Post-Newtonian γ-like parameters and the slip

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FAPES
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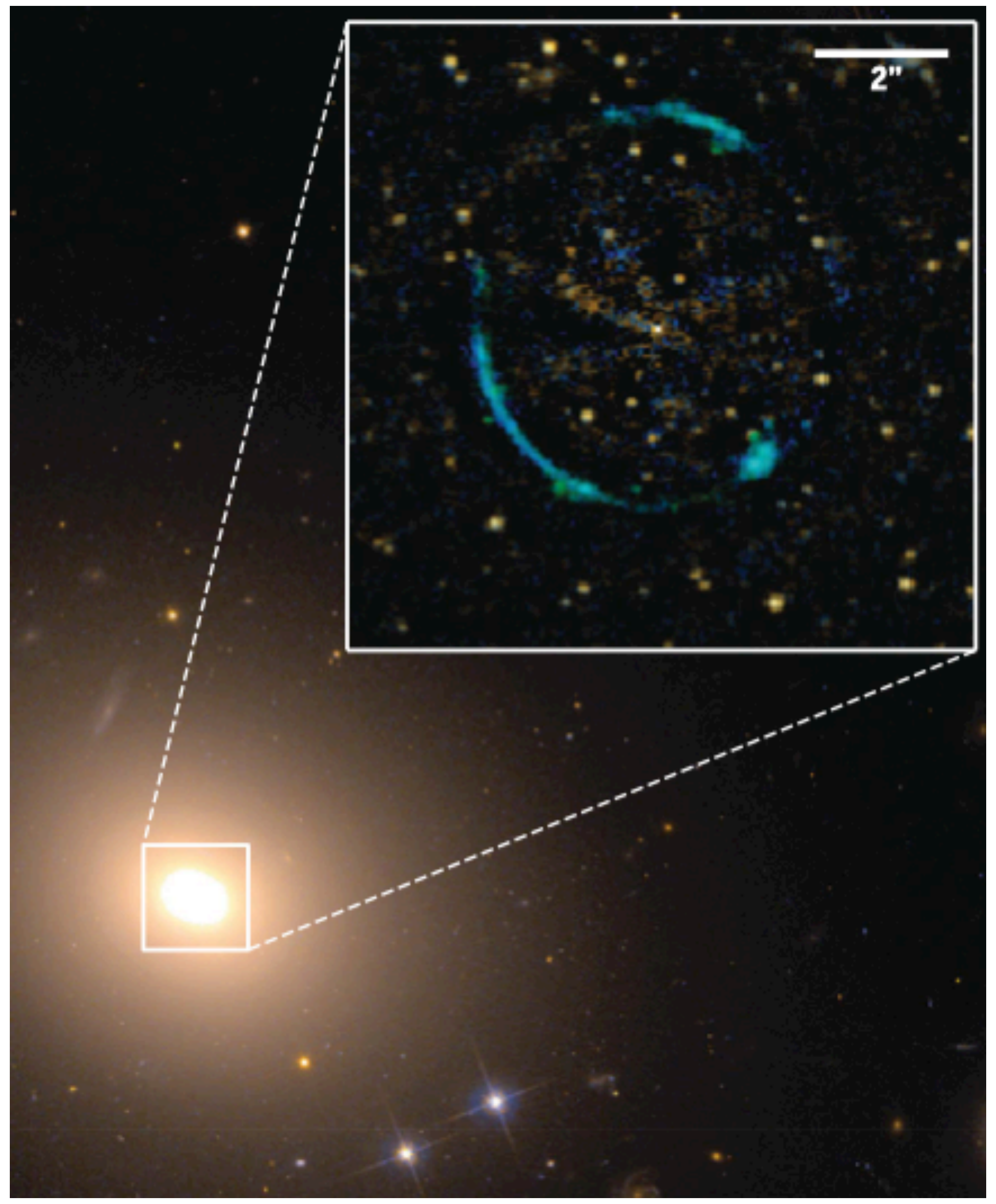


From [Collet et al Science 2018]

$$M_{\rm dyn} = \frac{1+\gamma}{2} M^{\rm GR}_{\rm lensing}$$

γ = 0.97 ± 0.09 at 68%

Introduction. Several works report "γ_{PPN}" parameters derived for different gravitational theories and observations. There are measurements within the solar system, other stellar systems, distant galaxies, cluster of galaxies...

However, there are different definitions being used and they are subjected to different physical bounds.

The γ parameter, within the PPN formalism due to Will and Nordtvedt, is *defined* as a constant in the following *post-Newtonian* metric expansion (up to v^2) [Will, Living Rev.Rel. (2014)]:

$$ds^2_{(PPN)} \approx -(1-2U)dt^2 + (1+2\gamma U)d\mathbf{x}^2, \text{ with } U(x,t) \equiv \int \frac{\rho(\mathbf{x}',t)}{|\mathbf{x}-\mathbf{x}'|} d^3x' \,.$$

This expansion is not a full first order post-Newtonian expansion, it includes the Newtonian contribution and the dynamics of light. Other 9 PPN parameters appear at higher orders.

Slip vs Gamma. The gravitational slip (η) is commonly defined in a cosmological context. A common definition is the following (in physical space):

$$ds^2 = a^2(\tau)\left[-(1-2\psi)d\tau^2 + (1+2\eta\psi)d\mathbf{x}^2\right], \text{ with } \eta \equiv \phi/\psi.$$

Assuming that: i) $a(\tau) = \text{constant}$, ii) $\psi = U$ and iii) $\eta = \text{constant}$ $\implies$ $\eta = \gamma$.
In general, however, $\eta \neq \gamma$ and these quantities are associated to ***different physical phenomena***.

The physical meaning of γ. It measures both *light bending* and the *Shapiro time delay*. Its best bound in the solar system reads $|\gamma - 1| \lesssim 10^{-5}$. If γ is not a constant, it cannot parametrize these phenomena ($\partial_i\gamma$ becomes locally relevant as well).

A simple and useful extension of γ based on its observational meaning: γ_Σ. Consider the parametrization

$$ds^2 = -(1-2\alpha_e U)dt^2 + (1+2\gamma_e U)d\mathbf{x}^2,$$

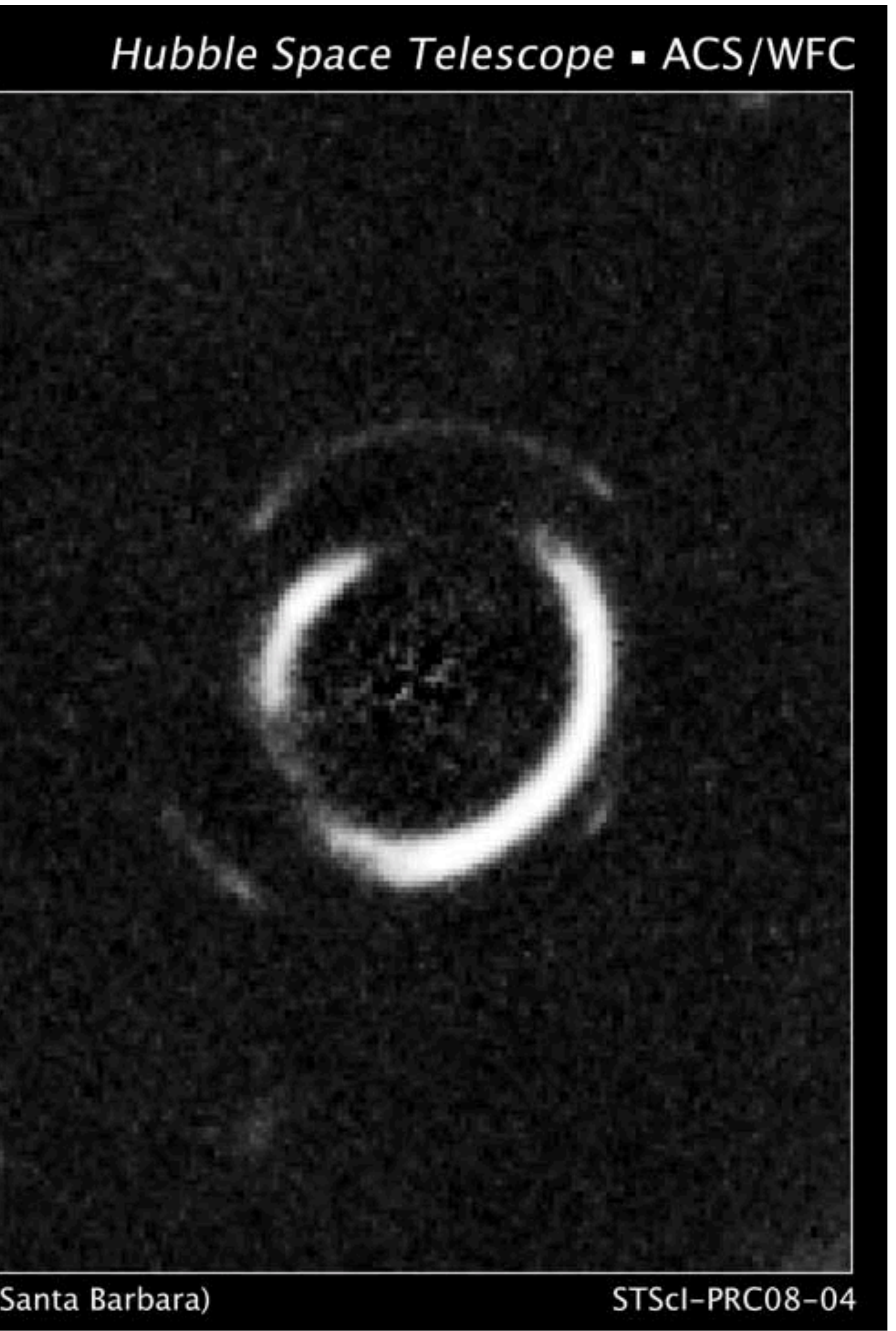
where α_e and γ_e are arbitrary *functions*. In general, $\gamma_e \neq \gamma \neq \eta$. In [Toniato, Rodrigues PRD (2021)], inspired by [Berry, Gair, PRD (2011)], we proposed the use of

$$\gamma_\Sigma \equiv \alpha_e + \gamma_e - 1.$$

<u>If γ_Σ is a constant, then it parametrizes both the Shapiro time delay and the bending of light</u> (this is shown from the equations of motion). Also, we show that several models do have this property of constant γ_Σ.

Deflection of light expression: it has the same form of PPN, but with γ_Σ in place of γ.

$$\delta\theta = \left(\frac{1+\gamma_\Sigma}{2}\right)\frac{4M}{d}$$



Horndeski application. We have shown that, if the Horndeski potentials can be *locally approximated* by analytical functions, then γ_Σ reads

$$\gamma_\Sigma = \begin{cases} \dfrac{\kappa}{4\pi G_{4(0,0)}} - 1\,, & \text{constant in general (independent from Newtonian limit),} \\ \gamma = \begin{cases} \dfrac{W - G^2_{4(1,0)}}{W + G^2_{4(1,0)}}\,, & \text{valid Newtonian limit due to scalar field small mass}\,, \\ 1\,, & \text{valid Newtonian limit due to scalar field large mass}\,. \end{cases} \end{cases}$$

$$\eta = \frac{b_3 - b_2\varphi/U}{b_1 + b_2\varphi/U} \neq \text{const.}$$

The results with a Newtonian limit agree with [Hohmann PRD 2015, Shaoqi, Yungui EPJC 2018]. However, in the absence of a Newtonian limit (which can be relevant for studying distant galaxies), we do not agree with the results of these references (which use η in place of γ).

Conclusions and consequences.

- Care should be taken on the meaning of "γ_{PPN}". The true γ that appears in PPN formalism cannot be approximated by the slip (the error can be either negligible or arbitrarily large). The coincidence between the slip and γ is only true if a Newtonian limit is assumed to exist and η is a constant.
- If $\gamma_\Sigma \equiv \alpha_e + \gamma_e - 1$ is a constant, then this quantity parametrizes the same physical phenomena of γ, even in the absence of a Newtonian limit.
- In general, $\gamma \neq \eta \neq \gamma_e \neq \gamma_\Sigma\,.$
- In [Toniato, Rodrigues PRD (2021)] we pointed out that, in principle, it is possible to test if γ_Σ is indeed a constant in distant galaxies, even without a Newtonian limit. For instance, from double Einstein-ring systems.