$m \neq 0$

$p^2 = m^2$ 4-momenta:

$p = k + q$

lightlike $\quad ( )^2 = 0$

$|k\rangle, |k]$; $|q\rangle, |q]$

note: for given p, choose k ; $2k\cdot q = 2k\cdot p = m^2$

$\Rightarrow$ just $\tilde{k}$

let's introduce more terminology:

$p = p^{I=1} + p^{I=2}$ $\quad SU(2)$ rotations

$p_{\alpha\dot{\alpha}} = \varepsilon_{IJ} \lambda^I_\alpha \tilde{\lambda}^J_{\dot{\alpha}} = \lambda^I_\alpha \tilde{\lambda}_{I\dot{\alpha}} = |p^I\rangle [p_I$

so can write mom. in terms of 4 massless spinors (2 indep for real mom.)

polarization:

$s=\frac{1}{2}$ fermions:
$$u^I(p) = \begin{pmatrix} \lambda^I_\alpha \\ \tilde{\lambda}^{I\dot{\alpha}} \end{pmatrix}$$
$$\bar{u}_{\underline{I}}(p) = \left(-\lambda^\alpha_{\underline{I}} \;\; \tilde{\lambda}_{I\dot{\alpha}}\right)$$

with $(\not{p}-m)\,u^I(p) = 0$

indeed have 2 sol'n's corresponding to 2 spin states.

$s=1$ massive vectors:

$$\varepsilon^+_\mu = \frac{\langle p^1|\sigma_\mu|p^1]}{\sqrt{2}m} \qquad (\equiv \varepsilon^{11}_\mu)$$

$$\varepsilon^-_\mu = \frac{\langle p^2|\sigma_\mu|p^2]}{\sqrt{2}m} \qquad (\equiv \varepsilon^{22}_\mu)$$

(no more freedom: no gauge sym)

$$\varepsilon^0 = \frac{1}{2m}\left(\langle p^1|\sigma_\mu p^2] + (1\leftrightarrow 2)\right) \qquad (\equiv \varepsilon^{12}_\mu = \varepsilon^{21}_\mu)$$

$$\varepsilon^{IJ}_\mu = \frac{\langle \underline{p}|\sigma_\mu|\underline{p}]}{\sqrt{2}m} \qquad \varepsilon^{IJ}_{\alpha\dot{\alpha}} = \sqrt{2}\,\frac{|\underline{p}\rangle[\underline{p}|}{m}$$

where bold (underline) = sym over I,J

not surprising; can construct any $SU(2)$ rep from

sym comb of spin $1/2$ (antisym - singlet)

so: A of massive/massless particles in terms of

just massless (2 comp) spinors.

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what is this new $SU(2)$? $I=1,2$

$p = p^{I=1} + p^{I=2}$

rotations of p^1, p^2 with p invariant

choice of $\hat{p}^1$ direction

Little group $(m \neq 0)$

recall: $(m,0,0,0)$: $SO(3) \sim SU(2)$

so $I=1,2$ is little group index (LG)

what about $m=0$? there $p = \epsilon(1,0,0,1)$ $SO(2) \sim U(1)$

and indeed $p_{\alpha\dot{\alpha}} = |p\rangle[p$: $|p\rangle \to e^{i\theta}|p\rangle$ $|p] \to e^{-i\theta}$

(θ is real for real mom; complex otherwise)

Under LG transf: momenta — unchanged
spin direction does

scalar legs ✗

$s=\frac{1}{2}$ fermion legs: $m\neq 0$: carry LG index I

$m=0$: +hel: $|p\rangle \to e^{i\theta}|p\rangle$ "weight" +1

-hel: $|p] \to e^{-i\theta}|p]$ " -1

$s=1$ leg: $m\neq 0$ carry LG indices I,J (sym)

$m=0$: +: weight +2: $\frac{|r\rangle[k|}{\langle kr\rangle}$ ✓

-: " -2

Will see: will be very useful in constraining ampl's.

"charges" — selection rules

It's useful to have some explicit forms:

$$p = E(1, \sin\theta\cos\varphi, \sin\theta\sin\varphi, \cos\theta)$$

$$|p\rangle_\alpha = \frac{e^{i\alpha}}{\sqrt{p^0 - p^3}} \begin{pmatrix} p_1 - ip_2 \\ p_0 - p_3 \end{pmatrix} \underset{\text{essm}}{\sim} e^{i\alpha}\sqrt{2E} \begin{pmatrix} \sin\theta/2 \\ \sin\theta/2 \, e^{-i\theta/2} \end{pmatrix}$$

$$|p]^{\dot\alpha} = \frac{e^{-i\alpha}}{\sqrt{p_0 - p_3}} \begin{pmatrix} p_0 - p_3 \\ -p_1 - ip_2 \end{pmatrix}$$

A will be given in terms of massless spinor products

$\lambda_\alpha(p_i)\lambda^\alpha(p_j)$ or $\langle p_i p_j\rangle \equiv \langle ij\rangle$ angle brackets

and similarly $[p_i p_j] \equiv [ij]$ square

for real momenta: $\langle ij\rangle^* = [ij]$

also: $2p_i \cdot p_j = p_i^{\alpha\dot\alpha} p_{j\,\dot\alpha\alpha} = \mathrm{tr}\left(|p_i\rangle[p_i p_j]\langle p_j|\right) =$

$= + \langle ij\rangle[ji]$ (note: $\sigma^\mu_{\alpha\dot\alpha}\sigma_\mu^{\dot\beta\beta} = 2\delta_\alpha^\beta\delta_{\dot\alpha}^{\dot\beta}$)

$\Rightarrow$ for real mom: $\langle ij\rangle = \sqrt{s_{ij}}\, e^{i\varphi}$ $[ij] = -\sqrt{s_{ij}}\, e^{-i\varphi}$

$s_{ij} = (p_i + p_j)^2 = 2p_i \cdot p_j$ for $m=0$

2d spinors are 2d ;

$$|k\rangle = a|i\rangle + b|j\rangle \qquad a,b \in \mathbb{C}$$

indep

$$\times \langle i| \qquad \langle ik\rangle = b\langle ij\rangle \qquad b = \frac{\langle ij\rangle}{\langle ik\rangle}$$

$\vdots$

$$\boxed{|i\rangle\langle jk\rangle + |j\rangle\langle ki\rangle + |k\rangle\langle ij\rangle = 0}$$

mult by some spinor $\langle l|$

$$\left.\begin{aligned} &\langle li\rangle\langle jk\rangle + \langle lj\rangle\langle ki\rangle + \langle lk\rangle\langle ij\rangle = 0 \\ &\text{and sim'ly:} \\ &[li][jk] + [lj][ki] + [lk][ij] = 0 \end{aligned}\right\} \text{Schouten id's.}$$

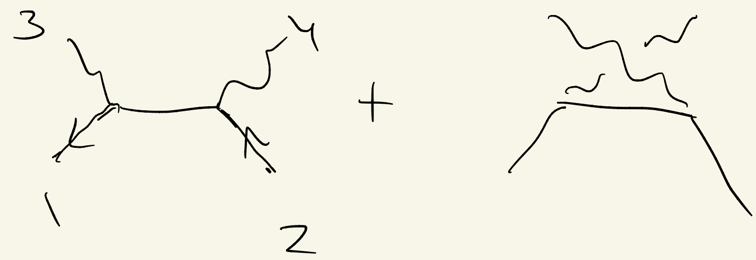
lets do an example to

a. demonstrate

b. see the use in simplifying Feyn diag comp.

<u>Compton scattering</u> $\quad e\gamma \to e\gamma$

$\mathcal{M}(\bar f f \gamma\gamma)$
e^+e^-



$$= \# e^2 \, \bar v_1 \not\epsilon_4 \frac{\not p_1 + \not p_3}{(p_1+p_3)^2} \not\epsilon_3 u_2 + \quad 3 \to 4$$

3 γ's : (helicity preserving) $\quad \bar f^+ f^-$ or $\bar f^- f^+$

take eg $\langle p_1|$ $\qquad\qquad |p_3]$

$\qquad\quad \langle 1|$ $\qquad\qquad |3]$

recall: $\quad \epsilon^+_{\alpha\dot\alpha}(k) = \sqrt{2}\, \frac{|r\rangle [k|}{\langle kr\rangle} \qquad \epsilon^-_{\alpha\dot\alpha} = \sqrt{2}\, \frac{|k\rangle [r|}{[kr]}$

choose $r_3 = r_4 = p_1$; $\langle 1|\epsilon_3^+$, $\langle 1|\epsilon_4^+ \to 0$

$++$ photons $\Rightarrow 0 \qquad \Rightarrow$ opposite hel. photons take eg 3^+4^-:

$$\varepsilon_3^+ \propto \frac{|1\rangle[3|}{\langle 31\rangle} \qquad \varepsilon_4^- \propto \frac{|4\rangle[1|}{[41]}$$

$$\frac{1}{s_{13}}\langle 1|\left(\frac{|4\rangle[1|}{[41]}\underbrace{(1+3)}_{\downarrow\, 0}\frac{|1\rangle[3|}{\langle 31\rangle}\right)2] + \# \underbrace{\langle 1|\big(|1\rangle[3|+\dots\big)}_{=\, 0}$$

$$= \frac{1}{s_{13}}\frac{\langle 14\rangle[13]}{[41]}\cdot\frac{\cancel{\langle 31\rangle}[32]}{\cancel{\langle 31\rangle}} =$$

$$= \frac{1}{\langle 31\rangle\cancel{[13]}}\cdot\frac{\langle 14\rangle\cancel{[13]}[32]}{[14]} =$$

$$= \# \frac{\langle 14\rangle[23]}{\langle 13\rangle[14]}$$

LG weights OK:

$2+$ ✓ $\quad 1-$ ✓

$3+$ ✓ $\quad 4-$ ✓

can rewrite more: $\langle 13\rangle[14] = -\langle 31\rangle[14] =$

$= \cancel{-}\langle 314] = \langle 3(2+3+4)4]$

$= \langle 324] = \langle 32\rangle[24]$

mult by $[23]$

$$= \# \frac{\langle 14\rangle[23]}{\langle 23\rangle[24]} = \# \frac{\langle 14\rangle[23]^2}{s_{23}[24]} = \# \frac{[23]^2}{[14][24]}$$

s_{14}

to get the cross section:

polariz. : $|M|^2$: here just a complex #!

$$\left|\frac{[23]^2}{[14][24]}\right|^2 = \frac{s_{23}^2}{s_{14}s_{24}} = \frac{s_{23}}{s_{24}}$$

* gauge sym $\Rightarrow$ let's us get rid of certain diag's [gauge redundancy less of a problem]

* manipulation easy

* ampl just a complex # - easy to square.

$\to$ one of the 1st clues for power:

for QCD (color-color ordering): Mangano-Parke

$gg \to ng$	2	3	4	..	8
	4	25	220		10,525,900

yet in some cases: final result is simple:

n-gluons : all + : 0 ; $- + \cdots +$: 0

MHV $- - + \cdots - +$: # $\dfrac{\langle 12\rangle^4}{\langle 12\rangle\langle 23\rangle\cdots\langle n1\rangle}$

again just a #!

(count dim: $\langle ij \rangle$, $[ij]$ dim -1

$4-n$ ✓ dim of M_n

so constant - gauge coupling $= \cancel{d} = 0$

suggests; there must be a better way to do this ... avoid gauge redundancy?

let's concentrate on the int. of $m=0$ spin-1