An Algorithm to Build the Auslander-Reiten Quiver of Some Equipped Posets.

Workshop on Representation Theory and Applications
São Paulo Brazil
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This talk is dedicated to all the Ukrainian people
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In this talk, we describe an algorithm to build the Auslander-Reiten Quiver of Some Equipped Posets.
The representations theory of partially ordered sets or posets was introduced and developed in the early 1970s by Nazarova, Roiter and their students in Kiev.

To do that, Nazarova and Roiter introduced an algorithm known as the algorithm of differentiation with respect to a maximal point which allowed to Kleiner in 1972 to obtain a classification of posets of finite representation type.
The categorical properties of such an algorithm was given by Gabriel in 1973 [2, 3], founding in this way a new line of research in representation theory of posets which consists of proving that algorithms of differentiation induce a categorical equivalence between the category of representations of the original poset and the corresponding category of the derived poset [8, 9, 12].

Such functors constitute the main tool to classify posets of different types, for instance, the algorithm of differentiation with respect to a maximal point was also used by Nazarova to give a tame representation type criterion for ordinary posets.
The categorical properties of such an algorithm was given by Gabriel in 1973 [2, 3], founding in this way a new line of research in representation theory of posets which consists of proving that algorithms of differentiation induce a categorical equivalence between the category of representations of the original poset and the corresponding category of the derived poset [8, 9, 12].

Such functors constitute the main tool to classify posets of different types, for instance, the algorithm of differentiation with respect to a maximal point was also used by Nazarova to give a tame representation type criterion for ordinary posets.
Soon afterwards between 1974 and 1977, Zavadskij defined the more general algorithm I (D-I) with respect to a suitable pair of points, this algorithm was used in 1981 by Nazarova and Zavadskij in order to give a criterion for the classification of posets of finite growth representation type [4, 5, 8].
**Introduction: Ordinary posets**

**Definition**

A partially ordered set (*Poset*) is an ordered pair \((P, \leq)\) which consists of a not empty set \(P\) and a binary relation contained in \(P \times P\), called order, such that \(\leq\) is reflexive, antisymmetric and transitive.

**Example**

\[ k_1 : (1, 1, 1, 1) \quad k_2 : (2, 2, 2) \quad G_1 \]
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Introduction: Ordinary posets

**Theorem (Nazarova-Zavadskij 1981)**

A poset $P$ of tame representation type is of finite growth representation type if and only if it does not contain the following subposet:

![Diagram of poset $G_1$]

An Algorithm to Build the Auslander-Reiten Quiver of Some Equipped Posets. April 29 2022. São Paulo BR.
The theory of representation of posets was developed in the 1980's and 1990's for posets with additional structures, for example, for posets with involution or for equipped posets by Bondarenko, Nazarova, Roiter, Zabarilo and Zavadskij among others.
Equipped posets
Equipped posets


A poset $(P \leq)$ is said to be **equipped** if the following three conditions hold:

- **i** The points of the set $P$ can be either strong or weak;
- **ii** The order relations between points can be either weak or strong;
- **iii** If $x < y$ is a weak relation, then both points $x$ and $y$ are weak, and if $x < t < y$ in this case, then the relations $x < t$ and $t < y$ are also weak (and the point $t$ is automatically weak).
Equipped posets

Definition

A poset \((P, \leq)\) is called *equipped* if all the order relations between its points \(x \leq y\) are separated into strong (denoted \(x \leq y\)) and weak (denoted \(x \lessdot y\)) in such a way that

\[
x \leq y \lessdot z \quad \text{or} \quad x \lessdot y \leq z \quad \text{implies} \quad x \lessdot z.
\]

i.e., a composition of a strong relation with any other relation is strong [6].
Equipped posets

\[ A \rightarrow X \rightarrow B \]

- \( A \) is a weak point.
- \( B \) is a strong point.
- \( X \) is a weak point.
- \( c \) and \( b \) represent strong points.

Strong relations:
- From \( A \) to \( X \)
- From \( X \) to \( B \)

Weak relations:
- From \( a \) to \( X \)
- From \( X \) to \( b \)
These equipped posets were introduced and classified by Zabarilo and Zavadskij, and they reflected the case of "order relation multiplicities" not exceeding 2.

The correlation of their representations with representations of ordinary posets is analogous to the correlation of representations of valued graphs with representations of ordinary graphs.
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The complexification of a real vector space can be generalized to the case \((F, G)\) where \(G = F(u)\) is a quadratic extension of \(F\). In this case, we assume that \(u\) is a root of the minimal polynomial \(m(t) = t^2 + \alpha t + \beta, \beta \neq 0, (\alpha, \beta \in F)\). In particular if \(U_0\) is a \(F\)-space then the corresponding complexification is the \(G\)-vector space denoted \(\widetilde{U}_0\). As in the case \((\mathbb{R}, \mathbb{C})\), we write \(U_0 + uU_0 = \widetilde{U}_0\).

To each \(G\)-subspace \(W\) of \(\widetilde{U}_0\) it is possible to associate the following \(F\)-subspaces of \(U_0\),

\[
W^+ = \text{Re } W = \text{Im } W \quad \text{and}
\]
\[
W^- = \text{span}\{\alpha \in U_0 \mid (\alpha, 0)^t \in W\} \subset W^+,
\]

and for a \(G\)-space \(Z\) we have the following property

\[
\widetilde{Z}^+ = F(Z), \text{ is called the } F\text{-}hull \text{ of } Z \text{ such that } Z \subset F(Z),
\]
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The Category of representations \( \text{rep } \mathcal{P} \)

The category of representations of an equipped poset \( \text{rep } \mathcal{P} \) over a pair of fields \((F, G)\) (where \(G\) is a quadratic extension of \(F\)) has the following objects and morphism.

**Definition**

A representation \( U \in \text{rep } \mathcal{P} \) is a system of vectors spaces of the form:

\[
U = (U_0 ; U_x \mid x \in \mathcal{P}),
\]

where \(U_0\) is a finite dimensional \(F\)-space and for each \(x \in \mathcal{P}\), \(U_x\) is a \(G\)-subspace of \(\widetilde{U}_0\), such that,

\[
x \preceq y \implies U_x \subset U_y
\]

\[
x \preceq y \implies F(U_x) \subset U_y.
\]

In this case, a morphism \(\varphi\) is defined in such a way that \(\varphi : (U_0 ; U_x \mid x \in \mathcal{P}) \rightarrow (V_0 ; V_x \mid x \in \mathcal{P})\) is an \(F\)-linear map \(\varphi : U_0 \rightarrow V_0\) such that:

\[
\bar{\varphi}(U_x) \subset V_x, \text{ for each } x \in \mathcal{P}.
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\[
\begin{align*}
x \preceq y & \implies U_x \subset U_y \\
x \trianglelefteq y & \implies F(U_x) \subset U_y.
\end{align*}
\]

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\widetilde{\varphi}(U_x) \subset V_x, \text{ for each } x \in \mathcal{P}.
\]
The Category of representations \((\text{rep } \mathcal{P})\)

**Definition**

The sum of two representations \(U, V \in \text{rep } \mathcal{P}\) is given by the formula:

\[
U \oplus V = (U_0 \oplus V_0; U_x \oplus V_x \mid x \in \mathcal{P})
\]

A representation \(U \neq 0\) is indecomposable if \(U \cong U_1 \oplus U_2\) implies that \(U_1 = 0\) or \(U_2 = 0\).

Thus, \(\text{rep } \mathcal{P}\) have the property of decomposition unique, i.e. \(\text{rep } \mathcal{P}\) is a Krull-Schmidt category. \(\text{Ind } \mathcal{P}\) denotes the set of all indecomposable representations of \(\mathcal{P}\).
Matrix problem

Each equipped poset $\mathcal{P}$ naturally defines a matrix problem of mixed type over the pair $(F, G)$. Consider a rectangular matrix $M$ separated into vertical stripes $M_x, x \in \mathcal{P}$, with $M_x$ being over $F$ (over $G$) if the point $x$ is strong (weak):

$$M = \begin{array}{cccc}
G & G & F & F \\
\otimes & \otimes & \bigcirc & \bigcirc \\
x & \rightarrow & y
\end{array},$$

such partitioned matrices $M$ are called \textit{matrix representations of $\mathcal{P}$ over $(F, G)$}.

The Main Problem

The main problem regarding equipped posets consists of giving a complete description of indecomposable representations and irreducible morphisms of the category of representations $\text{rep} \mathcal{P}$ of a given equipped poset $\mathcal{P}$.
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\[
\begin{array}{c}
\otimes & \otimes & \circ & \circ \\
G & G & F & F \\
\end{array}
\]

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\section*{The Main Problem}

The main problem regarding equipped posets consists of giving a complete description of indecomposable representations and irreducible morphisms of the category of representations $\text{rep } \mathcal{P}$ of a given equipped poset $\mathcal{P}$.
Here we describe some examples of indecomposable representations in the category $\text{rep } \mathcal{P}$, where $\mathcal{P}$ is an equipped poset.

If $\mathcal{P}$ is an equipped poset and $A \subset \mathcal{P}$ then
$$P(A) = P(\min A) = (F = \mathbb{R} ; \ P_x \mid x \in \mathcal{P}), \ P_x = G = \mathbb{C} \text{ if } x \in A^\vee \text{ and } P_x = 0 \text{ otherwise.} \text{ In particular, } \ P(\emptyset) = (F ; \ 0, \ldots, 0).$$
If $a, b \in \mathcal{P} \otimes$, $c \in \mathcal{P}^\circ$ then $T(a)$, $T(a, b)$, $G_1(a, c)$, $G_2(a, c)$ denote indecomposable objects with matrix representation of the following form:

$$T(a) = \begin{bmatrix} a \\ 1 \\ \otimes \end{bmatrix}, \quad a \in \mathcal{P} \otimes,$$

$$T(a, b) = \begin{bmatrix} a & b \\ 1 & 0 \\ \otimes & 1 \\ \otimes & a \end{bmatrix}, \quad \text{with } a < b.$$

$$G_1(a, c) = \begin{bmatrix} a & c \\ 1 & 0 \\ u & 1 \end{bmatrix}$$

$$G_2(a, c) = \begin{bmatrix} a & c \\ 1 & 0 & 1 \\ u & 1 & 0 \end{bmatrix}$$
If \( a, b \in \mathcal{P} \otimes, c \in \mathcal{P}^\circ \) then \( T(a), T(a, b), G_1(a, c), G_2(a, c) \) denote indecomposable objects with matrix representation of the following form:

\[
T(a) = \begin{pmatrix} a \\ \otimes \\ 1 \end{pmatrix}, \quad a \in \mathcal{P} \otimes,
\]

\[
T(a, b) = \begin{pmatrix} a & b \\ \otimes & 0 \\ 1 & 1 \end{pmatrix}, \quad \text{with } a < b.
\]

\[
G_1(a, c) = \begin{pmatrix} a & c \\ \otimes & \otimes \\ 1 & 0 \\ 1 & 1 \end{pmatrix}
\]

\[
G_2(a, c) = \begin{pmatrix} a & c \\ \otimes & \otimes \\ 1 & 0 \\ 1 & 0 \end{pmatrix}
\]
Some Indecomposable Representations

\[
\begin{align*}
T(1) &= ( (1, u)^t )_3 \\
&\quad ( (1, u)^t )_2 \\
&\quad ( (1, u)^t )_1 \\
\end{align*}
\]

\[
P(4) =
\begin{align*}
&\quad 0 \\
&\quad 0 \\
&\quad 0 \\
\end{align*}
\]

\[
\begin{align*}
(\mathbb{R}, \mathbb{C}) &\quad u = i \\
\end{align*}
\]
Some Indecomposable Representations

\[
T(1) = \langle (1, u)^t \rangle^5 \oplus 4 \oplus 3 \oplus 2 \oplus 1
\]

\[
G^2 \oplus 7 \oplus 6
\]

\[
P(4) = 0 \oplus 0 \oplus 0 \oplus 0 \oplus 3 \oplus 2 \oplus 1
\]

\[
\langle (1, u)^t \rangle^1 \oplus 2 \oplus 3 \oplus 4
\]

\[
(\mathbb{R}, \mathbb{C}) \quad u = i
\]
Some Indecomposable Representations

\[ T(1) = \langle (1, u)^t \rangle \]

\[ P(4) = \]

\[ (\mathbb{R}, \mathbb{C}) \quad u = i \]
The *evolvent* of an equipped poset $\mathcal{P}$ is an ordinary poset:

$$\hat{\mathcal{P}} = \bigcup_{x \in \mathcal{P}} \{x', x''\}$$

where $x' = x'' = x$ for a strong point $x \in \mathcal{P}$, and $x' \neq x''$ is a pair of new strong mutually incomparable points (replacing a weak point $x$), with the following order relations:

1. If $x \prec y$, then $x' < y'$ and $x'' < y''$;
2. If $x \triangleleft y$, then $x' < y'$; $x' < y''$; $x'' < y'$ and $x'' < y''$. 
$\mathcal{P} = \begin{array}{cccccc}
5 & 4 & \times 3 & \times 2 & \times 1 \\
\end{array}$

$\hat{\mathcal{P}} = \begin{array}{cccccc}
5' & 4' & 3' & 2' & 1' \\
5'' & 4'' & 3'' & 2'' & 1'' \\
\end{array}$
It was necessary to define a new class of algorithms to classify posets with these additional structures. In fact, Zavadskij introduced 17 algorithms. Algorithms, I-V (and some additional differentiations) were used by him and Bondarenko to classify posets with involution, whereas algorithms I, VII-XVII and completion were used to classify equipped posets with involution in 2003 [10].

In particular, algorithms I, VII, VIII, IX and completion were used to classify equipped posets of finite growth representation type without paying attention to the behavior of the morphisms of the corresponding categories.
The following theorems regard the classification of equipped posets [10, 11].

Theorem (Zavadskij 2003)

An equipped poset $\mathcal{P}$ is tame (wild) if its evolvent $\hat{\mathcal{P}}$ is tame (wild).
Theorem (Zavadskij 2005)

For an equipped poset \( P \), the following statements are equivalent.

- \( P \) is of finite growth (over an arbitrary field \( k \)).
- The evolvent \( \hat{P} \) is of finite growth.
- The set \( P \) is not wild and contains none of the following equipped posets:

\[
G_1 \quad G_2 \quad G_3
\]
\[
G_4 \quad G_5 \quad G_6 \quad G_7
\]

- \( P \) does not contain subsets of the form \( N_1 \ldots, N_6, W_1 = (\tilde{1}, 1, 1, 1), W_2 = (\tilde{1}, \tilde{1}, 1), W_3 = (\tilde{1}, \tilde{1}, \tilde{1}), W_4 = (\tilde{1}, 1, 2), W_5 = (\tilde{2}, 1, 1), W_6 = (\tilde{1}, \tilde{2}), W_7 = (\tilde{2}, 3), W_8 = (\tilde{3}, 2), W_9 = (\tilde{4}, 1), \) and posets \( G_1, \ldots, G_7 \).
In 1991 Zavadskij followed following Gabriel’s ideas regarding the categorical properties of the algorithm of differentiation with respect to a maximal point [2] described the structure of the Auslander-Reiten quiver for ordinary posets of finite growth representation type [9].

To do that, he proved that the algorithm with respect to a suitable pair of points induces a categorical equivalence between some quotient categories.
Lifting Algorithm
We recall that according to Arnold \( P(x) = (k; P(x)_y \mid x \in \mathcal{P}) \)\[^1\]

\[
P(x)_y = \begin{cases} 
k & \text{if } y \in x^\triangledown, 
0 & \text{otherwise}.
\end{cases}
\]  

\[
I(x)_y = \begin{cases} 
0 & \text{if } y \in x^\triangle, 
k & \text{otherwise}.
\end{cases}
\]

Following to Simson \[^7\], arrows in \( \Gamma(\mathcal{P}) \) are valued arrows \([U] \xrightarrow{(d_{UV}, d'_{UV})} [V]\) where \(d'_{UV} = \dim \text{Irr } (U, V)_{\overline{U}}\) and \(d_{UV} = \dim_{(V)} \text{Irr } (U, V)\), and \(\overline{U} = \text{End } U/ \text{rad End } U\). This means that there are irreducible morphisms \(U \rightarrow V^d_{UV}\) and \(U^d_{UV} \rightarrow V\), we write \(U \rightarrow V\) if \(d_{UV} = d'_{UV} = 1\).

Henceforth, the following notations will be assumed in valued arrows of an Auslander-Reiten quiver.

\[
\text{and}
\]

\[
\text{means}
\]

\[
\text{and}
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We recall that according to Arnold \( P(x) = (k; P(x)_y \mid x \in \mathcal{P}) \) \cite{1}.

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Henceforth, the following notations will be assumed in valued arrows of an Auslander-Reiten quiver.

\[
\begin{align*}
2 & \quad \xrightarrow{} \quad 2 \\
\quad \xrightarrow{} & \quad \xrightarrow{} \\
\rightarrow & \quad \rightarrow
\end{align*}
\]

and means

\[
\begin{align*}
(1, 2) & \quad \xrightarrow{} \quad (2, 1) \\
\quad \xrightarrow{} & \quad \xrightarrow{} \\
\rightarrow & \quad \rightarrow
\end{align*}
\]

and
We recall that according to Arnold $P(x) = (k; P(x)_y \mid x \in \mathcal{P})$[1]

$$P(x)_y = \begin{cases} 
  k & \text{if } y \in x^\vee, \\
  0 & \text{otherwise.}
\end{cases}$$

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\end{cases}$$

Following to Simson [7], arrows in $\Gamma(\mathcal{P})$ are valued arrows $[U] \xrightarrow{(d_{UV}, d'_{UV})} [V]$ where $d'_{UV} = \dim \text{Irr } (U, V)(U)$ and $d_{UV} = \dim (V) \text{Irr } (U, V)$, and $\overline{U} = \text{End } U/\text{rad } \text{End } U$. This means that there are irreducible morphisms $U \rightarrow V^{d_{UV}}$ and $U^{d'_{UV}} \rightarrow V$, we write $U \rightarrow V$ if $d_{UV} = d'_{UV} = 1$.

Henceforth, the following notations will be assumed in valued arrows of an Auslander-Reiten quiver.

\[\begin{array}{c}
\text{2} \\
\text{---} \\
\text{---} \\
\text{2}
\end{array}\] and
\[\begin{array}{c}
\text{2} \\
\text{---} \\
\text{---} \\
\text{2}
\end{array}\] means
\[\begin{array}{c}
\text{(1, 2)} \\
\text{---} \\
\text{---} \\
\text{(2, 1)}
\end{array}\] and
\[\begin{array}{c}
\text{(2, 1)} \\
\text{---} \\
\text{---} \\
\text{(1, 2)}
\end{array}\]
The following algorithm is a way to compress the information (in terms of the dimension vectors of the indecomposable representations) given by the Auslander-Reiten quiver of the evolvent of an equipped poset.

**Lifting Algorithm and the Auslander-Reiten quiver.**

Input. An equipped poset of type $\mathcal{P} \in \{F_{13}, \ldots, F_{18}\}$ or $\mathcal{A} \in \{k_6, k_8\}$.

Output. The Auslander-Reiten quiver $\Gamma(\mathcal{P})$ or $\mathcal{P}(\mathcal{A}), \mathcal{I}(\mathcal{A})$. 
choose one of the following equipped posets $F_{13}, \ldots, F_{18}, k_6, k_8$, and denoted it by $P$,

compute the evolvent $\hat{P}$ of $P$,

compute the Auslander-Reiten quiver $\Gamma(\hat{P})$ of the ordinary poset $\hat{P}$, associating its corresponding dimension vectors,

fix a vertex $U = (U_0; U_x \mid x \in \hat{P})$ in $\Gamma(\hat{P})$ with vector dimension $d$ of the form $d_U = (d_0; d_x \mid x \in \hat{P})$,

(lifting; $\rtimes \Gamma(\hat{P})$)

(a) $\rtimes U$ for each $x \in P^\otimes$ with associated evolvent points $x'$ and $x''$, do $d_x = d_{x'} + d_{x''}$

(b) $\rtimes f$ for each arrow $f : U \rightarrow V \in \Gamma(\hat{P})$ define the arrow $\rtimes f : \rtimes U \rightarrow \rtimes V$ in $\Gamma(P)$, keeping neighbors and arrows orientation of $\Gamma(\hat{P})$. 
Lifting Algorithm

6. fix two vertices $U$ and $V$ in $\rtimes \Gamma(\hat{P})$,

7. (weak gluing; $\rtimes \Gamma(\hat{P})$) if $d_U = d_V = d = (d_0; d_x \mid x \in P)$ then do

   \[ 2d = (2d_0; 2d_x \mid x \in P) \]

   and identify $U$ and $V$ else keep $d_U$ and $d_V$ invariants,

8. fix a vertex $W$ in $\rtimes \Gamma(\hat{P})$ with vector dimension $s = (s_0; s_x \mid x \in P)$ given in step 7,

9. define $d = (d_0; d_x \mid x \in P)$ and do $d_0 = s_0$,

   \[ d_x = \begin{cases} 
   \frac{s_x}{2} & \text{if } x \in P^\otimes, \\
   s_x & \text{if } x \in P^\circ.
   \end{cases} \]

10. assign vector $d$ to a unique vertex $U$ of $\Gamma(P)$ keeping neighbors and arrows orientation of $\rtimes \Gamma(\hat{P})$,

11. define the dimension vector $l = (l_0 = d_0; l_x = \frac{d_x}{\dim \rad U_x} \mid x \in P)$. 
1. $\ni \tau^{-l} P(x') = \ni \tau^{-l} P(x'')$ for any positive integer $l$.

2. Note that, $\ni P(\emptyset) = P(\emptyset)$ and $\ni I(\widehat{P}) = I(P)$.

3. Arrows $\ni(f) : \ni(U) \to \ni(V)$ correspond to irreducible morphisms, were $f$ is a system of arrows in $\Gamma(\widehat{P})$ as we previously described.

4. If $P(x)$ is an indecomposable projective representation of $\widehat{P}$ then $\ni(P(x)) = P(x)$ is an indecomposable projective representation of $P$.

5. If $P(x')$ and $P(x'')$ are indecomposable projective representations of $\widehat{P}$, then $\ni(P(x') \oplus P(x'')) = T(x)$ is an indecomposable projective representation of $P$. 
Auslander-Reiten quiver $\nabla \Gamma(\hat{F}_{13})$

$F_{13}: \quad \otimes \quad \Rightarrow \quad \Gamma(\hat{F}_{13}) = F_{7}: \quad \circ \quad \circ''$

Figure: The Auslander-Reiten quiver $\nabla \Gamma(F_{13})$ of the equipped poset $F_{13}$ obtained from $\nabla \Gamma(F_{7})$ via the lifting algorithm $\nabla$. 

Auslander-Reiten quiver $\Gamma(\hat{F}_{15})$

$F_{15} : \circ_1 \rightarrow \hatsym{F}_{15} = F_3 : \circ_1$
Auslander-Reiten quiver $\Gamma(\widehat{F}_{15})$
Auslander-Reiten quiver $\Gamma(\widehat{F}_{15})$
Auslander-Reiten quiver $\Gamma(\hat{F}_{18})$

$F_{18} =$

$\infty(F) :$

Preprojective component of the Auslander-Reiten quiver $\Gamma(\widehat{k}_6)$
Theorem

Let $\mathcal{P}$ be a sincere equipped poset of finite representation type. Then the Auslander-Reiten quiver of $\mathcal{P}$ is $\xleftarrow{\Delta}(\hat{\mathcal{P}})$.

Theorem

Let $\mathcal{P} \in \{k_i \mid i \in 6, 8\}$ be a sincere equipped poset of infinite representation type. If $\mathcal{P}(\hat{\mathcal{P}})$, (resp. $\mathcal{I}(\hat{\mathcal{P}})$) is the preprojective (resp. preinjective) component of $\hat{\mathcal{P}}$, then the preprojective (resp. preinjective) component of $\mathcal{P}$ is $\xleftarrow{\Delta}(\mathcal{P}(\hat{\mathcal{P}}))$ (resp. $\xleftarrow{\Delta}(\mathcal{I}(\hat{\mathcal{P}}))$).
Theorem

Let $\mathcal{P}$ be a sincere equipped poset of finite representation type. Then the Auslander-Reiten quiver of $\mathcal{P}$ is $\triangledown \prec (\hat{\mathcal{P}})$.

Theorem

Let $\mathcal{P} \in \{k_i \mid i \in 6, 8\}$ be a sincere equipped poset of infinite representation type. If $\mathbb{P}(\hat{\mathcal{P}})$, (resp. $\mathbb{I}(\hat{\mathcal{P}})$) is the preprojective (resp. preinjective) component of $\hat{\mathcal{P}}$, then the preprojective (resp. preinjective) component of $\mathcal{P}$ is $\triangledown (\mathbb{P}(\hat{\mathcal{P}}))$ (resp. $\triangledown (\mathbb{I}(\hat{\mathcal{P}}))$).


Thank You