Mapping f(R) Into GR

01 Field Equations of Palatini f(R)

$$f'(R)R_{\mu\nu} - \frac{1}{2}f(R)g_{\mu\nu} = \kappa T_{\mu\nu},$$

$$\nabla_\rho(\sqrt{-g}g^{\mu\nu}f'(\mathcal{R})) = 0.$$

02 Theoretical Framework of the Mapping

$$h_{\mu\nu} = \sqrt{\det\hat{\Sigma}}(\Sigma^{-1})^{\alpha}{}_{\nu}g_{\mu\alpha} = \Omega^{\alpha}{}_{\nu}g_{\mu\alpha},$$

$$\begin{aligned}\mathcal{G}_{\mu}{}^{\nu} &= \mathcal{R}_{\mu}{}^{\nu} - \frac{1}{2}\delta^{\nu}_{\mu}\mathcal{R} \\ &= \frac{\kappa}{\sqrt{\det\hat{\Omega}}}\left(T_{\mu}{}^{\nu} - \frac{1}{2}\delta^{\nu}_{\mu}\left(\frac{f(R)}{\kappa} + T\right)\right) \\ &= \kappa\mathcal{T}_{\mu}{}^{\nu},\end{aligned}$$

03 Mapping of a Non-Free Complex Scalar Field

$$K_Z Z_{\mu}{}^{\nu} - \frac{1}{2}\delta^{\nu}_{\mu}D(Z,U) = \frac{1}{\sqrt{\det\hat{\Omega}}}\left(F_X X_{\mu}{}^{\nu} - \frac{1}{2}\delta^{\nu}_{\mu}\left(\frac{f(\mathcal{R})}{\kappa} + F_X X - C(X,V)\right)\right).$$