Coupled dynamics of agents and link states on the random geometric graph: phase and topological transitions

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Introduction

• Social dynamics is the study of the social macroscopic properties of human communities determined by the interactions between people within this group.
• Usually a group of individuals in a model have different states that change via interactions with his/her neighbors, defined by a network where the individuals are the nodes and interaction between them is the edges (or links).
• In this work, we also consider the time evolution of the state of the link between two agents, not only the agent state. The coupling between these two properties defines the dynamics of the ensemble [1].

Theoretical Model

• We consider a set with $M$ agents connected by a RGG (see Fig. 1) with two different opinions: blue and red. Each pair of agents can be connected by a friendly or non-friendly link. Both agents and links have a time evolution for its states and the control parameter is the probability $p$.
• There are 6 different pair combinations of agents and links, named as satisfying and unsatisfying as depicted in Fig. 1.
• The density of satisfying and unsatisfying pairs are $\rho_s$ and $\rho_u$ respectively, but the latter is a parameter of the model. They evolve with time and the system comes to a halt when $p_\infty = 0.0$, which is an absorbing state. The active state occurs when the absorbing state is not reached at all, when $p_\infty$ remains greater than zero for $t \to \infty$.
• We propose the modularity (or assortativity coefficient) as another order parameter. It is defined as Eq. (1.1) of Ref. [3].

$$\Lambda = \frac{1}{2M} \sum_{i \neq j} \left( \delta(e_i, c_i) - \frac{\kappa_i(\delta_i)}{2M} \right) \delta(c_i, c_j),$$

where $\delta(i, j)$ is the Kronecker delta, $\kappa_i$ is the degree of vertex $i$ and $c_i$ is the class or the type of vertex $i$. Here we have two classes: red ($c_i = 1$) and blue ($c_i = 2$) agents. This definition is such that $\Lambda < 1.0$ and it is positive if there are more edges between vertices of the same type than expected by chance, and negative if there are less.

Results

Phase diagram

• The density $\rho_s$ is the immediate order parameter, from which we can build the phase diagram on the space $(\alpha, p)$ shown of Fig. 3. The radius $\alpha$ is the interaction distance on the definition of the RGG, which becomes the other control parameter of the system.
• Larger $\alpha$ value (more updates on the states) and low connectivity (low $\alpha$, less interacting neighbors) enable the system to easily get the absorbing state.
• The absorbing state is composed of only satisfying pairs (see Fig. 2). In this phase all friendly edges connect equal nodes and non-friendly ones connect different ones. This feature is captured by the modularity $\Lambda$.

Figure 4: Phase diagrams in the $(\alpha, p)$ space of the modularities $\Lambda_f$ of the friendly network and $\Lambda_n$ of the non-friendly network. The colorbar on the right works for both graphs.

- Our strategy to identify the phase transition using short time simulations is to find the points $(\alpha_c, p_c)$ where the unsatisfying density follows a power law $\rho_u(t) \sim t^{-\lambda}$.
- The quality of the fit is measured by the squared of the sample’s Pearson correlation coefficient (see Eq. 1.1 of Ref. [3]).

$$c_r = \frac{\left[ \sum_i x_i y_i - \left( \frac{1}{N} \sum_i x_i \right) \left( \frac{1}{N} \sum_i y_i \right) \right]^2}{\left[ \sum_i x_i^2 - \frac{1}{N} \sum_i x_i \right] \left[ \sum_i y_i^2 - \frac{1}{N} \sum_i y_i \right]}$$

where the sum $\sum$ goes through $i = 1, 2, \ldots, N$, $x_i = \ln t$ and $y_i = \ln \rho_u$.

- We found that the phase transition can be roughly estimated by the right border of the region where $c_r \lesssim 1.0$ (see Fig. 5b).

Conclusions

- In summary, we have performed steady-state and out of equilibrium Monte Carlo simulations in order to investigate the phase transitions of the edge node coupled dynamics on the Random Geometric Graph.
- We showed that the modularity for each associated network can be used as order parameter.
- We also showed that the phase transition can be detected using short-time simulations.

Acknowledgments

References