Testing Gravitational lensing in an Expanding Universe

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Frame Title

- Introduction
- Lens system and bending angle
- Bending angle in Λ+matter models
- Testing with Einstein rings
- Concluding remarks

Does Λ contribute to the bending of light?

Schwarschild-de Sitter metric

- Islam, 1986 (and others): **No, Λ does not enter the geodesic equation!**
- Rindler & Ishak, 2007: **Yes, we just need to correct for the definition of the bending angle! SdS is not asymptotically flat!**
- After Rindler & Ishak, new derivations: through lensing and gravitational potentials, Fermat principle...

Kottler, perturbed FLRW metrics

- Faraoni & Lapierre-Léonard 2016: **We also need to define the lens mass, as there is no standard Schwarzshild lens!**

Does Λ contribute to the bending of light?

Standard result

At lowest order, for Schwarzschild-de Sitter spacetime, for impact parameter and lens mass b, M the correction is

$$\Delta\alpha = -\frac{\Lambda b^3}{12M} \implies \Lambda\text{"}\textbf{bends outwards}\text{"}$$

However...

- Simpson et al, 2010; Park 2009: **When one takes into accout the Hubble flow, there is no observable Λ effect!**
- Piattella 2016: **In the most general metric including a lensing object in an expanding Universe, the Hubble flow must be taken into account**

Does Λ contribute to the bending of light?

What is the current status of the question?

- When both Hubble flow, MSH mass and the local correction is taken into account, there is a subdominant correction, at next to leading order (Piattella 2016)
- In general, the correction os of order $\mathcal{O}(10^{-6})$
- The correction is sensitive to different equation of state parameters ω for dark energy (He Zhang)

Models, however, do not take into account the matter-Λ phase of the Universe, assuming either a matter or Λ dominated (SdS, Kottler)

The need to go beyond Λ

- Current lens systems have sources with redshift $z_S > 1.0$ and lenses with $z_L > 0.7$, both higher the transition redshift $z_\Lambda \approx 0.66$ for the ΛCDM model.
- For non constant H, there should be effects on the bending angle beyond the purely Λ dominated
- To which order do these effects are comparable to the already small Λ correction?
- How do we model these effects?

McVittie metric

Isotropic form

$$ds^2 = \frac{(1-\mu(t))^2}{(1+\mu(t))^2}dt^2 + (1+\mu(t))^4 a^2(t)\delta_{ij}dx^i dx^j,$$

where the mass parameter is defined as the central mass of the lens divided by the scale factor $a(t)$

Small mass parameter limit

$$ds^2 = -(1-4\mu)dt^2 + (1+4\mu)a(t)^2\gamma_{ij}dx^i dx^j$$

- Arbitrary scale factor $a(t)$
- At the lens, the Schwarzschild mass matches the Misner-Sharp-Hernandez mass $\mu(t) \equiv M/a(t) = m_{MSH}/a(t)|_{z_L}$

Angular diameter distance

For Matter+Λ universes, one can obtain the angular and comoving distances in terms of the elementary hypergeometric functions, which have an exact series expansion:

$$H_0\eta_{SO} = \frac{(1+z)}{\Omega_\Lambda^{1/2}}\frac{\Gamma(\frac{4}{3})}{\Gamma(\frac{1}{2})\Gamma(\frac{1}{3})}\times$$

$$\times\sum_{n=0}^{\infty}\frac{\Gamma(\frac{1}{2}+n)\Gamma(\frac{1}{3}+n)}{\Gamma(\frac{4}{3}+n)n!}\left[-\frac{\Omega_m}{\Omega_\Lambda}(1+z)^3\right]^n\Bigg|_O^S, \qquad (1)$$

$$H_0 D_{SO} = \frac{1}{\Omega_\Lambda^{1/2}}\frac{\Gamma(\frac{4}{3})}{\Gamma(\frac{1}{2})\Gamma(\frac{1}{3})}\times$$

$$\times\sum_{n=0}^{\infty}\frac{\Gamma(\frac{1}{2}+n)\Gamma(\frac{1}{3}+n)}{\Gamma(\frac{4}{3}+n)n!}\left[-\frac{\Omega_m}{\Omega_\Lambda}(1+z)^3\right]^n\Bigg|_O^S. \qquad (2)$$

Lens configuration

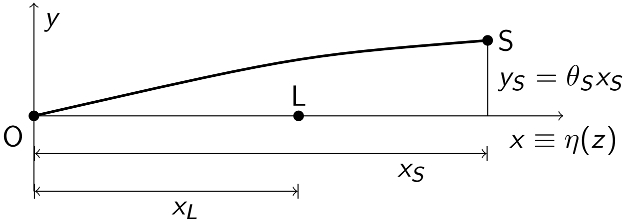

Figure: Lensing configuration. The comoving distance to the lens **L** is taken as a characteristic scale. The actual position of the source **S** is y_S.

We define characteristic scales and the mass-dependent expansion parameter α

$$X \equiv x/x_L, \quad Y \equiv y/x_L, \quad \alpha \equiv 2M/x_L$$

Bending angle

The trajectory of the light ray in this metric is described by the equation

$$\frac{d^2Y}{dX^2} = -\alpha \frac{Y}{a(X)[(X-1)^2 + Y^2]^{3/2}} \,, \tag{3}$$

Keeping the leading order in α, we assume Y to be given by the zeroth order solution $Y = Y_S$ For small angles one can approximate $\tan\theta \approx \theta$, and from the small impact paremeter relation $dy/dx \approx \tan\theta$, one obtains

$$\frac{d\theta}{dX} = -\alpha \frac{Y_S}{a(X)[(X-1)^2 + Y_S^2]^{3/2}} \,. \tag{4}$$

The bending angle is then defined as

Full bending angle

$$\delta = \int_{X_S}^{0} \frac{d\theta}{dX} \,.$$

Bending angle in Λ+matter models

From the angular diameter distance we can obtain an explicit, analytical relation between the redshift and the distances, and use the Power Series to calculate the bending angle to arbitrary order.

$$\frac{d\theta}{dX} = \frac{-\alpha Y_S}{[(X-1)^2+Y_S^2]^{3/2}}\Big[1+(z_L+b_1z_L^2+b_2z_L^3)X + (a_1z_L^2+2a_1b_1z_L^3)X^2+a_2(z_LX)^3\Big], \tag{5}$$

where the coefficients are given by the power series and its inverse power series (1)

Higher order corrections

Full expression is not pretty, but analytic!

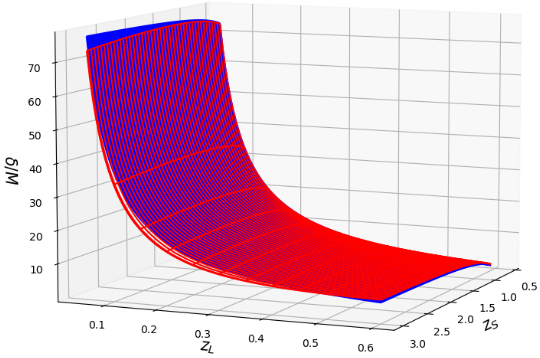

Figure: Plots for the ratio δ/M for the different approximations.

Higher order corrections

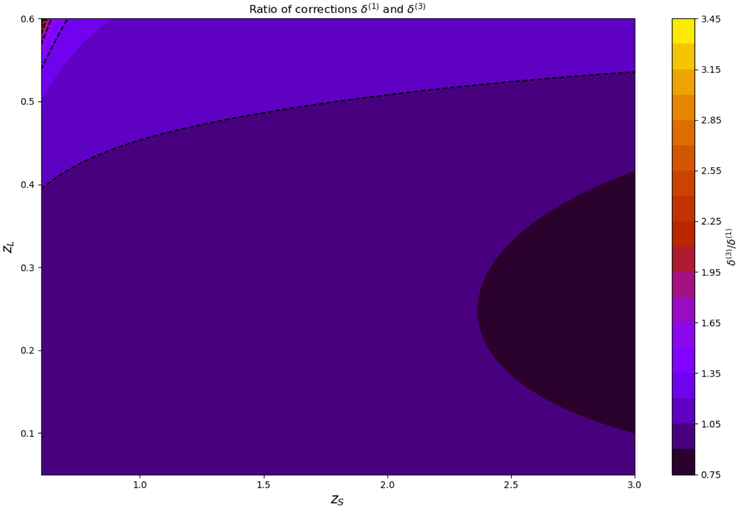

Figure: Ratio of the plots in figure 2. There is a pronounced increase as $z_S/z_L \to 1$

Einstein ring visual selection

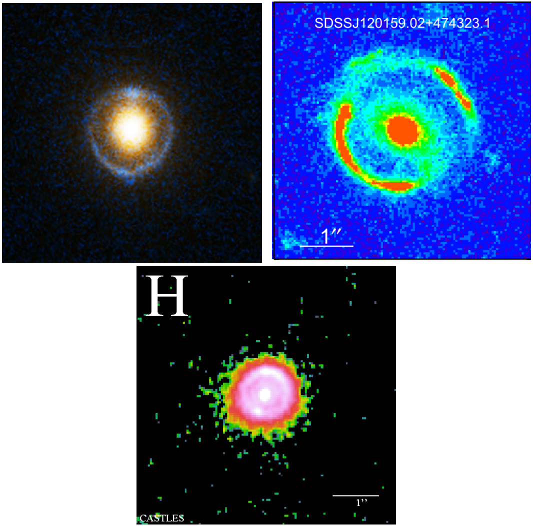

Figure: Grade A lens objects used as references for the visual inspection.

Mass estimate

One can check if these corrections are meaningful through the predicted mass of the lens object in Einstein ring systems.

$$M_E^{(0)} = \frac{D_L D_S}{D_{LS}} \frac{\theta_E^2}{4}, \tag{6}$$

$$M_E^{(1)} = \frac{D_L D_S}{D_{LS}} \frac{\theta_E^2}{4} \left(1 + \delta^{(1)}\right), \tag{7}$$

$$M_E^{(3)} = \frac{D_L D_S}{D_{LS}} \frac{\theta_E^2}{4} \left(1 + \delta^{(1)} + \delta^{(2)} + \delta^{(3)}\right), \tag{8}$$

soon...

Data

- Einstein rings with z_S, z_L and M_L
- Visual inspection as to match idealized ER, Schwarzschild lenses.
- CASTLES and SDSS
- About 70 observed "ideal" Einstein rings, recuced to about 20.

Remarks

- Mass estimates are highly dependent on the surface distribution of the lens - ER are ideal approximations!
- Systematic errors are to be expected from surveys
- The order of the correction is still of $\mathcal{O}(\theta_O^2) \approx 10^{-11}$, even if it's comparable to the 1rst order correction. Observations cannot probe!

Conclusion

- There are not only Λ dependence but also full cosmology dependence of the bending angle.
- The corrections due to the effects of matter-Λ transition are of the same order as the ones obtained for Λ effects...
- ... which are really small in relation to the characteristic angles in lensing systems.
- The usual (weak) lensing paradigm should not be changed before extremely precise observations.