Efimov Scenario In The Presence Of Spin-Orbit Coupling

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Lecture 3

Efimov effect:
Three identical bosons with short-range interactions (beautifully introduced in Pascal Naidon's lectures)

- unequal-mass three-boson and three-fermion systems
- larger equal-mass bosonic systems with short-range interactions
- nuclear systems with finite range (beautiful lectures by Alejandro Kievsky)
- three equal-mass bosons with 1D spin-orbit coupling (this lecture)

Discussion of one few-body technique:

Stochastic variational approach with explicitly correlated Gaussians.

Overview Of Lecture 3

Efimov scenario of three equal-mass bosons with 1D spin-orbit coupling

Review of Efimov effect for equal-mass bosons with short-range interactions.
What is 1D spin-orbit coupling (system lives in three-dimensional space) and how does it differ from "conventual" situation?

- One-body problem.
- Intermezzo: Numerical approach.
- Two-body problem.
- Three-body problem.

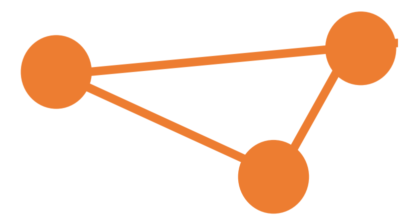
Unlike lectures 1 and 2:

- Consider stationary system (no time dependence).
- No experimental data yet on generalized radial scaling law, even though spin-orbit coupling has been realized in cold atom systems.

Braaten, Hammer, Physics Reports 428, 259 (2006).

Review: Three-Boson Hamiltonian

$$H = \frac{\vec{p}_{12}^{\,2}}{2\mu_{12}} + \frac{\vec{p}_{12,3}^{\,2}}{2\mu_{12,3}} + \sum_{j<k} g_2 \delta(\vec{r}_{jk}) + g_3 \delta(\vec{r}_1 - \vec{r}_2)\delta(\vec{r}_2 - \vec{r}_3).$$



Question: What units do g_2 and g_3 have?

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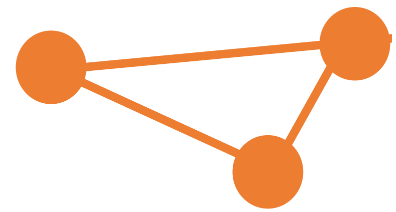

Question: What units do g_2 and g_3 have?

g_2: energy times length^3.
g_3: energy times length^6.

Follow-up question: Given this answer, what are the most general expressions for g_2 and g_3?

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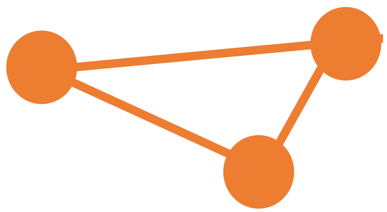

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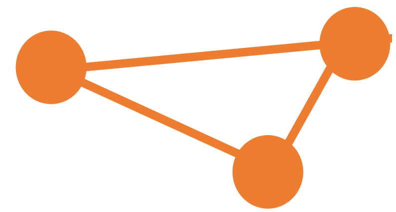
$$g_2 = \#\frac{\hbar^2\,\text{length}}{m};\ g_3 = \#\frac{\hbar^2\text{length}^4}{m}\ \text{(since energy} = \frac{\hbar^2}{m\,\text{length}^2}\text{)}.$$

Braaten, Hammer, Physics Reports 428, 259 (2006).

Peculiar Three-Boson Efimov States

$$H = \frac{\vec{p}_{12}^{\,2}}{2\mu_{12}} + \frac{\vec{p}_{12,3}^{\,2}}{2\mu_{12,3}} + \sum_{j<k} g_2 \delta\left(\vec{r}_{jk}\right) + g_3 \delta(\vec{r}_1 - \vec{r}_2)\delta(\vec{r}_2 - \vec{r}_3).$$



$g_2 = \frac{4\pi\hbar^2 a_s}{m}$ and $g_3 = \frac{\#\,\hbar^2\kappa_*^{-4}}{m}$, where $E_{unit} = \frac{\hbar^2\kappa_*^2}{m}$.

Time-dependent SE for H possesses continuous scaling symmetry:

$t \to \lambda^2 t;\ \vec{r} \to \lambda\vec{r};\ a_s \to \lambda a_s;\ E \to \lambda^{-2}E;\ \kappa_* \to \lambda^{-1}\kappa_*.$

Time-dependent SE for H also possesses discrete scaling symmetry ($\lambda_0 \approx 22.7$):

$t \to \lambda_0^2 t;\ \vec{r} \to \lambda_0\vec{r};\ a_s \to \lambda_0 a_s;\ E \to \lambda_0^{-2}E;\ \kappa_* \to \kappa_*.$

Let s-Wave Scattering Length Be Infinitely Large

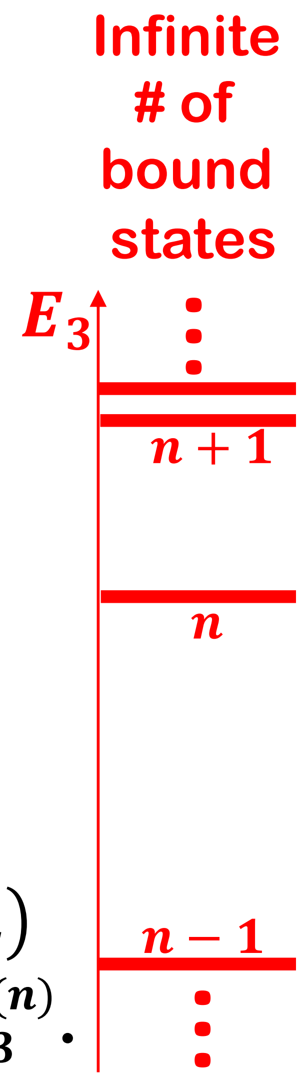
Hyperradial and hyperangular motion separate exactly:
$\Psi = F(R_{hyper})\Phi(\vec{\Omega})$; $R_{hyper}^2 \propto r_{12}^2 + r_{13}^2 + r_{23}^2$.

$L^{\Pi} = 0^+$ hyperangular equation yields eigenvalue ιs_0, where $s_0 = 1.006...$

Hyperangular eigenvalue enters into Schroedinger-like hyperradial equation: $H_{radial}F(R_{hyper}) = E_3 F(R_{hyper})$,

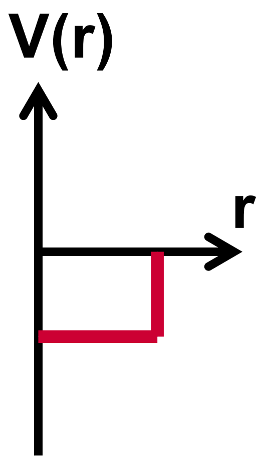
where $H_{radial}(R_{hyper}) = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial R_{hyper}^2} + \frac{\hbar^2((\iota s_0)^2 - \frac{1}{4})}{2mR_{hyper}^2}$.

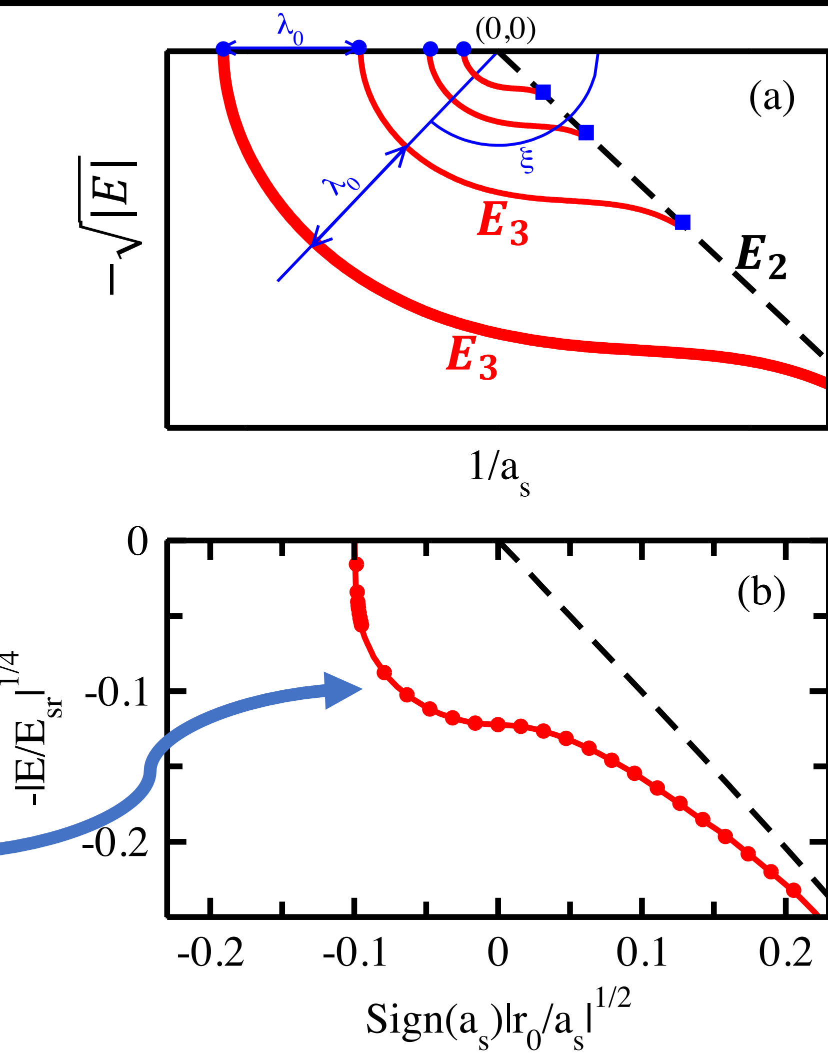
If $F(R_{hyper})$ is a solution with energy $E_3^{(n)}$, then $F(\lambda_0 R_{hyper})$ with $\lambda_0 = \exp\left(\frac{\pi}{s_0}\right) = 22.7 \ldots$ is a solution with energy $\lambda_0^{-2} E_3^{(n)}$.

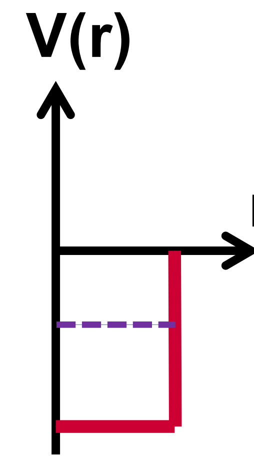


Finite s-Wave Scattering Length: Universally Linked States

stronger attraction



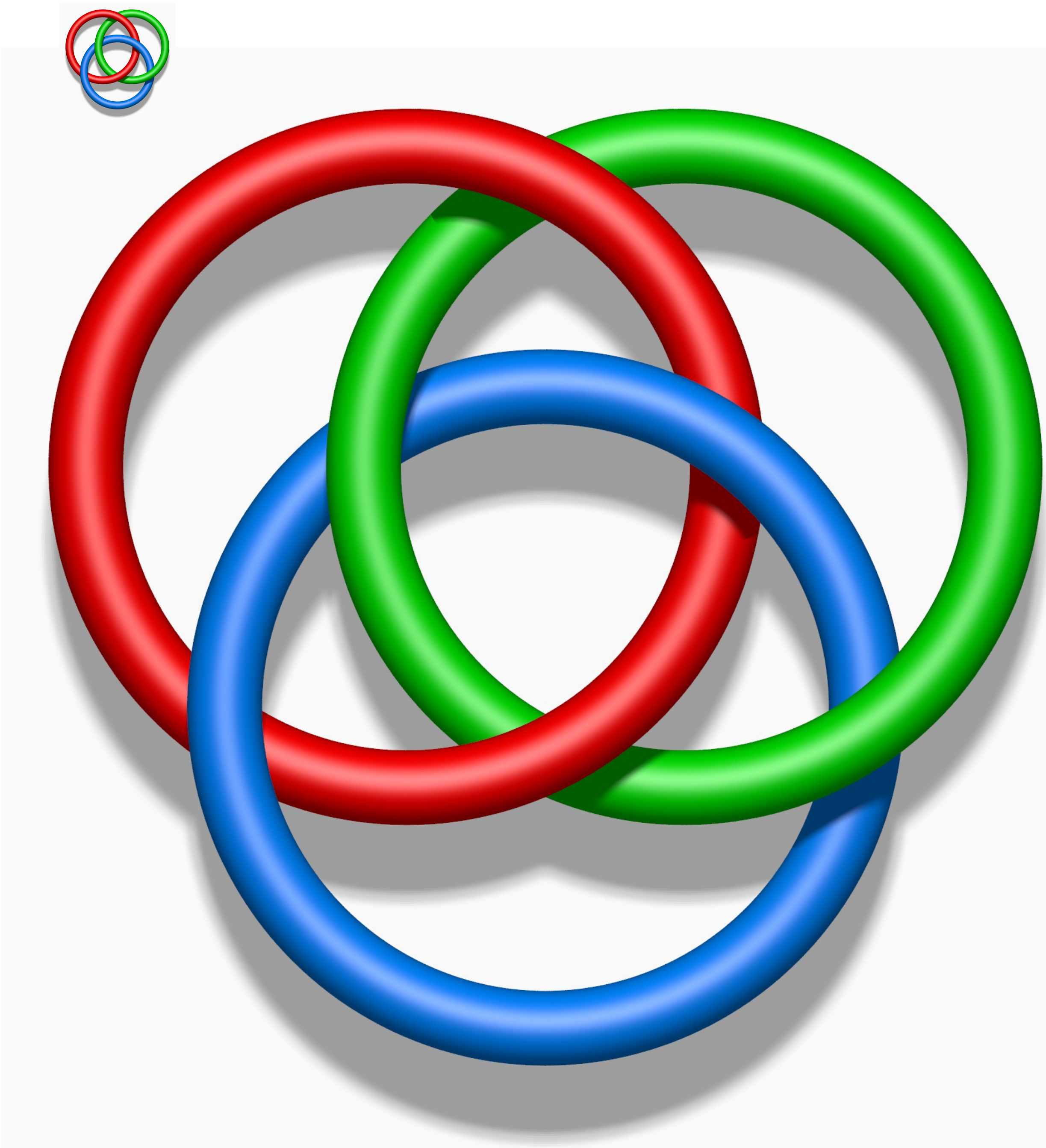




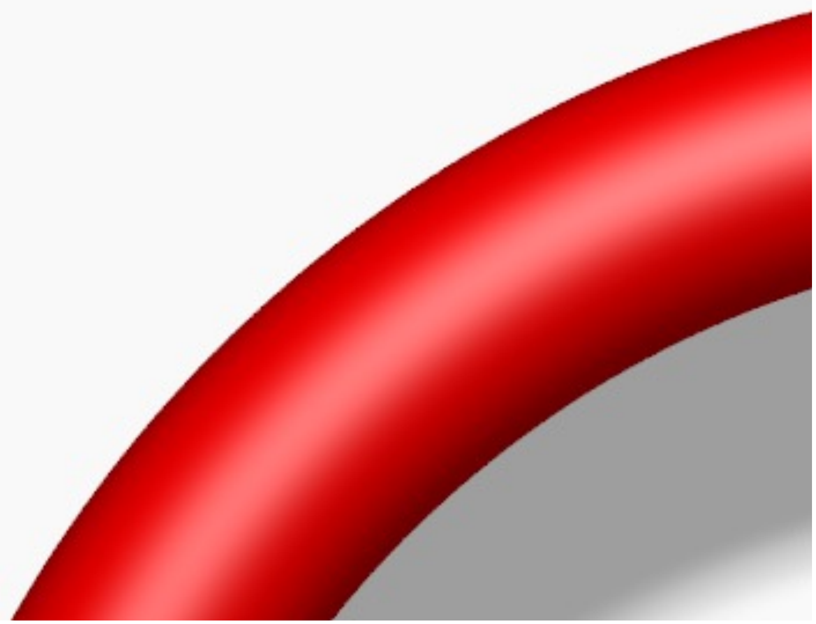
Numerical test for two-body plus three-body Gaussian potential: Perfect "collapse" of neighboring energy levels (see lectures by Naidon and Kievsky).

Spectrum is determined by a_s and three-body parameter κ_* (radial scaling law).

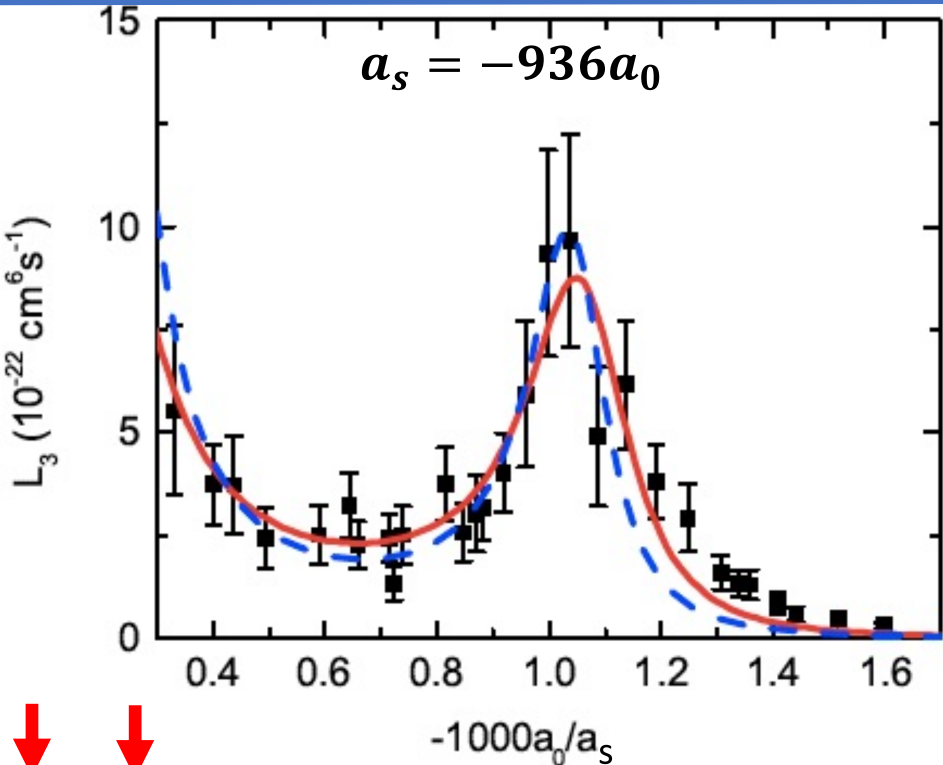
Borromean rings:

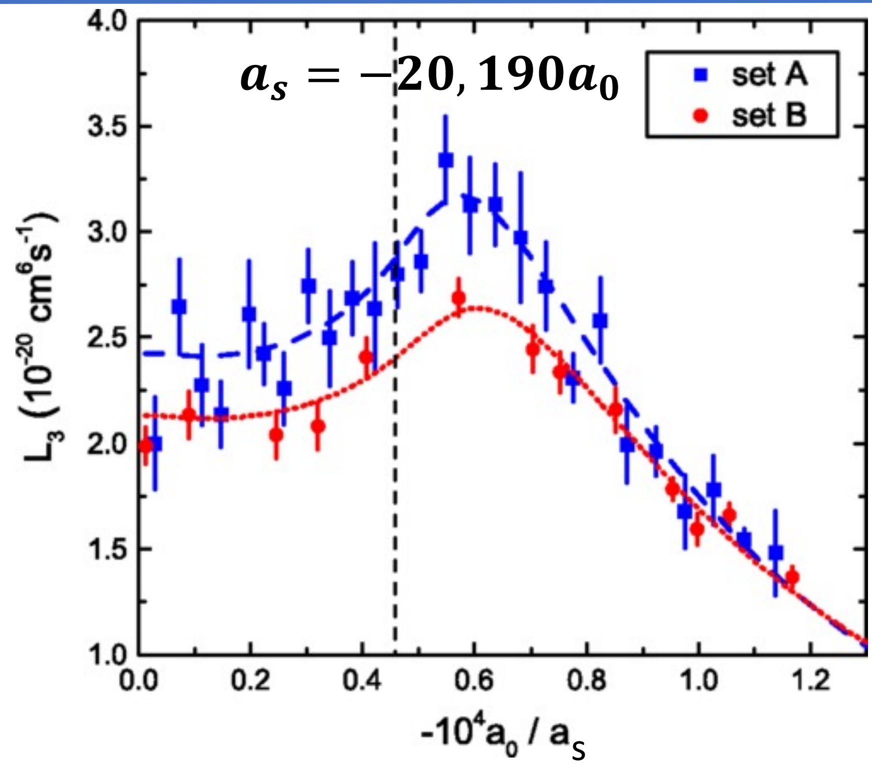


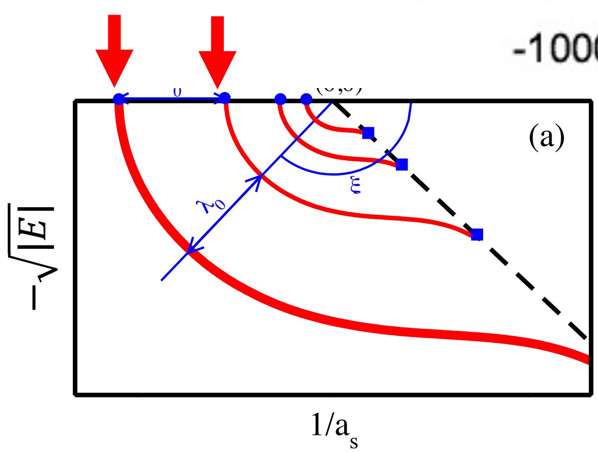
The blue ring lies under the green ring (the “blue-green dimer” is unbound). If the red ring is cut open, the trimer flies apart.



Measurement Of Loss Rate For Non-Degenerate ^{133}Cs Gas







Huang et al., PRL 112, 190401 (2014).

Enhanced losses when trimer is degenerate with three free atoms.

Ratio of $\lambda_0 = 21.6$ (compared to 22.7)! Confirmation of discrete scaling symmetry.

Efimov Scenario In The Presence Of Spin-Orbit Coupling?

If the single-particle dispersion is modified and the s-wave scattering length is large, what happens to the discrete scaling symmetry/Efimov physics?

Fermions with 3D SOC:
Shi et al., PRL 112, 013201 (2014); PRA 91, 023618 (2015).

Two-parameter radial scaling law does not hold.

Generalized multiple-parameter radial scaling law holds. Discrete scaling symmetry survives.

BBB with 1D SOC:
Guan, Blume: PRX 8, 021057 (2018).

Conjecture: Should hold for any type of SOC.

Looking Ahead...

$$H = \left(\frac{\vec{p}_{12}^{\,2}}{2\mu_{12}} + \frac{\vec{p}_{12,3}^{\,2}}{2\mu_{12,3}} + \sum_{j<k} g_2\delta(\vec{r}_{jk}) + g_3\delta(\vec{r}_1 - \vec{r}_2)\delta(\vec{r}_2 - \vec{r}_3)\right) I_8 + \frac{\hbar k_{so}}{m}(\dots) + \Omega(\dots) + \tilde{\delta}(\dots).$$

changes and extra terms due to SOC

Continuous scaling symmetry (easy to check)!

$t \to \lambda^2 t$; $\vec{r} \to \lambda\vec{r}$; $a_s \to \lambda a_s$; $k_{so} \to \lambda^{-1}k_{so}$; $\Omega \to \lambda^{-2}\Omega$; $\tilde{\delta} \to \lambda^{-2}\tilde{\delta}$; $E \to \lambda^{-2}E$; $\kappa_* \to \lambda^{-1}\kappa_*$

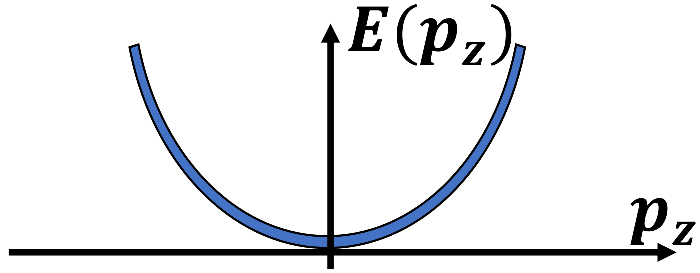
Discrete scaling symmetry?

$t \to \lambda_0^2 t$; $\vec{r} \to \lambda_0\vec{r}$; $a_s \to \lambda_0 a_s$; $k_{so} \to \lambda_0^{-1}k_{so}$; $\Omega \to \lambda_0^{-2}\Omega$; $\tilde{\delta} \to \lambda_0^{-2}\tilde{\delta}$; $E \to \lambda_0^{-2}E$; $\kappa_* \to \kappa_*$; $\lambda_0 \approx 22.7$

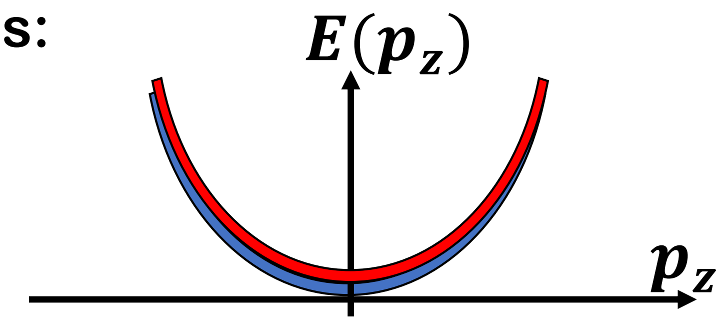
Start With Single-Particle Dispersion

Conventionally, kinetic energy:

$H = \frac{1}{2m}\vec{p}^2 = \frac{1}{2m}\left(p_x^2 + p_y^2 + p_z^2\right).$

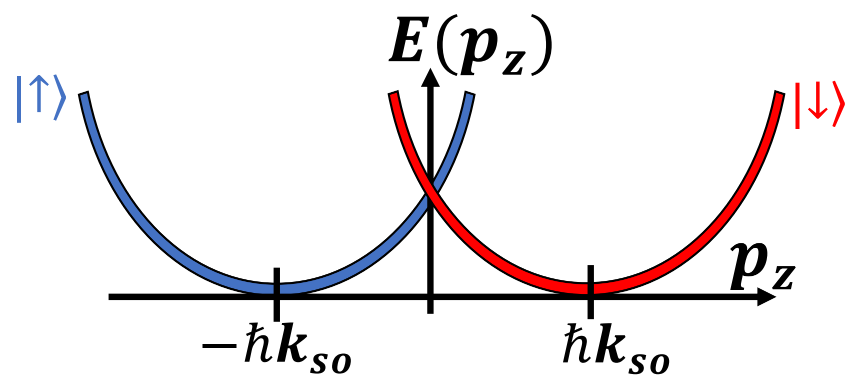


Say, the particle has two spin states:



Now let the spin states have different momenta:

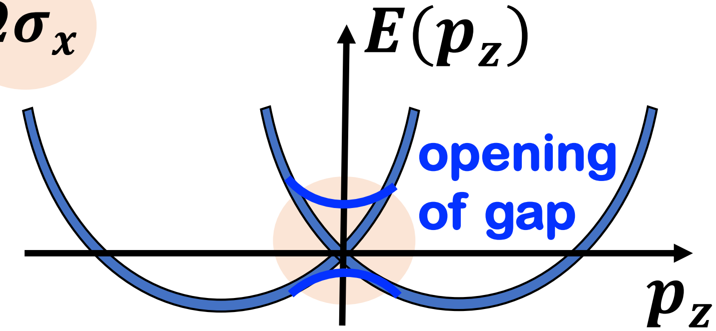
For this to be interesting, must couple states.



Single-Particle Dispersion

For 1D spin-orbit coupling (equal mixture of Rashba and Dresselhaus coupling): $H = (\frac{p_x^2+p_y^2}{2m} + \frac{p_z^2}{2m})I_2 + \frac{\hbar k_{so}}{m} p_z\sigma_z + \Omega\sigma_x$

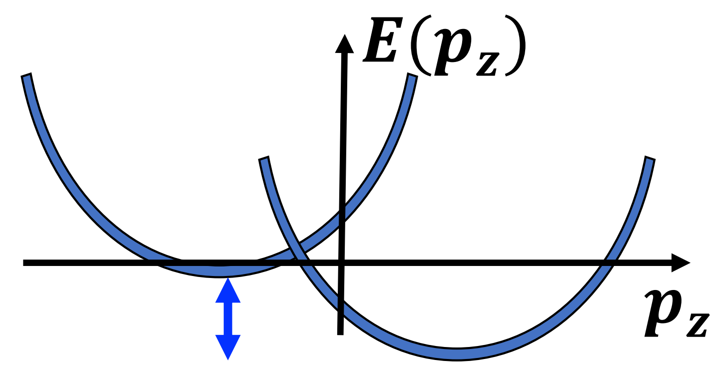
$$E_\pm = \frac{p_x^2+p_y^2}{2m} + \frac{p_z^2}{2m} \pm \sqrt{(\frac{\hbar k_{so} p_z}{m})^2 + \frac{\Omega^2}{4}}$$



Add detuning δ:

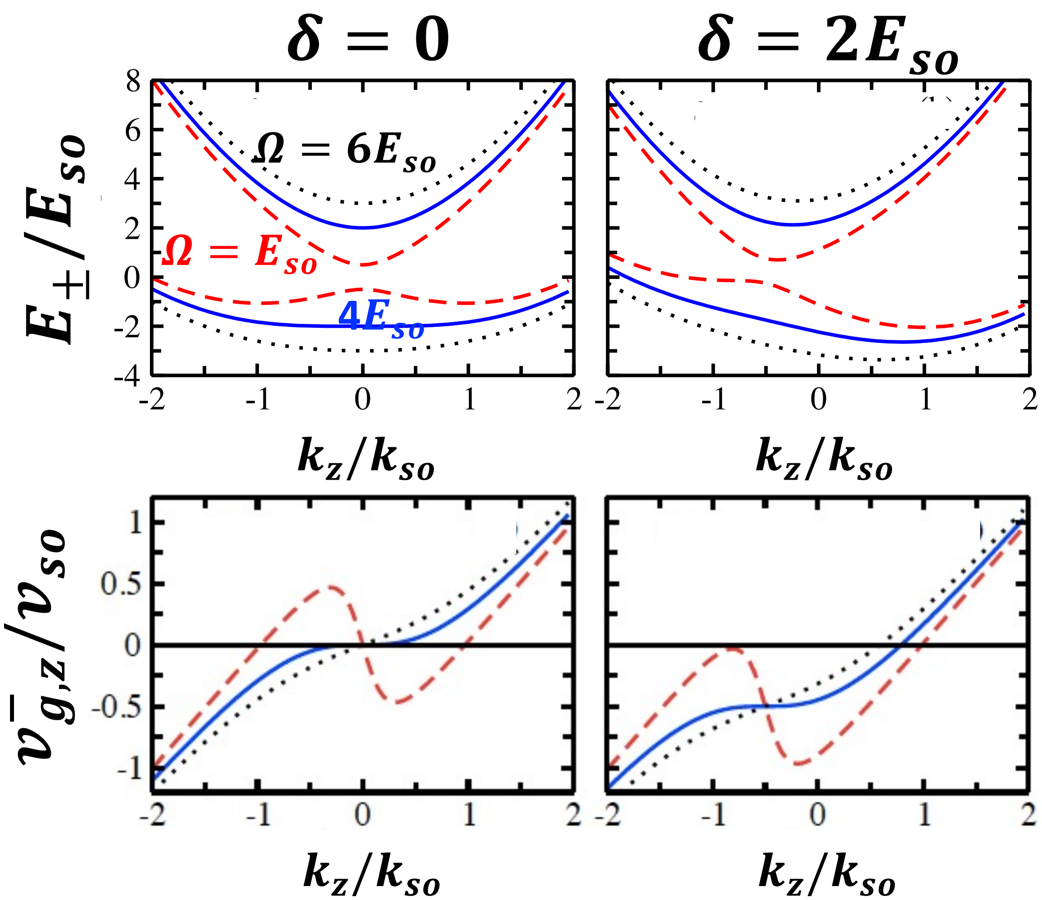
$H = (\frac{p_x^2+p_y^2}{2m} + \frac{p_z^2}{2m})I_2 + \frac{\hbar k_{so}}{m} p_z\sigma_z + \delta\sigma_z.$

$$E_\pm = \frac{p_x^2+p_y^2}{2m} + \frac{p_z^2}{2m} \pm \sqrt{(\frac{\hbar k_{so} p_z}{m} + \frac{\delta}{2})^2}$$



Modified Single-Particle Dispersion → New Physics

Dirac-like term in Hamiltonian.

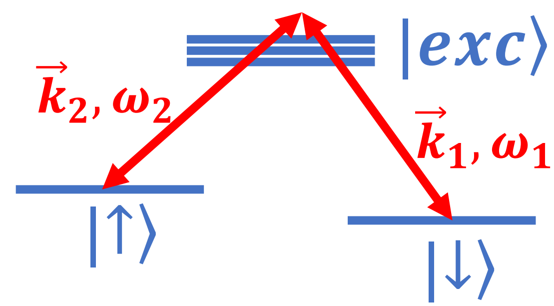


$$E_{so} = \frac{\hbar^2 k_{so}^2}{2m}$$

$$t_{so} = \hbar / E_{so}$$

$$v_{so} = \hbar k_{so}/m$$

$$\vec{v}_g = \frac{1}{\hbar}\langle \vec{\nabla}_{\vec{k}} H(\vec{k}) \rangle$$



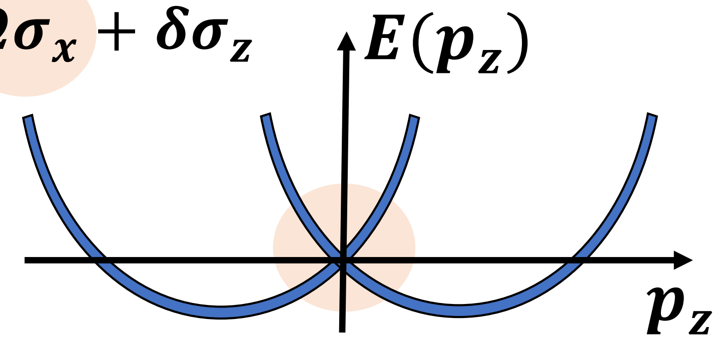
Two hyperfine states coupled by lasers.

Typical parameters:
$(k_{so})^{-1} \approx 1,810\text{Å}$;
$\frac{E_{so}}{\hbar} \approx 2\pi \times 1.775$ kHz;
$\Omega, \delta \in [0, 10E_{so}]$.

Modified Single-Particle Dispersion → New Physics

For 1D spin-orbit coupling (equal mixture of Rashba and Dresselhaus coupling): $H = \left(\frac{p_x^2+p_y^2}{2m} + \frac{p_z^2}{2m}\right)I_2 + \frac{\hbar k_{so}}{m} p_z\sigma_z + \Omega\sigma_x + \delta\sigma_z$

$$E_\pm = \frac{p_x^2+p_y^2}{2m} + \frac{p_z^2}{2m} \pm \sqrt{\left(\frac{\hbar k_{so} p_z}{m} + \frac{\delta}{2}\right)^2 + \frac{\Omega^2}{4}}$$



Why is this of interest? Expand E_- for large Ω and $\delta \neq 0$ around $p_{z,min}^-$:

$$E_- = const + \frac{\left(p_z - p_{z,min}^-\right)^2}{2m} + \cdots$$

Like a charged particle in a uniform vector potential $\vec{A}^*$: $e\vec{A}^* = p_{z,min}^- \, \vec{e}_z$!

Possibility to simulate physics of charged particles (e.g., fractional quantum Hall effect) with neutral atoms!

Two Bosons With One-Dimensional Spin-Orbit Coupling

3D system with 1D SOC (spin-orbit coupling + Raman coupling + detuning) → Two-body bound state or not?

Rewrite Hamiltonian in relative coordinates ($\vec{r}$ and $\vec{p}$ with reduced mass μ) and center-of-mass coordinates ($\vec{R}$ and $\vec{P}$ with total mass M):

$$H = H_{rel} + H_{cm}$$

coupling

$$H_{rel}(P_z) = \frac{p_x^2 + p_y^2 + p_z^2}{2\mu} I_2^{(1)} \otimes I_2^{(2)} + \frac{\hbar k_{so} p_z}{\mu}\left(\sigma_z^{(1)} \otimes I_2^{(2)} - I_2^{(1)} \otimes \sigma_z^{(2)}\right) + \Omega\left(\sigma_x^{(1)} \otimes I_2^{(2)} + I_2^{(1)} \otimes \sigma_x^{(2)}\right) + \left(\delta + \frac{\hbar k_{so} P_z}{M}\right)\left(\sigma_z^{(1)} \otimes I_2^{(2)} + I_2^{(1)} \otimes \sigma_z^{(2)}\right) + V_{2b}(r) I_2^{(1)} \otimes I_2^{(2)}$$

$$[H_{rel}, P_z] = 0$$

parametric dependence on CoM momentum (call $\tilde{\delta}$)

Demands On Numerical Method

- Must be…
  - …able to treat multiple channels.
  - …able to describe small and large length scales.
  - …able to describe eigen energies and eigen states that vary by orders of magnitude.
  - …applicable to two and three particles.

Our method of choice:
Stochastic variational approach with explicitly correlated Gaussian basis function (see also contributed talk yesterday by Roy Yaron).

Basis Set Expansion: Variational Approach

Let Φ_j with $j = 0, 1, \cdots$ be an orthonormal complete set.

Any eigen state φ_l with energy E_l of H can be expanded as
$\varphi_l = \sum_{j=0}^{\infty} c_j^{(l)} \Phi_j$.

In reality: $\phi_l = \sum_{j=0}^{N_b} c_j^{(l)} \Phi_j$ ($N_b < \infty$; ϕ_l is an approximation to φ_l).

Form matrix $\overleftrightarrow{C}$ with matrix elements $C_{jl} = c_j^{(l)}$.

Eigenvalues ε_l of matrix equation $\overleftrightarrow{H}\,\overleftrightarrow{C} = \overleftrightarrow{\varepsilon}\,\overleftrightarrow{C}$ have the following property:

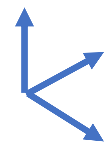
$E_0 \leq \varepsilon_0, E_1 \leq \varepsilon_1, \cdots$ (variational upper bounds).

Basis Set Expansion: Variational Approach

Question: What changes if Φ_j with $j = 0, 1, \cdots$ are not orthogonal?

Basis Set Expansion: Variational Approach

Now: Allow Φ_j with $j = 0, 1, \cdots$ to be linearly dependent (but not too much).



Expand $\phi_l = \sum_{j=0}^{N_b} c_j^{(l)} \Phi_j$ ($N_b < \infty$; ϕ_l is an approximation to exact eigen state φ_l).

Form matrix $\overleftrightarrow{C}$ with matrix elements $C_{jl} = c_j^{(l)}$.

The eigenvalues ε_l of generalized eigen value equation $\overleftrightarrow{H}\,\overleftrightarrow{C} = \overleftrightarrow{\varepsilon}\,\overleftrightarrow{O}\,\overleftrightarrow{C}$, where $O_{jl} = \langle \Phi_j | \Phi_l \rangle$, have the following property:

$E_0 \leq \varepsilon_0, E_1 \leq \varepsilon_1, \cdots$ (variational upper bounds).

Basis Set Expansion: Variational Approach

Take advantage of the fact that the basis functions $\boldsymbol{\Phi_j}$ can be "anything".

Pick $\boldsymbol{\Phi_j}$ such that integrals have compact analytical expressions.

Pick $\boldsymbol{\Phi_j}$ such that the different length scales of the system are covered.

Take advantage of the fact that low-energy Hamiltonian can be constructed using different functional forms for interaction potential:

$$H = \sum_j T_j + V_{soc,j} + \sum_{j<k} V_{2b,jk} + \sum_{j<k<l} V_{3b,jkl}$$

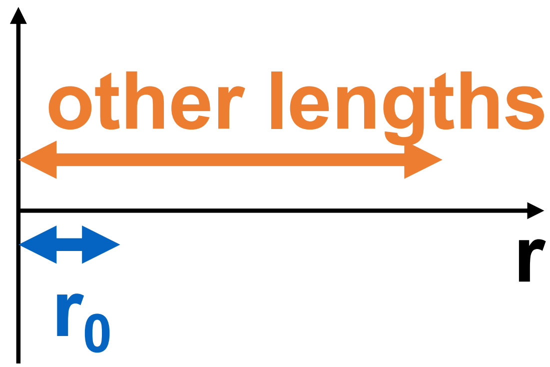
purely repulsive Gaussian (see also talks by Kievsky)

$$V_{2b,jk} = v_0 exp\left(-\frac{r_{jk}^2}{2r_0^2}\right)$$

Require $r_0 \ll$ other scales:
Need to resolve multiple scales.
Use $\boldsymbol{\Phi_j}$ with "different widths."

Basis Set Expansion: Stochastic Variational Approach

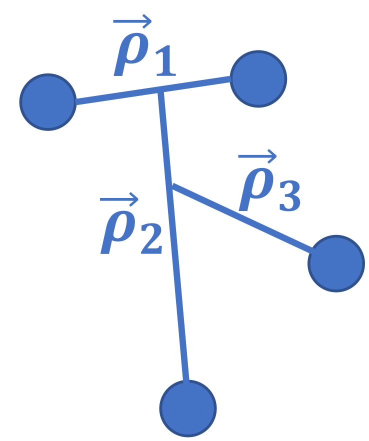
Method first introduced to cold atom community for bosons by Sorensen, Fedorov and Jensen, AIP Conf. Proc. No. 777, p. 12 (2005). See also work on fermions by von Stecher and Greene, PRL 99, 090402 (2007). For details see: Suzuki and Varga (Springer, 1998); von Stecher, Greene, Blume, PRA 77, 043619 (2008).



Idea:

Use basis functions that involve Gaussians with different widths in interparticle distances (correlations).

Large number of non-linear parameters that are being optimized semi-stochastically.



Simplest case: Basis functions with $L = 0$ and $\Pi = +1$.

$$\Phi_j = \exp\left(-\sum_{s<t}^{N} \frac{r_{st}^2}{2d_{j,st}^2}\right) = \exp\left(-\frac{1}{2}\vec{x}^T \overleftrightarrow{A}\ \vec{x}\right).$$

$\vec{x}$: Denotes Jacobi vectors $\vec{\rho}_1, \vec{\rho}_2, \cdots$.

$\overleftrightarrow{A}$: $(N-1)\times(N-1)$ matrix with $N(N-1)/2$ independent parameters.

Stochastic Variational Approach: Outline of Algorithm

- Pick basis function $\boldsymbol{\Phi_1}$ and calculate $\boldsymbol{\varepsilon_1}$.

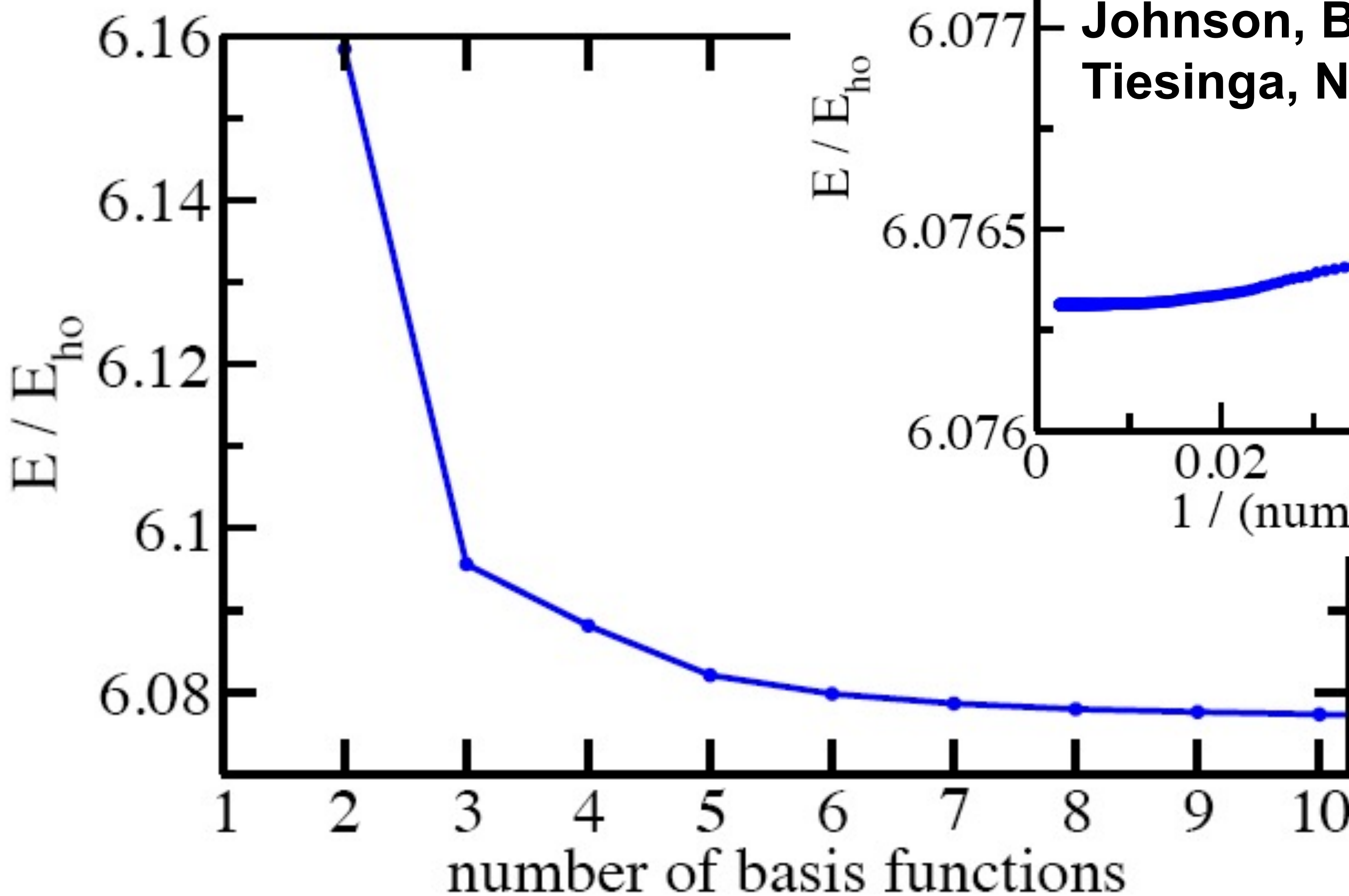
- Goal: Add $\boldsymbol{\Phi_2}$. Procedure:
  - Pick $\boldsymbol{\Phi_{2,1},\ldots,\Phi_{2,p}}$ $(\boldsymbol{p \sim 1 - 10000})$.
  - Calculate $\boldsymbol{\varepsilon_{2,1},\ldots,\varepsilon_{2,p}}$. $\boldsymbol{\varepsilon_{2,j}}$ is eigen value of target state if basis function $\boldsymbol{\Phi_{2,j}}$ is added to basis $(\boldsymbol{j = 1, \cdots, p})$.
  - Determine $\boldsymbol{\Phi_2 = \Phi_{2,j}}$ such that $\boldsymbol{\varepsilon_2 = \varepsilon_{2,j} = \min(\varepsilon_{2,1},\ldots,\varepsilon_{2,p})}$.
  - Diagonalize Hamiltonian matrix to obtain eigenvalues and eigenvectors.

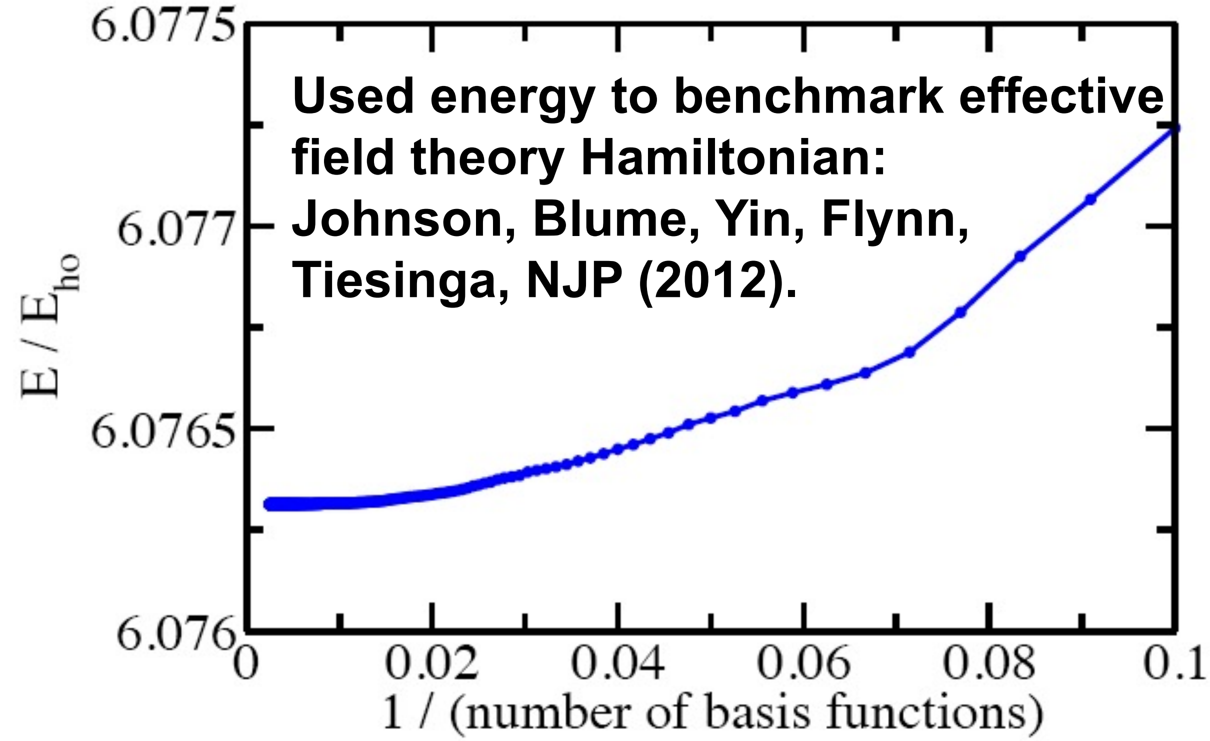
- To add $\boldsymbol{\Phi_3}$, proceed as above.
- Once basis set is “complete”, calculate structural properties.

- Can optimize ground or excited state.
- Can optimize multiple states simultaneously.

Harmonically Trapped Five-Boson System: Convergence

$r_0 = 0.01a_{ho}$
$a_s = 0.0096a_{ho}$



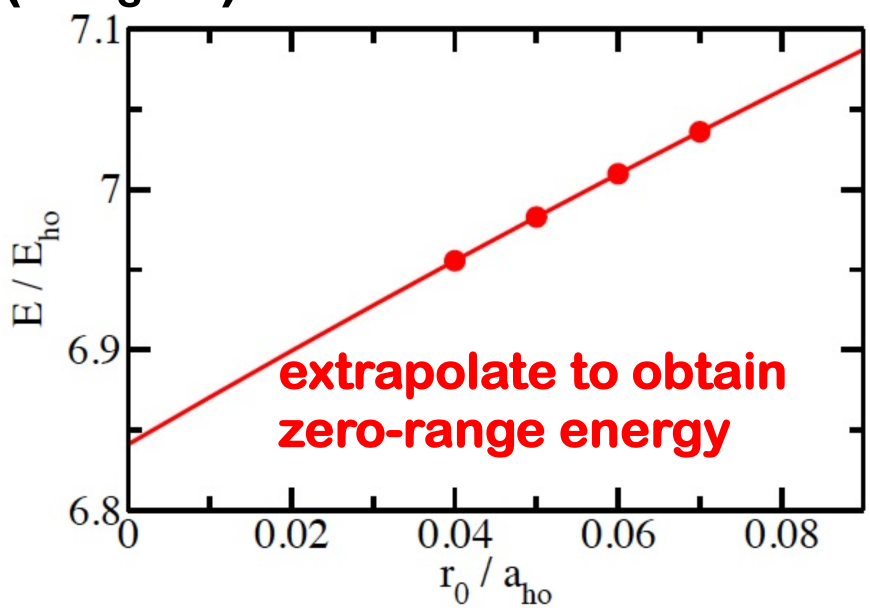


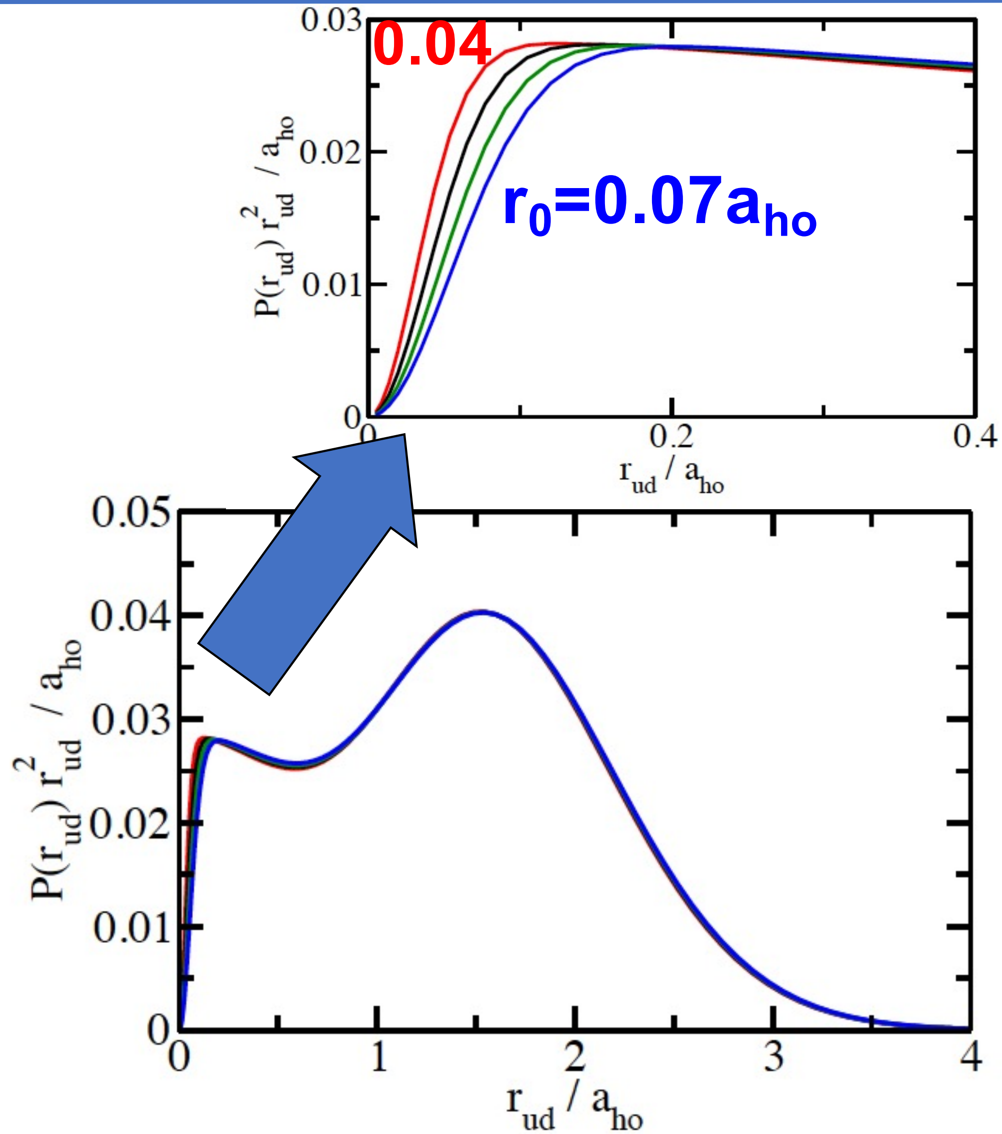
Used energy to benchmark effective field theory Hamiltonian: Johnson, Blume, Yin, Flynn, Tiesinga, NJP (2012).

For each N_b, try a few 1000 and keep the best.

Trapped (3,3) System: Energy And Pair Distribution Function

Lowest energy at unitarity ($1/a_s=0$):



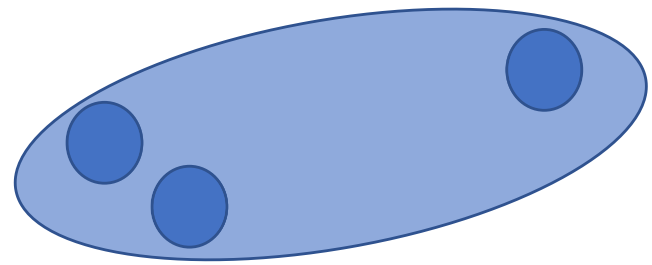


Two-peak structure of up-down pair distribution function:
Small r_{ud} peak: pair formation.
Large r_{ud} peak: unpaired.

A Few More Comments

Basis functions need to be symmetrized: Five identical bosons implies 5!=120 permutations.

Use physical insight to choose $d_{j,st}$ efficiently:
E.g., “2+1” or “1+1+1” configuration.



If parameter windows for non-linear variational parameters are not set properly, a non-converged energy may appear converged…

Basis sets tend to be small (a few 1000); but we work hard to select the basis functions we want.

Beyond $L^{\Pi} = 0^{+}$ states? Many possibilities… Global vector approach is quite convenient.

Spin-Orbit Coupling: Need To Account For Spin

$$\Phi_j = \exp\left(-\sum_{s<t}^{N} \frac{r_{st}^2}{2d_{j,st}^2} + \sum_{t=1}^{N-1} \iota \vec{s}_{j,t} \cdot \vec{\rho}_t \right)$$

Spatial two-body correlations

Correlation between spin and spatial degrees of freedom. Can be rewritten as $\sum_{t=1}^{N} \iota \vec{S}_{j,t} \cdot \vec{r}_t$

$\Psi_{rel} = \sum_{j=1}^{N_b} c_j \psi_j$ and $\psi_j = \mathcal{S}(\Phi_j(\vec{\rho}_1, \ldots, \vec{\rho}_{N-1})\chi_j)$

Matrix elements have compact analytical expressions.

Bound state:
Energy of dimer with CM momentum P_z is more negative than that of two free atoms with the same P_z.
Energy of trimer with CM momentum P_z is more negative than that of three free atoms with the same P_z and that of a dimer and an atom with the same P_z.

Two Bosons With One-Dimensional Spin-Orbit Coupling

3D system with 1D SOC (spin-orbit coupling + Raman coupling + detuning) → Two-body bound state or not?

Rewrite Hamiltonian in relative coordinates ($\vec{r}$ and $\vec{p}$ with reduced mass μ) and center-of-mass coordinates ($\vec{R}$ and $\vec{P}$ with total mass M):

$$H = H_{rel} + H_{cm}$$

coupling

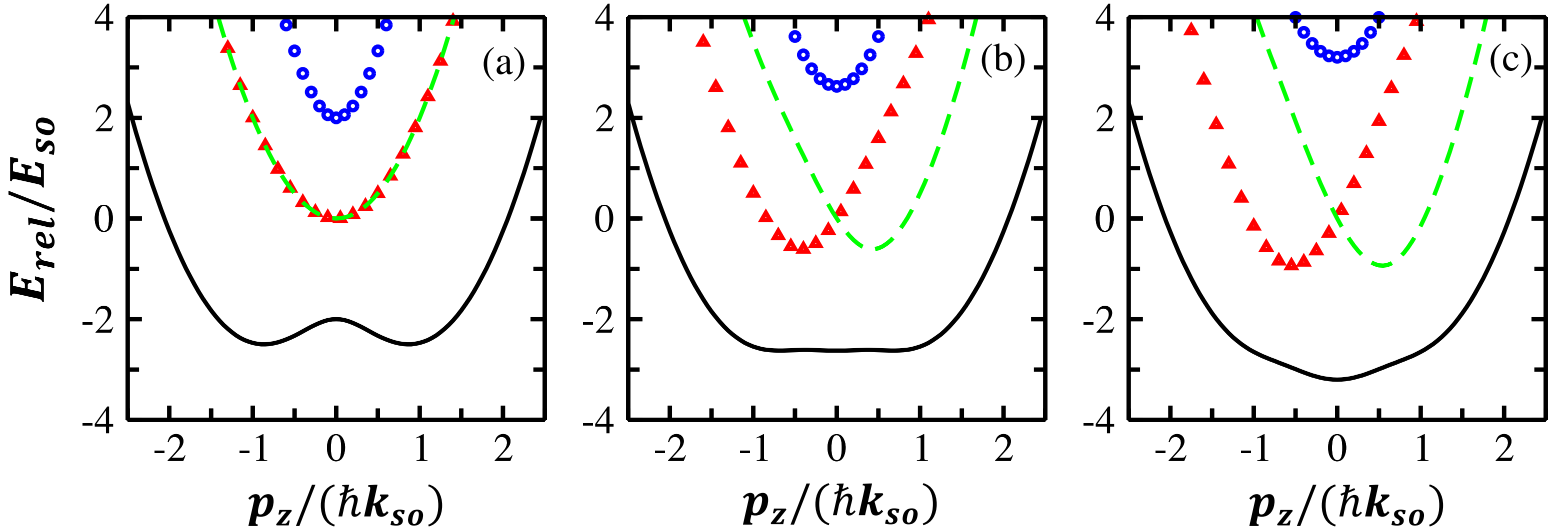
$$H_{rel}(P_z) = \frac{p_x^2 + p_y^2 + p_z^2}{2\mu} I_2^{(1)} \otimes I_2^{(2)} + \frac{\hbar k_{so} p_z}{\mu}\left(\sigma_z^{(1)} \otimes I_2^{(2)} - I_2^{(1)} \otimes \sigma_z^{(2)}\right) + \Omega\left(\sigma_x^{(1)} \otimes I_2^{(2)} + I_2^{(1)} \otimes \sigma_x^{(2)}\right) + \left(\delta + \frac{\hbar k_{so} P_z}{M}\right)\left(\sigma_z^{(1)} \otimes I_2^{(2)} + I_2^{(1)} \otimes \sigma_z^{(2)}\right) + V_{2b}(r) I_2^{(1)} \otimes I_2^{(2)}$$

$$[H_{rel}, P_z] = 0$$

parametric dependence on CoM momentum (call $\tilde{\delta}$)

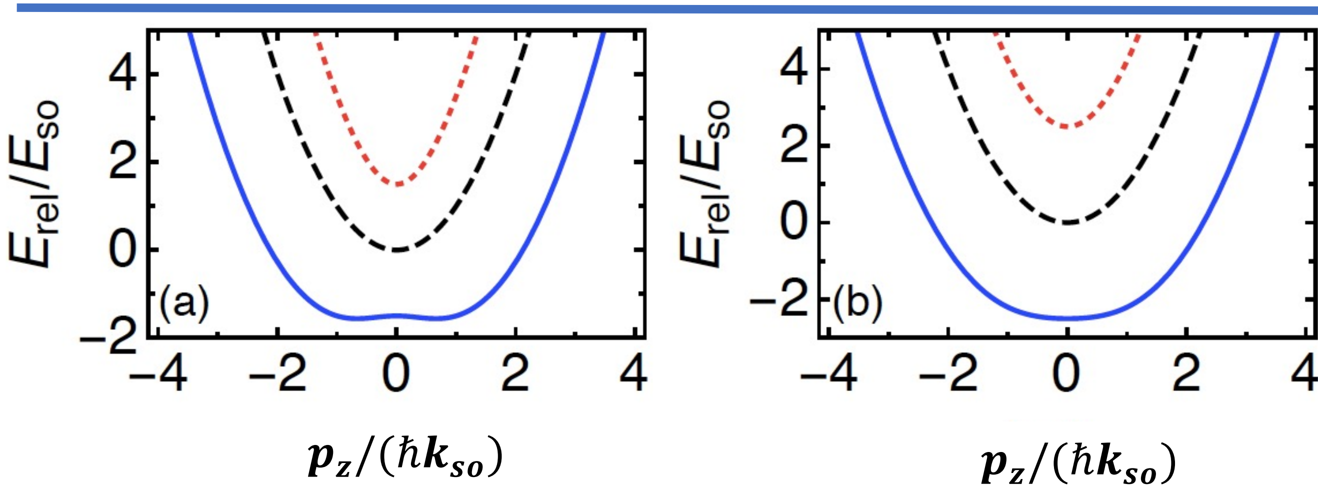
Non-Interacting Relative Dispersion Curves Along z

increase $\tilde{\delta} = \delta + \frac{\hbar k_{so} P_z}{M}$.



fixed Raman coupling

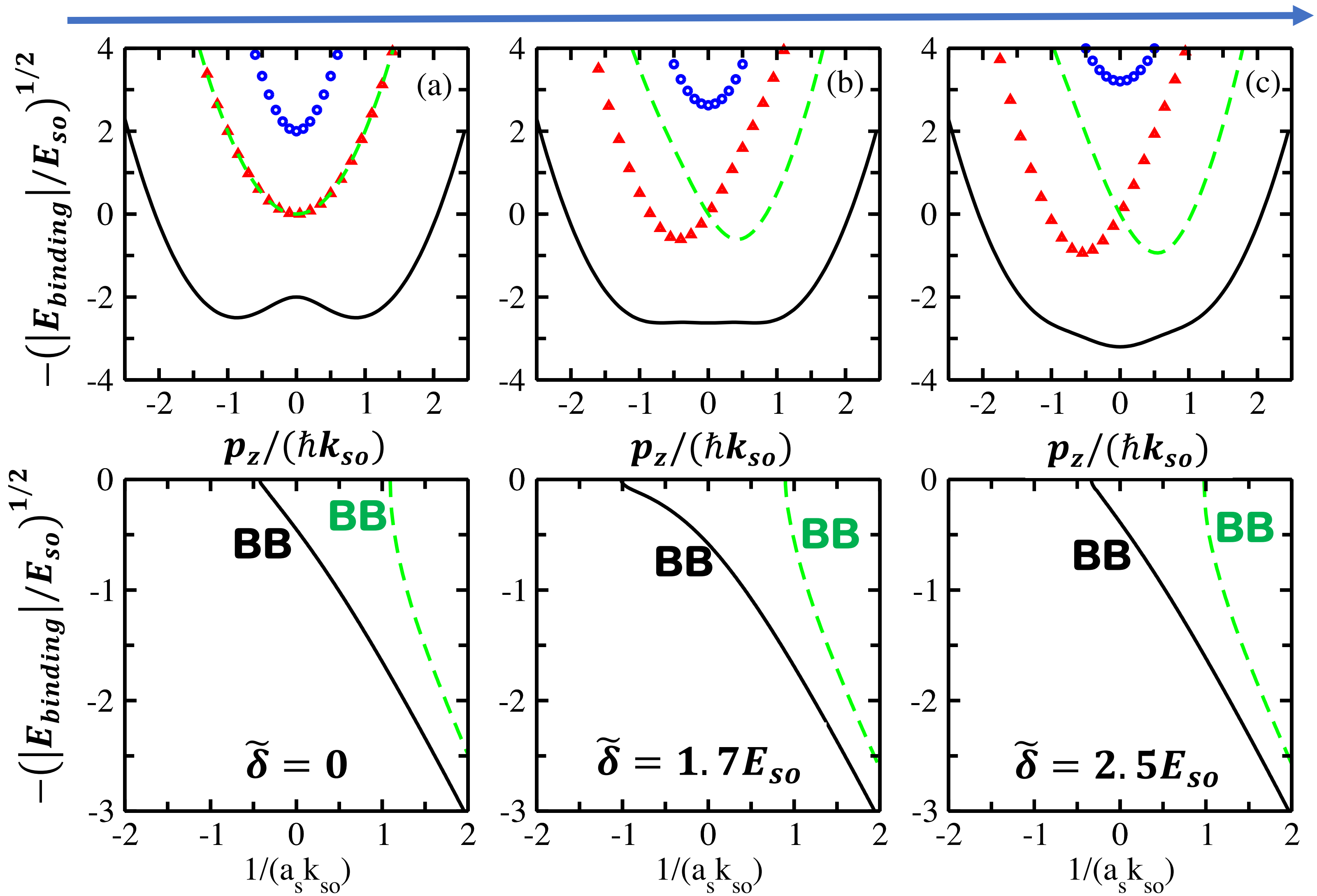
increase Ω.



Transition from double- to single-minimum regime.

Two Identical Bosons:

$\Omega = 2E_{so}; \tilde{\delta} \geq 0$ $(a_{\uparrow\uparrow} = a_{\uparrow\downarrow} = a_{\downarrow\uparrow} = a_{\downarrow\downarrow})$



increase

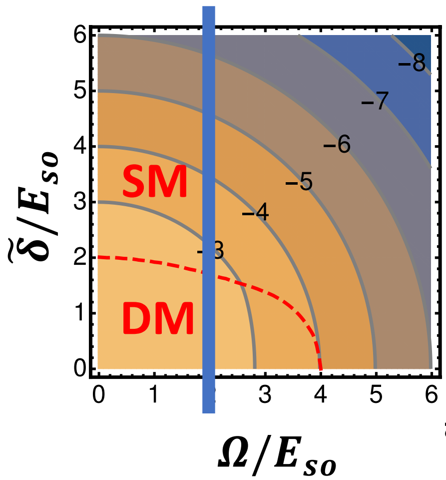
$\tilde{\delta} = \delta + \frac{\hbar k_{so} P_z}{M}$.

NI two-particle dispersion

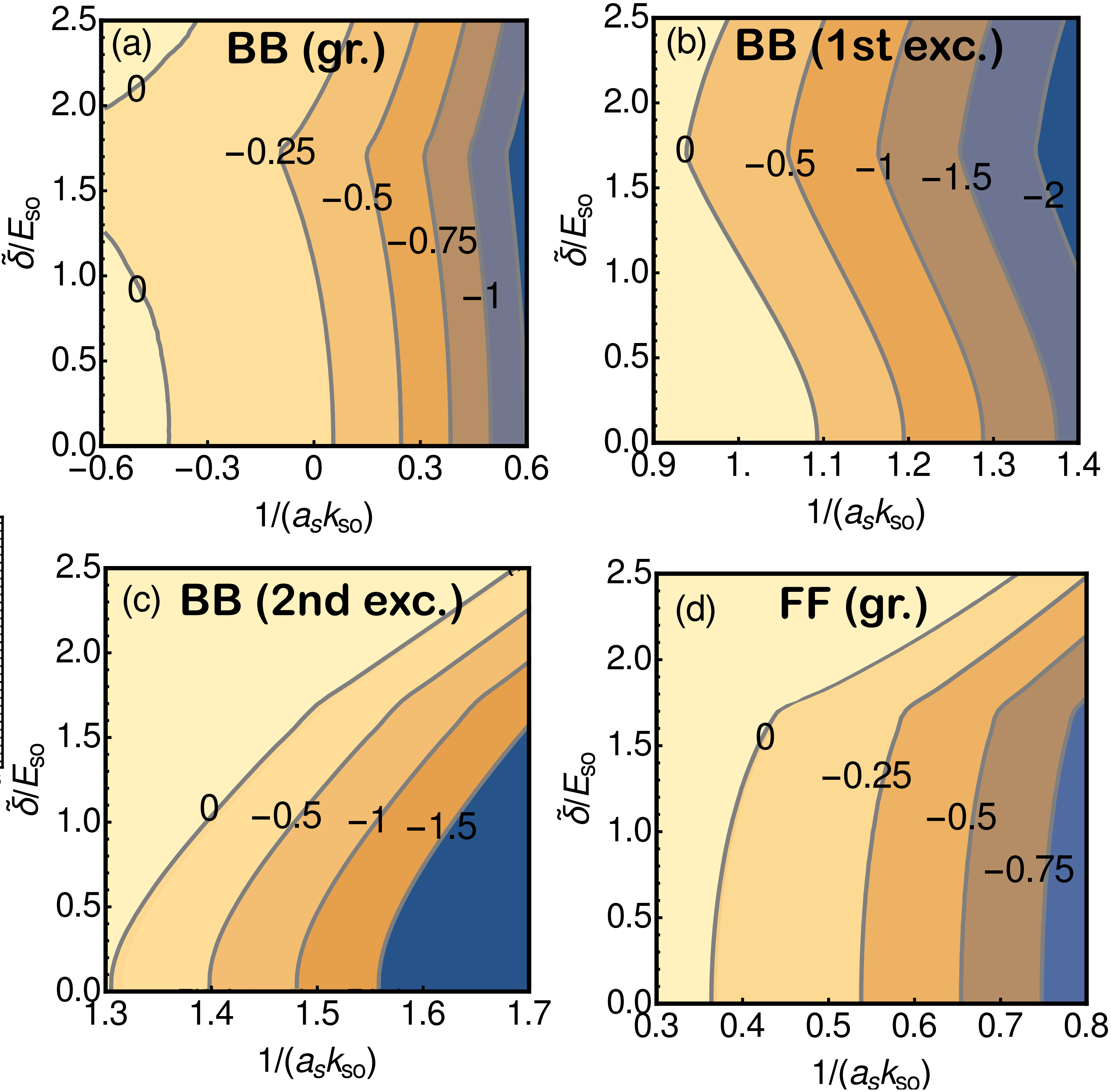
Negative of binding energy; deviations from $\frac{-\hbar^2}{m a_s^2}$

Three BB, one FF bound states ($\Omega = 2E_{so}$)

Scattering threshold:

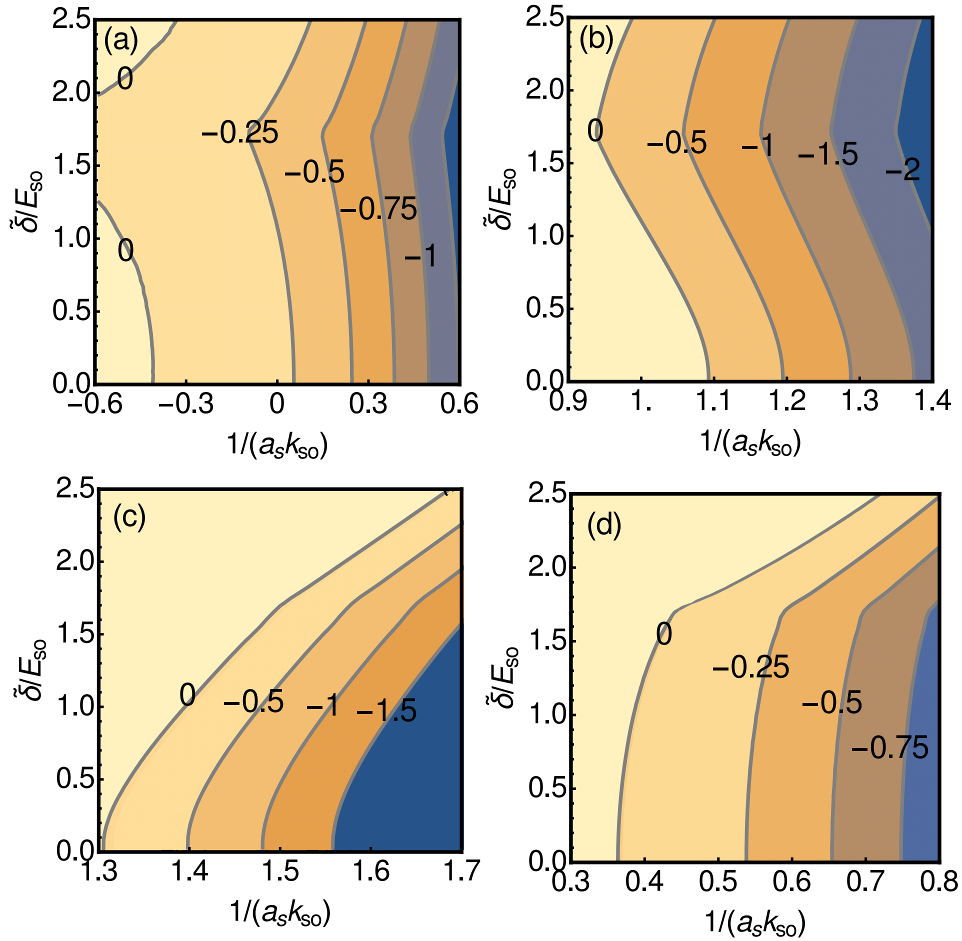


Where does the “shape” come from?

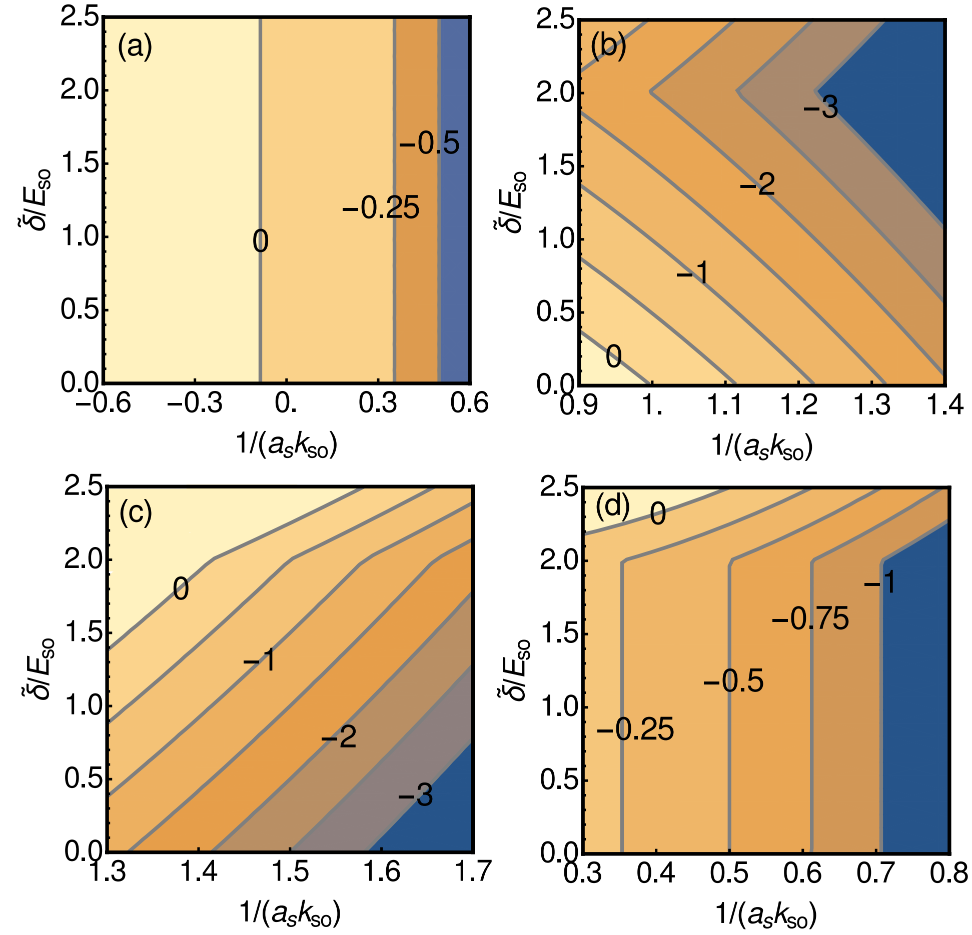


“Shape”? Simple Qualitative Picture

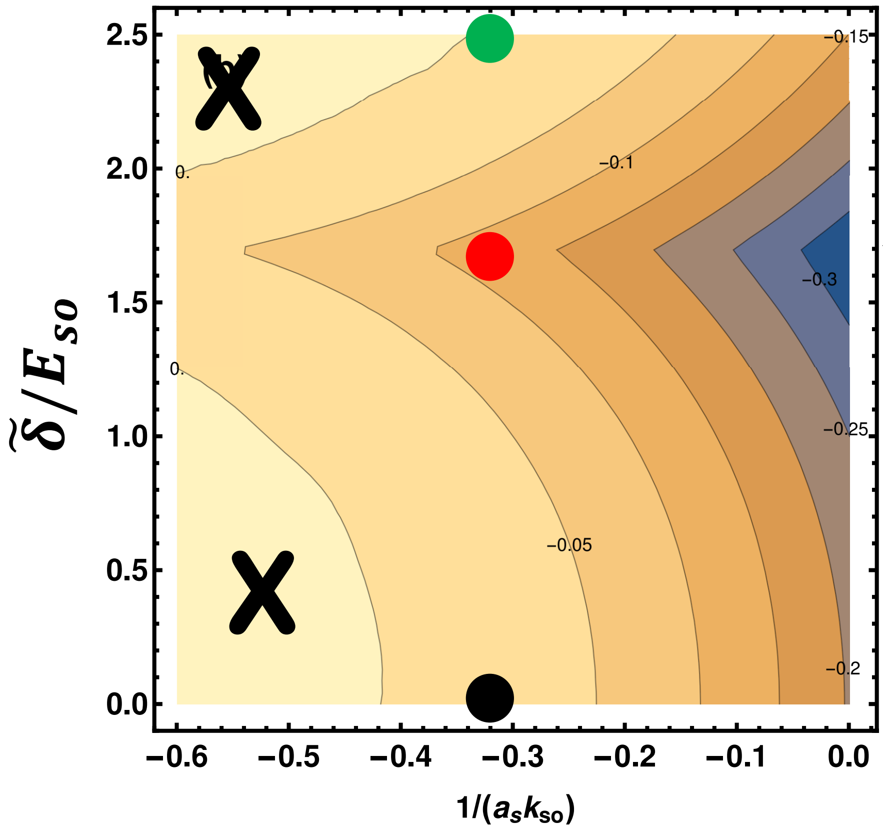
$\Omega = 2E_{so}$ (numerical)



$\Omega = 0$ (analytical)

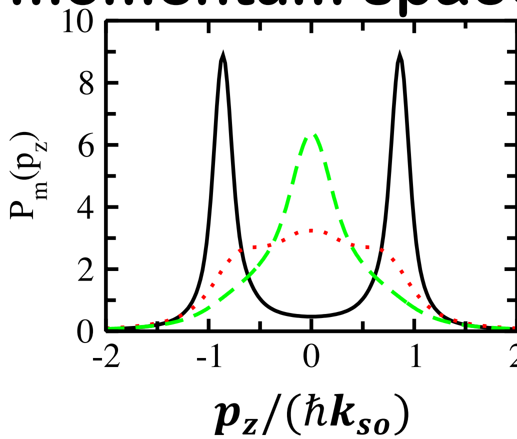


Binding Energy For $\Omega = 2E_{so}$: Lowest BB State

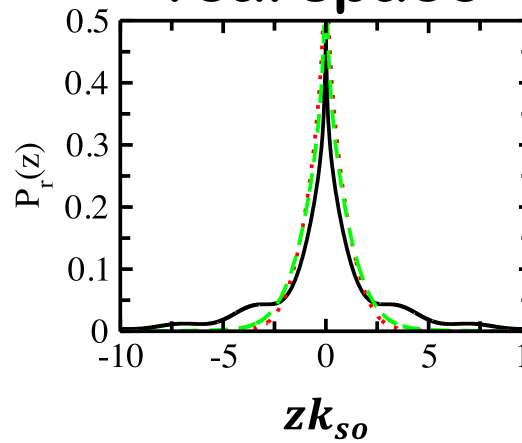


Maximum binding roughly where the dispersion has three global minima

momentum space



real space



Weakly-bound state for certain negative free-space s-wave scattering lengths.

For FF, see: Shenoy, PRA 88, 033609 (2013).
Dong et al., PRA 87, 043616 (2013).

With SOC: Fate Of Three-Boson Efimov States?

$$H = \left(\frac{\vec{p}_{12}^{\,2}}{2\mu_{12}} + \frac{\vec{p}_{12,3}^{\,2}}{2\mu_{12,3}} + \sum_{j<k} g_2\delta(\vec{r}_{jk}) + g_3\delta(\vec{r}_1 - \vec{r}_2)\delta(\vec{r}_2 - \vec{r}_3)\right) I_8$$

$$+ \frac{\hbar k_{so}}{m}(\dots) + \Omega(\dots) + \tilde{\delta}(\dots).$$

changes and extra terms due to SOC

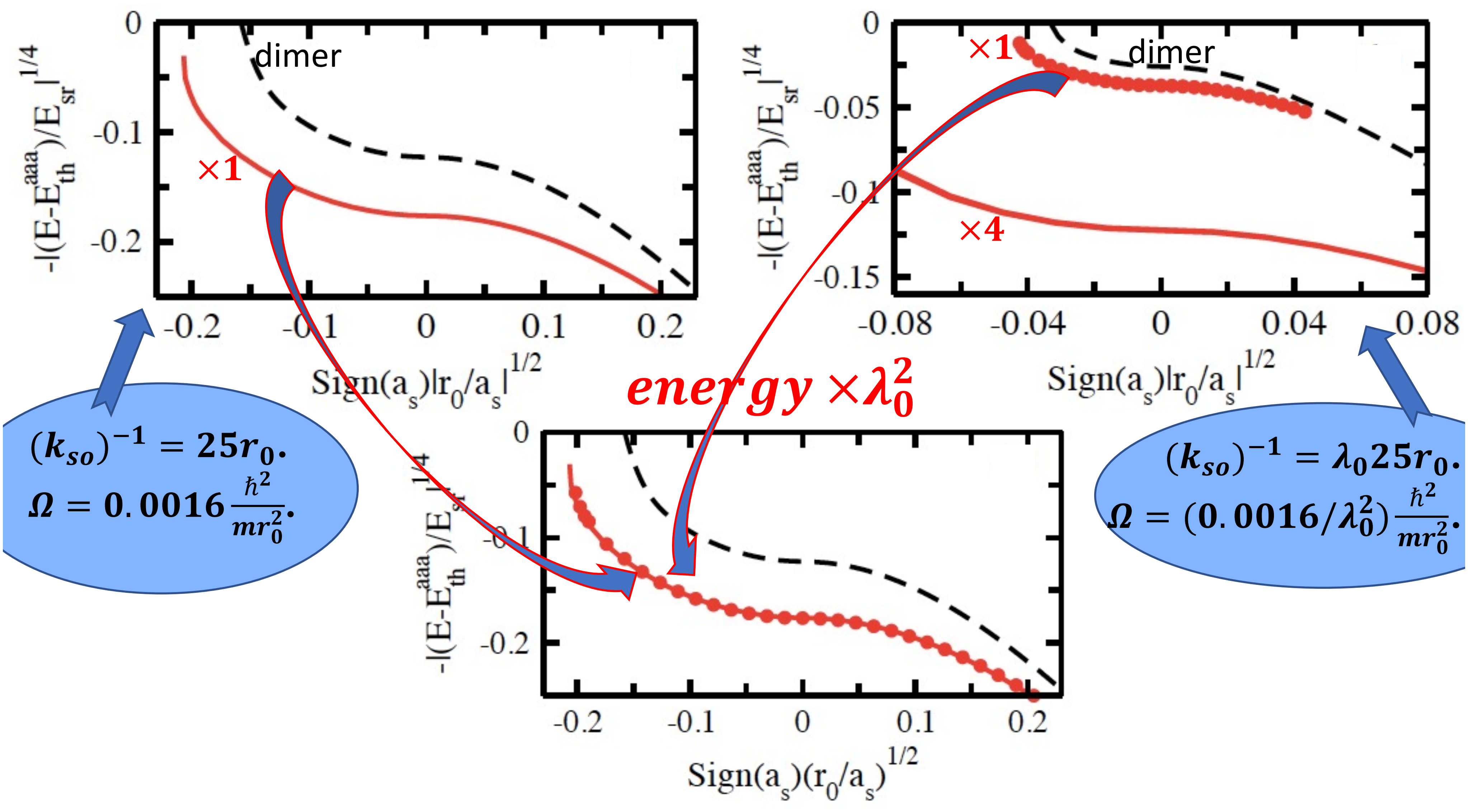
Continuous scaling symmetry (easy to check)!

$t \to \lambda^2 t;\ \vec{r} \to \lambda\vec{r};\ a_s \to \lambda a_s;\ k_{so} \to \lambda^{-1}k_{so};\ \Omega \to \lambda^{-2}\Omega;$
$\tilde{\delta} \to \lambda^{-2}\tilde{\delta};\ E \to \lambda^{-2}E;\ \kappa_* \to \lambda^{-1}\kappa_*$

Discrete scaling symmetry?

$t \to \lambda_0^2 t;\ \vec{r} \to \lambda_0\vec{r};\ a_s \to \lambda_0 a_s;\ k_{so} \to \lambda_0^{-1}k_{so};\ \Omega \to \lambda_0^{-2}\Omega;$
$\tilde{\delta} \to \lambda_0^{-2}\tilde{\delta};\ E \to \lambda_0^{-2}E;\ \kappa_* \to \kappa_*;\ \lambda_0 \approx 22.7$

Generalized Radial Scaling Law? $\tilde{\delta} = 0$ And $(\kappa_*)^{-1} = 66r_0$



Generalized Radial Scaling Law (Five Instead Of Two Axes)

Discrete scaling symmetry ($\lambda_0 \approx 22.7$)!

$a_s \rightarrow \lambda_0 a_s$;

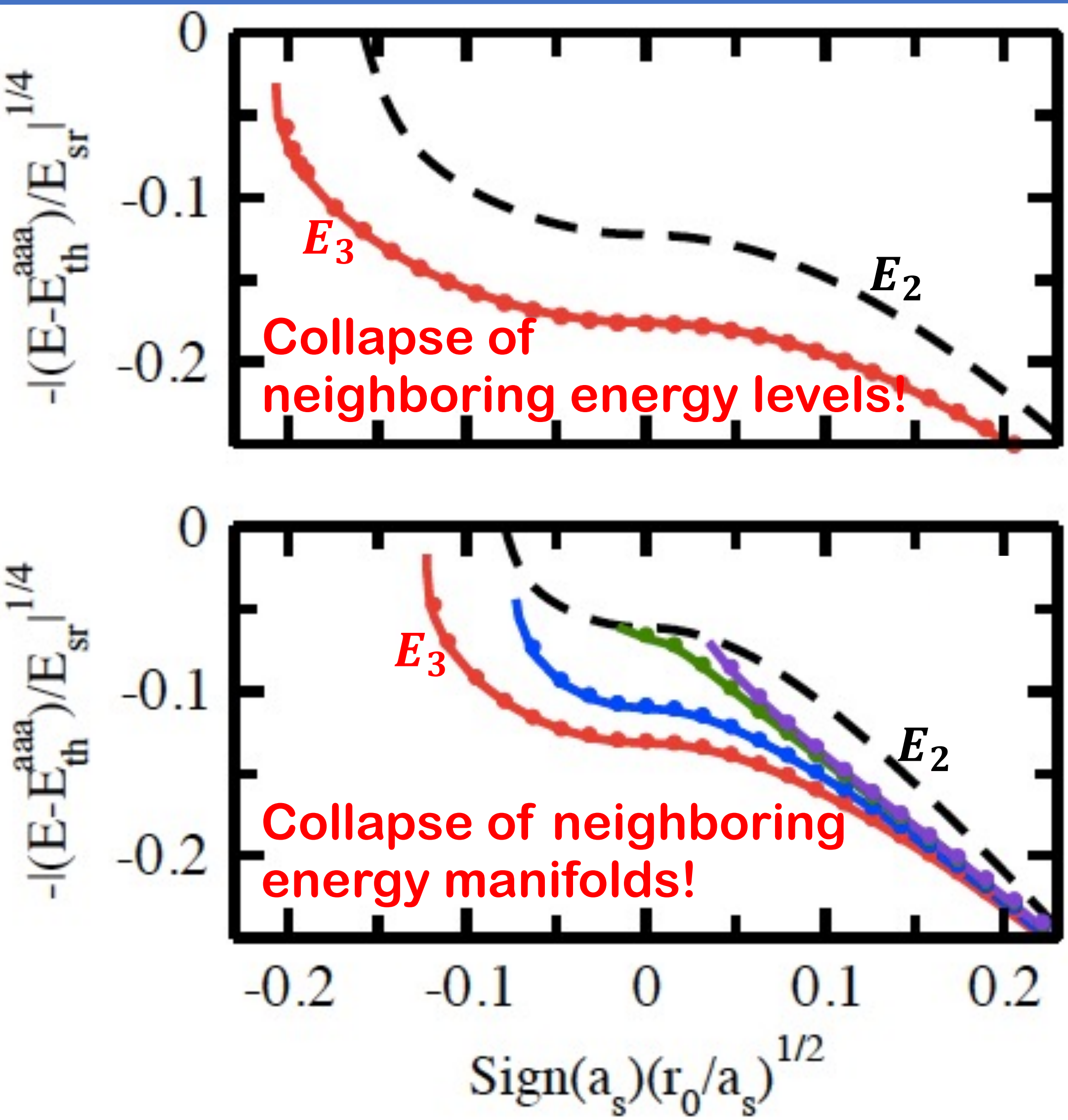
$k_{so} \rightarrow \lambda_0^{-1} k_{so}$;

$\Omega \rightarrow \lambda_0^{-2} \Omega$;

$\tilde{\delta} \rightarrow \lambda_0^{-2} \tilde{\delta}$;

$E \rightarrow \lambda_0^{-2} E$.

$\kappa_* \rightarrow \kappa_*$.



Solid line (gr. st.):
$(\kappa_*)^{-1} = 66 r_0$.
$(k_{so})^{-1} = 25 r_0$.
$\Omega = 2E_{so}$; $\tilde{\delta} = 0$.
Dots (exc. st. of H with scaled parameters).

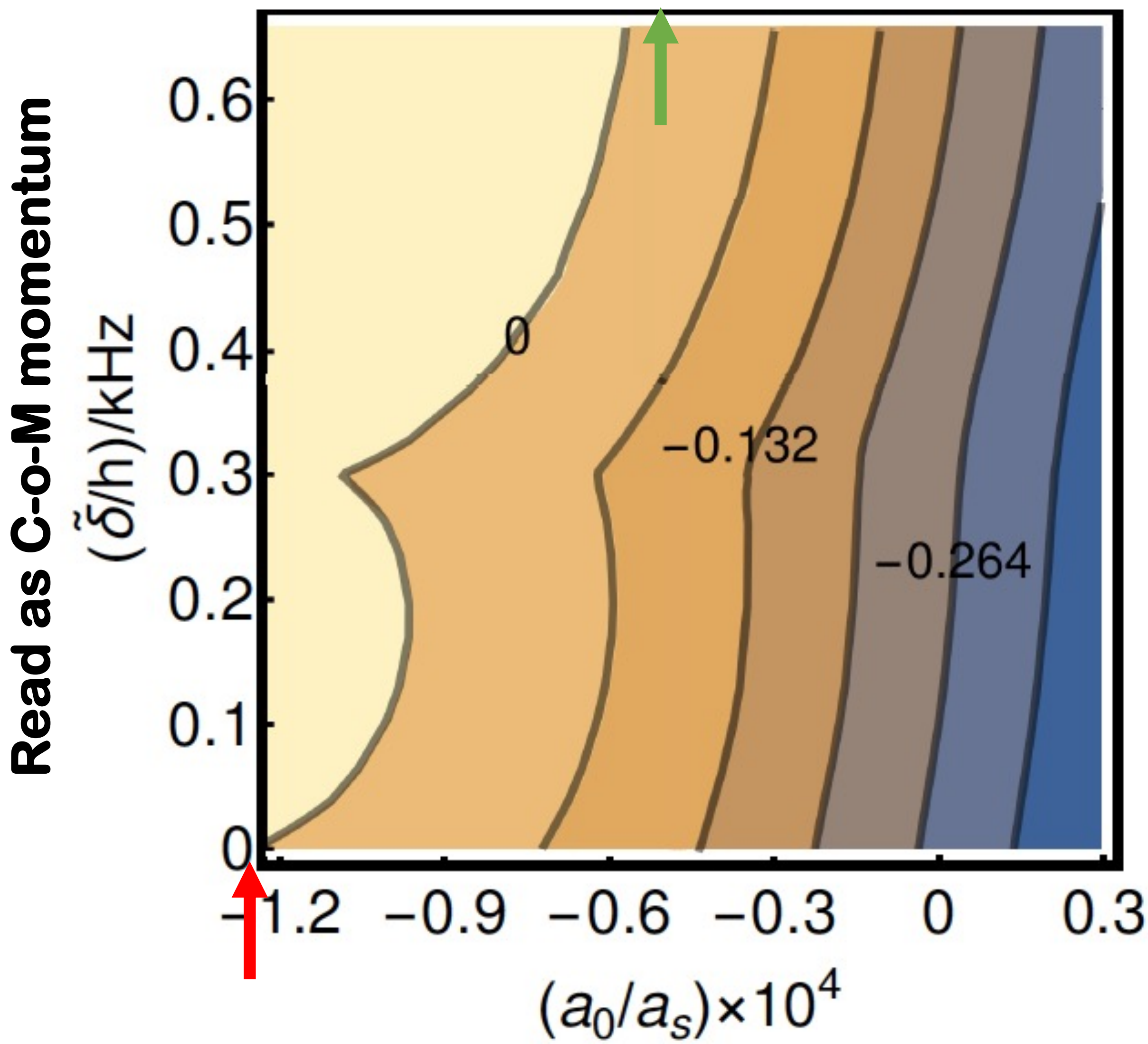
Solid lines (gr. st. manifold):
$(\kappa_*)^{-1} = 66 r_0$.
$(k_{so})^{-1} = 100 r_0$.
$\Omega = 2E_{so}$. $\tilde{\delta} = 0$.
Dots (exc. st. manifold of H with scaled parameters).

Proposal: Experimental Observability

Using three-body parameter for ^{133}Cs.
Lowest state in excited state manifold.
$(k_{so})^{-1} \approx 10,160 a_0$.
$\frac{k_{so}}{\kappa_*} \approx 1.32$ (exc. state).
$\Omega = 2E_{so}$.

Ground state resonance mostly unchanged.

Excited state resonance: Enhanced losses between $a_s \approx -7,790 a_0$ and $a_s \approx -20,190 a_0$.
Scattering length window!



Summary

Discussion of one few-body technique:
Stochastic variational approach with explicitly correlated Gaussians.

Application of this approach to bosons in the presence of 1D spin-orbit coupling.

Generalized radial scaling law for three identical bosons.

Many Thanks To Collaborators

Debraj Rakshit, Xiangyu (Desmond) Yin, Qingze Guan: ECG approach.

Qingze Guan:
ECG approach and generalized radial scaling law.

PHYSICAL REVIEW X **8**, 021057 (2018)

Three-Boson Spectrum in the Presence of 1D Spin-Orbit Coupling: Efimov's Generalized Radial Scaling Law

Q. Guan and D. Blume

PHYSICAL REVIEW A **100**, 042708 (2019)

Energetics and structural properties of two- and three-boson systems in the presence of one-dimensional spin-orbit coupling

Q. Guan and D. Blume

Thank You!