Finding true Kugel-Khomskii systems





50 years of Kugel-Khomskii model

Crystal structure and magnetic properties of substances with orbital degeneracy

K. I. Kugel' and D. I. Khomskii P. N. Lebedev Physics Institute (Submitted November 13, 1972) Zh. Eksp. Teor. Fiz. 64, 1429-1439 (April 1973) KCuF₃ LaMnO₃

Exchange interaction in magnetic substances containing ions with orbital degeneracy is considered. It is shown that, among with spin ordering, superexchange also results in cooperative ordering of Jahn-Teller ion orbitals, which, generally speaking, occurs at a higher temperature and is accompanied by distortion of the lattice (which is a secondary effect here). Concrete studies are performed for substances with a perovskite structure (KCuF₃, LaMnO₃, MnF₃). The effective spin Hamiltonian is obtained for these substances and the properties of the ground state are investigated. The orbital and magnetic structure tures obtained in this way without taking into account interaction with the lattice are in accord with the structures observed experimentally. The approach employed also permits one to explain the strong anisotropy of the magnetic properties of these compounds and to obtain a reasonable estimate for the critical temperatures.



50 years of Kugel-Khomskii model

$$H = -\sum_{imjm'\sigma} t_{imjm'} c^{\dagger}_{im\sigma} c_{jm'\sigma} + U \sum_{im\sigma jm'\sigma'} n_{im\sigma} n_{jm'\sigma'}$$

perturbation t/U

super-exchange Hamiltonian



 $H = J_{SS}S_1 \cdot S_2 + J_{OO}\tau_1\tau_2 + J_{SO}[\tau_1S_2 + \tau_2S_1]$

 τ : pseudospins in orbital space

order without distortions

no static crystal-field splitting



Kugel Khomskii physics sed superconductors

Orbital Physics in Transition-Metal Oxides

Y. Tokura^{1,2} and N. Nagaosa¹

An electron in a solid, that is, bound to or nearly localized on the specific atomic site, has three attributes: charge, spin, and orbital. The orbital represents the shape of the electron cloud in solid. In transition-metal oxides with anisotropic-shaped d-orbital electrons, the Coulomb interaction between the electrons (strong electron correlation effect) is of importance for understanding their metal-insulator transitions and properties such as high-temperature superconductivity and colossal magnetoresistance. The orbital degree of freedom occasionally plays an important role in these phenomena, and its correlation and/or order-disorder transition causes a variety of phenomena through strong coupling with charge, spin, and lattice dynamics. An overview is given here on this "orbital physics," which will be a key concept for the science and technology of correlated electrons.

TRANSITION METAL OXIDES Ferroelectricity driven by orbital order

The discovery that the rotation of the orbital arrangement in manganites induces ferroelectricity exposes an intriguing phase transition that could serve as a blueprint for novel applications.

BERNHARD KEIMER is at the Max Planck Institute for Solid State R Heisenbergstr. 1, 70569 Stuttgart, Germany

ransition metal oxides have fascinated scientists since the 1950s, when the newly developed technique of neutron diffraction was used to show that the compound La_{1-x}Ca₄MnO₃ exhibits a rich variety of structural and magnetic phases as the Ca concentration is tuned¹. The fascination has increased in the wake of the discovery of high-temperature superconductivity in a chemically similar compound,



show that the compound La_{1-x}Ca_xMnO₃ exhibits a rich variety of structural and magnetic phases as the Ca concentration is tuned¹. The fascination has increased in the wake of the discovery of high-temperature

When more than two orbitals are involved, a variety of situations can be realized, and this quantum mechanical process depends on the orbitals (4, 5). In this way, the spin \tilde{S} and the orbital pseudospin \tilde{T} are coupled. In more general cases, the transfer integral t_{ii} depends on the direction of the

pair of the two orbital: $(3z^2 - r^2)$. This gives of the Hamiltonian in t well as in the real sp transfer integral betwee Mn atoms in the cryst



by SAJ Kimber - 2008 - Cited by 24 - Related articles shows a coupling of Ru4+ 4d **orbital order** to distortions from Pb2+ 6s6p orbital hybridization. The Pnma to Imma transition is an unconventional reversal of the ...

Preemptive nematic order, pseudogap, and orbital order in ... arxiv.org > cond-mat v arXiv v

by RM Fernandes - 2011 - Cited by 132 - Related articles Oct 9, 2011 - Furthermore, due to the distinct orbital character of each Fermi pocket, the nematic transition also induces **orbital order**. We compare our results ...

Exotic Spin Order due to Orbital Fluctuations

trong competition between different types of spin and orbital phases at the crossover ...

rder in transition-metal oxides

.pdf ▼ Max Planck Society ▼ oxides. Eva Pavarini (FZ Jülich, Germany). **Orbital** sicis of trasition-metal oxides, and yet its.

first principles study - ScienceDirect

../S0038109812004413 ScienceDirect Related articles al order in the layered triangular lattice system ecent experiments of McQueen et al. [Phys.

al models of transition-metal ...

tte... University of California, Los Angeles v ed articles is in which the 3d orbital order is detected by its case for orbital ordering has been ...

Surface science is an important and well-established branch of materials science involving the study of changes in material

Department of Physics, Columbia University 538 West 120th Street, New York,

Electronic reconstruction at

an interface between a Mott

insulator and a band insulator



Satoshi Okamoto & Andrew J. Millis

New York 10027, USA

ectricity in manganites with ...

 .rsc.org > ... > Physical Chemistry Chem...
 Royal Society of Chemistry ▼

 SN Figueiras - 2014 - Cited by 1 - Related articles
 I bias induced ferroelectricity in manganites with competing charge and orbital r states. Fábio G. N. Figueiras, Igor K. Bdikin, Vitor B. S. Amaral and ...

ral Charge and Orbital Order,' SIMES Seminar March 28 ...

://news.slac.stanford.edu/.../c... SLAC National Accelerator Laboratory > 26, 2014 - Jasper van Wezel, a lecturer in condensed matter theory at Bristol ersity, will deliver this week's SIMES Seminar, "Chiral charge and **orbital** ...

earch?client=safari&rls=en&q=andreas+...lich&ie=UTF-8&oe=UTF-8#q=%22orbital+o



orbital liquid

ME 85, NUMBER 18

PHYSICAL REVIEW LETTERS

30 Осто

Orbital Liquid in Three-Dimensional Mott Insulator: LaTiO₃

G. Khaliullin^{1,2} and S. Maekawa²

¹Max-Planck-Institut für Festkörperforschung, Heisenbergstrasse 1, D-70569 Stuttgart, Germany ²Institute for Materials Research, Tohoku University, Sendai 980-8577, Japan (Received 5 June 2000)

We present a theory of spin and orbital states in Mott insulator LaTiO₃. The spin-orbital superexchange interaction between $d^1(t_{2g})$ ions in cubic crystal suffers from a pathological degeneracy of orbital states at the classical level. Quantum effects remove this degeneracy and result in the formation of the coherent ground state, in which the orbital moment of t_{2g} level is fully quenched. We find a finite gap for orbital excitations. Such a disordered state of local degrees of freedom on unfrustrated, simple cubic lattice is highly unusual. Orbital liquid state naturally explains observed anomalies of LaTiO₃.



orbital fluctuations

PRL 99, 156401 (2007)

PHYSICAL REVIEW LETTERS

week ending 12 OCTOBER 2007

Superexchange Interaction in Orbitally Fluctuating RVO₃

J.-S. Zhou and J. B. Goodenough

Texas Materials Institute, University of Texas at Austin, Austin, Texas 78712, USA

J.-Q. Yan

Ames Laboratory, Ames, Iowa 50011, USA

Y. Ren

Advanced Photon Source, Argonne National Laboratory, Argonne, Illinois 60439, USA (Received 18 May 2007; published 8 October 2007)

Changes in pressure and magnetic field in the orbital and magnetic ordering temperatures of RVO_3 perovskites are reported; they reveal a competition between two magnetic orbitally ordered phases that have opposite preferences for the *e*-orbital component in the localized ${}^3T_{1g}$ ground state of the V³⁺ ion. This competition is shown to be biased by the VO_{6/2} site distortion intrinsic to the orthorhombic structure. A remarkable enhancement of T_N with pressure is found where the competition leads to enhanced orbital critical fluctuations.

DOI: 10.1103/PhysRevLett.99.156401

PACS numbers: 71.70.Ej, 75.10.Dg, 75.30.Et, 75.50.Ee



spin-orbital entanglement

IOP PUBLISHING

JOURNAL OF PHYSICS: CONDENSED MATTER

J. Phys.: Condens. Matter 24 (2012) 313201 (28pp)

doi:10.1088/0953-8984/24/31/313201

TOPICAL REVIEW

Fingerprints of spin–orbital entanglement in transition metal oxides

Andrzej M Oleś

Marian Smoluchowski Institute of Physics, Jagellonian University, Reymonta 4, PL-30059 Kraków, Poland and Max-Planck-Institut für Festkörperforschung, Heisenbergstrasse 1, D-70569 Stuttgart, Germany

E-mail: a.m.oles@fkf.mpg.de



manipulation of orbitals degrees of freedom

Optical Stabilization of Fluctuating High Temperature Ferromagnetism in YTiO₃

A.S. Disa¹, J. Curtis², M. Fechner¹, A. Liu¹, A. von Hoegen¹, M. Först¹, T.F. Nova¹, P. Narang², A. Maljuk³, A.V. Boris⁴, B. Keimer⁴, A. Cavalleri^{1,5} ¹Max Planck Institute for the Structure and Dynamics of Matter, Hamburg, Germany ²John A. Paulson School of Engineering and Applied Sciences, Harvard University, Cambridge, USA ³Leibniz Institute for Solid State and Materials Research Dresden, Germany ⁴Max Planck Institute for Solid State Research, Stuttgart, Germany ⁵Clarendon Laboratory, Department of Physics, Oxford University, Oxford, UK

In quantum materials, degeneracies and frustrated interactions can have a profound impact on the emergence of long-range order, often driving strong fluctuations that suppress functionally relevant electronic or magnetic phases. Engineering the atomic structure in the bulk or at heterointerfaces has been an important research strategy to lift these degeneracies, but these equilibrium methods are limited by thermodynamic, elastic, and chemical constraints. Here, we show that all-optical, mode-selective manipulation of the crystal lattice can be used to enhance and stabilize high-temperature ferromagnetism in YTiO₃, a material that exhibits only partial orbital polarization, an unsaturated low-temperature magnetic moment, and a suppressed Curie temperature, $T_c = 27$ K.

arXiv:2111.13622



50 years of Kugel-Khomskii model

Crystal structure and magnetic properties of substances with orbital degeneracy

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Exchange interaction in magnetic substances containing ions with orbital degeneracy is considered. It is shown that, among with spin ordering, superexchange also results in cooperative ordering of Jahn-Teller ion orbitals, which, generally speaking, occurs at a higher temperature and is accompanied by distortion of the lattice (which is a secondary effect here). Concrete studies are performed for substances with a perovskite structure (KCuF₃, LaMnO₃, MnF₃). The effective spin Hamiltonian is obtained for these substances and the properties of the ground state are investigated. The orbital and magnetic structures obtained in this way without taking into account interaction with the lattice are in accord with the structures observed experimentally. The approach employed also permits one to explain the strong anisotropy of the magnetic properties of these compounds and to obtain a reasonable estimate for the critical temperatures.



where is the evidence?

co-operative Jahn-Teller distortion partially field *e*_g levels

KCuF₃





The Normal Mode Q₂ (Q₂>0)



 $t_{2g}^{6}e_{g}^{3}$

the hallmark of orbital order

co-operative Jahn-Teller distortion partially field e_g levels

KCuF₃

LaMnO₃





 $t_{2g}^{3}e_{g}^{1}$



Jahn-Teller: splitting generates order

Crystal Distortion in Magnetic Compounds

JUNJIRO KANAMORI^{*} Institute for the Study of Metals, University of Chicago, Chicago 37, Illinois

The crystal distortion which arises from the Jahn-Teller effect is discussed in several examples. In the case of compounds containing Cu^{2+} or Mn^{3+} at octahedral sites, the lowest orbital level of these ions is doubly degenerate in the undistorted structure, and there is no spin-orbit coupling in this level. It is shown that, introducing a fictitious spin to specify the degenerate orbital states, we can discuss the problem by analogy with the magnetic problems. The "ferromagnetic" and "antiferromagnetic" distortions are discussed in detail. The transition from the distorted to the undistorted structure is of the first kind for the former and of the second kind for the latter. Higher approximations are discussed briefly. In compounds like FeO, CoO, and CuCr₂O₄, the lowest orbital level is triply degenerate, and the spin-orbit coupling is present in this level. In this case the distortion is dependent on the magnitude of the spin-orbit coupling relative to the strength of the Jahn-Teller effect term. The distortion at absolute zero temperature and its temperature dependence are discussed.

electron-phonon coupling



The Normal Mode Q₂ (Q₂>0)

static crystal-field splitting

 $H = -q\sqrt{C}(\tau_z Q_3 + \tau_x Q_2)$

 $\boldsymbol{\tau}$: pseudospins in orbital space

$$|\theta\rangle = \sin\frac{\theta}{2}|3z^2 - 1\rangle + \cos\frac{\theta}{2}|x^2 - y^2\rangle$$



a chicken-and-egg problem



how to disentangle the two?



LDA+U: KK-like mechanism

Density-functional theory and strong interactions: Orbital ordering in Mott-Hubbard insulators

A. I. Liechtenstein

Max-Planck-Institut für Festkörperforschung, D-70506 Stuttgart, Germany

V. I. Anisimov Institute of Metal Physics, GSP-170 Ekaterinburg, Russia

J. Zaanen Lorentz Institute for the Theoretical Physics, Leiden University, Leiden, The Netherlands (Received 15 May 1995)



The situation changes drastically if we allow for orbital polarization. Because *U* exceeds the bandwidth, the orbital sector is already strongly polarized (as are the spins) before the lattice is allowed to react. Overlooking some unimportant details concerning the coherence of the intermediate states, the well-known rule that electronic MFT in strong coupling maps onto the classical "spin" problem holds also in this case. In other words, we find the quadrupolar orbital-ferromagnetic spin phase to be most stable (for the same reasons as Kugel and Khomskii^o). Obviously the cubic lattice is unstable in the presence of this orbital order parameter. In fact, despite large-scale changes in the electronic system the deformation is modest, indicating a rather weak electron-phonon coupling.





something not quite right





orbital-order transition at Too in LaMnO₃

*T*₀₀ ~ 800 *K*





20 JULY 1998

something not quite right





50 years of Kugel and Khomskii model

$$H = -\sum_{imjm'\sigma} t_{imjm'} c^{\dagger}_{im\sigma} c_{jm'\sigma} + U \sum_{im\sigma jm'\sigma'} n_{im\sigma} n_{jm'\sigma'}$$

super-exchange Hamiltonian



order without distortions

no static crystal-field splitting



LDA+DMFT with Wannier functions



LDA+DMFT with Wannier functions

$$H = -\sum_{ii'} \sum_{mm'} \sum_{\sigma} t^{ii'}_{mm'} c^{\dagger}_{im\sigma} c_{i'm'\sigma}$$

+ $U \sum_{im} n_{im\uparrow} n_{im\downarrow}$
+ $\frac{1}{2} \sum_{im\neq m'\sigma\sigma'} (U - 2J - J\delta_{\sigma\sigma'}) n_{im\sigma} n_{im'\sigma'}$
- $J \sum_{m\neq m'} (c^{\dagger}_{m\uparrow} c^{\dagger}_{m'\downarrow} c_{m'\uparrow} c_{m\downarrow} + c^{\dagger}_{m\uparrow} c^{\dagger}_{m\downarrow} c_{m'\uparrow} c_{m'\downarrow})$

DMFT and cDMFT

quantum impurity solvers: general HF QMC general CT-INT QMC general CT-HYB QMC

 A. Flesch, E. Gorelov, E. Koch and E. Pavarini Phys. Rev. B 87, 195141 (2013)





the KK mechanism in eg³ KCuF₃

 $p=n_1-n_2$



$T_{KK} \ll T_{00} > 1400 \, K$

LDA+DMFT reminder: mean field theory *overestimates* T_{KK}



but also JT alone is not the full story





more experiments

PHYSICAL REVIEW B 87, 014109 (2013)

Unusual structural evolution in KCuF₃ at high temperatures by neutron powder diffraction

Luke G. Marshall,¹ Jianshi Zhou,^{1,*} Jianzhong Zhang,² Jiantao Han,² Sven C. Vogel,² Xiaohui Yu,² Yusheng Zhao,^{2,3} M. T. Fernández-Díaz,⁴ Jinguang Cheng,¹ and John B. Goodenough¹

¹*The Materials Science and Engineering Program and Texas Materials Institute, University of Texas at Austin, Austin, Texas 78712, USA*

²LANSCE Division, Los Alamos National Laboratory, Los Alamos, New Mexico 87545, USA

³Department of Physics and Astronomy, University of Nevada, Las Vegas, Nevada 89154, USA

⁴Institute Laue-Langevin (ILL) 156X, F-38042 Grenoble Cedex 9, France

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excellent agreement with GGA+U



(same result of GGA+DMFT)

GGA+U energy curve



(same result of GGA+DMFT)

effects of distortion: a simple model

change in e_g crystal-field splitting change in Ewald term

$$E_{\text{ionJT}}(\Delta) = \frac{\Gamma - \sqrt{\Gamma^2 + (g\Delta)^2} - C_{\text{Ewald}}\Delta^2/2}{+2Be^{-a/\rho 2\sqrt{2}} [\cosh(\Delta/\rho) - 1]},$$

Born-Mayer potential

the T-dependence is via the lattice constant!

PHYSICAL REVIEW B 96, 054107 (2017)

Thermally assisted ordering in Mott insulators

Hunter Sims,¹ Eva Pavarini,^{2,3} and Erik Koch^{1,2,3,*}

¹Computational Materials Science, German Research School for Simulation Sciences, 52425 Jülich, Germany ²Institute for Advanced Simulation, Forschungszentrum Jülich, 52425 Jülich, Germany ³JARA High-Performance Computing, 52425 Jülich, Germany

(Received 16 November 2016; revised manuscript received 19 July 2017; published 8 August 2017)

Landau theory describes phase transitions as the competition between energy and entropy: The ordered phase has lower energy, while the disordered phase has larger entropy. When heating the system, ordering is reduced entropically until it vanishes at the critical temperature. This picture implicitly assumes that the energy difference between the ordered and disordered phases does not change with temperature. We show that for orbital ordering in the Mott insulator KCuF₃, this assumption fails qualitatively: entropy plays a negligible role, while thermal expansion energetically stabilizes the orbitally ordered phase to such an extent that no phase transition is observed. To understand this strong dependence on the lattice constant, we need to take into account the Born-Mayer repulsion between the ions. It is the latter, and not the Jahn-Teller elastic energy, which determines the magnitude of the distortion. This effect will be seen in all materials where the distortion expected from the Jahn-Teller mechanism is so large that the ions would touch. Our mechanism explains not only the absence of a phase transition in KCuF₃, but even suggests the possibility of an *inverted* transition in closed-shell systems, where the ordered phase emerges only at high temperatures.

Kugel-Khomskii candidates







 $t_{2g}{}^3e_{g}{}^1$



orbital-order transition at Too in LaMnO₃

$T_{00} \sim 800 \ K$





20 JULY 1998

LaMnO₃ : *T_{KK}* ~ 600 K !!







Phys. Rev. Lett. 104, 086402 (2010)

Too: orbital order-to-disorder transition

$T_{OO} \sim 800 \ K$

PRL 94, 177203 (2005)

PHYSICAL REVIEW LETTERS

week ending 6 MAY 2005

Orbital Correlations in the Pseudocubic O and Rhombohedral R Phases of LaMnO₃

Xiangyun Qiu,¹ Th. Proffen,² J. F. Mitchell,³ and S. J. L. Billinge¹

¹Department of Physics and Astronomy, Michigan State University, E. Lansing, Michigan 48824, USA ²Los Alamos National Laboratory, LANSCE-12, MS H805, Los Alamos, New Mexico 87545, USA ³Material Science Division, Argonne National Laboratory, Argonne, Illinois 60439, USA (Received 12 August 2004; revised manuscript received 24 February 2005; published 5 May 2005)

The local and intermediate structure of stoichiometric LaMnO₃ has been studied in the pseudocubic and rhombohedral phases at high temperatures (300–1150 K). Neutron powder diffraction data were collected and a combined Rietveld and high real space resolution atomic pair distribution function analysis was carried out. The nature of the Jahn-Teller (JT) transition around 750 K is confirmed to be orbital order to disorder. In the high-temperature orthorhombic (*O*) and rhombohedral (*R*) phases, the MnO₆ octahedra are still fully distorted locally. More importantly, the intermediate structure suggests the presence of local ordered clusters of diameter ~16 Å (~4 MnO₆ octahedra) implying strong nearest-neighbor JT antiferrodistortive coupling. These clusters persist well above the JT transition temperature even into the high-temperature *R* phase.

DOI: 10.1103/PhysRevLett.94.177203

PACS numbers: 75.47.Lx, 61.12.-q, 75.47.Gk

 $T_{.IT} > 1150 K$



orbital order melting in ReMnO₃









KK critical temperatures

T_{KK} decreases with IR



from total energy

Phys. Rev. B 85, 035124 (2012)

Phys. Rev. B 87, 195141 (2013)



Kugel-Khomskii candidates

eg systems











 $t_{2g}^{3}e_{g}^{1}$





perhaps we looked in the wrong place..

t_{2g} titanates at low temperature?

larger orbital degeneracy, smaller electron-lattice coupling, smaller crystal-field coupling



orbital liquid?

UME 85, NUMBER 18

PHYSICAL REVIEW LETTERS

30 Octobe

Orbital Liquid in Three-Dimensional Mott Insulator: LaTiO₃

G. Khaliullin^{1,2} and S. Maekawa²

¹Max-Planck-Institut für Festkörperforschung, Heisenbergstrasse 1, D-70569 Stuttgart, Germany ²Institute for Materials Research, Tohoku University, Sendai 980-8577, Japan (Received 5 June 2000)

We present a theory of spin and orbital states in Mott insulator LaTiO₃. The spin-orbital superexchange interaction between $d^1(t_{2g})$ ions in cubic crystal suffers from a pathological degeneracy of orbital states at the classical level. Quantum effects remove this degeneracy and result in the formation of the coherent ground state, in which the orbital moment of t_{2g} level is fully quenched. We find a finite gap for orbital excitations. Such a disordered state of local degrees of freedom on unfrustrated, simple cubic lattice is highly unusual. Orbital liquid state naturally explains observed anomalies of LaTiO₃.

d¹ electronic configuration



Coulomb-enhanced crystal field

Volume 92, Number 17

PHYSICAL REVIEW LETTERS

week ending 30 APRIL 2004

Mott Transition and Suppression of Orbital Fluctuations in Orthorhombic $3d^1$ Perovskites

E. Pavarini,¹ S. Biermann,² A. Poteryaev,³ A. I. Lichtenstein,³ A. Georges,² and O. K. Andersen⁴



but what about low T, right above T_N?



change of orbitals at low T?



Ψ = occupied orbital



General Super Exchange Hamiltonians

PHYSICAL REVIEW B 102, 035113 (2020)

Origin of orbital ordering in YTiO₃ and LaTiO₃

Xue-Jing Zhang, Erik Koch, and Eva Pavarini Institute for Advanced Simulation, Forschungszentrum Jülich, 52425 Jülich, Germany



(Received 14 April 2020; accepted 15 June 2020; published 6 July 2020)

The origin of orbital order in correlated transition-metal compounds is strongly debated. For the paradigmatic e_g systems KCuF₃ and LaMnO₃, it has been shown that the electronic Kugel'-Khomskii mechanism alone is not sufficient to drive the orbital-ordering transition up to the high temperatures at which it is experimentally observed. In the case of t_{2g} compounds, however, the role played by the superexchange interaction remains unclear. Here we investigate this question for two representative systems, the $3d t_{2g}^1$ Mott insulators LaTiO₃ and YTiO₃. We show that the Kugel'-Khomskii superexchange transition temperature T_{KK} is unexpectedly large, comparable to the value for the e_g^3 fluoride KCuF₃. By deriving the general form of the orbital superexchange Hamiltonian for the t_{2g}^1 configuration, we show that the GdFeO₃-type distortion plays a key part in enhancing T_{KK} to about 300 K. Still, orbital ordering above 300 K can be ascribed only to the presence of a static crystal-field splitting.



General Super Exchange Hamiltonians

PHYSICAL REVIEW B 105, 115104 (2022)

General superexchange Hamiltonians for magnetic and orbital physics in e_g and t_{2g} systems

Xue-Jing Zhang,¹ Erik Koch,^{1,2} and Eva Pavarini^{1,2,*}

¹Institute for Advanced Simulation, Forschungszentrum Jülich, 52425 Jülich, Germany ²JARA High-Performance Computing, 52062 Aachen, Germany.



(Received 6 December 2021; accepted 16 February 2022; published 3 March 2022)

Material-specific super-exchange Hamiltonians are the key to studying spin and orbital physics in strongly correlated materials. Recently, via an irreducible-tensor operator representation, we derived the orbital superexchange Hamiltonian for t_{2g}^1 perovskites and successfully used it, in combination with many-body approaches, to explain orbital physics in these systems. Here, we generalize our method to e_g^n and t_{2g}^n systems at arbitrary integer filling *n*, including both spin and orbital interactions. The approach is suitable for numerical implementations based on *ab initio* hopping parameters and realistic screened Coulomb interactions and allows for a systematic exploration of superexchange energy surfaces in a realistic context.

DOI: 10.1103/PhysRevB.105.115104



derivation

II. FORMALISM

The superexchange Hamiltonian has the form:

$$\hat{H}_{\rm SE} = \frac{1}{2} \sum_{ij} \hat{H}_{\rm SE}^{ij},\tag{1}$$

where *i* and *j* are neighboring sites coupled via hopping integrals. This Hamiltonian acts in the subspace of states with $|n_i, n_j\rangle$, where n_i and n_j are the site occupations with the constraint $n_i+n_j = N = 2n$, where *n* is the number of electrons per site. From strong-coupling second-order perturbation theory, Eq. (1) can be written as

$$\hat{H}_{\rm SE} = -\hat{H}_{\rm T}(\hat{H}_U - E_0)^{-1}\hat{H}_{\rm T},$$

so that

$$\hat{H}_{\rm SE}^{ij} = -\hat{H}_{\rm T}(\hat{P}_{ij} + \hat{P}_{ji})\hat{H}_{\rm T}$$

Here, \hat{P}_{ij} is an operator which projects, with an energy denominator, to atomic excited states of type $|n_i+1, n_j-1\rangle$, and \hat{H}_T is the hopping part of the Hubbard Hamiltonian from which the superexchange interaction is derived, while \hat{H}_U is the electron-electron repulsion.

Let us start from the well-known case of magnetic exchange for the single-band Hubbard model:

$$\hat{H} = -\sum_{\sigma} \sum_{i,j} t^{i,j} c^{\dagger}_{i\sigma} c_{j\sigma} + U \sum_{i} \hat{n}_{i\uparrow} \hat{n}_{i\downarrow}, \qquad (2)$$

where $\hat{n}_{i\sigma} = c^{\dagger}_{i\sigma}c_{i\sigma}$, $t^{i,j}$ is the hopping integral and U the screened Coulomb parameter. Since the atomic limit of the half-filled Hubbard Hamiltonian has only spin degrees of freedom, one can write the associated exchange Hamiltonian in an

irreducible tensor basis as

$$\hat{H}_{\rm SE}^{ij} = \sum_{qq'} \sum_{\nu\nu'} \hat{s}_i^{q,\nu} D_{q\nu,q'\nu'}^{ij} \hat{s}_j^{q',\nu'},\tag{3}$$

where q = 0, 1 is the rank of the operators and v the associated components. For q = 0, the only component is v = s, while for q = 1, we have instead v = x, y, z in the real harmonics representation. For convenience, we normalize the tensors such that

$$\sum_{\sigma} \langle 0|c_{i\sigma} \left(\hat{s}_{i}^{q,\nu}\right)^{2} c_{i\sigma}^{\dagger}|0\rangle = 1.$$
(4)

With this convention, the irreducible tensors are

$$\hat{s}_{i}^{0,s} = \frac{1}{\sqrt{2}} \sum_{\sigma} c_{i\sigma}^{\dagger} c_{i\sigma}, \qquad (5)$$

$$\hat{s}_{i}^{1,\nu} = \frac{1}{\sqrt{2}} \sum_{\sigma\sigma'} c_{i\sigma}^{\dagger} \langle \sigma | \hat{\sigma}^{\nu} | \sigma' \rangle c_{i\sigma'}, \qquad (6)$$

where $\hat{\sigma}_{\nu}$ is the $\nu = x, y, z$ Pauli matrix. At half-filling $(n_i = n_j = 1)$, we define the projectors as

$$\hat{P}_{ij} = \sum_{\alpha_+\alpha_-} \frac{|\alpha_+\rangle_i |\alpha_-\rangle_{jj} \langle \alpha_-|_i \langle \alpha_+|}{E_{\alpha_+} + E_{\alpha_-} - 2E_0},\tag{7}$$

where $|\alpha_{\pm}\rangle_i$ are atomic (site *i*) multiplets with $n_i \pm 1$ electrons, quantum number α_{\pm} and energy $E_{\alpha_{\pm}}$. In the case of the single-band Hubbard model, $|\alpha_{+}\rangle_i$ and $|\alpha_{-}\rangle_i$ are, respectively, the doubly occupied and the vacuum state; in general, however, α_{+} and α_{-} will label several excited states with different energies. Here, E_0 is the energy of the ground state with $N = n_i + n_j = 2$ electrons in the atomic limit, here, $E_0 = 0$. The tensor elements in Eq. (3) are obtained using the orthogonality properties of irreducible tensors. To this end, we multiply by a pair of irreducible operators, one for site *i* and one for site *j*, and trace over all states in the atomic ground multiplet. This yields

$$D_{0s,0s}^{ij} = -\frac{\text{Tr}[\hat{s}_i^{0,s} \hat{s}_j^{0,s} \hat{H}_{\mathrm{T}}(\hat{P}_{ij} + \hat{P}_{ji}) \hat{H}_{\mathrm{T}}]}{\text{Tr}[(\hat{s}_i^{0,s})^2 (\hat{s}_j^{0,s})^2]}$$
$$= -2\frac{|t^{i,j}|^2}{U},$$

$$D_{1\nu,1\nu'}^{ij} = -\frac{\mathrm{Tr}[\hat{s}_{i}^{1,\nu}\hat{s}_{j}^{1,\nu'}\hat{H}_{\mathrm{T}}(\hat{P}_{ij} + \hat{P}_{ji})\hat{H}_{\mathrm{T}}]}{\mathrm{Tr}[(\hat{s}_{i}^{1,\nu})^{2} (\hat{s}_{j}^{1,\nu'})^{2}]}$$
$$= 2\frac{|t^{i,j}|^{2}}{U}\delta_{\nu,\nu'}.$$



and

irreducible tensor decomposition





representation of orbital



 $|\theta, \phi\rangle = - |\pi - \theta, \phi \pm \pi\rangle$ $= \sin\theta \cos\phi |xz\rangle + \cos\theta |xy\rangle + \sin\theta \sin\phi |yz\rangle.$



representation of orbital



$$\begin{aligned} |\theta, \phi\rangle &= -|\pi - \theta, \phi \pm \pi\rangle \\ &= \sin\theta \cos\phi |xz\rangle + \cos\theta |xy\rangle + \sin\theta \sin\phi |yz\rangle. \end{aligned}$$

CH

FORSCHUNGSZENTRUM

titanates: T_{KK}~300 K



nothing changes at T_{KK}!

50 years of Kugel and Khomskii

Crystal structure and magnetic properties of substances with orbital degeneracy

K. I. Kugel' and D. I. Khomskii P. N. Lebedev Physics Institute (Submitted November 13, 1972) Zh. Eksp. Teor. Fiz. 64, 1429-1439 (April 1973)

Exchange interaction in magnetic substances containing ions with orbital degeneracy is considered. It is shown that, among with spin ordering, superexchange also results in cooperative ordering of Jahn-Teller ion orbitals, which, generally speaking, occurs at a higher temperature and is accompanied by distortion of the lattice (which is a secondary effect here). Concrete studies are performed for substances with a perovskite structure (KCuF₃, LaMnO₃, MnF₃). The effective spin Hamiltonian is obtained for these substances and the properties of the ground state are investigated. The orbital and magnetic structure tures obtained in this way without taking into account interaction with the lattice are in accord with the structures observed experimentally. The approach employed also permits one to explain the strong anisotropy of the magnetic properties of these compounds and to obtain a reasonable estimate for the critical temperatures.

— Kugel-Khomskii systems where are they?



irreducible tensor decomposition





what about vandatates?

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PHYSICAL REVIEW LETTERS

week ending 12 OCTOBER 2007

Superexchange Interaction in Orbitally Fluctuating RVO₃

J.-S. Zhou and J.B. Goodenough

Texas Materials Institute, University of Texas at Austin, Austin, Texas 78712, USA

J.-Q. Yan

Ames Laboratory, Ames, Iowa 50011, USA

Y. Ren

Advanced Photon Source, Argonne National Laboratory, Argonne, Illinois 60439, USA (Received 18 May 2007; published 8 October 2007)

Changes in pressure and magnetic field in the orbital and magnetic ordering temperatures of RVO_3 perovskites are reported; they reveal a competition between two magnetic orbitally ordered phases that have opposite preferences for the *e*-orbital component in the localized ${}^3T_{1g}$ ground state of the V³⁺ ion. This competition is shown to be biased by the VO_{6/2} site distortion intrinsic to the orthorhombic structure. A remarkable enhancement of T_N with pressure is found where the competition leads to enhanced orbital critical fluctuations.

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suppression of orbital fluctuations

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PHYSICAL REVIEW LETTERS

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Orbital Fluctuations in the Different Phases of LaVO₃ and YVO₃

M. De Raychaudhury,^{1,2} E. Pavarini,^{3,4} and O. K. Andersen¹

¹Max-Planck-Institut für Festkörperforschung, Heisenbergstrasse 1, D-70569 Stuttgart, Germany
 ²S. N. Bose National Centre for Basic Sciences, Kolkata 700098, India
 ³Institut für Festkörperforschung, Forschungzentrum Jülich, D-52425 Jülich, Germany
 ⁴CNISM-Dipartimento di Fisica "A. Volta", Università di Pavia, Via Bassi 6, I-27100 Pavia, Italy (Received 13 February 2007; published 17 September 2007)



FIG. 1 (color). Temperature-dependent structural and magnetic phases of LaVO₃ and YVO₃. The lines show LDA + DMFT (quantum Monte Carlo [21]) results for the occupations, n, of the three t_{2g} crystal-field orbitals, 1, 2, and 3 (Table I). Black lines: orthorhombic phases. Green and blue lines: monoclinic, sites 1 and 3 (see Fig. 2). For each structure we calculated the occupations down the temperature at which the orbital



what about vanadates?



H

LaVO₃ — similar to cubic!





we should find magnetic fluctuations..

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FIG. 3 (color online). Top: the *IR* dependence of the coefficient $d \ln T_N/dP$ for the *R*VO₃. T_N is determined in the ZFC measurement. The solid symbol for LaVO₃ is obtained in the FC measurement. Middle: the *IR* dependence of bulk modulus B_0 and the coefficient ds/dP for the *R*VO₃. Bottom: the phase diagram of transition temperatures versus *IR* for *R*VO₃, data from Ref. [8].

shortening the bond length. This extra gain of T_N under pressure is turned off sharply for the phase in 1.10\AA < IR < 1.19 Å. In order to identify the possible mechanism responsible for the extra gain of T_N , we will elaborate the pressure effect on the structure and the structural evolution versus IR for these regions of IR. For IR < 1.10 Å, the orbitals undergo G-type ordering at T_{00} and an orbitalflipping transition to C-type ordering at T_{CG} . In both types of orbital ordering, the long axis of the octahedra alternates direction within the (001) planes; there are changes from out-of-phase in the type-G to in-phase in type-C orbital order along the c axis and structural symmetry from $P2_1$ to *Pbnm* [10,14]. Therefore, an abrupt volume change of the octahedra at T_{CG} cannot be explained by a simple switching from the in-phase to the out-of-phase configuration. The octahedral-site volume in the G_{00} phase larger than that in the C_{00} phase obtained from the structural studies [10] suggests a larger percentage of e orbital is present in the ${}^{3}T_{1g}$ ground state in the G_{OO} phase. More interestingly, the intrinsic component of the site distortion, which can be well-resolved in the orbitally disordered phase and in the G_{00} phase, disappears in the C_{00} phase of smaller volume where each $VO_{6/2}$ site has one long and two equally short V-O bonds. It is clear, therefore, that the orbital-flipping transition results from a competition between a spin-orbital interaction and the structural bias. Pressure favors the C_{00} phase since it has a smaller cell volume and a smaller octahedral-site distortion, which explains the giant pressure dependence of T_{CG} in Fig. 2. As for the phase with IR > 1.19 Å, the orbital-ordering temperature T_t to the G_{00} phase drops below T_N [8,15]. It is important to note



yes, **T**_N > **T**_{KK} !





50 years of Kugel-Khomskii model

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K. I. Kugel' and D. I. Khomskii

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the first clear case: LaVO₃

X.-J. Zhang, E. Koch and E. Pavarini, under review





X.-J. Zhang, E. Koch and E. Pavarini

Institute for Advanced Simulation, Forschungszentrum Jülich

Juelich Supercomputing Centre





thank you!