On the double of the Jordan plane

Nicolás Andruskiewitsch

Universidad of Córdoba, Argentina

ICTP-SAIFR Workshop on Representation Theory and Applications, April 26, 2022

Report on joint work with François Dumas, Héctor Martín Peña Pollastri, Dirceu Bagio, Saradia Della Flora, Daiana Flôres

I. Preliminaries.

Let $\Bbbk = \overline{\Bbbk}$ be an algebraically closed field, char $\Bbbk \neq 2$.

The **Jordan plane** is the (graded, quadratic) algebra

$$J = \Bbbk\langle x, y | yx - xy + \frac{1}{2}x^2 \rangle.$$

- (M. Artin & W. F. Schelter) The classification of the AS-regular algebras of rank 2 consists of the quantum planes and the Jordan plane.

Notice that GK-dim $J = 2$.

• (D. Gurevich) Let V be a vector space with basis $\{x, y\}$ and let $c \in GL(V \otimes V)$ be given by

$$c(x \otimes x) = x \otimes x, \qquad c(y \otimes x) = x \otimes y,$$
$$c(x \otimes y) = (y + x) \otimes x, \qquad c(y \otimes y) = (y + x) \otimes y.$$

Then c (called the braiding) satisfies the braid equation

$$(c \otimes \mathrm{id})(\mathrm{id} \otimes c)(c \otimes \mathrm{id}) = (\mathrm{id} \otimes c)(c \otimes \mathrm{id})(\mathrm{id} \otimes c).$$

From this it is easy to see that J is a braided Hopf algebra.

Actually $V \in {}^{\Bbbk\Gamma}_{\Bbbk\Gamma}\mathcal{YD}$, where $\Gamma = \langle \mathbf{g} \rangle \simeq \mathbb{Z}$ acts on V by $\mathbf{g} \cdot x = x$, $\mathbf{g} \cdot y = y + x$. Thus

$$\begin{aligned}\widetilde{H} := J\#\Bbbk\Gamma = \Bbbk\langle x, y, \mathbf{g}^{\pm 1} |\ & \mathbf{g}^{\pm 1}\mathbf{g}^{\mp 1} - 1, \\ & \mathbf{g}x - x\mathbf{g},\ \mathbf{g}y - y\mathbf{g} - x\mathbf{g},\ yx - xy + \frac{1}{2}x^2\rangle\end{aligned}$$

is a Hopf algebra with

$$\begin{aligned}\Delta(\mathbf{g}^{\pm 1}) &= \ \mathbf{g}^{\pm 1} \otimes \mathbf{g}^{\pm 1}, \\ \Delta(x) &= x \otimes 1 + \mathbf{g} \otimes x, \\ \Delta(y) &= y \otimes 1 + \mathbf{g} \otimes y.\end{aligned}$$

The Hopf algebra $\widetilde{H}$ is the bosonization of the Jordan plane.

Assume that $\operatorname{char} \Bbbk = 0$. Then $J \simeq \mathscr{B}(V)$ is a Nichols algebra (Cibils, Lauve & Witherspoon); that is, all primitives are in degree 1. Indeed, given $\epsilon \in \Bbbk^{\times}$ and $\ell \in \mathbb{N}_{\geq 2}$, the **block** $\mathcal{V}(\epsilon, \ell)$ is a vector space with a basis $(x_i)_{i \in \mathbb{I}_\ell}$ and a braiding

$$c(x_i \otimes x_j) = \begin{cases} \epsilon x_1 \otimes x_i, & j = 1, \\ (\epsilon x_j + x_{j-1}) \otimes x_i, & j \geq 2, \end{cases} \qquad i \in \mathbb{I}_\ell.$$

Theorem. (A.–Angiono–Heckenberger)
$\operatorname{GK-dim} \mathcal{B}(\mathcal{V}(\epsilon, \ell)) < \infty \iff \ell = 2$ and $\epsilon \in \{\pm 1\}$.

- $\mathcal{B}(\mathcal{V}(1,2)) \simeq J$ is the **Jordan plane**.

- $\mathcal{B}(\mathcal{V}(-1,2)) = \Bbbk\langle x_1, x_2 | x_1^2, x_2 x_{21} - x_{21} x_2 - x_1 x_{21} \rangle$ is the **super Jordan plane**. Here $x_{21} = x_2 x_1 + x_1 x_2$.

This result is important for the question of classifying Hopf algebras with finite Gelfand-Kirillov dimension. Indeed, we have

Theorem. (A.–Angiono–Heckenberger) Assume that $\operatorname{char} \Bbbk = 0$. The classification of the braided vector spaces V which are direct sums of blocks and points and that have $\operatorname{GK-dim} \mathscr{B}(V) < \infty$ is known (up to a Conjecture on diagonal type).

Remark. (A.–Angiono–Heckenberger), (A.–Bagio–Della Flora–Flôres) The analogous braided vector spaces have often *finite-dimensional* Nichols algebras.

II. The restricted Jordan plane (odd characteristic).

Assume now that $p = \operatorname{char} \Bbbk > 2$. Recall that V has a basis $\{x, y\}$ and braiding given by

$$
\begin{aligned}
c(x \otimes x) &= x \otimes x, & c(y \otimes x) &= x \otimes y, \\
c(x \otimes y) &= (y + x) \otimes x, & c(y \otimes y) &= (y + x) \otimes y.
\end{aligned}
$$

Theorem. (Cibils, Lauve & Witherspoon) The Nichols algebra $\mathscr{B}(V)$ is the quotient of $T(V)$ by the ideal generated by

$$
x^p, \qquad y^p, \qquad yx - xy + \frac{1}{2}x^2.
$$

That is, $\mathscr{B}(V) \simeq J/\langle x^p,\ y^p\rangle$; this is the *restricted* Jordan plane (in characteristic p).

Actually $V \in {}^{\Bbbk C_p}_{\Bbbk C_p}\mathcal{YD}$, where $C_p = \langle g \rangle$ has order p. Thus

$$H := \mathscr{B}(V)\#\Bbbk C_p = \Bbbk\langle x, y, g |\ g^p - 1,\ gx - xg,\ gy - yg - xg,\\ x^p,\ y^p,\ yx - xy + \frac{1}{2}x^2\rangle$$

is a Hopf algebra (of dimension p^3) with

$$\begin{aligned} \Delta(g) &= \ g \otimes g & & (g \in G(H)), \\ \Delta(x) &= x \otimes 1 + g \otimes x, & & (x \in \mathcal{P}_{g,1}(H)) \\ \Delta(y) &= y \otimes 1 + g \otimes y & & (x \in \mathcal{P}_{g,1}(H)). \end{aligned}$$

H is the bosonization of the restricted Jordan plane.

Clearly there is a surjective map of Hopf algebras $\widetilde{H} \twoheadrightarrow H$.

Let L be a finite-dimensional Hopf algebra. V. G. Drinfeld introduced the double of L, a quasitriangular Hopf algebra denoted by $D(L)$. Actually ${}^{L}_{L}\mathcal{YD} \simeq {}_{D(L)}\mathcal{M}$. The underlying coalgebra is $L \otimes L^{*\,\mathsf{op}}$ and the algebra is a kind of bi-semidirect product.

Here *quasitriangular* means that any L-module comes with a solution of the braid equation.

Proposition. (A.–Peña Pollastri) The algebra $D(H)$ is generated by

$$\underbrace{g, x, y,}_{H} \qquad \underbrace{u, v, \zeta}_{H^*}$$

with defining relations

$$g^p = 1,\ \zeta^p = 0,\ x^p = 0,\ y^p = 0,\ u^p = 0,\ v^p = 0. \tag{1}$$

$$\zeta g = g\zeta, \tag{2}$$

$$\begin{aligned} &gx = xg,\ gy = yg + xg,\ \zeta y = y\zeta + y,\ \zeta x = x\zeta + x, \\ &ug = gu,\ vg = gv + gu,\ v\zeta = \zeta v + v,\ u\zeta = \zeta u + u, \end{aligned} \tag{3}$$

$$\begin{aligned} &yx = xy - \frac{1}{2}x^2, && vu = uv - \frac{1}{2}u^2, \\ &ux = xu, && vx = xv + (1 - g) + xu, \\ &uy = yu + (1 - g), && vy = yv - g\zeta + yu. \end{aligned} \tag{4}$$

The comultiplication is defined by

$$\begin{aligned} g &\in G(D(H)), \\ u, \zeta &\in \mathcal{P}(D(H)), \\ x, y &\in \mathcal{P}_{g,1}(D(H)), \\ \Delta(v) &= v \otimes 1 + 1 \otimes v + \zeta \otimes u. \end{aligned} \tag{5}$$

Recall that a short exact sequence of Hopf algebras is a collection

$$A \overset{\iota}{\hookrightarrow} C \overset{\pi}{\twoheadrightarrow} B$$

where

(i) ι is injective.

(ii) π is surjective.

(iii) $\ker \pi = C\iota(A)^+$.

(iv) $\iota(A) = C^{\mathrm{co}\,\pi}$.

One also says that C is an extension of B by A.

When A is commutative and B is cocommutative, one says that the extension is *abelian*.

Let $\{h, e, f\}$ be the Cartan generators of $\mathfrak{sl}_2(\mathbb{k})$, whose restricted enveloping algebra is denoted $\mathfrak{u}(\mathfrak{sl}_2(\mathbb{k}))$.

Proposition. (A.–Peña Pollastri) The subalgebra $\mathbf{R}$ of $D(H)$ generated by g, x and u is a normal local commutative Hopf subalgebra of $D(H)$ of dimension p^3 with defining relations

$$g^p = 1, \qquad x^p = 0, \qquad u^p = 0. \tag{6}$$

It gives rise to the (abelian) exact sequence of Hopf algebras

$$\mathbf{R} \overset{\iota}{\hookrightarrow} D(H) \overset{\pi}{\twoheadrightarrow} \mathfrak{u}(\mathfrak{sl}_2(\mathbb{k}))$$

where $\pi(\zeta) = h$, $\pi(y) = \frac{1}{2}e$ and $\pi(v) = f$.

Since $\mathbf{R}$ is normal and local, we conclude

Theorem. (A.–Peña Pollastri) $\operatorname{Irrep} D(H) \simeq \operatorname{Irrep} \mathfrak{u}(\mathfrak{sl}_2(\mathbb{k}))$. Thus there are exactly p isomorphism classes of simple $D(H)$-modules which have dimensions $1, 2, \ldots, p$.

These simple modules can be constructed as quotients of Verma modules.

III. The double of the Jordan plane, $\operatorname{char} \Bbbk \neq 2$

Definition. (A.–Peña Pollastri) The double of the Jordan plane is the algebra $\mathcal{D}$ generated by

$$\underbrace{\mathbf{g}^{\pm 1}, x, y,}_{\widetilde{H}} \qquad \qquad \underbrace{u, v, \zeta}_{\widetilde{K}}$$

with defining relations (2), (3), (4) and

$$\mathbf{g}^{\pm 1}\mathbf{g}^{\mp 1} = 1 \tag{7}$$

The comultiplication is defined by (5).

Consider the algebraic groups

- $\mathbf{G}_a =$ additive group $(\Bbbk, +)$,
- $\mathbf{G}_m =$ multiplicative group $(\Bbbk^\times, \cdot)$,
- $\mathbf{H}_3 =$ Heisenberg group of dimension 3,
- $\mathbf{G} = (\mathbf{G}_a \times \mathbf{G}_a) \rtimes \mathbf{G}_m$,
- $\mathbf{B} = ((\mathbf{G}_a \times \mathbf{G}_a) \rtimes \mathbf{G}_m) \times \mathbf{H}_3$,

with suitable semidirect products.

The algebra of regular functions on $\mathbf{G}$ is denoted by $\mathcal{O}(\mathbf{G})$, etc.

- The subalgebra of $\mathcal{D}$ generated by x, u and $\mathbf{g}$ is a normal Hopf subalgebra isomorphic to $\mathcal{O}(\mathbf{G})$ (as Hopf algebras).

- Let $\pi\colon \mathcal{D} \longrightarrow U(\mathfrak{sl}_2(\Bbbk))$ be the Hopf algebra map given by

$$x \mapsto 0, \qquad u \mapsto 0, \qquad \mathbf{g} \mapsto 1, \qquad y \mapsto \frac{1}{2}e, \qquad v \mapsto f, \qquad \zeta \mapsto h.$$

Proposition. (A.–Peña Pollastri) There is an exact sequence

$$\mathcal{O}(\mathbf{G}) \overset{\iota}{\hookrightarrow} \mathcal{D} \overset{\pi}{\twoheadrightarrow} U(\mathfrak{sl}_2(\Bbbk)).$$

Assume that $\operatorname{char}\Bbbk = p > 2$.

Proposition. (A.–Peña Pollastri) There is a commutative diagram

$$\begin{tikzcd}
\mathcal{O}(\mathbf{G}) \arrow[r, hook] \arrow[d, hook, "\mathrm{Fr}"'] & \mathcal{O}(\mathbf{B}) \arrow[r, two heads] \arrow[d, hook] & \mathcal{O}(\mathbf{G}_a^3) \arrow[d, hook] \\
\mathcal{O}(\mathbf{G}) \arrow[r, hook] \arrow[d, two heads] & \mathcal{D} \arrow[r, two heads] \arrow[d, two heads] & U(\mathfrak{sl}_2(\Bbbk)) \arrow[d, two heads] \\
\mathbf{R} \arrow[r, hook] & D(H) \arrow[r, two heads] & \mathfrak{u}(\mathfrak{sl}_2(\Bbbk))
\end{tikzcd}$$

where all columns and rows are exact sequences.

Properties of $\mathcal{D}$, char $\Bbbk = 0$

- The algebra $\mathcal{D}$ admits an exhaustive ascending filtration $(\mathcal{D}_n)_{n\in\mathbb{N}_0}$ such that $\operatorname{gr}\mathcal{D} \simeq \Bbbk[T^{\pm}]\otimes\Bbbk[X_1,\dots,X_5]$.

- $\mathcal{D}$ is a noetherian domain.

- $\mathcal{D}$ is an Ore extension, hence strongly noetherian, AS-regular and Cohen-Macaulay.

- $\mathcal{D}$ is a PI-algebra.

- $\mathcal{D}$ satisfies the Gelfand-Kirillov property i. e. its skew field of fractions $\operatorname{Frac}\mathcal{D}$ is $\Bbbk$-isomorphic to a Weyl skew field $\mathscr{D}_{n,s}(\Bbbk)$.

Theorem. (A.–Dumas–Peña Pollastri) $\operatorname{Irrep}\mathcal{D} \simeq \operatorname{Irrep} U(\mathfrak{sl}_2(\Bbbk))$.

For the proof we use the following result. Let A be an algebra. Let $\mathcal{F} \subset A$ be a family of elements satisfying

- the elements of $\mathcal{F}$ commute with each other;
- any $x \in \mathcal{F}$ acts nilpotently on any $M \in {}_A\mathcal{M}$, $\dim M < \infty$;
- $\mathcal{F}$ is normal: $A\mathcal{F} = \mathcal{F}A$.

Let $L \in \operatorname{Irrep} A$. Then the representation $\rho : A \to \operatorname{End} L$ factorizes through $A/A\mathcal{F}$. Thus the projection $A \to A/A\mathcal{F}$ induces a bijection

$$\operatorname{Irrep} A \simeq \operatorname{Irrep} A/A\mathcal{F}.$$

Then we check that $\mathcal{F} = \{x, u, \mathrm{g} - 1\}$ fulfills the preceding hypothesis.

Recall that the algebra $\mathcal{D}$ generated by $\mathrm{g}^{\pm 1}, x, y, u, v, \zeta$. Let

$$q = ux + 2(1+g), \;\; s = xv + uy + (-\frac{1}{2}ux + g - 1)\zeta - 2(1+g).$$

Then we can show that the following elements belong to $\mathcal{Z}(\mathcal{D})$:

$$z = q^2 g^{-1}, \qquad \theta = s^2 g^{-1}, \qquad \omega = qg^{-1}s.$$

Theorem. (A.–Dumas–Peña Pollastri) The center of $\mathcal{D}$ is the commutative subalgebra generated by z, ω and θ, which is isomorphic to the quotient $\Bbbk[X, Y, Z]/(XZ - Y^2)$.

V. The super Jordan plane (& its restricted version in odd char).

Assume that $\operatorname{char} \Bbbk \neq 2$. Let V have a basis $\{x, y\}$ and braiding

$$
\begin{aligned}
c(x \otimes x) &= -x \otimes x, & c(y \otimes x) &= -x \otimes y,\\
c(x \otimes y) &= (-y + x) \otimes x, & c(y \otimes y) &= (-y + x) \otimes y.
\end{aligned}
$$

Let $x_{21} = x_2 x_1 + x_1 x_2$. The super Jordan plane is the algebra

$$
sJ = \Bbbk\langle x_1, x_2 | x_1^2,\, x_2 x_{21} - x_{21} x_2 - x_1 x_{21}\rangle
$$

Theorem. (A.–Angiono–Heckenberger)

- If $\operatorname{char} \Bbbk = 0$, then the Nichols algebra $\mathscr{B}(V) \simeq sJ$.
- If $\operatorname{char} \Bbbk = p > 2$, then $\mathscr{B}(V) \simeq sJ/\langle x_2^{2p}, x_{21}^p\rangle$.

This is the restricted super Jordan plane in characteristic p; it has dimension $4p^2$.

Assume that $\operatorname{char}\Bbbk = p > 2$.

Now $V \in {}^{\Bbbk C_{2p}}_{\Bbbk C_{2p}}\mathcal{YD}$. Let H be the bosonization of $\mathscr{B}(V)$, i. e.

$$H = \mathscr{B}(V)\#\Bbbk C_{2p} = \Bbbk\langle x, y, \gamma |\ \gamma^{2p} - 1,\ \gamma x + x\gamma,\ \gamma y + y\gamma - x\gamma,\\ x_1^2,\ x_2^{2p} = 0,\ x_2x_{21} - x_2x_{21} - x_{21}x_1,\ x_2^{2p}, x_{21}^p.\rangle$$

is a Hopf algebra (of dimension $4p^3$) with

$$\gamma \in G(H), \qquad\qquad x, y \in \mathcal{P}_{\gamma,1}(H).$$

Facts.

- The Drinfeld double $D(H)$ can be computed explicitly.

- $D(H)$ has a normal Hopf subalgebra $Z_0 =\simeq \Bbbk C_2$; the quotient is $D := D(H)/D(H)Z_0^+$.

- There is a Hopf superalgebra $\mathscr{D}$ such that $D \simeq \mathscr{D}\#\Bbbk C_2$.

- Let $\mathbf{R}$ be the super commutative Hopf superalgebra

$$\mathbf{R} := \Bbbk[X_1, X_2, T]/(X_1^p, X_2^p, T^p - 1) \otimes \Lambda(Y_1, Y_2)$$

with $|X_1| = |X_2| = |T| = 0$, $|Y_1| = |Y_2| = 1$, and comultiplication

$$\Delta(X_1) = X_1 \otimes 1 + T^2 \otimes X_1 + Y_1 T \otimes Y_1, \;\; \Delta(T) = T \otimes T,$$
$$\Delta(X_2) = X_2 \otimes 1 + 1 \otimes X_2 + Y_2 \otimes Y_2,$$
$$\Delta(Y_2) = Y_2 \otimes 1 + 1 \otimes Y_2, \;\; \Delta(Y_1) = Y_1 \otimes 1 + T \otimes Y_1.$$

Theorem. (A.–Peña Pollastri) There exist Hopf superalgebra maps ι and π such that

$$\mathbf{R} \xhookrightarrow{\iota} \mathscr{D} \xrightarrow{\pi} \mathfrak{u}(\mathfrak{osp}(1|2)) \tag{8}$$

is an exact sequence of Hopf superalgebras.

Since $\mathbf{R}$ is normal and local, we conclude

Theorem. (A.–Peña Pollastri) $\operatorname{Irrep} \mathscr{D} \simeq \operatorname{Irrep} \mathfrak{u}(\mathfrak{osp}(1|2))$. Hence there are p isomorphism classes of simple $\mathscr{D}$-modules which have dimensions $1, 3, 5, \ldots, 2p-1$.

Proposition. (A.–Peña Pollastri) There is a commutative diagram of Hopf superalgebras

$$\begin{tikzcd}
\mathcal{O}(\mathbf{G}) \arrow[r, hook] \arrow[d, hook] & \mathcal{O}(\mathbf{G}) \arrow[r, two heads] \arrow[d, hook] & \mathcal{O}(\mathbf{G}_a^3) \arrow[d, hook] \\
\mathcal{O}(\mathfrak{G}) \arrow[r, hook] \arrow[d, two heads] & \widetilde{\mathscr{D}} \arrow[r, two heads] \arrow[d, two heads] & U(\mathfrak{osp}(1|2)) \arrow[d, two heads] \\
\mathbf{R} \arrow[r, hook] & \mathscr{D} \arrow[r, two heads] & \mathfrak{u}(\mathfrak{osp}(1|2))
\end{tikzcd}$$

where all columns and rows are exact sequences.

V. The restricted Jordan plane (even characteristic).

Assume now that $p = \operatorname{char} \Bbbk = 2$. Recall that V has a basis $\{x, y\}$ and braiding given by

$$\begin{aligned} c(x \otimes x) &= x \otimes x, & c(y \otimes x) &= x \otimes y, \\ c(x \otimes y) &= (y + x) \otimes x, & c(y \otimes y) &= (y + x) \otimes y. \end{aligned}$$

Theorem. (Cibils, Lauve & Witherspoon) The Nichols algebra $\mathscr{B}(V)$ is the quotient of $T(V)$ by the ideal generated by

$$x_1^2 = 0, \ x_2^4 = 0, \ x_2^2 x_1 = x_1 x_2^2 + x_1 x_2 x_1, \ x_1 x_2 x_1 x_2 = x_2 x_1 x_2 x_1.$$

This is the restricted Jordan plane in characteristic 2; it has dimension 16. It is closer to the restricted Jordan plane in odd characteristic.

Now $V \in {}^{\Bbbk C_2}_{\Bbbk C_2}\mathcal{YD}$. Let H be the bosonization of $\mathscr{B}(V)$, i. e.

$$H = \mathscr{B}(V)\#\Bbbk C_2 = \Bbbk\langle x, y, g |\ g^2 - 1,\ gx - xg,\ gy - yg - xg,$$
$$x_1^2,\ x_2^4,\ x_2^2x_1 - x_1x_2^2 + x_1x_2x_1,\ x_1x_2x_1x_2 = x_2x_1x_2x_1.\rangle$$

is a Hopf algebra (of dimension 32) with

$$g \in G(H), \qquad\qquad x, y \in \mathcal{P}_{g,1}(H).$$

Let $\mathfrak{s}$ be the derived Lie algebra of the 4-dimensional Witt Lie algebra. Then $\mathfrak{s}$ has a basis $\{a, b, c\}$ and bracket

$$[a, b] = c, \qquad [a, c] = a, \qquad [b, c] = b. \tag{9}$$

$\mathfrak{s}$ is the unique (up to isomorphism) simple Lie algebra of dim. 3.

The Lie algebra $\mathfrak{s}$ is not restricted.

The minimal 2-envelope of $\mathfrak{s}$ is the Lie algebra $\mathfrak{m}$ with basis $\{b', b, c, a, a'\}$, bracket (9) and

$[a', b] = a,\ [a', b'] = c,\ [a, b'] = b,\ [a', a] = [a', c] = [b', b] = [b', c] = 0;$

and 2-operation $(\ )^{[2]} : \mathfrak{m} \to \mathfrak{m}$ given by

$$(a')^{[2]} = (b')^{[2]} = 0, \qquad c^{[2]} = c, \qquad a^{[2]} = a', \qquad b^{[2]} = b'.$$

The restricted enveloping algebra $\mathfrak{u}(\mathfrak{m})$ is isomorphic to $\Bbbk\langle a, b, c\rangle / I$ where I is generated by the relations

$$ab + ba = c,\ ac + ca = a,\ bc + cb = b,\ a^4 = b^4 = 0,\ c^2 + c = 0.$$

That is,

$$\mathfrak{u}(\mathfrak{m}) \simeq U(\mathfrak{s})/\langle a^4, b^4, c^2 + c\rangle.$$

Proposition. (A.–Bagio–Della Flora–Flôres) The algebra $D(H)$ is generated by x, y, g, u, v, γ with defining relations explicitly described.

Proposition. (A.–Bagio–Della Flora–Flôres) There is a normal local commutative Hopf subalgebra $\mathbf{T}$ of $D(H)$ such that $D(H)$ fits in an abelian extension

$$\mathbf{T} \overset{\iota}{\hookrightarrow} D(H) \overset{\pi}{\twoheadrightarrow} \mathfrak{u}(\mathfrak{m}),$$

Since $\mathbf{T}$ is normal and local, we conclude

Theorem. (A.–Bagio–Della Flora–Flôres) $\operatorname{Irrep} D(H) \simeq \operatorname{Irrep} \mathfrak{u}(\mathfrak{m})$.

Even more the functor $\pi^* : {}_{\mathfrak{u}(\mathfrak{m})}\mathcal{M} \to {}_{D(H)}\mathcal{M}$ is tensor and sends indecomposables to indecomposables.

References.

N. A., I. Angiono, I. Heckenberger. *On finite GK-dimensional Nichols algebras over abelian groups*. Mem. Amer. Math. Soc. **271** (2021).

_________ *Examples of finite-dimensional pointed Hopf algebras in positive characteristic*. In Progr. Math. **340** (2021).

N. A., D. Bagio, S. Della Flora, D. Flores. *Examples of finite-dimensional pointed Hopf algebras in characteristic 2*. Glasg. Math. J. **64** 65–78 (2022).

_________ *On the Drinfeld double of the restricted Jordan plane in characteristic* 2, in preparation.

N. A., F. Dumas, H. Peña Pollastri. *On the double of the Jordan plane*. Ark. Mat., to appear.

N. A. and H. Peña Pollastri. *On the restricted Jordan plane in odd characteristic*. J. Algebra Appl. **20** (1), 2140012 (2021).

_________ *On the double of the (restricted) super Jordan plane*. New York J. Math., to appear.