OnLine – December 14th, 2021

Reuven Opher Workshop on Challenges of New Physics in Space

FAPESP ICTP unesp IFT - UNESP

On a Poynting vector for gravitational waves using gravitoelectromagnetism

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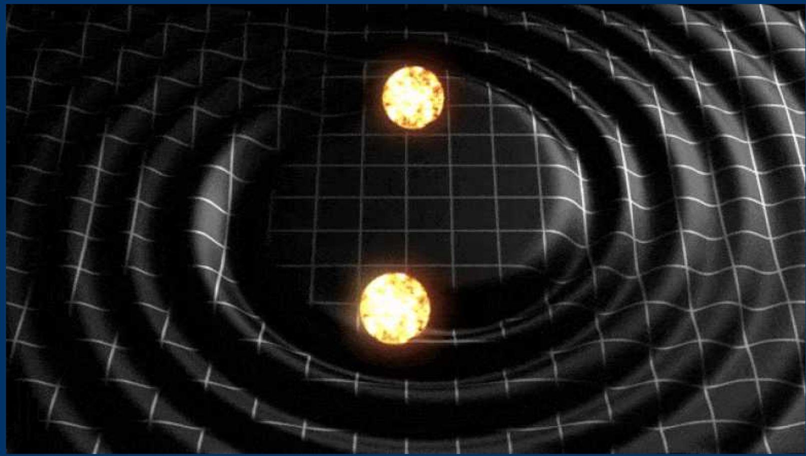
Outline of this poster

- Gravitational waves
- Gravitoelectromagnetism
- Poynting vector in GR and in GEM
- An illustration with GW detectors
- Final remarks

Gravitational waves



Phenomena predicted by general relativity

Weak fields: $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$ $\quad |h_{\mu\nu}| \ll 1$

$$[\eta_{\mu\nu}] = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Low speeds: $v \ll c$

Linear approximation: only keeps $O(h_{\mu\upsilon})$ terms

Einstein equations + gauge TT $\rightarrow$ wave equation for $h_{\mu\upsilon}$.

https://scitechdaily.com/images/Gravitational-Waves.gif

Gravitational waves

TT gauge → [$h_{\mu\upsilon}$] has 5 independent components

These components allow the determination of all the wave's parameters:
direction (q, f), polarization (h+, hx) and amplitude.

$$\boldsymbol{h}^{\mathrm{TT}} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & h_+ & h_\times & 0 \\ 0 & h_\times & -h_+ & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\Box \bar{h}_{\mu\nu} = -\frac{16\pi G}{c^4} T_{\mu\nu}$$

https://scitechdaily.com/images/Gravitational-Waves.gif

Gravitoelectromagnetism (GEM)

Similarities between electromagnetism (EM) and gravitation are known for a long time.

1918: Thirring pointed out an analogy between GR (in first approximation) and EM.
He and Lense calculated a “magnetic” effect (Lense-Thirring precession)
that was tested with satellites.

LAGEOS satellite

https://upload.wikimedia.org/wikipedia/commons/7/78/LAGEOS-NASA.jpg

Gravitoelectromagnetism (GEM)

1961: Forward presented a direct analogy between the equations of EM and GR to help experimentalists deal with GR experiments.

He recognized that this was an approximation to the tensorial formalism.

TABLE I

	EM Symbol		Gravitational Symbol	Value or Definition
Force Vector	$-\boldsymbol{E}$	$\rightarrow$	$\boldsymbol{G}$	$= -\nabla\chi - \dfrac{\partial \boldsymbol{K}}{\partial t}$
Solenoidal Force Vector	$-\boldsymbol{B}$	$\rightarrow$	$\boldsymbol{P}$	$= \nabla\times\boldsymbol{K}$
Scalar Potential	$-\phi$	$\rightarrow$	χ	$\approx -\dfrac{1}{4\pi\gamma}\int_V \dfrac{\mu}{r}\,dV$
Vector Potential	$-\boldsymbol{A}$	$\rightarrow$	$\boldsymbol{K}$	$\approx -\dfrac{\eta}{4\pi}\int_V \dfrac{\mu \boldsymbol{v}}{r}\,dV$
Source Density	ρ	$\rightarrow$	μ	$= \dfrac{dM}{dV}$
Source Quantity	Q	$\rightarrow$	M	$= \int_V \mu\,dV$

Gravitoelectromagnetism (GEM)

1977: Braginski, Caves & Thorne presented a similar analogy, based on the PPN expansion.

Over the years, various applications of this analogy – the gravitoelectromagnetism – have resulted.
The subject is present in textbooks on gravity, like those by Ohanian & Ruffini, Rindler and Moore.

Mashoon (2008) wrote a brief review about it.
Costa & Herdeiro (2008) proposed a new approach to a physical analogy between GR and EM, based on tidal tensors of both theories and wrote a covariant form for the gravitational analogues of the Maxwell equations.

Research using GEM and on GEM is ongoing.

GEM applied to the GW detection system of LIGO

After decades of technological improvement (since 1960's), the announcement of the first direct detection of GW occurred in 2015 using the interferometric detectors LIGO.

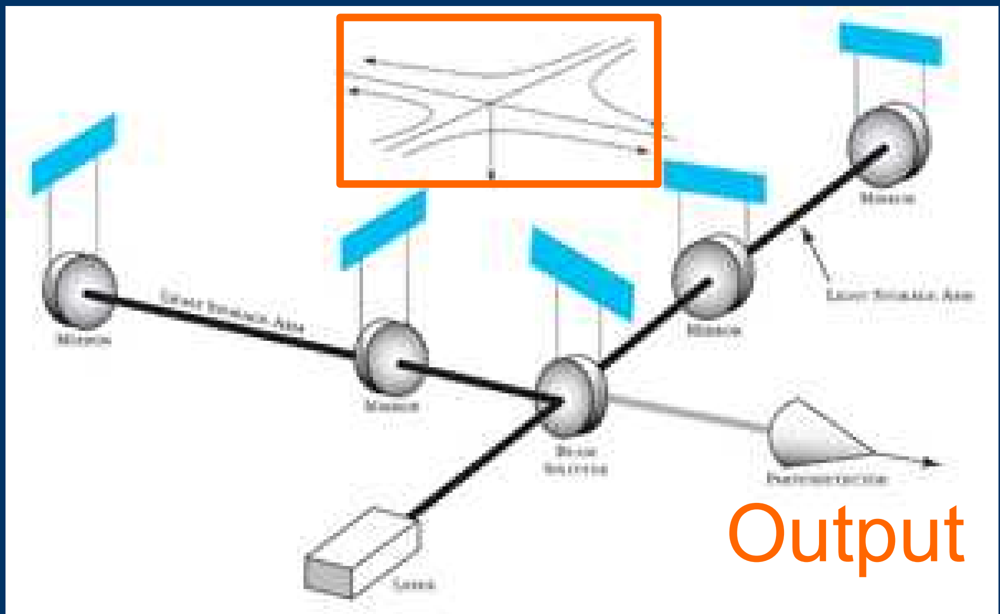
https://www.ligo.caltech.edu/system/media_files/binaries/180/large/LIGOs_Dual_Detectors.jpg?1431726608

GEM applied to the detection of GW by LIGO

A detector has a response function that depends on the input signal

Iorio & Crosta (2011) showed that the total response fuction of the LIGO had relevant contribution at higher frequencies (e.g. 8kHz) from the (gravito)magnetic component.

Input: GW



https://www.aps.org/publications/apsnews/200306/images/ligo_detector.jpg

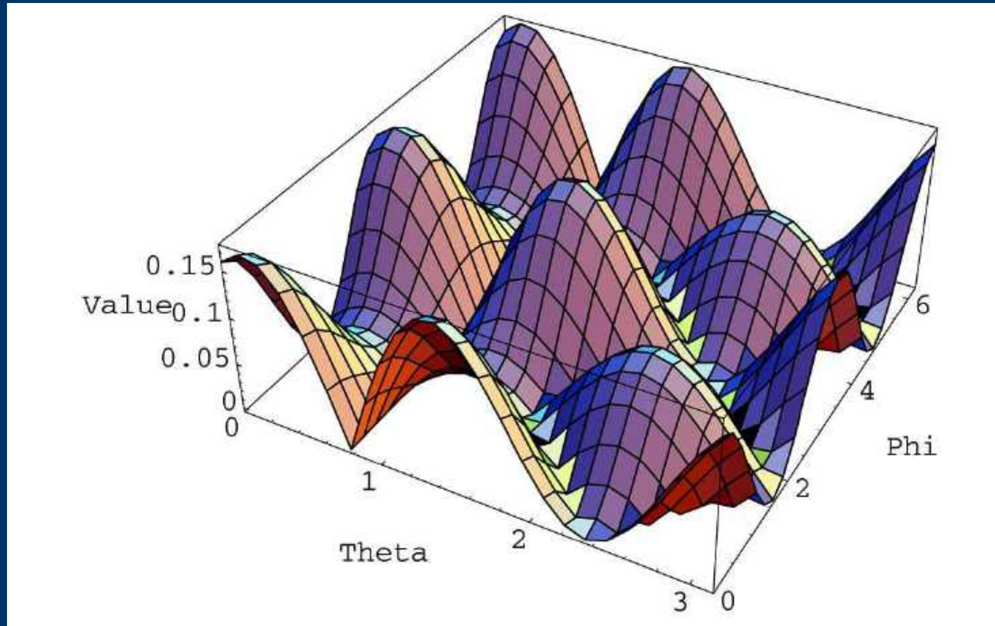


Fig. (3). the angular dependence of the response function of the LIGO interferometer to the magnetic component of the + polarization for $f = 8000$ Hz.

Poynting vector in GEM

From Mashoon (2008), the GEM field equations: $O(h_{\mu\upsilon})$

$$\nabla \times \mathbf{E} = -\frac{1}{c}\frac{\partial}{\partial t}\left(\frac{1}{2}\mathbf{B}\right), \quad \nabla \cdot \left(\frac{1}{2}\mathbf{B}\right) = 0.$$

$$\nabla \cdot \mathbf{E} = 4\pi G\rho, \quad \nabla \times \left(\frac{1}{2}\mathbf{B}\right) = \frac{1}{c}\frac{\partial}{\partial t}\mathbf{E} + \frac{4\pi G}{c}\mathbf{j}.$$

The GEM fields defined by: $\mathbf{E} = -\nabla\Phi - \frac{1}{c}\frac{\partial}{\partial t}\left(\frac{1}{2}\mathbf{A}\right), \quad \mathbf{B} = \nabla \times \mathbf{A},$

The GEM potentials defined by: $\bar{h}_{00} = 4\Phi/c^2$ $\bar{h}_{0i} = -2A_i/c^2$

Trace-reversed amplitude: $\bar{h}_{\mu\nu} = h_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}h,$ where $h = \eta^{\mu\nu}h_{\mu\nu}$

Poynting vector in GEM

From Mashoon (2008), the Landau-Lifshitz pseudotensor ($t_{\mu\upsilon}$) is employed to determine the local stress-energy content of the GEM fields. For a stationary configuration:

$$\partial\Phi/\partial t = 0$$
$$\partial\mathbf{A}/\partial t = 0$$

$$4\pi G t_{0i} = 2(\mathbf{E}\times\mathbf{B})_i$$

The GEM Poynting vector defined by:

$$\mathcal{S} = -\frac{c}{2\pi G}\mathbf{E}\times\mathbf{B}.$$

$O(h_{\mu\upsilon})$

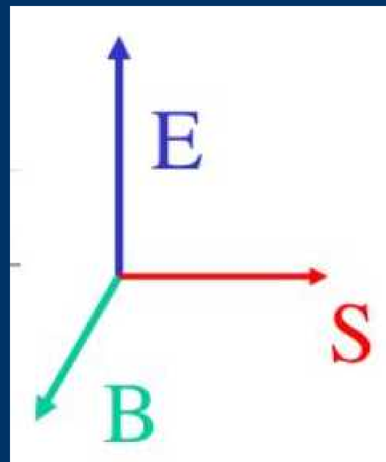
$$\bar{h}_{00} = 4\Phi/c^2$$
$$\bar{h}_{0i} = -2A_i/c^2$$

Electromagnetic Poynting vector

From classical electromagnetism, the electromagnectic flux is characterized by the Poynting vector:

$$\mathbf{S} = \frac{c}{4\pi}\mathbf{E} \times \mathbf{H}$$

It represents the EM energy that flows per unit time (power) through a unit area in a direction perpendicular to the electric field E and the magnetic field H.



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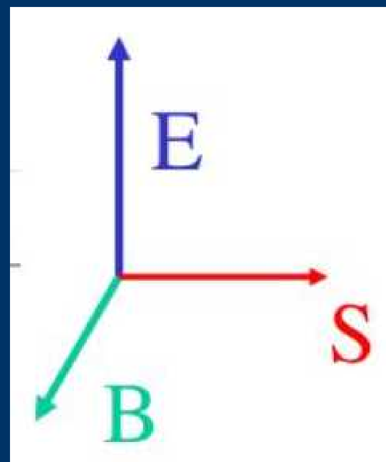

The GEM Poynting vector:

$$\boxed{\mathcal{S} = -\frac{c}{2\pi G}\mathbf{E}\times\mathbf{B}}$$

Poynting vector in EM and in GEM

EM: $\mathbf{S} = \frac{c}{4\pi}\mathbf{E}\times\mathbf{H}$

GEM: $\boxed{\mathcal{S} = -\frac{c}{2\pi G}\mathbf{E}\times\mathbf{B}}$

It represents the energy that flows per unit time (power) through a unit area in a direction perpendicular to the electric field and the magnetic field.

GEM: $4\pi G t_{0i} = 2(\mathbf{E}\times\mathbf{B})_i$

The component of the Landau-Lifshitz pseudotensor ($t_{\mu\upsilon}$) represents this energy flux.

Energy flux in general relativity

In GR the energy flux of a system is given by the stress-energy tensor ($T_{\mu\upsilon}$), specifically its T_{0k} component.

For a gravitational wave this component is given by

MTW 1973 § 35.7. c=G=1.

< > denotes an average over several wavelengths.

$$T_{0k} = 1/(32\pi) \langle h^{TT}{}_{ij,0}\, h^{TT}{}_{ik,k} \rangle \ .$$

Calculating this average using t_{0k} yields the same as above.

No Poynting vector is defined in this context.

Energy flux in general relativity: an illustration

Magalhaes et al. (1997)*, studying the detection of GW by spherical resonant-mass detectors, analysed the influence of a GW on a spherical distribution of free masses.

*Magalhaes et al. ApJ 475, 462 (1997)

Energy flux in general relativity: an illustration

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In the next images I present the use of a spherical geometry in the detection of GW

*Magalhaes et al. ApJ 475, 462 (1997)

Spherical detectors of GW

The last generation of resonant-mass GW detector was under study in the early 1990's, when the LIGO detectors got funds to be constructed.

The US projects for resonant-mass detector were shut down. A Brazilian and a Dutch prototypes with spherical geometry were developed.

The Brazilian prototype evolved to the Schenberg dector and is still under study.

Spherical detectors of GW: SCHENBERG

One only, well-tuned spherical GW detector should be capable of yielding all 5 independent components of $h_{\mu\upsilon}$.

The Schenberg dector was designed to detect GW around 3200 Hz, with narrow bandwidth and limited by quantum noise.

It is presently disassembled.

Optimization studies are ongoing aiming at coincidence operation with interferometers.

Bortoli, Frajuca, Magalhes et al. BPJ 50, 541 (2020)

Spherical detectors of GW:
The Laser Gravitational Compass

The laser gravitational compass:
A spheroidal interferometric gravitational observatory

It is shown that a minimum of four non-coplanar mass probes are necessary for a Michelson-Morley interferometer to fully detect gravitational waves within the context of GR.

With fewer probes, some alternative theories of gravitation could also explain the observations.

The conversion of the existing gravitational wave detectors to four probes is also suggested.

Ferreira, Magalhaes et al. IJMPA 35, 2040020 (2020).

Back to

Energy flux in general relativity: an illustration

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Gravitational wave in GR

The wave of space-time (GW) distorts the background space-time (Minkowskian) where the particles are located.

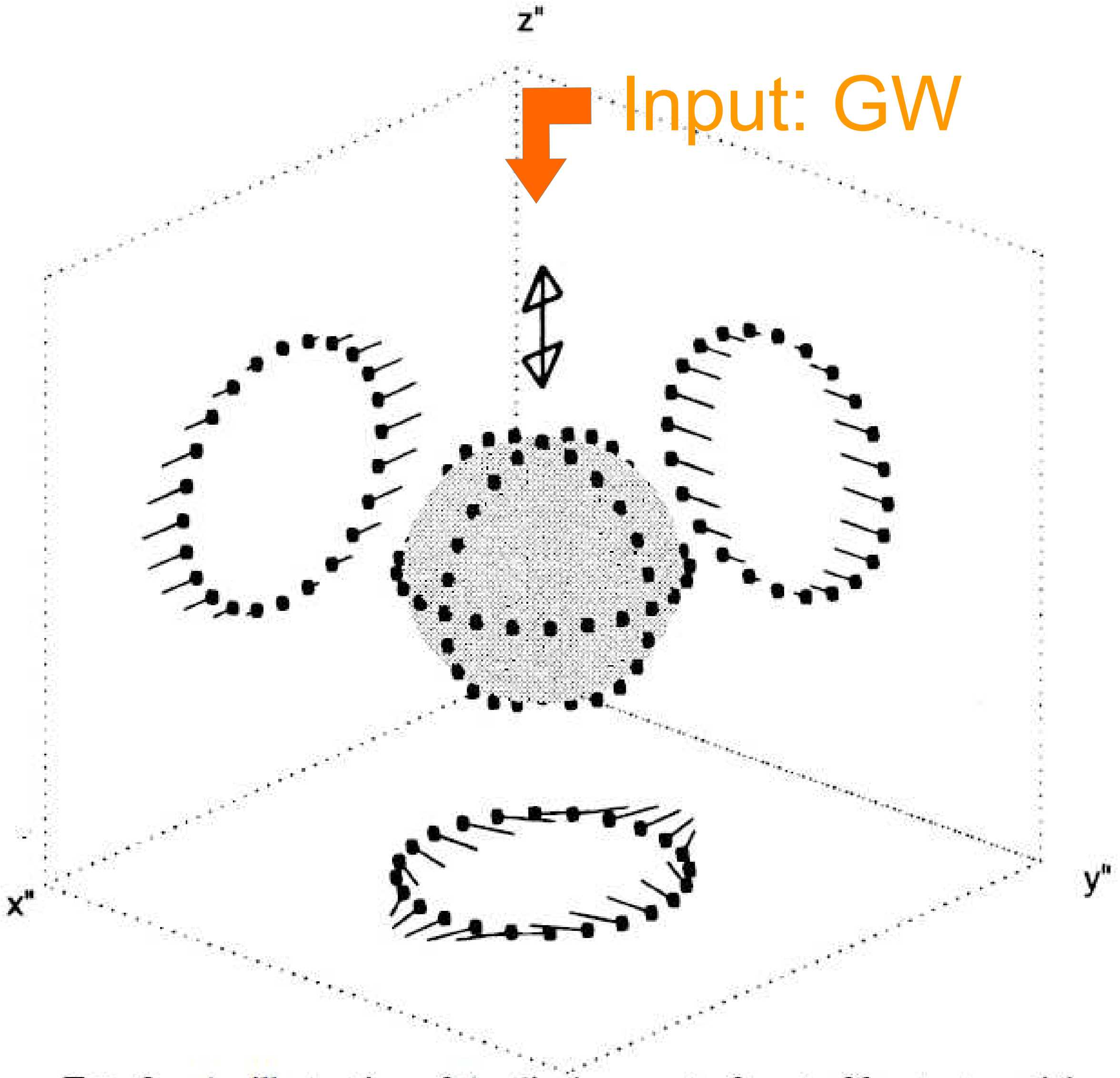


FIG. 2.—An illustration of the displacement of a set of free test particles by a gravitational wave traveling in the z'' direction. The particles are initially arranged in three orthogonal concentric rings. Each undeformed ring of particles is projected onto a plane. Their subsequent gravitational displacements are shown by the lines, which all lie in a plane normal to the direction of propagation.

Magalhaes et al. ApJ 475, 462 (1997)

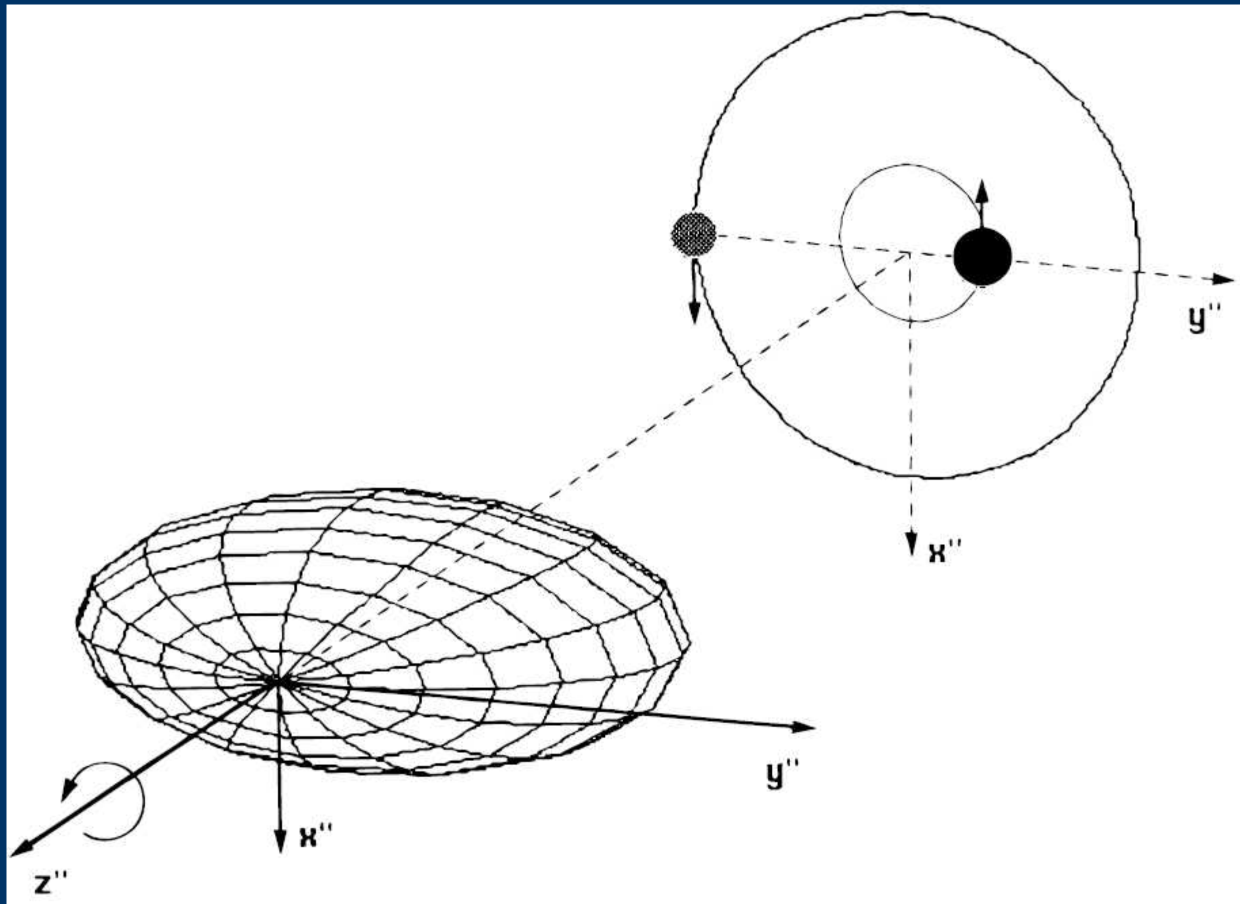
Gravitational wave in GR

The wave of space-time distorts the background space-time while it travels.

The volume of 3-space depends on g = Det(**g**).

In the TT gauge

$$g = 1 - h_t^2$$



$$\boldsymbol{h}'' = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & h_t & 0 & 0 \\ 0 & 0 & -h_t & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

In the diagonal frame of the ellipsoid

Magalhaes et al. ApJ 475, 462 (1997)

Gravitational wave flux in GR

In the TT gauge $g = 1 - h_t^2$

This shows that the particles move relative to the center of the sphere.

We can show that the energy flux, $T_{0k} = 1/(32\pi) \langle h^{TT}_{ij,0}\, h^{TT}_{ik,k} \rangle$, is not zero.

Consider the simple example of a monochromatic wave:

$$h_{t''} = \mathscr{A} \cos \omega(t'' - z''/c)$$

Gravitational wave flux in GEM

The GR energy flux, $T_{0k} = 1/(32\pi) \langle h^{TT}_{ij,0}\, h^{TT}_{ik,k} \rangle$, is not zero.

$$\boldsymbol{h}'' = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & h_t & 0 & 0 \\ 0 & 0 & -h_t & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

GEM: $$4\pi G t_{0i} = 2(\mathbf{E} \times \mathbf{B})_i$$

$$\boxed{\partial\Phi/\partial t = 0}$$
$$\boxed{\partial\mathbf{A}/\partial t = 0}$$

$$\mathbf{E} = -\nabla\Phi - \frac{1}{c}\frac{\partial}{\partial t}\left(\frac{1}{2}\mathbf{A}\right), \quad \mathbf{B} = \nabla \times \mathbf{A},$$

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$$\bar{h}_{\mu\nu} = h_{\mu\nu} - \tfrac{1}{2}\eta_{\mu\nu} h, \text{ where } h = \eta^{\mu\nu} h_{\mu\nu}$$

The GEM Poynting vector is null for the GW.

Mashoom's GEM & the Poynting vector

- Mashoon's GEM provides insights on the Poyinting vector for statationary space-times.
- Mashoon's GEM does not yield a Poynting vector for gravitational waves.
- The work by Magalhaes et al. (1997) suggests, for gravitational waves, an object similar to the Poynting vector of electromagnetic waves.

Does it means that GEM is inadequate

to describe GW?

Broadening GEM

New works have broadened the initial GEM.

1a) A new appoach by F. Costa and collaborators, based on tidal tensors, yields a covariant form for the analogy Gravity ↔ EM.

PHYSICAL REVIEW D **78,** 024021 (2008)

Gravitoelectromagnetic analogy based on tidal tensors

L. Filipe O. Costa* and Carlos A. R. Herdeiro+

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(Received 21 May 2007; revised manuscript received 29 April 2008; published 11 July 2008)

We propose a new approach to a physical analogy between general relativity and electromagnetism, based on *tidal tensors* of both theories. Using this approach we write a covariant form for the gravitational analogues of the Maxwell equations, which makes transparent both the similarities and key differences between the two interactions. The following realizations of the analogy are given. The first one matches linearized gravitational tidal tensors to exact electromagnetic tidal tensors in Minkowski spacetime. The second one matches exact magnetic gravitational tidal tensors for ultrastationary metrics to exact magnetic tidal tensors of electromagnetism in curved spaces. In the third we show that our approach leads to a two-step exact derivation of Papapetrou's equation describing the force exerted on a spinning test particle. Analogous scalar invariants built from tidal tensors of both theories are also discussed.

1b) Under the covariant appoach, F. Costa and collaborators explore the concept of a “super Poynting vector”.

Gravito-electromagnetic analogies

L. Filipe O. Costa · José Natário

Gen Relativ Gravit (2014) 46:1792
DOI 10.1007/s10714-014-1792-1

Abstract We reexamine and further develop different gravito-electromagnetic analogies found in the literature, and clarify the connection between them. Special emphasis is placed in two exact physical analogies: the analogy based on inertial fields from the so-called “1+3 formalism”, and the analogy based on tidal tensors. Both are reformulated, extended and generalized. We write in both formalisms the Maxwell and the full exact Einstein field equations with sources, plus the algebraic Bianchi identities, which are cast as the source-free equations for the gravitational field. New results within each approach are unveiled. The well known analogy between linearized gravity and electromagnetism in Lorentz frames is obtained as a limiting case of the exact ones. The formal analogies between the Maxwell and Weyl tensors are also discussed, and, together with insight from the other approaches, used to physically interpret gravitational radiation. The precise conditions under which a similarity between gravity and electromagnetism occurs are discussed, and we conclude by summarizing the main outcome of each approach.

2) F. Rocha and collaborators investigate perturbations beyond GEM's.

Beyond gravitomagnetism with applications to Mercury's perihelion advance and the bending of light

Flavia Rocha*, Rubens Marinho Jr. and Manuel Malheiro
Geanderson Araújo Carvalho Gerson Otto Ludwig

International Journal of Modern Physics D
(2021) 2150073 (19 pages)

The expansion of both sides of Einstein's field equations in the weak-field approximation, up to terms of order $1/c^4$ is derived. This new approach leads to an extended form of gravitomagnetism (GEM) properly named as Beyond Gravitomagnetism (BGEM). The metric of BGEM includes a quadratic term in the gravitoelectric potential n the time and also space metric functions in contrast with first post-Newtonian 1PN approximation where the quadratic term appears only in the time metric function. This nonlinear term does not appear in conventional GEM, but is essential in achieving the exact value of Mercury's perihelion advance as we explicitly show. The new BGEM metric is also applied to the classical problem of light deflection by the Sun, but the contribution of the new nonlinear terms produce higher-order terms in this problem and can be neglected, giving the correct result obtained already in the Lense–Thirring (GEM) approximation. The BGEM approximation also provides new terms that depend on the dynamics of the system, which may bring new insights into galactic and stellar physics.

Final remarks

The analogy between gravity and eletromagnetism may still yield new insights on gravitational phenomena.

We are particularly interested in its applications to gravitational radiation.

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Thank you