omega_{c} \sum_{n=1}^{N} \hat{a}_{n}^{\dagger} \hat{a}_{n} - J \sum_{n=1}^{N} \left(\hat{a}_{n}^{\dagger} \hat{a}_{n+1} + \hat{a}_{n+1}^{\dagger} \hat{a}_{n} \right) + \frac{U}{2} \sum_{n=1}^{N} \hat{a}_{n}^{\dagger} \hat{a}_{n}^{\dagger} \hat{a}_{n} \hat{a}_{n}.$$

Group velocity:
$$v_g = \hbar^{-1} \frac{\partial E_k}{\partial k}$$

 $\Rightarrow v_g = \frac{2Ja}{\hbar} \sin(ka).$

Question: How far does single photon travel during characteristic time?

Sub-question: What is the characteristic time of the cavity array?

Tight-binding + non-linearity:

$$\hat{H}_{b} = \hbar\omega_{c} \sum_{n=1}^{N} \hat{a}_{n}^{\dagger} \hat{a}_{n} - J \sum_{n=1}^{N} \left(\hat{a}_{n}^{\dagger} \hat{a}_{n+1} + \hat{a}_{n+1}^{\dagger} \hat{a}_{n} \right) \\ + \frac{U}{2} \sum_{n=1}^{N} \hat{a}_{n}^{\dagger} \hat{a}_{n} \hat{a}_{n} \hat{a}_{n}.$$

Question: What is the effective mass of the photons "trapped" in the cavity array?

Group velocity:
$$v_g = \hbar^{-1} \frac{\partial E_k}{\partial k}$$

 $\Rightarrow v_g = \frac{2Ja}{\hbar} \sin(ka).$

Single Emitter: Single-Excitation Bound State

$$\hat{H} = \hat{H}_s + \hat{H}_b + \hat{H}_{sb}$$

$$\hat{H}_s = \frac{\hbar\omega_e}{2} \sum_{j=1}^{N_e} \left(\hat{\sigma}_j^z + \hat{1}_j\right)$$

$$\hat{H}_{sb} = g \sum_{j=1}^{N_e} \left(\hat{a}_{n_j} \hat{\sigma}_j^+ + \hat{a}_{n_j}^\dagger \hat{\sigma}_j^- \right)$$

FIG. 1. Single impurity with energy Δ coupled through Ω to a bath with dispersion relation ε_k and a bandwidth W. A bosonic bound state (in red) localizes around the impurity.

$$\hat{H}_{b} = \boxed{\hbar\omega_{c}\sum_{n=1}^{N}\hat{a}_{n}^{\dagger}\hat{a}_{n} - J\sum_{n=1}^{N}\left(\hat{a}_{n}^{\dagger}\hat{a}_{n+1} + \hat{a}_{n+1}^{\dagger}\hat{a}_{n}\right)} \\ + \frac{U}{2}\sum_{n=1}^{N}\hat{a}_{n}^{\dagger}\hat{a}_{n}^{\dagger}\hat{a}_{n}\hat{a}_{n}.$$
tight-binding + non-linearity

Figure taken from Shi, Wu, Gonzalez-Tudela, and Cirac, PRX 6, 021027 (2016).

Emitter-photon bound state! Superposition of $|e\rangle$ and $|g\rangle$.

$$\begin{split} \mathbf{Tight-binding} + \mathbf{non-linearity:} \\ \hat{H}_{b} &= \boxed{\hbar \omega_{c} \sum_{n=1}^{N} \hat{a}_{n}^{\dagger} \hat{a}_{n} - J \sum_{n=1}^{N} \left(\hat{a}_{n}^{\dagger} \hat{a}_{n+1} + \hat{a}_{n+1}^{\dagger} \hat{a}_{n} \right)} \\ &+ \frac{U}{2} \sum_{n=1}^{N} \hat{a}_{n}^{\dagger} \hat{a}_{n}^{\dagger} \hat{a}_{n} \hat{a}_{n}. \end{split}$$

We have k_1 and k_2 .

Just as in the "usual" case, it turns out to be convenient to switch to relative and center-ofmass wave vectors: *q* and *K*. *K* is a good quantum number. For U = 0:

$$E_{q,K} = E_{k_1} + E_{k_2} =$$

$$2\hbar\omega_c - 4J\cos\left(\frac{Ka}{2}\right)\cos(qa) =$$

$$2\hbar\omega_c - 2J_K\cos(qa)$$

 $E_{q,K}$ depends parametrically on K.

For each *K*, *q* can take a range of values: energy band or scattering continuum.



Tight-binding + non-linearity:

$$\hat{H}_{b} = \frac{\hbar\omega_{c}\sum_{n=1}^{N}\hat{a}_{n}^{\dagger}\hat{a}_{n} - J\sum_{n=1}^{N}\left(\hat{a}_{n}^{\dagger}\hat{a}_{n+1} + \hat{a}_{n+1}^{\dagger}\hat{a}_{n}\right)}{+\frac{U}{2}\sum_{n=1}^{N}\hat{a}_{n}^{\dagger}\hat{a}_{n}^{\dagger}\hat{a}_{n}\hat{a}_{n}\hat{a}_{n}}$$

For $U \neq 0$: Scattering continuum is unchanged (does not depend on U).

However, there also exists exactly one two-photon bound state:

$$E_{K,b} = 2\hbar\omega_c + \operatorname{sign}(U) \left[U^2 + 16J^2\cos^2\left(\frac{Ka}{2}\right) \right]^{1/2}$$



Negative *U*:

 $\frac{\frac{U}{J}}{\frac{U}{J}} = -4$

Tight-binding + non-linearity:

$$\hat{H}_{b} = \boxed{\hbar\omega_{c}\sum_{n=1}^{N}\hat{a}_{n}^{\dagger}\hat{a}_{n} - J\sum_{n=1}^{N}\left(\hat{a}_{n}^{\dagger}\hat{a}_{n+1} + \hat{a}_{n+1}^{\dagger}\hat{a}_{n}\right)} + \frac{U}{2}\sum_{n=1}^{N}\hat{a}_{n}^{\dagger}\hat{a}_{n}^{\dagger}\hat{a}_{n}\hat{a}_{n}\hat{a}_{n}.$$

Question:

What is the binding energy of "red" state? What is the binding energy of "blue" state?



Negative *U*:

$$\frac{\frac{U}{J}}{\frac{U}{J}} = -1$$

Tight-binding + non-linearity:

$$\hat{H}_{b} = \frac{\hbar\omega_{c}\sum_{n=1}^{N}\hat{a}_{n}^{\dagger}\hat{a}_{n} - J\sum_{n=1}^{N}\left(\hat{a}_{n}^{\dagger}\hat{a}_{n+1} + \hat{a}_{n+1}^{\dagger}\hat{a}_{n}\right)}{+\frac{U}{2}\sum_{n=1}^{N}\hat{a}_{n}^{\dagger}\hat{a}_{n}^{\dagger}\hat{a}_{n}\hat{a}_{n}\hat{a}_{n}.}$$

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However, there also exists exactly one two-photon bound state:

$$E_{K,b} = 2\hbar\omega_c + \operatorname{sign}(U) \left[U^2 + 16J^2\cos^2\left(\frac{Ka}{2}\right) \right]^{1/2}$$



= 1

Cavity Array With Two Excitations: Two-Photon Wave Function

From Valiente and Petrosyan, JPB 41, 161002 (2008).



Three-Body Bound States?

- Same tight-binding Hamiltonian as discussed here supports deeply-bound and weakly-bound three-body states ("3" and "2+1" states). See M. Valiente, D. Petrosyan, and A. Saenz, PRA 81, 011601(R) (2010) for details.
- Anisotropic Heisenberg model: Collective excitations in quantum magnets (magnons) exhibit Efimov effect. See Y. Nishida, Y. Kato, and C. D. Batista, Nat. Phys. 9, 93 (2013) for details.

Coupling Emitter(s) To Cavity Array

 \hat{H}_s

$$\hat{H} = \hat{H}_s + \hat{H}_b + \hat{H}_{sb}$$
$$= \frac{\hbar\omega_e}{2} \sum_{j=1}^{N_e} \left(\hat{\sigma}_j^z + \hat{1}_j \right)$$
$$\hat{H}_{sb} = g \sum_{j=1}^{N_e} \left(\hat{a}_{n_j} \hat{\sigma}_j^+ + \hat{a}_{n_j}^\dagger \hat{\sigma}_j^- \right)$$

$$\hat{H}_{b} = \boxed{\hbar\omega_{c}\sum_{n=1}^{N}\hat{a}_{n}^{\dagger}\hat{a}_{n} - J\sum_{n=1}^{N}\left(\hat{a}_{n}^{\dagger}\hat{a}_{n+1} + \hat{a}_{n+1}^{\dagger}\hat{a}_{n}\right)} \\ + \frac{U}{2}\sum_{n=1}^{N}\hat{a}_{n}^{\dagger}\hat{a}_{n}^{\dagger}\hat{a}_{n}\hat{a}_{n}.$$
tight-binding + non-linearity

Two-Level Emitter in Resonance with Two-Photon Bound State



Comparison With Dicke Model (Single Radiation Mode)

$$\hat{H} = \left| \hat{H}_s \right| + \left| \hat{H}_b \right| + \left| \hat{H}_{sb} \right|$$
$$\hat{H}_s = \frac{\hbar\omega_e}{2} \sum_{i=1}^{N_e} \left(\hat{\sigma}_i^z + \hat{1}_j \right)$$

j=1

 \boldsymbol{Z}

$$\hat{H}_{sb} = g \sum_{j=1}^{N_e} \left(\hat{a}_{n_j} \hat{\sigma}_j^+ + \hat{a}_{n_j}^\dagger \hat{\sigma}_j^- \right)$$

$$\hat{H}_{b} = \hbar \omega_{c} \sum_{n=1}^{N} \hat{a}_{n}^{\dagger} \hat{a}_{n} - J \sum_{n=1}^{N} \left(\hat{a}_{n}^{\dagger} \hat{a}_{n+1} + \hat{a}_{n+1}^{\dagger} \hat{a}_{n} \right) + \frac{U}{2} \sum_{n=1}^{N} \hat{a}_{n}^{\dagger} \hat{a}_{n}^{\dagger} \hat{a}_{n} \hat{a}_{n}.$$

$$\hat{H} = \hat{H}_{s} + \hat{H}_{b} + \hat{H}_{sb}$$
$$\hat{H}_{s} = \frac{\hbar\omega_{e}}{2} \sum_{j=1}^{N_{e}} (\hat{\sigma}_{j}^{z} + \hat{1}_{j})$$
$$\hat{H}_{sb} = g \sum_{j=1}^{N_{e}} (\hat{a} \ \hat{\sigma}_{j}^{+} + \hat{a}^{+} \ \hat{\sigma}_{j}^{-})$$
$$\hat{H}_{b} = \hbar\omega_{c} \ \hat{a}^{+} \hat{a}$$

R. H. Dicke, Phys. Rev. 93, 99 (1954)

Dicke Model: Superradiance

$$\hat{H} = \hat{H}_s + \hat{H}_b + \hat{H}_{sb}$$
$$\hat{H}_s = \frac{\hbar\omega_e}{2} \sum_{j=1}^{N_e} \left(\hat{\sigma}_j^z + \hat{1}_j\right)$$

$$\widehat{H}_{sb} = g \sum_{j=1}^{N_e} (\widehat{a} \ \widehat{\sigma}_j^+ + \widehat{a}^+ \ \widehat{\sigma}_j^-)$$

 $\widehat{H}_b = \hbar \omega_c \ \widehat{a}^\dagger \widehat{a}$

R. H. Dicke, Phys. Rev. 93, 99 (1954)

Superradiance occurs when a group of N_e emitters interact with a common light field.

If the wavelength of the light is much greater than the separation of the emitters, then the emitters interact with the light in a collective and coherent fashion. This causes the group to emit light as a high intensity pulse (with rate proportional to N_e^2).

This is a surprising result, drastically different from the expected exponential decay (with rate proportional to N_e) of a group of independent atoms.

Text adapted from Wikipedia.

Superradiance Vs. Subradiance

Superradiance: Enhancement of spontaneous emission by constructive interatomic interference.

Subradiance: Cooperative inhibition of spontaneous emission by a destructive interatomic interference.

Coupling Emitter(s) To Cavity Array

 \hat{H}_s

$$\hat{H} = \hat{H}_s + \hat{H}_b + \hat{H}_{sb}$$
$$= \frac{\hbar\omega_e}{2} \sum_{j=1}^{N_e} \left(\hat{\sigma}_j^z + \hat{1}_j \right)$$
$$\hat{H}_{sb} = g \sum_{j=1}^{N_e} \left(\hat{a}_{n_j} \hat{\sigma}_j^+ + \hat{a}_{n_j}^\dagger \hat{\sigma}_j^- \right)$$

$$\hat{H}_{b} = \boxed{\hbar\omega_{c}\sum_{n=1}^{N}\hat{a}_{n}^{\dagger}\hat{a}_{n} - J\sum_{n=1}^{N}\left(\hat{a}_{n}^{\dagger}\hat{a}_{n+1} + \hat{a}_{n+1}^{\dagger}\hat{a}_{n}\right)} \\ + \frac{U}{2}\sum_{n=1}^{N}\hat{a}_{n}^{\dagger}\hat{a}_{n}^{\dagger}\hat{a}_{n}\hat{a}_{n}.$$
tight-binding + non-linearity

Four- and Many-Emitter Case Studied Using Master Equation

PRL 124, 213601 (2020).

Figures taken from Wang, Jaako, Kirton, Rabl,



1D waveguide QED setup:

Two-level systems (emitters: atom, quantum dot, superconducting qubit) coupled to nanophotonic or microwave waveguide.

Can also think about this in context of dissipative system dynamics.

Parameter Space for Fixed U/J and Fixed $\hbar(\omega_c - \omega_e)/J$



Wang, Jaako, Kirton, (and Rabl: PRL 124, 213601 (2020): Supercorrelated Radiance in Nonlinear Photonic Waveguides (results based on master equation)



Solve Time-Dependent Schroedinger Equation for \widehat{H}

Time-dependent Schroedinger equation: $\imath\hbar\frac{\partial\Psi(t)}{\partial t}=\hat{H}\Psi(t)$

Ansatz (we are working in two-excitation sub-space):

Plugging ansatz into time-dependent
Schroedinger equation yields
differential equations for
$$\dot{c}_{ee}(t), \dot{c}_{1k}(t), \dot{c}_{2k}(t), \dot{c}_{K,b}(t), \dot{c}_{K,q}(t).$$

$$\begin{split} |\Psi\rangle(t) &= \exp(-2\iota\omega_{e}t)[c_{ee}(t)|g,g,\mathrm{vac}\rangle + \\ &\sum_{k}c_{1k}(t)\hat{a}_{k}^{\dagger}|e,g,\mathrm{vac}\rangle + \\ &\sum_{k}c_{2k}(t)\hat{a}_{k}^{\dagger}|g,e,\mathrm{vac}\rangle + \\ &\sum_{K}c_{K,b}(t)\hat{P}_{K,b}^{\dagger}|g,g,\mathrm{vac}\rangle + \\ \end{split}$$
endent
$$\begin{split} &\sum_{K,q}c_{K,q}(t)\hat{P}_{K,q}^{\dagger}|g,g,\mathrm{vac}\rangle], \end{split}$$

Solve Time-Dependent Schroedinger Equation for \widehat{H}

For suitable parameter combination, one finds $\dot{c}_{1k}(t) \approx 0$ and $\dot{c}_{2k}(t) \approx 0$.

If these coefficients are set equal to zero, then $c_{1k}(t)$ and $c_{2k}(t)$ can be (adiabatically) eliminated from equations.

$$\hat{H}_{sb} = g \sum_{j=1}^{N_e} \left(\hat{a}_{n_j} \hat{\sigma}_j^+ + \hat{a}_{n_j}^\dagger \hat{\sigma}_j^- \right)$$

Physical picture:





Example Of Dynamics: Initial State |*ee*>



Example Of Dynamics: Initial State |*ee*>



Example Of Dynamics: Initial State |*ee*>





Summary and Outlook

Discrete lattice systems support universal states.

Coupling of emitters to cavity array provides unique opportunity to study impact of non-trivial mode structure.

Interesting radiation dynamics.

Opportunity to study dissipative dynamics in different regimes.

Alternative framework: Master equation.

Thank You!

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