

MIRROR FERMION AS DARK MATTER CANDIDATE IN LEFT-RIGHT SYMMETRIC MODEL

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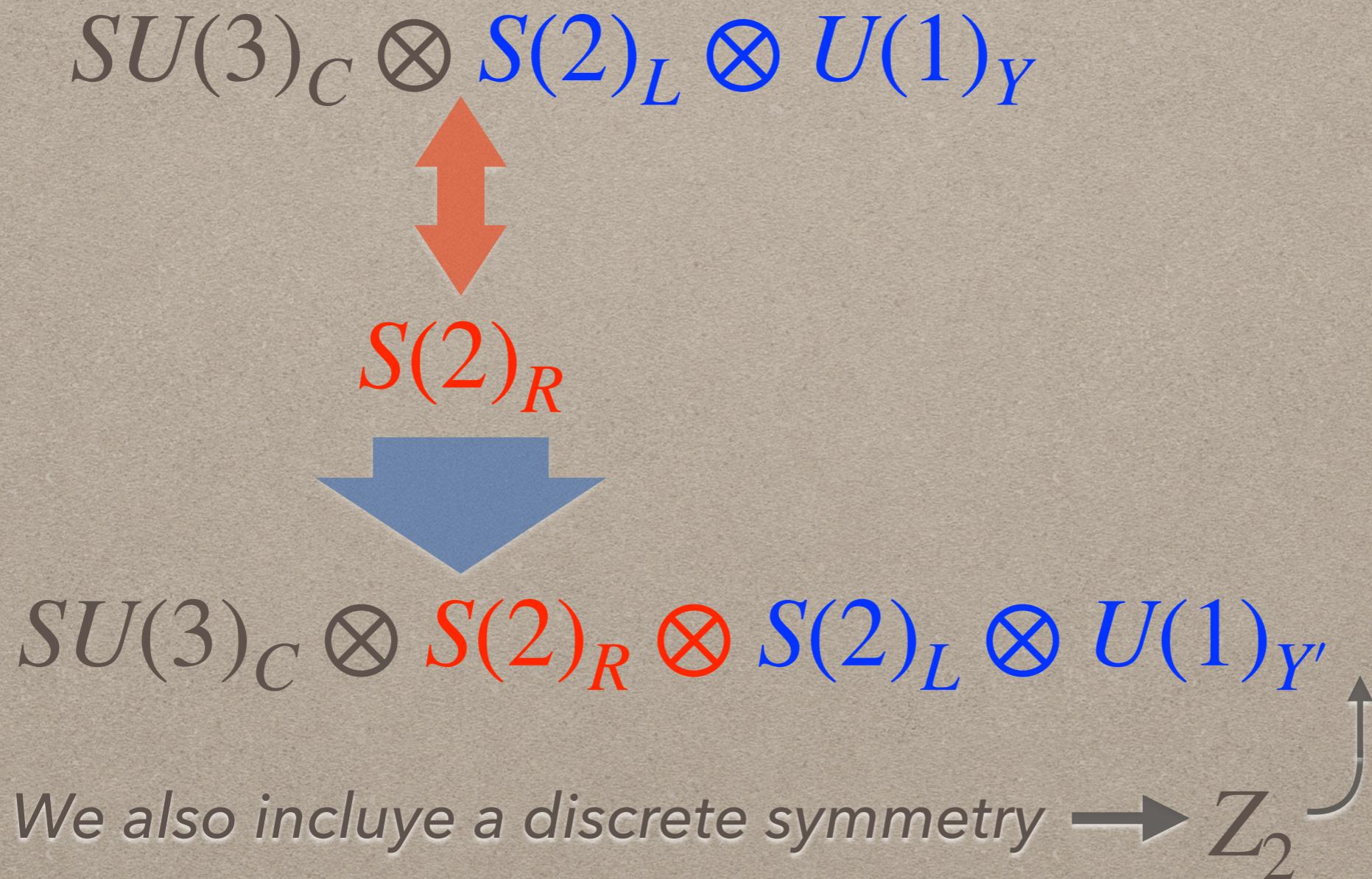
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LEFT-RIGHT MIRROR SYMMETRY



Fields in the Model

The irreducible representations are presented for fermion and scalar fields. The bold numbers denote the dimensions of representations under the gauge group, meanwhile the last entry is for the new hypercharge.

Leptons

Field	$SU(3)_C \otimes SU(2)_L \otimes SU(2)_R \otimes U(1)_{Y'}$
ℓ_{iL}^0	(1, 2, 1, -1)
ν_{iR}^0	(1, 1, 1, 0)
e_{iR}^0	(1, 1, 1, -2)
$\hat{\nu}_{iL}^0$	(1, 1, 1, 0)
\hat{e}_{iL}^0	(1, 1, 1, -2)
\hat{l}_{iR}^0	(1, 1, 2, -1)

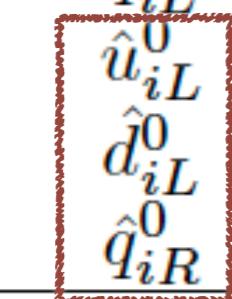
Mirror → ← **Gauge eigenstate**

Fields in the Model

The irreducible representations are presented for fermion and scalar fields. The bold numbers denote the dimensions of representations under the gauge group, meanwhile the last entry is for the new hypercharge.

Quarks

Field	$SU(3)_C \otimes SU(2)_L \otimes SU(2)_R \otimes U(1)_Y'$
u_{iR}^0	(3, 1, 1, 4/3)
d_{iR}^0	(3, 1, 1, 2/3)
q_{iL}^0	(3, 2, 1, 1/3)
\hat{u}_{iL}^0	(3, 1, 1, 4/3)
\hat{d}_{iL}^0	(3, 1, 1, 2/3)
\hat{q}_{iR}^0	(3, 1, 2, 1/3)



Mirror

Fields in the Model

The irreducible representations are presented for fermion and scalar fields. The bold numbers denote the dimensions of representations under the gauge group, meanwhile the last entry is for the new hypercharge.

Scalars

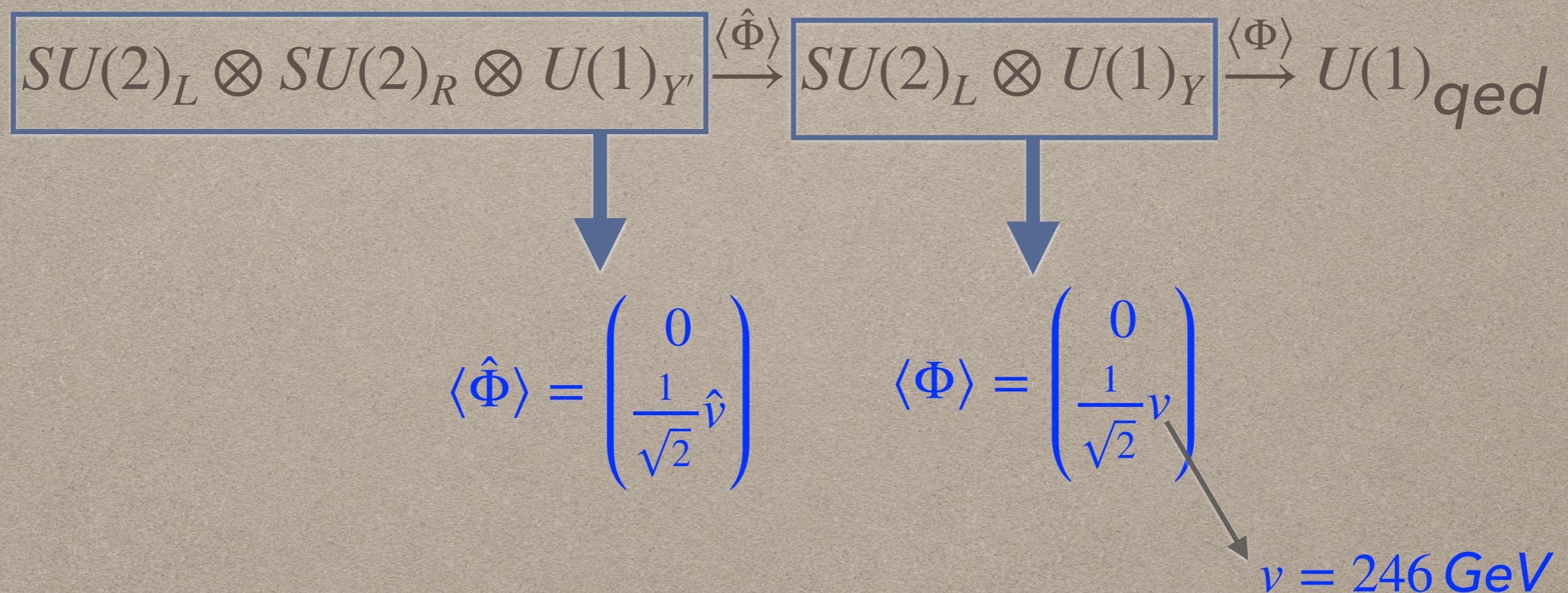
Field	$SU(3)_C \otimes SU(2)_L \otimes SU(2)_R \otimes U(1)_{Y'}$
Φ	(1, 2, 1, -1)
$\hat{\Phi}$	(1, 1, 2, -1)

Gauges

Field	$SU(3)_C \otimes SU(2)_L \otimes SU(2)_R \otimes U(1)_{Y'}$
W_L^μ	(1, 3, 1, 0)
B^μ	(1, 1, 1, 0)
W_R^μ	(1, 1, 3, 0)

SCALAR FIELDS AND SSB

The symmetry breaking pattern should be as follows



POTENTIAL FOR SCALAR FIELDS

The scalar potential is

$$V = - \left(\mu_1^2 \Phi^\dagger \Phi + \mu_2^2 \hat{\Phi}^\dagger \hat{\Phi} \right) + \frac{\lambda_1}{2} \left[(\Phi^\dagger \Phi)^2 + (\hat{\Phi}^\dagger \hat{\Phi})^2 \right] + \lambda_2 (\Phi^\dagger \Phi) (\hat{\Phi}^\dagger \hat{\Phi})$$

After the symmetry breaking, the neutral Higgs boson squared mass matrix is

$$M_{H^0}^2 = \begin{pmatrix} 2\lambda_1 v^2 & 2\lambda_2 v \hat{v} \\ 2\lambda_2 v \hat{v} & 2\lambda_2 \hat{v}^2 \end{pmatrix}$$

Thus, the neutral physical states are

$$\begin{pmatrix} H \\ \hat{H} \end{pmatrix} = \begin{pmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{pmatrix} \begin{pmatrix} Re[\phi^0] \\ Re[\hat{\phi}^0] \end{pmatrix}$$

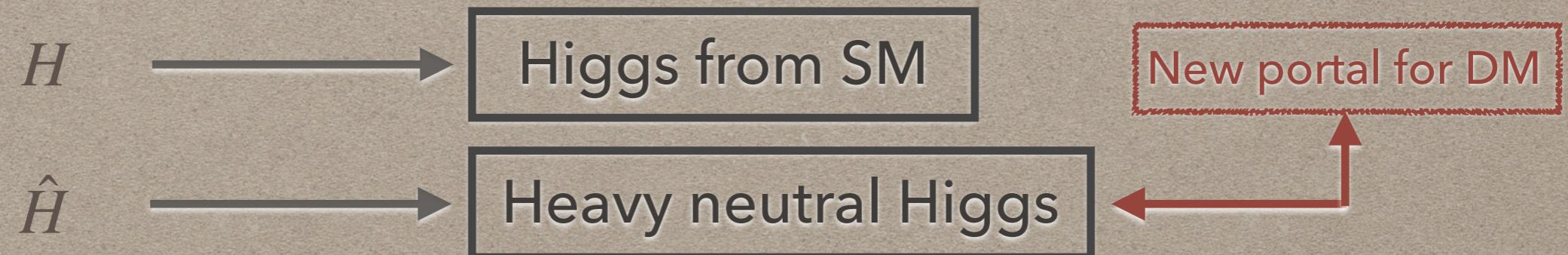
In this case the mixing angle for neutral scalar is given by

$$\tan(2\alpha) = \frac{2\lambda_2 v \hat{v}}{\lambda_1 (v^2 - \hat{v}^2)}$$

and the neutral scalar masses are

$$m_H^2 = \lambda_1 (v^2 + \hat{v}^2) - \sqrt{\lambda_1^2 (v^2 - \hat{v}^2)^2 + 4\lambda_2 v^2 \hat{v}^2}$$

$$m_{\hat{H}}^2 = \lambda_1 (v^2 + \hat{v}^2) + \sqrt{\lambda_1^2 (v^2 - \hat{v}^2)^2 + 4\lambda_2 v^2 \hat{v}^2}$$



FERMIOS AND SCALAR FIELDS

The renormalizable and gauge invariant interactions of the scalar doublets with the leptons are described by the Yukawa interactions, which takes the form

$$\mathcal{L}_Y^\ell = \sum_{i,j} \lambda_{ij} \bar{\ell}_{iL}^o \phi e_{jR}^o + \sum_{i,j} \lambda'_{ij} \bar{\ell}_{iR}^o \phi' \hat{e}_{jL}^o + \sum_{i,j} \mu_{ij} \bar{\ell}_{iL}^o e_{jR}^o + h.c.$$

The VEV's of the neutral scalars produce the fermion mass terms, which in the gauge eigenstate basis read

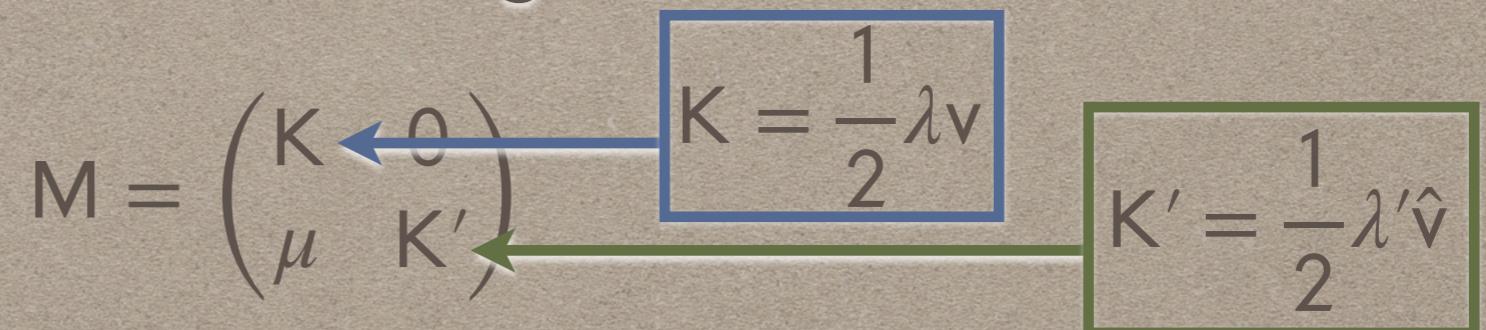
$$\mathcal{L}_{\text{mass}} = \bar{\psi}_L^o M \psi_R^o + h.c.$$

For the lepton sector, the non-diagonal mass matrix M , takes the form

$$M = \begin{pmatrix} K & 0 \\ \mu & K' \end{pmatrix}$$

$K = \frac{1}{2} \lambda v$

$K' = \frac{1}{2} \lambda' \hat{v}$



Thus, the mass matrices can be diagonalized through unitary matrices U_a , for $a = L, R$; as

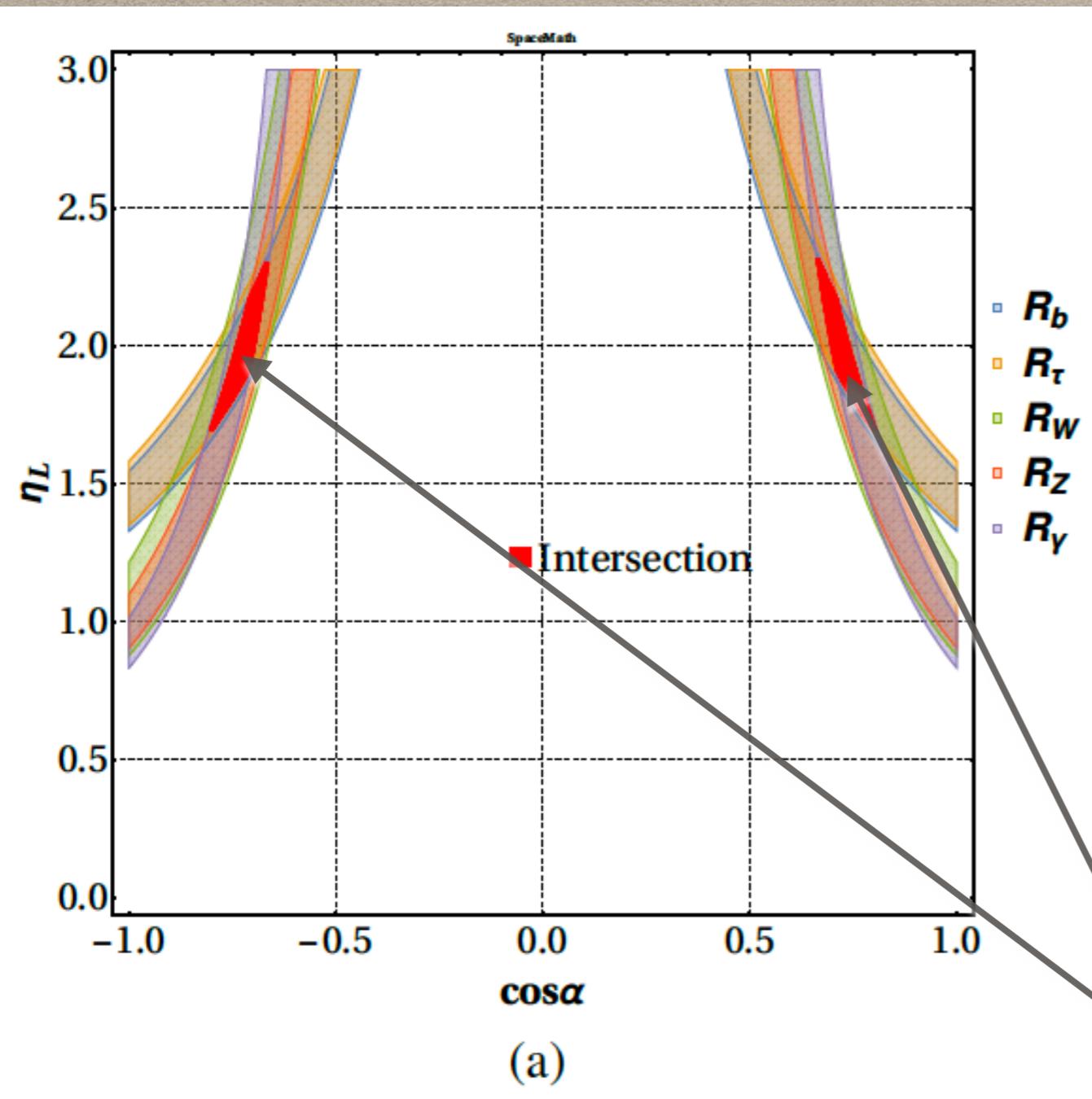
$$M_D = U_L^\dagger M U_R.$$

We write U_a as

$$U_a = \begin{pmatrix} A_a & E_a \\ F_a & G_a \end{pmatrix}.$$

Thus, the tree-level interactions of the neutral Higgs bosons H and H^{\wedge} with the light fermions are given by

$$\begin{aligned} \mathcal{L}_Y^l = & \frac{g_2}{2\sqrt{2}} \bar{f}_L^i (A_L^\dagger A_L)_{ij} \frac{m_l}{M_W} f_R^j \left(H \cos \alpha - \hat{H} \sin \alpha \right) \\ & + \frac{g'_2}{\sqrt{2}} \bar{f}_L^i \frac{m_l}{M_{W'}} (F_R^\dagger F_R)_{ij} f_R^j \left(H \sin \alpha + \hat{H} \cos \alpha \right) + h.c. \end{aligned}$$



$$\left(A_L^\dagger A_L \right)_{f_i f_j} \equiv \left(\eta_L \right)_{f_i f_j}$$

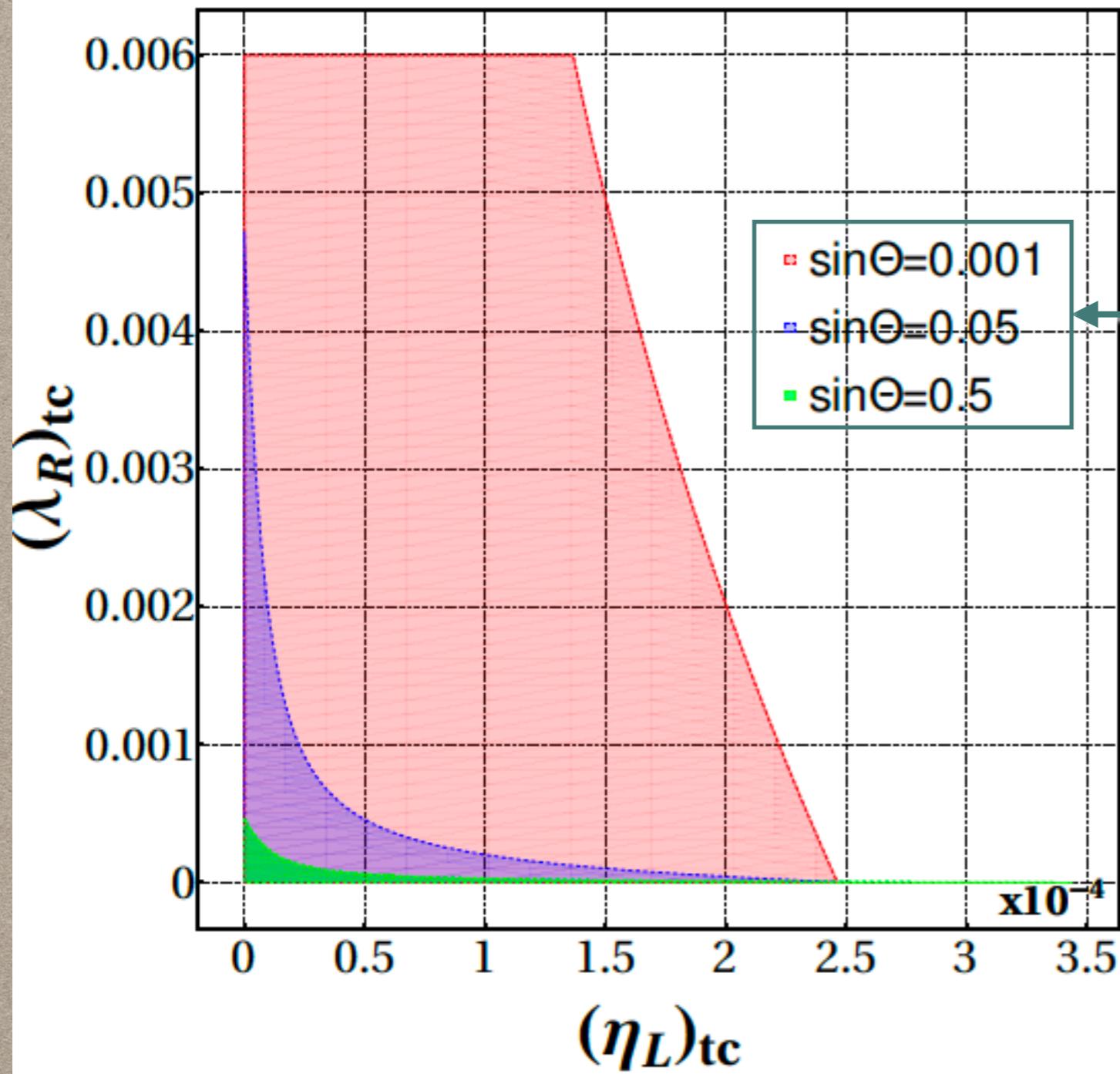
$$\mathcal{R}_X = \frac{\sigma(pp \rightarrow H) \cdot \mathcal{BR}(H \rightarrow X)}{\sigma(pp \rightarrow H^{SM}) \cdot \mathcal{BR}(H^{SM} \rightarrow X)}$$

$X = b\bar{b}, \tau^-\tau^+, \mu^-\mu^+, WW^*, ZZ^*, \gamma\gamma$

$\cos \alpha \approx \pm 0.7$
 $1.9 \lesssim \eta_L \lesssim 2.2$

$$\mathcal{BR}(t \rightarrow Zc) \lesssim 5 \times 10^{-6}$$

Mixing parameter
for Z and Z'



NEUTRINO MASSES AND MIXING

With the fields of fermions introduced in the model, we may write the gauge invariant Yukawa couplings for the neutral sector:

$$\begin{aligned}\mathcal{L}_\nu = & h_{ij} \bar{\hat{\nu}}_{iL} \nu_{jR} + \chi_{ij} \bar{\nu}_{iR} \left(\nu_{jR} \right)^c + \hat{\chi}_{ij} \bar{\hat{\nu}}_{iL} \left(\hat{\nu}_{jL} \right)^c + \sigma_{ij} \bar{l}_{iL} \tilde{\Phi} \left(\hat{\nu}_{jL} \right)^c \\ & + \hat{\sigma}_{ij} \bar{\hat{l}}_{iR} \tilde{\tilde{\Phi}} \left(\nu_{jR} \right)^c + \lambda_{ij} \bar{l}_{iL} \tilde{\Phi} \nu_{jR} + \hat{\lambda}_{ij} \bar{\hat{l}}_{iR} \tilde{\tilde{\Phi}} \hat{\nu}_{jL} + h.c.\end{aligned}$$

When doublet scalar fields acquire VEV's we get the neutrino mass terms

$$\mathcal{L}_{\nu\text{-mass}} = (\bar{\Psi}_{\nu L}, \quad \bar{\Psi^c}_{\nu L}) \begin{pmatrix} M_L & M_D \\ M_D & M_R \end{pmatrix} \begin{pmatrix} \Psi_{\nu R} \\ \Psi^c_{\nu R} \end{pmatrix}$$

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$$\boxed{\Psi_{\nu(L,R)} = \begin{pmatrix} \nu_i \\ \hat{\nu}_i \end{pmatrix}_{(L,R)}}$$

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$$M_L = \boxed{\begin{pmatrix} 0 & \frac{v}{\sqrt{2}} \sigma_{ij} \\ \frac{v}{\sqrt{2}} \sigma_{ij}^T & \hat{\chi}_{ij} \end{pmatrix}}$$

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$$M_R = \boxed{\begin{pmatrix} \chi_{ij} & \frac{\hat{v}}{\sqrt{2}} \hat{\sigma}_{ij} \\ \frac{\hat{v}}{\sqrt{2}} \hat{\sigma}_{ij}^T & 0 \end{pmatrix}}$$

NEUTRINO MASSES AND MIXING

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$$M_D = \boxed{\begin{pmatrix} \frac{v}{\sqrt{2}} \lambda_{ij} & 0 \\ h_{ij} & -\frac{\hat{v}}{\sqrt{2}} \hat{\lambda}_{ij} \end{pmatrix}}$$

DARK MATTER AND NEUTRINOS

The charge under Z2 symmetry for the doublet scalar fields can generate two scenarios,

$$\Phi, \hat{\Phi} \xrightarrow[Z_2]{} \Phi, \hat{\Phi} \quad \Rightarrow \quad h_{ij} = \sigma_{ij} = \hat{\sigma}_{ij} = 0$$

$$\begin{aligned} \mathcal{L}_\nu = & h_{ij} \bar{\hat{\nu}}_{iL} \nu_{jR} + \chi_{ij} \bar{\nu}_{iR} \left(\nu_{jR} \right)^c + \hat{\chi}_{ij} \bar{\hat{\nu}}_{iL} \left(\hat{\nu}_{jL} \right)^c + c_{ij} \bar{l}_{iL} \tilde{\Phi} \left(\hat{\nu}_{jL} \right)^c \\ & + \hat{c}_{ij} \bar{\hat{l}}_{iR} \tilde{\tilde{\Phi}} \left(\nu_{jR} \right)^c + \lambda_{ij} \bar{l}_{iL} \tilde{\Phi} \nu_{jR} + \hat{\lambda}_{ij} \bar{\hat{l}}_{iR} \tilde{\tilde{\Phi}} \hat{\nu}_{jL} + h.c. \end{aligned}$$

DARK MATTER AND NEUTRINOS

The charge under Z2 symmetry for the doublet scalar fields can generate two scenarios,

$$\Phi, \hat{\Phi} \xrightarrow[Z_2]{} \Phi, \hat{\Phi} \quad \text{blue arrow} \quad h_{ij} = \sigma_{ij} = \hat{\sigma}_{ij} = 0$$

$$\Phi, \hat{\Phi} \xrightarrow[Z_2]{} \Phi, -\hat{\Phi} \quad \text{orange arrow} \quad h_{ij} = \sigma_{ij} = \hat{\lambda}_{ij} = 0$$

$$\begin{aligned} \mathcal{L}_\nu = & h_{ij} \bar{\hat{\nu}}_{iL} \nu_{jR} + \chi_{ij} \bar{\nu}_{iR} \left(\nu_{jR} \right)^c + \hat{\chi}_{ij} \bar{\hat{\nu}}_{iL} \left(\hat{\nu}_{jL} \right)^c + \cancel{\sigma_{ij} \bar{l}_{iL} \tilde{\Phi} \left(\hat{\nu}_{jL} \right)^c} \\ & + \hat{\sigma}_{ij} \bar{\hat{l}}_{iR} \tilde{\Phi} \left(\nu_{jR} \right)^c + \lambda_{ij} \bar{l}_{iL} \tilde{\Phi} \nu_{jR} + \cancel{\hat{\lambda}_{ij} \bar{\hat{l}}_{iR} \tilde{\Phi} \hat{\nu}_{jL}} + h.c. \end{aligned}$$

$$\Phi, \hat{\Phi} \xrightarrow[Z_2]{} \Phi, \hat{\Phi} \xrightarrow{} h_{ij} = \sigma_{ij} = \hat{\sigma}_{ij} = 0$$

In this case the ordinary neutrinos can be written separately from for mirror neutrinos in the matrix as follows

$$(\bar{\nu}_{iL}, \quad \bar{\nu^c}_{\nu R}) \begin{pmatrix} 0 & \frac{\nu}{\sqrt{2}} \lambda_{ij} \\ \frac{\nu}{\sqrt{2}} \lambda_{ij}^T & \chi_{ij} \end{pmatrix} \begin{pmatrix} \nu_{iL}^c \\ \nu_{jR} \end{pmatrix} \xleftarrow{\text{Ordinary}}$$

$$(\hat{\bar{\nu}}_{iL}, \quad \hat{\bar{\nu^c}}_{\nu R}) \begin{pmatrix} \hat{\chi}_{ij} & \frac{\hat{\nu}}{\sqrt{2}} \hat{\lambda}_{ij} \\ \frac{\hat{\nu}}{\sqrt{2}} \hat{\lambda}_{ij}^T & 0 \end{pmatrix} \begin{pmatrix} \hat{\nu}_{iL}^c \\ \hat{\nu}_{jR} \end{pmatrix} \xleftarrow{\text{Mirror}}$$

By assuming the natural hierarchy $|\lambda_{ij}| \ll |\chi_{ij}|$ among the mass terms, the mass matrix for ordinary neutrinos can approximately be diagonalized, yielding

$$\begin{pmatrix} 0 & \frac{\nu}{\sqrt{2}}\lambda_{ij} \\ \frac{\nu}{\sqrt{2}}\lambda_{ij}^T & \chi_{ij} \end{pmatrix} \sim \begin{pmatrix} M_\nu^{light} & 0 \\ 0 & M_\nu^{heavy} \end{pmatrix}$$

$M_\nu^{light} \approx -\frac{\nu^2}{2}\lambda\chi^{-1}\lambda^T$

$M_\nu^{heavy} \approx \chi$

We parameterize λ and χ matrices as

$$\lambda = yS \xrightarrow{S = S^T} S = S^T$$

$$\chi = mD^{-1}S \xrightarrow{D = \text{Diagonal}(y_1, y_2, y_3)}$$

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Then, the inverse matrix for χ is

$$\chi^{-1} = \frac{1}{m} S^{-1} D$$

and the matrix for light neutrinos is

$$M_\nu^{light} = \frac{\nu^2 y^2}{2m} S D$$

By assuming the natural hierarchy $|\lambda_{ij}| \ll |\chi_{ij}|$ among the mass terms, the mass matrix for ordinary neutrinos can approximately be diagonalized, yielding

$$\begin{pmatrix} 0 & \frac{\nu}{\sqrt{2}}\lambda_{ij} \\ \frac{\nu}{\sqrt{2}}\lambda_{ij}^T & \chi_{ij} \end{pmatrix} \sim \begin{pmatrix} M_\nu^{light} & 0 \\ 0 & M_\nu^{heavy} \end{pmatrix}$$

In order to diagonalize the matrix M we use the PNMS matrix

$$\text{Diagonal} [m_{\nu_1}, m_{\nu_2}, m_{\nu_3}] = \frac{\nu^2 y^2}{2m} U_\nu^T S U_\nu^T D$$

PNMS transpose matrix

DARK MATTER FROM MIRROR NEUTRINO

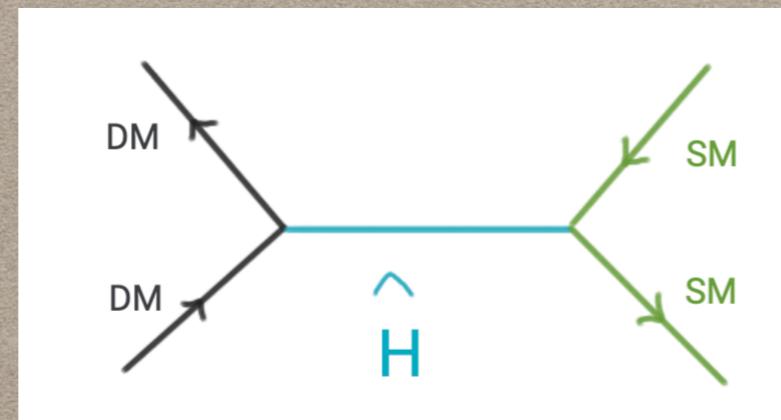
We consider the lightest mirror neutrino as Dark Matter candidate. The mass matrix for mirror neutrinos was introduced as

$$\begin{pmatrix} \bar{\hat{\nu}}_{iL}, & \bar{\hat{\nu}}^c_{\nu R} \end{pmatrix} \begin{pmatrix} \hat{\chi}_{ij} & \frac{\hat{\nu}}{\sqrt{2}} \hat{\lambda}_{ij} \\ \frac{\hat{\nu}}{\sqrt{2}} \hat{\lambda}_{ij}^T & 0 \end{pmatrix} \begin{pmatrix} \hat{\nu}_{iL}^c \\ \hat{\nu}_{jR} \end{pmatrix} \xleftarrow{\text{Mirror}}$$

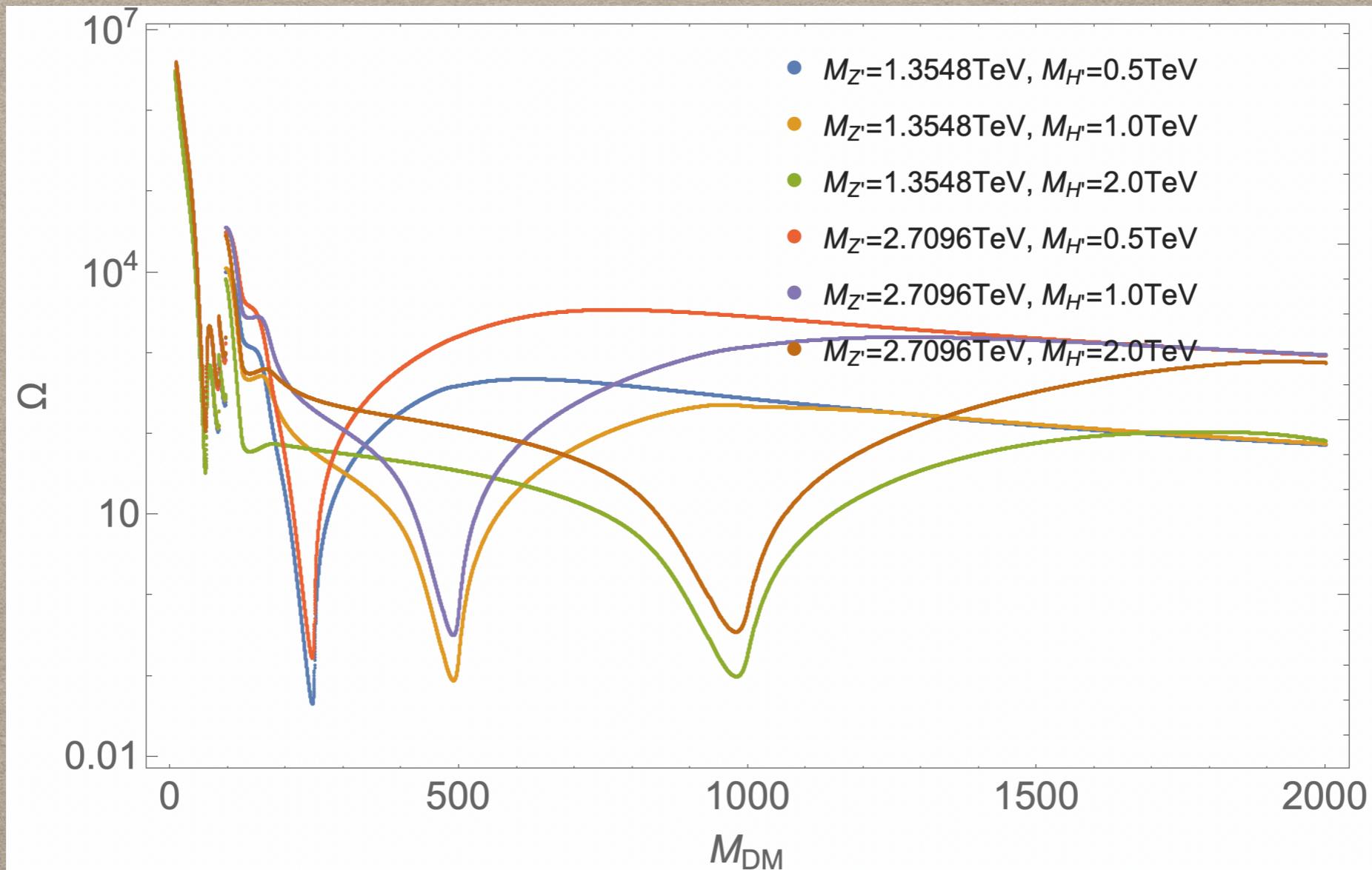
In the scenario

$$\Phi, \hat{\Phi} \xrightarrow[Z_2]{} \Phi, \hat{\Phi}$$

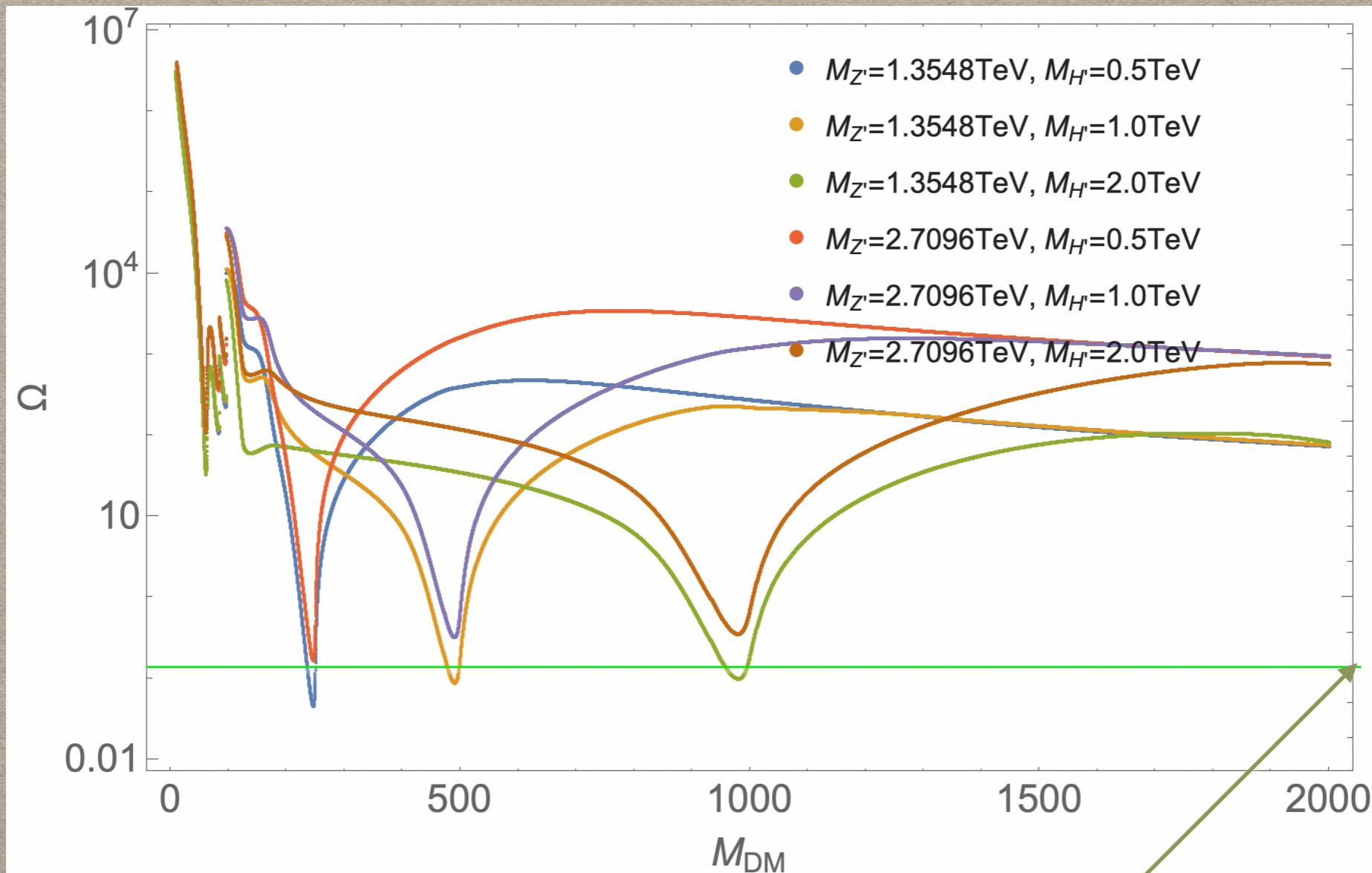
the candidate is linked with particles through the mix of the H^\wedge



RELIC DENSITY VS DM MASS [GeV]



RELIC DENSITY VS DM MASS [GEV]



$$\Omega_{DM} h^2 = 0.12 \pm 0.001$$

SUMMARY

- We explore DM in LRMM, assuming the lightest mirror neutrino as DM.
- Masses for SM neutrinos are included by see-saw type I.
- Some model parameters are constrained to explore a benchmark for DM relic density and SI cross section
- Under the Plank collaboration reported value for non baryonic relic density, we find that the heavy neutral scalar is viable as portal with mass $\sim 1\text{TeV}$ for the reported limit for Z' mass in the LRM.

REFERENCES

- P.A. Zyla et al. (PDG), *Prog. Theor. Exp. Phys.* 2020, 083C01 (2020) and 2021 update.
- Planck 2018 results. VI. Cosmological parameters, *Astron. Astrophys.* 641, A6, (2018).
- M. A. Arroyo-Ureña, R. Gaitan, R. Martinez, J. H. Montes de Oca Yemha; *Eur. Phys. J. C* 80 (2020) 8, 788.
- Semenov. A., *LanHEP, Nucl.Inst.&Meth.* A393 (1997) p. 293 .
- G. Bélanger, F. Boudjema, A. Goudelis, A. Pukhov, B. Zaldivár, *Comput.Phys.Commun.* 231 (2018) 173.
- K.S. Babu and Rabindra N. Mohapatra., *Phys. Rev. D* 41 (1990), p. 1286. DOI: 10.1103/PhysRevD. 41.1286
- V E. Ceron et al., *Phys. Rev. D* 57 (1998), pp. 1934-1939. DOI: 10.1103/PhysRevD.57.1934. arXiv: hep-ph/9705478.
- U. Cotti et al., *Phys. Rev. D* 66 (2002), p. 015004. DOI: 10.1103/PhysRevD.66.015004. arXiv: hep-ph/0205170.
- Gaitan, et. al., *Nucl.Part.Phys.Proc.* 267-269 (2015) 101-107, Contribution to: SILAFAE 2014, 101-107.
- Gaitan, et. al., *Eur.Phys.J.C* 72 (2012) 1859 , e-Print: 1201.3155 [hep-ph]
- Gaitan, et. al., *Int.J.Mod.Phys.A* 22 (2007) 2935-2943 , e-Print: hep-ph/0605249 [hep-ph].