Static universes

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1. Abstract

We consider conditions for existence and stability of a static cosmological solution in quadratic gravity. It appears that such a solution for a Universe filled by only one type of perfect fluid is possible in a wide range of the equation of state parameter w and for both positively and negatively spatially curved Universe. We show that the static solution for the negative curvature is always unstable if we require positive energy density of the matter content. On the other hand, a static solution with positive spatial curvature can be stable under certain restrictions. Stability of this solution with respect to isotropic perturbation requires that the coupling constant with the R^2 therm in the Lagrangian of the theory is positive, and the equations of state parameter w is located in a rather narrow interval. Nevertheless, the stability condition does not require violation of the Strong Energy Condition. Taking into account anisotropic perturbations leads to further restrictions on the values of coupling constants and the parameter w.

2. Introduction

Quadratic gravity provides an extension of General Relativity for sufficiently strong gravity, in the same sense as Maxwell theory is modified to the non linear Heisenberg-Euler electromagnetism. Apparently quadratic gravity was first proposed by H. Weyl in 1918 [1]. Only many years after it was addressed again by H. A. Buchdahl in 1962 [2]. Since it is a reasonable extension of GR for very intense gravity it should be a reasonable effective theory in the some neighborhood of a space time singularity. It is in this context that Starobinsky proposed his inflationary model in 1980 [3] which is consistent with the latest measurements in the CMBR by the Planck collaboration [4]. After the 1960s quadratic gravity was intensively investigated by many reasearches. In this poster we summerize recents results obtained by us and refere to [5] for further details.

3. General Relativity

Consider the metric of an isotropic spatially curved Universe filled with two types of fluids with energy densities ρ_1 and ρ_2 and the equation of state parameters w_1 and w_2 respectively. In order to have a static solution the spatial curvature of spacetime must be positive. Stability is obtained in the rather unphysical situation of energy densities with opposite signs. For its existence it requires either two types of matter with $w_{1,2} < -1/3$ or with $w_{1,2} > -1/3$. In the former case the condition for stability is $w_2 < w_1$ where w_1 corresponds to positive energy matter. In the latter case the condition has the opposite form $w_2 > w_1$. From this point of view there exists one particular interesting example of a *stable* static solution in GR, realising in a Universe filled by a positive density matter with $w < -1/3$ and a negative cosmological constant.

4. Quadratic Gravity

In 4 dimensions,

$$L = \frac{1}{16G\pi}\left\{\beta R^2 + \alpha\left(R_{ab}R^{ab} - \frac{1}{3}R^2\right) + R\right\}, \qquad (1)$$

is the most general lagrangian for quadratic gravity. For the spatial positively and negatively curved cases, $k = \pm 1$, if all derivatives of $a = \ln \mathcal{R}$ are zero, we have the following static solution

$$a_s = -\frac{1}{2}\ln\left(-\frac{1+3w}{6\beta k\,(-1+3w)}\right)$$

$$\rho_s = 1/8\,\frac{1+3\,w}{\pi\,G\beta\,(-1+3\,w)^2}.$$

Ruzmaikina-Ruzmaikin-Starobinsky's inflationay solution since it is one of the outcomes

$$H \approx H_0 - \frac{t}{36\beta}. \qquad (2)$$

Singularity is characterized by curvature scalars.

4.1 Linearized modes

- a masseless spin 2 field from GR (at the tree level, graviton)
- a massive scalar field with mass $m_s = 1/\sqrt{6\beta}$
- a ghost massive spin two field with mass $m_2 = 1/\sqrt{-\alpha}$ (the ghost mode has energy with opposite sign)

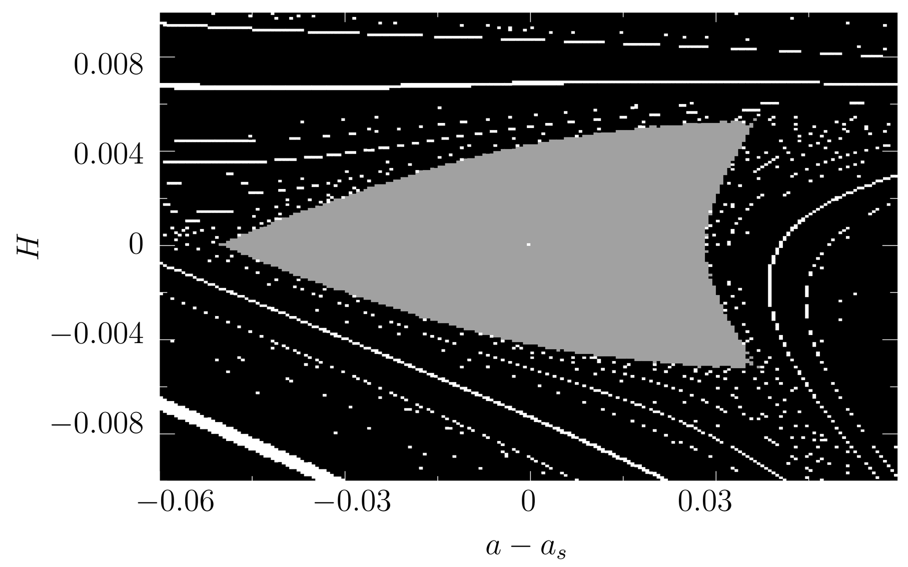
5. Stability in the isotropic case

In the quadratic counterpart, there's no need to consider a two fluid mixture for a stable and static solution. Also, in contrast to GR, it is possible to have static solution with negative spatial curvature, a detail that apparently was not known [5]. Anyway, reasonable substance does not lead to stable negatively curved universes.

We have the linearized frequencies

$$\lambda = \pm\frac{i\sqrt{3}\sqrt{9w+1\pm\sqrt{117w^2+6w-3+108w^3}}}{6\sqrt{\beta}\sqrt{-1+3w}}.$$

- Static solution for negative spatial curvature for a single fluid satisfying the strong energy condition and positive energy density $\rho > 0$. The solution is unstable. Stability can be achieved if $\rho < 0$.
- Static and stable solution for positive spatial curvature for a single fluid satisfying the strong energy condition and positive energy density $\rho > 0$. Stability is obtained for a narrow range of EOS parameter $w \simeq [-0.33, -0.21]$. Stability occurs also in the presence of the tachyon, $\beta < 0$.



A mesh of initial conditions for $\beta = 10$, $w = -0.22$ for the isotropic with positive spatial curvature case. We choose $\beta > 0$ since it does not exclude the infaltionary solution (2). In the x axis we have the displacement from the logarithmic of scale factor $a = \ln \mathcal{R}$, from its value for the static universe a_s. In the y axis the initial value of the Hubble parameter $H = \dot{a}$. Grey points is the the basin of stable oscillations, white points are solutions which appoach the oscillatory scaleron behavior $e^{it/\sqrt{6\beta}}$ and black points approach the singularity

6. Stability with shear

The non isotropic generalization of the positive curvature case is given by the appropriate Bianchi IX case, which can be seen in [5]. Shear must be non zero and defined as

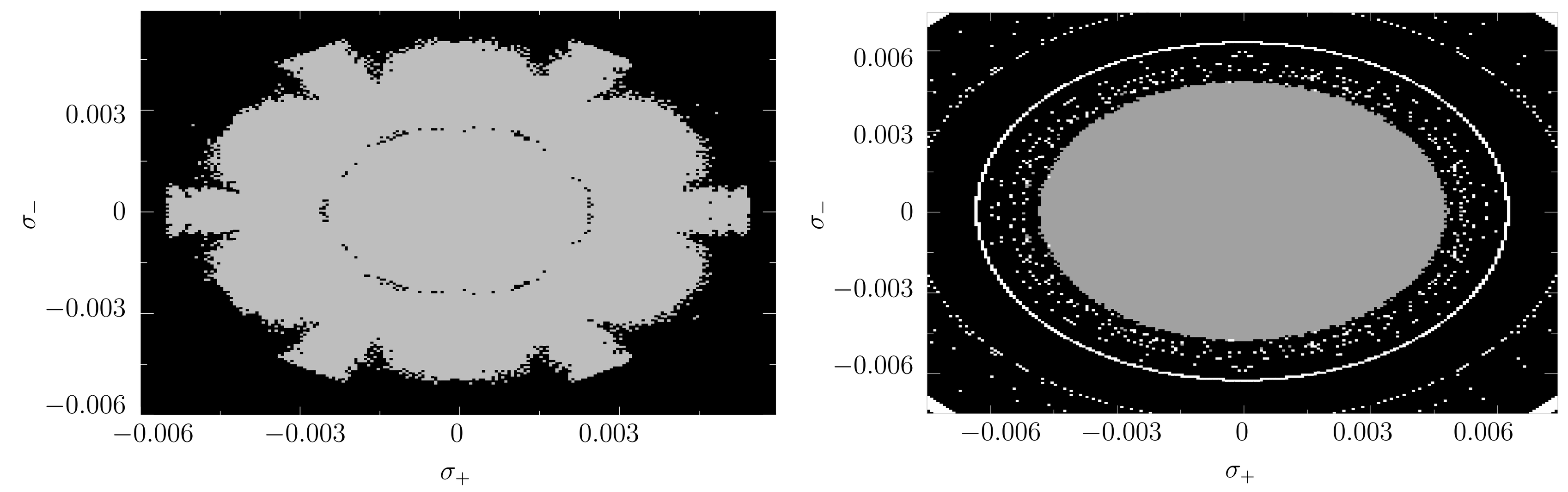
$$\sigma_{ij} = \begin{pmatrix} -2\sigma_+ & 0 & 0 \\ 0 & \sigma_+ + \sqrt{3}\sigma_- & 0 \\ 0 & 0 & \sigma_+ - \sqrt{3}\sigma_-. \end{pmatrix}$$

Besides the frequencies from the isotropic case, there are additional frequencies

$$\lambda = 0,$$
$$\lambda = \pm i/6\,\sqrt{6}\sqrt{\frac{-30\,\alpha\,w + 9\,\beta\,w + 9\,\beta - 10\,\alpha \pm 3\,\sqrt{\Delta}}{(3\,w-1)\,\alpha\,\beta}}$$

where

$$\Delta = 36\,\alpha^2w^2 - 12\,\alpha\,w^2\beta - 16\,\alpha\,w\beta + 24\,\alpha^2w + 9\,\beta^2w^2 + 18\,\beta^2w + 9\,\beta^2 - 4\,\alpha\,\beta + 4\,\alpha^2.$$



a) b)

In panel a) a mesh of initial conditions for $\beta > 0$ and $\alpha > 0$, regime in which the ghost becomes tachyonic. Grey points oscillate near the staticorbit and black points approach the singularity. While in panel b) a mesh which exclude the tachyon with both $\beta > 0$ and $\alpha < 0$. Grey points form the stable basin, black points approach the singularity and white points show both the scaleron and ghost linear oscillatory modes.

References

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[3] A. A. Starobinsky, Phys. Lett. **91B**, 99 (1980).
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