Quasinormal modes for Kerr black hole via Painlevé transcendents

João Paulo Cavalcante * and Bruno Carneiro da Cunha†

Department of Physics, Federal University of Pernambuco, 50670-901, Recife, Brazil

Introduction

The Teukolsky Master equation (TME) governs linear perturbations of the Kerr metric [2], where, for vaccum perturbations, one has

$$\frac{1}{\sin\theta}\frac{d}{d\theta}\left[\sin\theta\frac{dS}{d\theta}\right]+\left[a^2\omega^2\cos^2\theta-2a\omega s\cos\theta-\frac{(m+s\cos\theta)^2}{\sin^2\theta}+s+\lambda\right]S(\theta)=0,$$

$$\Delta^{-s}\frac{d}{dr}\left(\Delta^{s+1}\frac{dR(r)}{dr}\right)+\left(\frac{K^2(r)-2is(r-M)K(r)}{\Delta}+4is\omega r-{}_s\lambda_{\ell,m}-a^2\omega^2+2am\omega\right)R(r)=0,$$

where $\Delta = r^2 - 2Mr + a^2 = (r - r_+)(r - r_-)$, $K(r) = (r^2+a^2)\omega - am$. M and J = aM are the mass and angular momentum of the black hole. The spin-weight field s can assume the values 0, ±1, and ±2.

Riemann-Hilbert map

The Riemann-Hilbert map, between t_0 , c_{t0} and σ, η, is made possible by the isomonodromic τ function, which has a natural expansion in terms of monodromy data [3]. Thus, the RHm is expressed in terms of τ_V by

$$\tau_V(\vec{\theta};\sigma,\eta;t_0)=0,\qquad t_0\frac{d}{dt_0}\log\tau_V(\vec{\theta}_-;\sigma-1,\eta;t_0)-\frac{\theta_0(\theta_t-1)}{2}=t_0c_{t_0}\tag{3}$$

Where $\vec{\theta} = \{\theta_0\ \theta_t\ \theta_*\}$ are the parameters in the CHE associated to the local monodromy of solutions and $\vec{\theta}_- = \{\theta_0 , \theta_t -1, \theta_* +1\}$. In turn, the Riemann-Hilbert map associated to the DCHE is given by

$$\tau_{III}(\vec{\theta};\sigma,\eta;u_0)=0,\qquad u_0\frac{d}{du_0}\log\tau_{III}(\vec{\theta}_-;\sigma-1,\eta;u_0)-\frac{(\theta_\circ-1)^2}{8}-\frac{1}{2}=u_0k_0,\tag{4}$$

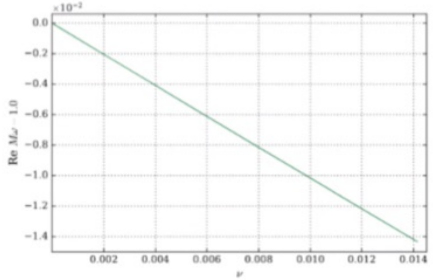
where $\vec{\theta} = \{\theta_\circ,\theta_*\}$ are the parameters in the DCHE associated to the local monodromy of solutions and $\vec{\theta}_- = \{\theta_\circ -1, \theta_* +1\}$.

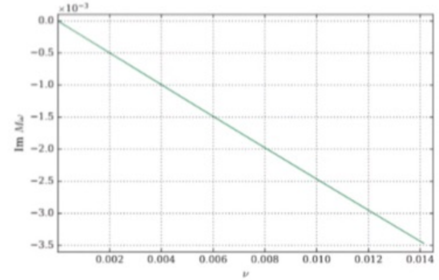
The function τ_V and τ_{III} can be expressed in terms of Fredholm determinant [4] or via Nekrasov partition function [5], while the parameters σ and η are functions of the monodromy parameters of the equations (CHE and DCHE).

Numerical Results

Extremal Limit a → M: **A.** We observed numerically that for the modes l = m, with m ≠ 0, the eigenfrequencies tend to m/(2M). In this case, the Riemann Hilbert map (3) actually solved the QNMs for a/M ∈ [0,1].

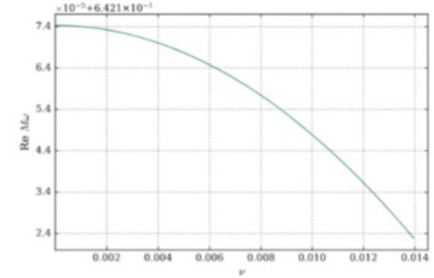
Near-extremal behavior for the fundamental quasi-normal frequency for s = −2, l = m = 2, where a/M = cos(ν).

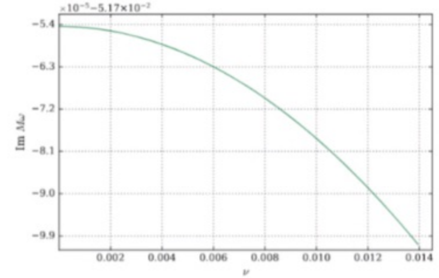




B. Mω does not go to m/2. All modes with l ≠ m, including those with negative m, will not tend to M ω = m/2 in the extremal limit. In this situation the modes for a =M are calculated using the Riemann-Hilbert map (4).

The near-extremal behavior for the fundamental quasi-normal frequency for s = −1, l = 2 and m = 1, where the mode calculated using τ_V converges to the frequency for τ_{III} as ν goes to 0.





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