HW # 1

Cosmology

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Curvature and the Einstein Equations

Fake fluid (use a Mathematica package! Or equiv.)

Consider a more general metric ansatz:

$$ds^2 = -dt^2 + a^2(t)\left[f(r)dr^2 + r^2 d\Omega_2^2\right]$$

Use your favorite GR package to show that the trace of the EE's (with no matter) yields $R = 0$,
giving an equation for $f(r)$

Show that to have the r-dependence of the LHS vanish requires that $f(r) = \dfrac{1}{1 - kr^2}$, presuming $f(r \ll k^{-1/2}) \approx 1$

Funny Fluids

Show that there are no fluids with energy density that scale like $\rho \propto 1/a^{\sigma}$ where $\sigma > 6$

Hint: Consider the sound speed in such a fluid

Confirm the Friedmann Eq's

Please use Mathematica (or equivalent)

$$ds^2 = a^2(\tau)(-d\tau^2 + d\vec{x}^2)$$

$$\mathscr{H} \equiv \frac{d}{d\tau} \ln a$$

$$G_{00} = 3\mathscr{H}^2 = \frac{1}{m_{\rm Pl}^2} \bar{\rho} a^2$$

$$G_{ii} = -2\dot{\mathscr{H}} - \mathscr{H}^2 = \frac{1}{m_{\rm Pl}^2} \bar{P} a^2$$

Derive the CνB temperature

(Hint - Daniel Baumann did it…)

Walk through the derivation of the neutrino background temperature starting from conservation of entropy, and from the contribution of e^+e^- annihilation:

$$sa^3 = \frac{\rho + P}{T} a^3$$

This is firmly in the radiation dominated era, by the way!