Revisiting Kleinnian Relativity

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The Klein Space-time is a flat and non-Minkowskian spacetime whose metric is

$$ds^2 = dt^2 + dx^2 - dy^2 - dz^2 \tag{1}$$

Motivations

- Change signature universe papers;
- The Klein metric appears in metamaterials papers;
- Some extra-dimensional scenarios;
- Question: Is the Special Relativity still valid for particles in this spacetime?
- The effect of covariance principle in Klein spaces.
- Motivated by the modified special relativity theories;

Results

- Klein transformations $\rightarrow$ Time contraction and space dilatation;
- New relativistic mechanics for a single particle ;
- Free massive Fermions without energy gap;

Kleinian Transformations

$$x' = \frac{x \mp vt}{\sqrt{1 + v^2/c^2}}, \qquad (2)$$

$$t' = \frac{t \pm \frac{vx}{c^2}}{\sqrt{1 + v^2/c^2}}. \qquad (3)$$

Space-dilatation and time contraction

$$L = L_0\sqrt{1+\frac{v^2}{c^2}}, \tag{4}$$

$$T = \frac{T_0}{\sqrt{1+\frac{v^2}{c^2}}}. \tag{5}$$

Muon experiment

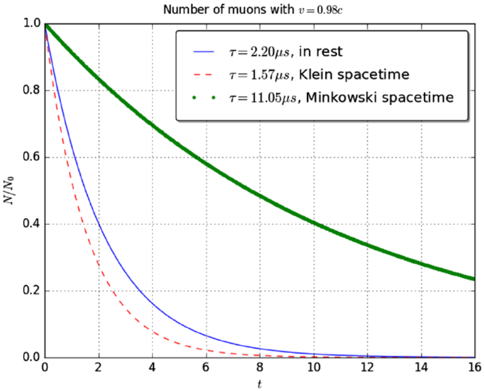

Figure: Comparação da relativiade especial e da relatividade de Klein quanto ao tempo de vida do múon.

The Kleinian Momentum

For a particle moving along the x-direction we have the momentum:

$$p = -\frac{m_0 v}{\sqrt{1 + \frac{v^2}{c^2}}}. \tag{6}$$

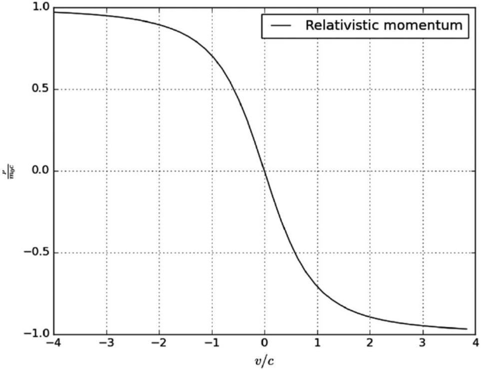

Figure: O momento é limitado, mas a velocidade não.

The Kleianian energy

$$E = \frac{m_0 c^2}{\sqrt{1 + \frac{v^2}{c^2}}}. \tag{7}$$

The Kleianian energy

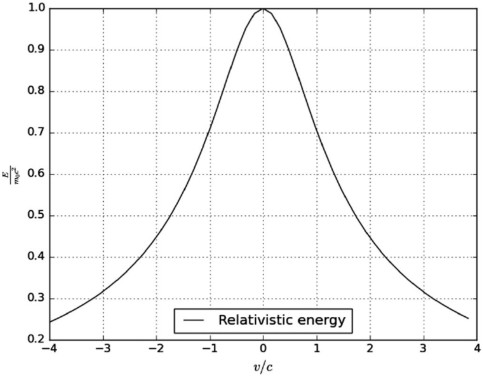

Figure: Quanto maior a velocidade da partícula livre menor sua energia total.

Searching for Fermions

Negative states:

$$\psi_{(-)}^{\mathrm{up}} = N_{(-)} e^{i(\vec{p}\cdot\vec{x} - E^{(-)}t)} \begin{bmatrix} 0 \\ \frac{-ip_x c}{E^{(-)} - m_0 c^2} \\ 1 \\ 0 \end{bmatrix}, \tag{8}$$

$$\psi_{(-)}^{\mathrm{down}} = N_{(-)} e^{i(\vec{p}\cdot\vec{x} - E^{(-)}t)} \begin{bmatrix} \frac{-ip_x c}{E^{(-)} - m_0 c^2} \\ 0 \\ 1 \\ 0 \end{bmatrix}, \tag{9}$$

Searching for Fermions

Positive states:

$$\psi^{\mathrm{up}}_{(+)} = N_{(+)} e^{i(\vec{p}\cdot\vec{x} - E^{(+)}t)} \begin{bmatrix} 1 \\ 0 \\ 0 \\ \frac{ip_x c}{E^{(+)} + m_0 c^2} \end{bmatrix}, \tag{10}$$

$$\psi^{\mathrm{down}}_{(+)} = N_{(+)} e^{i(\vec{p}\cdot\vec{x} - E^{(+)}t)} \begin{bmatrix} 0 \\ 1 \\ \frac{ip_x c}{E^{(+)} + m_0 c^2} \\ 0 \end{bmatrix}, \tag{11}$$

Searching for Fermions

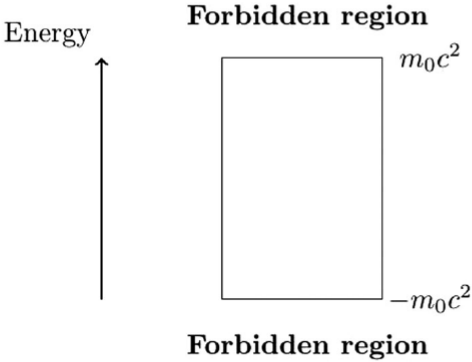

Figure: There is no energy gap.

Final Remarks

- As a future and pedagocial work we could verfify how rewrite the relativistics paradoxes for Klein spaces;
- It is intersting to note that the Dirac sea is finite for Klein in $1+1$;
- It is possible to realize some of these Fermions some metamaterial systems using eletromagnetism;

References

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