The impact of higher-modes into binary black hole parameter estimation with numerical waveforms

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ABSTRACT

Gravitational waves higher-modes improve parameter estimation and bring into play waveform additional features of source dynamics. In this project, we investigate which set of higher-modes is necessary to improve the precision of parameter estimation – in particular, the luminosity distance – of massive binary black holes. We rely exclusively on numerical waveforms for Bayesian estimation, interpolating them for all needed values of intrinsic parameters. Preliminary results for non-spinning waveforms show that the luminosity distance estimation improves dramatically, including modes $(l \leq 4, l-1 \leq |m| \leq l)$.

Introduction

In gravitational wave (GW) spectroscopy, we study the sensitivity of *waveform morphology*, *mode-mixing*, and *parameter estimation* (PE) regarding the modes from the GW decomposition into spherical/spheroidal harmonics [1, 2]. The more modes played into account in the *signal-to-noise ratio* (SNR) templates, the more accurately the PE is. In the parameter set of a binary black hole (BBH), we will focus on the luminosity distance measurement since it is a key for GW Astronomy and Cosmology research. To compute Bayesian inference, we interpolate numerical relativistic (NR) waveforms from the SXS catalog [3].

Methods

We employed 23 NR waveforms of spinless BBH. For each strain mode we interpolate its amplitude $A_{lm}(t)$ and phase $\Phi_{lm}(t)$ over the mass-ratio $q = [1:10]$. Those modes are time-series coefficient of the spin-weighted spherical harmonics decomposition, given by NR simulations,

$$r h_{lm}(t)/M = A_{lm}(t)\, e^{-i\Phi_{lm}(t)} = \int d\Omega\, h(t,\vec{x})\, {}_{-2}Y_{lm}(\theta,\phi)\,. \tag{1}$$

To compute the Bayesian estimation, we first defined the inner product between two arbitrary functions as

$$\langle a \,|\, b\rangle_k \equiv 4\Re\left\{\int \frac{\tilde{a}(f)^* \tilde{b}(f)}{S_k(f)}\, df\right\}, \tag{2}$$

where $S_k(f)$ is the *power spetral density* (PSD) of a given the detector k. The SNR of a detector network is then $SNR^2 = \sum_k SNR^2_{(k)} = \sum_k \langle h\,|h\rangle_k$.

From the Fourier Transformation of the Eq.1, we separate the waveform polarizations,

$$\tilde{h}_+(f) = \sum_{l,m} \frac{1}{2}\left(\tilde{h}_{lm}(f) + \tilde{h}^*_{lm}(-f)\right)\, {}_{-2}Y_{lm}(\iota,\phi)\,, \tag{3}$$

$$\tilde{h}_\times(f) = \sum_{l,m} \frac{i}{2}\left(\tilde{h}_{lm}(f) - \tilde{h}^*_{lm}(-f)\right)\, {}_{-2}Y_{lm}(\iota,\phi)\,, \tag{4}$$

and compile the strain observed by the detector k,

$$\tilde{h}^{(k)}(f) = \left\{F_+^{(k)}(\alpha,\delta,\psi)\,\tilde{h}_+(f) + F_\times^{(k)}(\alpha,\delta,\psi)\,\tilde{h}_\times(f)\right\} e^{-2i\pi f(\tau^{(k)}+t_0)}\,, \tag{5}$$

where $F_{+/\times}$ are the *antenna pattern functions.*

Our set of parameters Θ is divided into:

i) Intrinsic: symmetric mass-ratio $\eta = q/(1+q)^2$, and the chirp mass $M_c = \eta^{3/5}M$;

ii) Extrinsic: luminosity distance $r \to d_L$; coalescence time t_0; the GW propagation direction angles, from the orbital plane, inclination ι and phase ϕ; the angular sky coordinates right-ascension α and declination δ; and the GW polarization angle ψ.

For the posterior distribution $p(\Theta|data) \propto \mathcal{L}(data|\Theta)\,\pi(\Theta)$, we defined all priors $\pi(\Theta)$ as uniform, and the logarithm of the likelihood as

$$\log \mathcal{L}(\tilde{h}|\Theta) = -\frac{1}{2}\sum_k \langle \Delta \,|\, \Delta\rangle_k\,, \tag{6}$$

such that $\Delta \equiv \tilde{h}_{injection} - \tilde{h}_{template}$. Here, both injection and template are interpolated NR waveforms, but with different *modes set*:

i) Injections: $(l \leq 8,\ |m| \leq l)$;

ii) Templates: $(l = |m| = 2)$, $(l \leq 3,\ |m| = l)$, $(l \leq 4,\ l-1 \leq |m| \leq l)$, $(l \leq 4; |m| \leq l)$.

Preliminary results

In our preliminary results, we considered:

i) Single detector;

ii) Zero noise approximation;

iii) Parameter set $\Theta = (\eta = 0.16,\ M_c = 40M_\odot;\ d_L = 600Mpc,\ \cos\iota = 0.5,\ \phi = 0.0,\ t_0 = 0.4s)$;

iv) Bayesian inference: *dynesty* sampler [4].

Figure 1 shows the *relative bias* between the injection parameters and the estimative results, taken from the posterior distribution for different modes set used in the templates computation. The x-axis labels each parameter on Θ; the bias is flagged by the different dot shapes representing the modes set. The plot on the right is a zoom from the left one, such that the d_L bias values can be better distinguished from each modes set. For this particular parameter, we notice that the dominant modes do not guarantee enough accuracy for the estimative and bring degeneracy for the $\cos(\iota)$ estimative. It happens since the signal amplitude is sensitive to both parameters. However, for $l \leq 4$, and when the modes $|m| = l-1$ is introduced (see the green diamond dot shape), the estimative improves at its maximum. The additional subdominant modes in $(l \leq 4,\ |m| \leq l)$ saturates the estimative in this single detector test. The figure 2 presents similar analysis but shows the *error* between injection parameters and estimative results which conforms our conclusions.

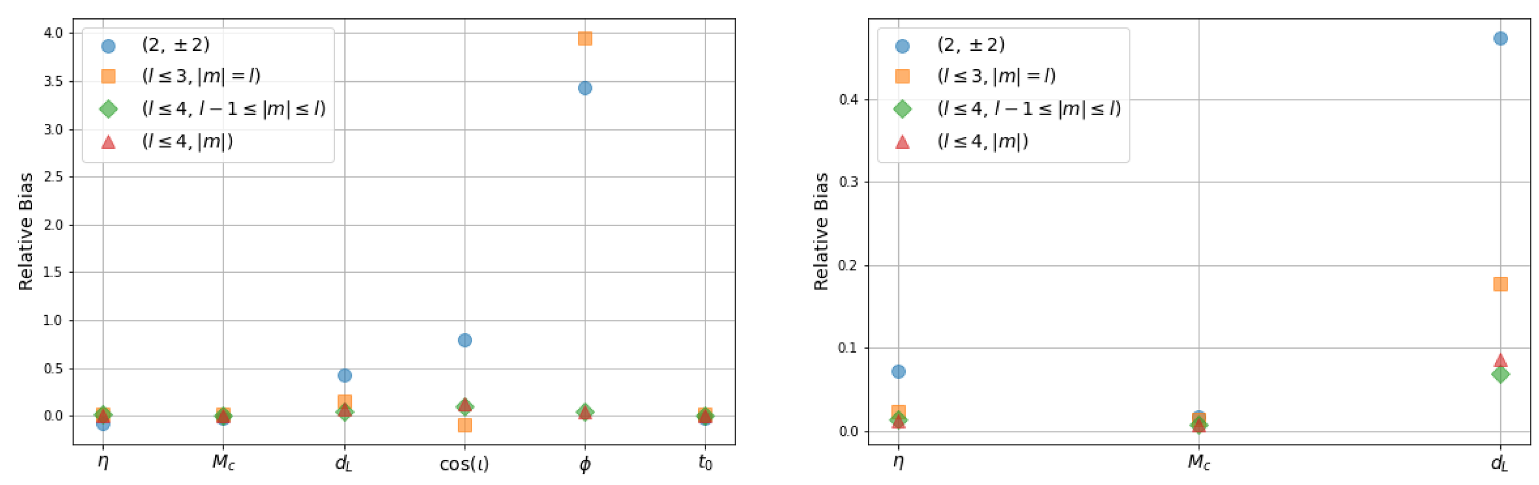

Figure 1: Relative bias.

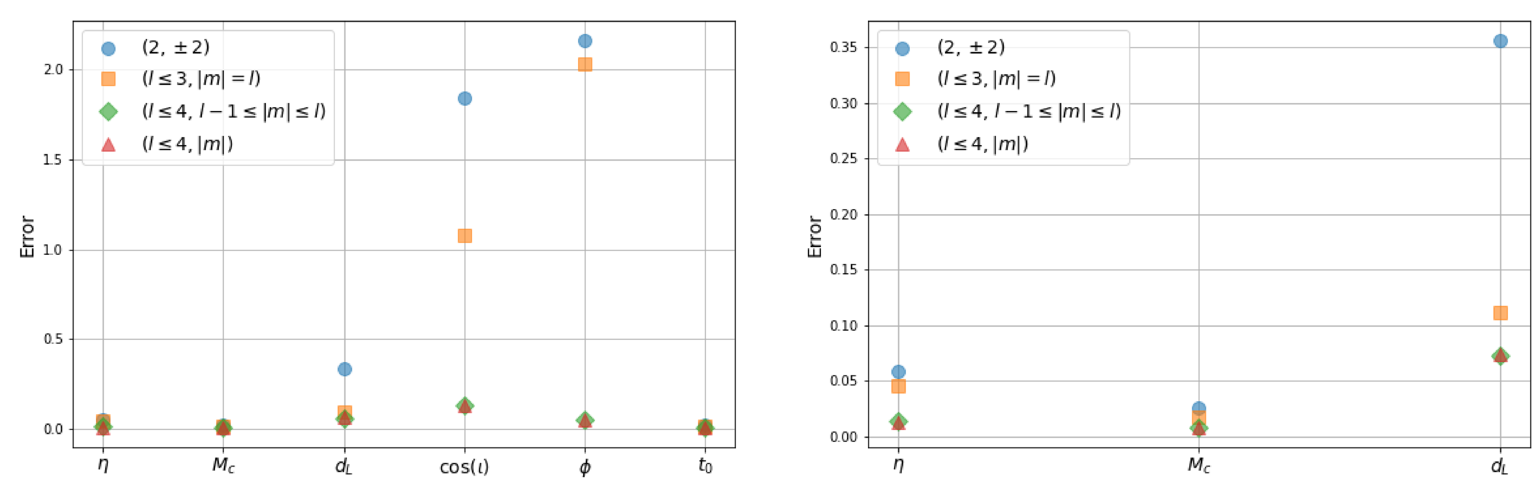

Figure 2: Error.

Forthcoming Steps

To improve our results, we will consider:

i) Four detectors with their respective pattern functions and PSD;

ii) Use the all extrinsic parameters in the PE;

iii) Non-precessing spinning black holes.

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References

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