Representations of Map Superalgebras

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If $\mathfrak{g}_{\bar{0}}$ is semisimple, then any simple cuspidal bounded $\mathcal{L}$-module is an evaluation module.

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$$[x \otimes t^m + \alpha c, y \otimes t^n + \beta c] = [x, y] \otimes t^{m+n} + n(x, y)\delta_{m,-n}c,$$

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- We can consider Harish-Chandra modules over $\mathcal{A}(\mathfrak{g})$ by assuming that the weight spaces with respect $\mathfrak{h} \otimes \mathbb{C} \oplus \mathbb{C}c$ have finite dimension.
- Any simple $L(\mathfrak{g})$-module is a simple $\mathcal{A}(\mathfrak{g})$-module. Conversely,

Theorem

If V is a simple Harish-Chandra module over $\mathcal{A}(\mathfrak{g})$, then V is a simple $L(\mathfrak{g})$-module.

Affine Kac-Moody Lie Superalgebras

- The *affine Kac-Moody Lie superalgebra* $\widehat{\mathfrak{g}} = \mathcal{A}(\mathfrak{g}) \oplus \mathbb{C}d$ associated to a basic classical Lie superalgebra $\mathfrak{g}$ is a one-dimensional extension of $\mathcal{A}(\mathfrak{g})$ by an element d with $[d, x \otimes t^k] = kx \otimes t^k$.

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- If V is a weight $L(\mathfrak{g})$-module, then $L(V) = V \otimes \mathbb{C}[t, t^{-1}]$ is a weight $\widehat{\mathfrak{g}}$-module, where

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- V is a (bounded) Harish-Chandra $L(\mathfrak{g})$-module if and only if $L(V)$ is a (bounded) Harish-Chandra $\widehat{\mathfrak{g}}$-module. Hence we get a functor

$$\left\{ \begin{matrix} \text{(Bounded) Harish-Chandra} \\ L(\mathfrak{g})\text{-modules} \end{matrix} \right\} \xrightarrow{L} \left\{ \begin{matrix} \text{(Bounded) Harish-Chandra} \\ \widehat{\mathfrak{g}}\text{-modules} \end{matrix} \right\}$$

Theorem

Let $\lambda \in \mathfrak{h}^*$, $F(\lambda)$ be the corresponding simple highest weight $\mathfrak{g}$-module, V_0 be a simple weight $\mathfrak{g}$-module and $a, b \in \mathbb{C}^\times$ with $a \neq b$. Then $V_0 \otimes F(\lambda)$ is a $L(\mathfrak{g})$-module, where $(x \otimes t^r)v \otimes w = a^r(xv) \otimes w + (-1)^{|x||w|} b^r v \otimes (xw)$. If there is $\lambda_0 \in \operatorname{Supp} V_0$ for which the map

$$\chi(h \otimes t^n) = (a^n \lambda_0(h) + b^n \lambda(h))\, t^n, \quad \chi(d) = 0.$$

satisfies $\operatorname{im} \chi = \mathbb{C}[t^r, t^{-r}]$, then we have an isomorphism of $\widehat{\mathfrak{g}}$-modules

$$L(V_0 \otimes F(\lambda)) \cong \bigoplus_{i=0}^{r-1} L^i\,(V_0 \otimes F(\lambda))$$

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Remark

In the case of Lie algebras, these are exactly the simple modules that appear in the unpublished work of Dimitrov and Grantcharov (2009).

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Theorem

Let $\mathfrak{g}$ be a basic classical Lie superalgebras of type I. Then the Kac induction functor gives a bijection between the sets of isomorphism classes of simple bounded Harish-Chandra $\widehat{\mathfrak{g}}$-modules and simple bounded Harish-Chandra $\widehat{\mathfrak{g}}_0$-modules.

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Thank you!