Quasinormal modes for Kerr black hole via Painlevé transcendents

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Introduction

The Teukolsky Master equation (TME) governs linear perturbations of the Kerr metric [2], where, for vacuum perturbations, one has

$$\frac{1}{\sin \theta} \frac{d}{d \theta} \left[ \sin \theta \frac{dS}{d \theta} \right] + \left[ a^2 \omega^2 \cos^2 \theta - 2 a \omega L \cos \theta - \frac{(m + s \cos \theta)^2}{\sin^2 \theta} + s + \lambda \right] S(\theta) = 0,$$

$$\Delta^{-s} \frac{d}{dr} \left( \Delta^{s+1} \frac{dR(r)}{dr} \right) + \left( \frac{K^2(r) - 2i\omega (r - M)K(r)}{\Delta} + 4i\omega r - s \lambda \right) R(r) = 0,$$

where $\Delta = r^2 - 2Mr + a^2 = (r - r_+)(r - r_-)$, $K(r) = (r^2 + a^2)\omega$ and $M$ and $J = aM$ are the mass and angular momentum of the black hole. The spin-weight field $s$ can assume the values $0$, $\pm 1$, and $\pm 2$.

Riemann-Hilbert map

The Riemann-Hilbert map, between $\tau_V$, $c_0$ and $\sigma$, $\eta$, is made possible by the isomonodromic $\tau$ function, which has a natural expansion in terms of monodromy data [3]. Thus, the RH map is expressed in terms of $\tau_V$ by

$$\tau_V(\bar{\theta}_-; \sigma, \eta; t_0) = 0, \quad t_0 \frac{d}{dt_0} \log \tau_V(\bar{\theta}_-; \sigma - 1, \eta; t_0) - \frac{\theta_0(t_0 - 1)}{2} = t_0 c_{t_0},$$

where $\bar{\theta} = \{\theta_0, \theta_1, \theta_2\}$ are the parameters in the CHE associated to the local monodromy of solutions and $\bar{\theta}_- = \{\theta_0, \theta_1 - 1, \theta_2 + 1\}$. In turn, the Riemann-Hilbert map associated to the DCHE is given by

$$\tau_{III}(\bar{\theta}_-; \sigma, \eta; u_0) = 0, \quad u_0 \frac{d}{du_0} \log \tau_{III}(\bar{\theta}_-; \sigma - 1, \eta; u_0) - \frac{(\theta_0 - 1)^2}{2} = u_0 k_{u_0},$$

where $\bar{\theta} = \{\theta_0, \theta_1, \theta_2\}$ are the parameters in the DCHE associated to the local monodromy of solutions and $\bar{\theta}_- = \{\theta_0 - 1, \theta_1, \theta_2 + 1\}$.

The function $\tau_V$ and $\tau_{III}$ can be expressed in terms of Fredholm determinant [4] or via Nekrasov partition function [5], while the parameters $\sigma$ and $\eta$ are functions of the monodromy parameters of the equations (CHE and DCHE).

Numerical Results

Extremal Limit $a = M$: A. We observed numerically that for the modes $l = m$, with $m \neq 0$, the eigenfrequencies tend to $m(2M)$. In this case, the Riemann Hilbert map (3) actually solved the QNM for $a/M \in [0, 1]$.

Near-extremal behavior for the fundamental quasi-normal frequency for $s = -2$, $l = m = 2$, where $a/M = \cos(v)$.

B. Mu does not go to $m/2$. All modes with $l \neq m$, including those with negative $m$, will not tend to $m = m/2$ in the extremal limit. In this situation the modes for $a = M$ are calculated using the Riemann-Hilbert map (4).

The near-extremal behavior for the fundamental quasi-normal frequency for $s = -1$, $l = 2$ and $m = 1$, where the mode calculated using $\tau_V$ converges to the frequency for $\tau_III$ as $v$ goes to 0.

Bibliography


