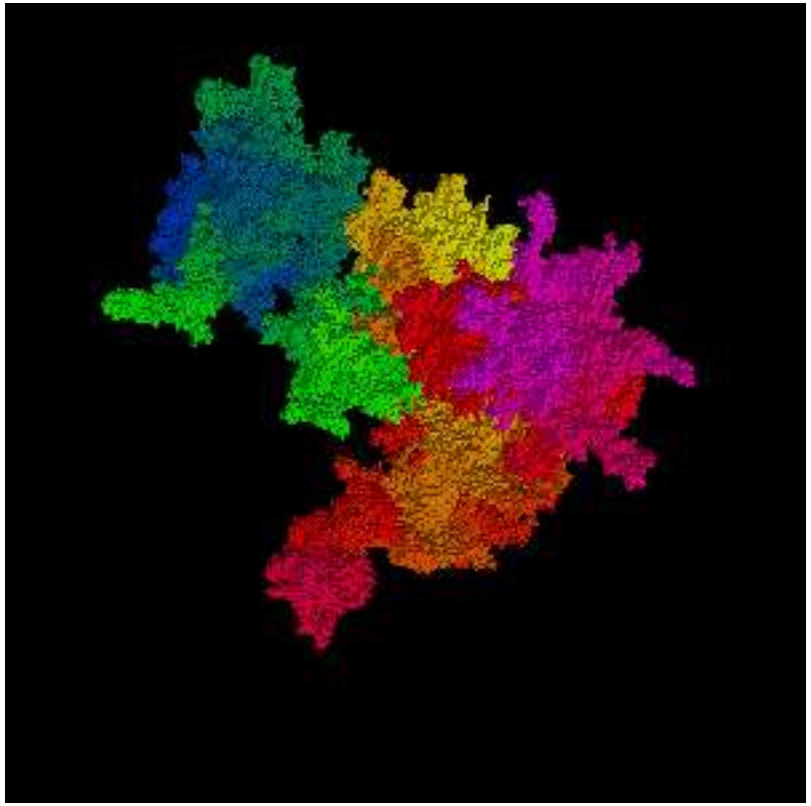
Avalanche Litany
Jumping between minima in materials

An avalanche

- Is a jump between two metastable states in a disordered system.
- Starts from a weak spot, that triggers the weaker of its neighbors in a cascade that finally terminates.
- Often is fractal in time and space

Avalanches come in all sizes (are scale invariant)

- Near the onset of flow or failure (POC)
- In cases 'self organized' to the onset of flow (SOC)
- In some entire phases (Generic scale invariance)



Fractal avalanche in 3D RFIM.
Colors indicate time sequence.

Pandemic Outbreaks & the Random Field Ising Model

Pandemic

Infected +1, Not yet -1; Each sick person infects R_0 healthy neighbors Critical point $R_c = 1$.

Mean field RFIM

Spins start -1. External field H triggers weakest spin, which triggers R_0 spins at random. Critical point H_c when R_0=1.

Lattice

1	2	3	4	5
6	7	8	9	10
11	12	13	14	15
16	17	18	19	20
21	22	23	24	25

Avalanche in 2D RFIM
13 weak spot, triggers shells of neighbors, temporal shape
V(t) = {1, 3, 5, 4}

Size distribution $P(S,R)$

$$r = (R_c - R_0)/R_0$$

Universal exponents

$$\tau = 3/2,\ \sigma = ½$$

Scaling form

$$P(S,R) = S^{-\tau}\ \mathscr{P}(S^{\sigma} r)$$

Universal scaling function

$$\mathscr{P}(X) = \exp(-X^2/2)$$

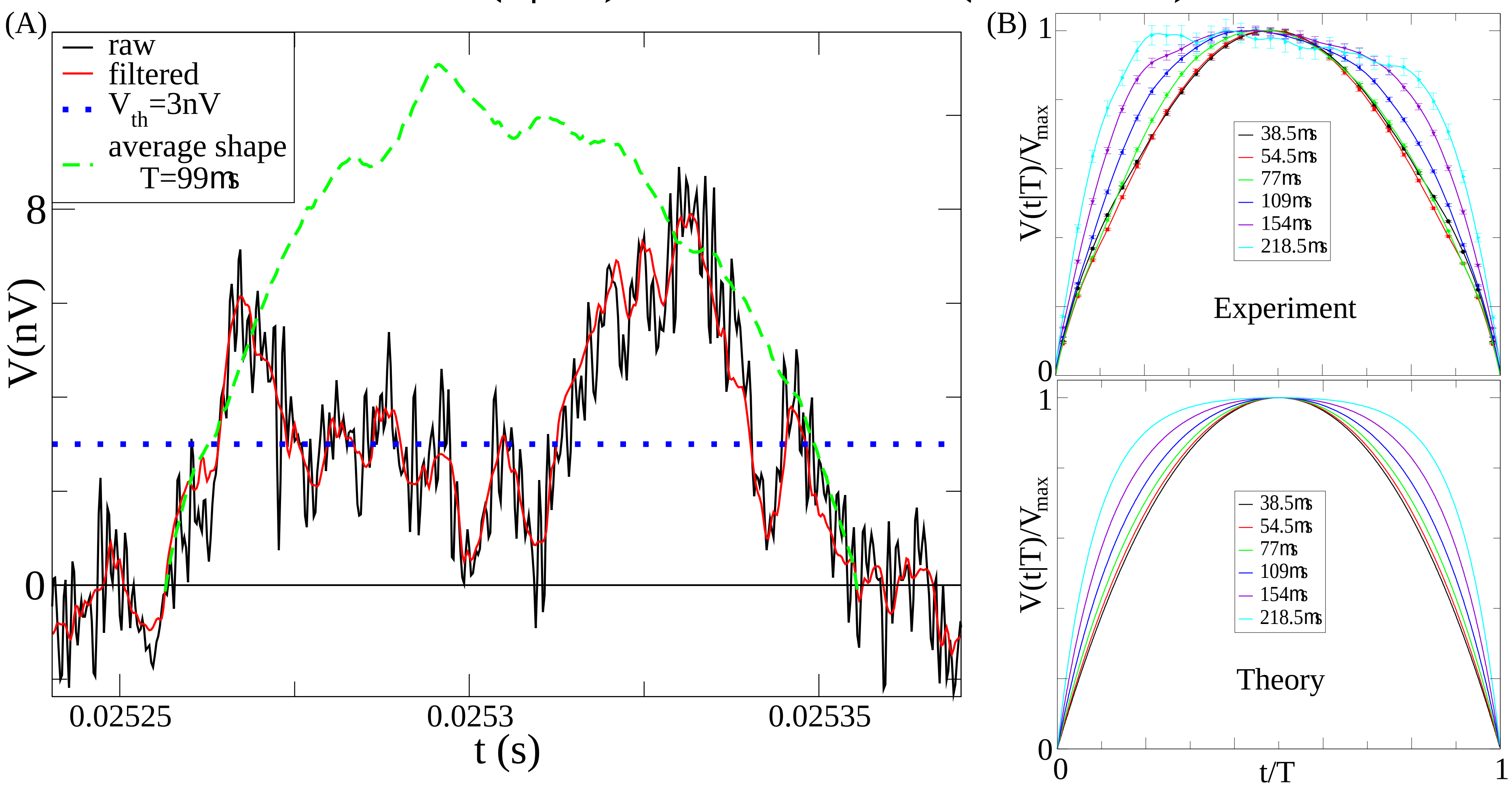
Kuntz

Avalanches in mean-field Ising expts

Avalanche temporal shape V(t|T) for fixed duration T predicted to scale as

$$\langle V\rangle(t|T,r) \sim T^{1-(1/\sigma\nu z)}\, \mathcal{V}(t/T,\, r/T^{-w})$$

Avalanche smoothed (red) and averaged (green)



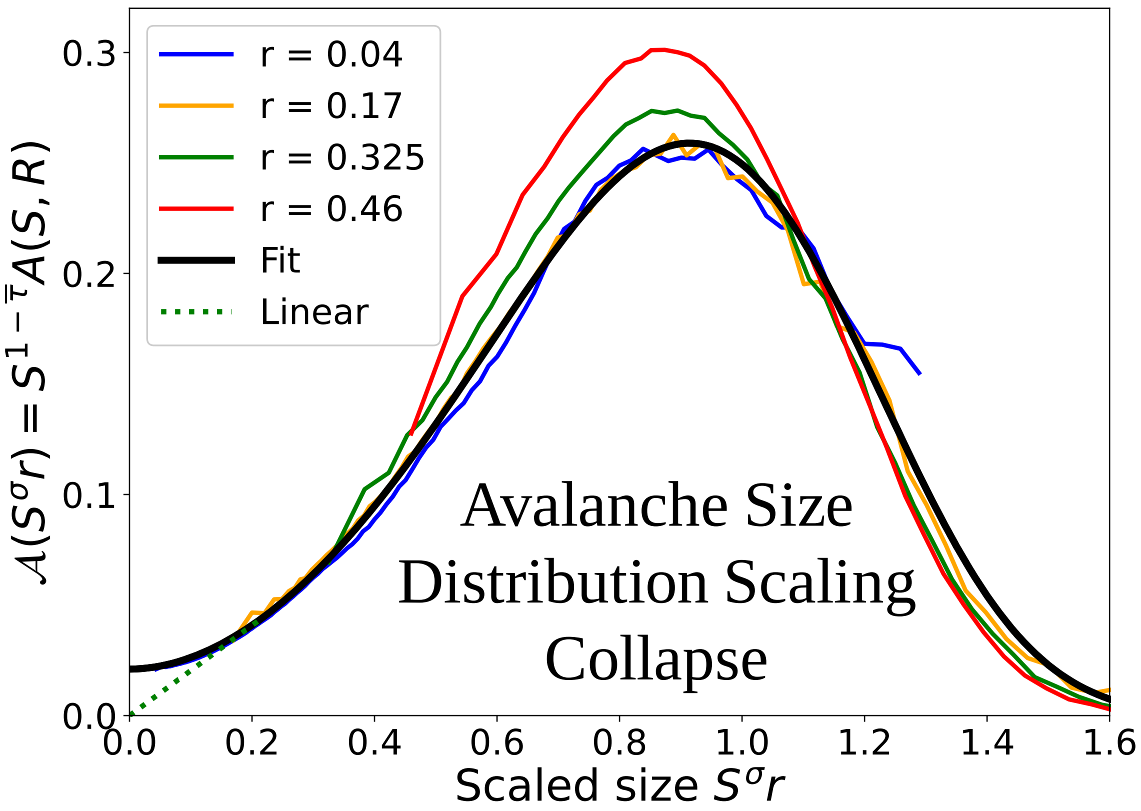
Scaling collapse $\mathcal{V}(t/T, r/T^{-w})$

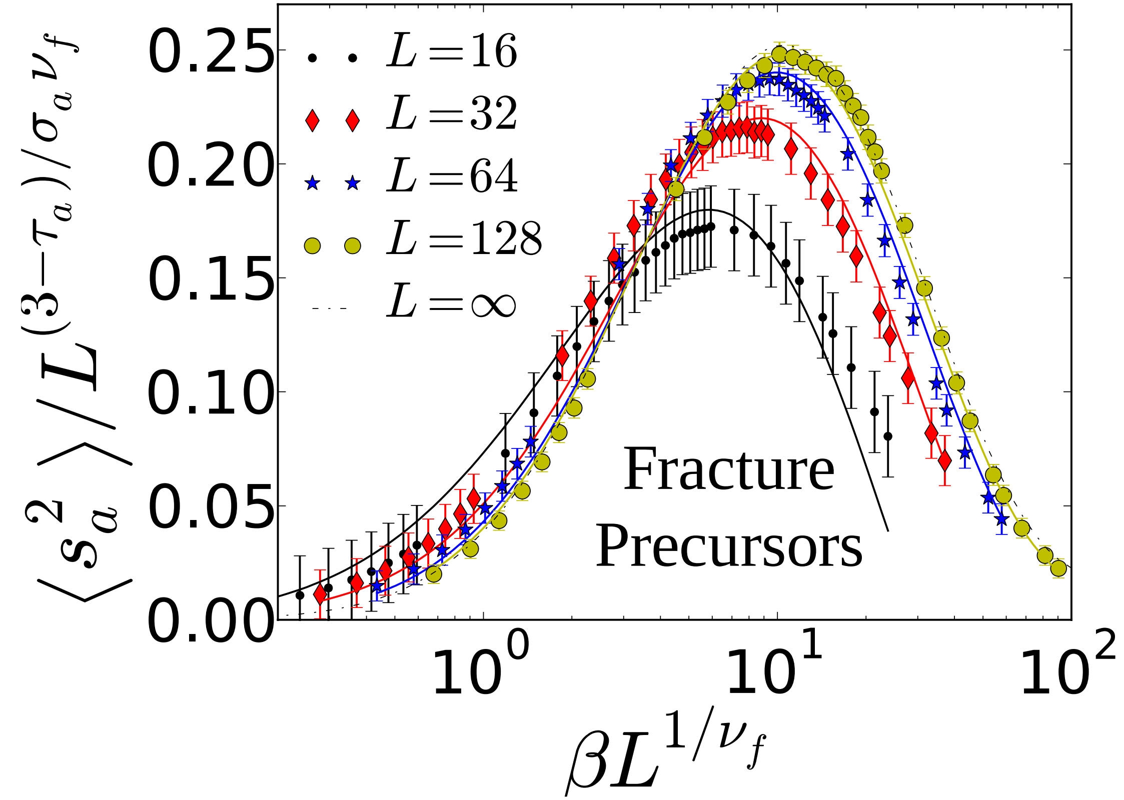
Centro Brasileiro de Pesquisas Físicas, Rio de Janeiro R

Brazil! Papanikolaou, **Bohn, Sommer**, Durin, Zapperi

Singular corrections to scaling*

What happens farther from the critical point?





Scaling collapses are supposed to make the curves lie atop one another. Far from the critical point, they stop overlapping! Can we find a universal scaling prediction for these ‘subdominant’ corrections to scaling?

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Singular Corrections to Scaling: Irrelevant Variables Matter Too

Consider $z(x, y, u)$ with x, y relevant (growing under coarse-graining), and u irrelevant (shrinking). (L and S shrink under coarse graining: $x = 1/S$ and $y = 1/L$ are relevant.)

- RG flow equations:

$$dx/d\ell = ax \quad dy/d\ell = by \quad dz/d\ell = cz \quad du/d\ell = -du.$$

- Invariant scaling combinations:

$$Y = y/x^{\beta\delta} = yx^{-b/a} \quad U = ux^{\omega} = ux^{d/a}.$$

- New universal critical exponent ω.
- Universal scaling function

$$z(x, y, u) = x^{\beta}\mathcal{Z}(Y, U) = x^{\beta}\mathcal{Z}(y/x^{\beta\delta}, u\, x^{\omega}),$$

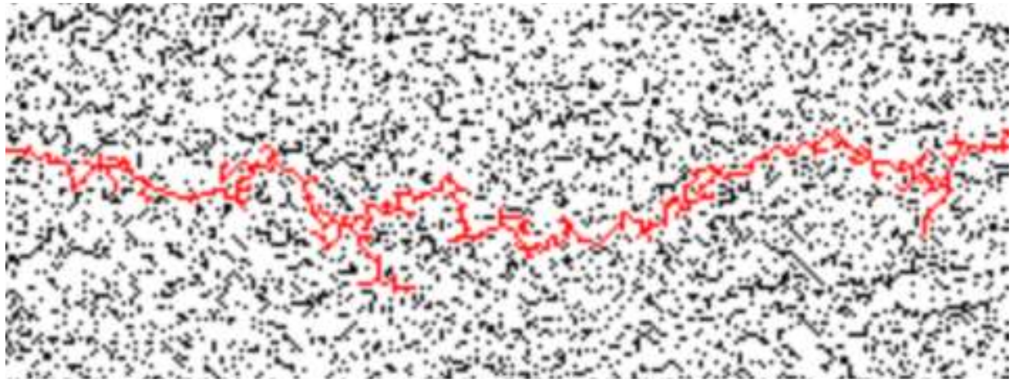
- Because U becomes small as $x \to 0$, *if* $\mathcal{Z}(Y, U)$ *is differentiable at* $U = 0$, we can expand

$$z(x, y, u) \approx x^{\beta}\mathcal{Z}(Y, 0) + x^{\beta}U\mathcal{Z}^{[0,1]}(Y, 0) = x^{\beta}\mathcal{Z}(y/x^{\beta\delta}, 0) + x^{\beta+\omega}u\mathcal{Z}^{[0,1]}(y/x^{\beta\delta}, 0).$$

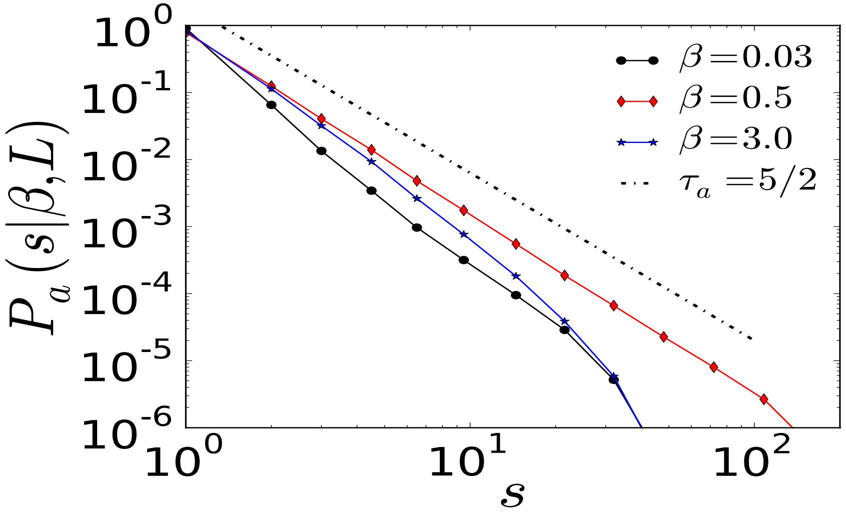
- New term is subdominant as $x \to 0$. It is a universal correction to scaling, $u\mathcal{Z}^{[0,1]}$ with nonuniversal amplitude u.

Example: Crackling Fracture

Power-law fracture precursors



Fuse model for fracture



Microfractures happen in bone, seashells; toughens

(Black) clusters of weak bonds break early, with a power law size distribution.

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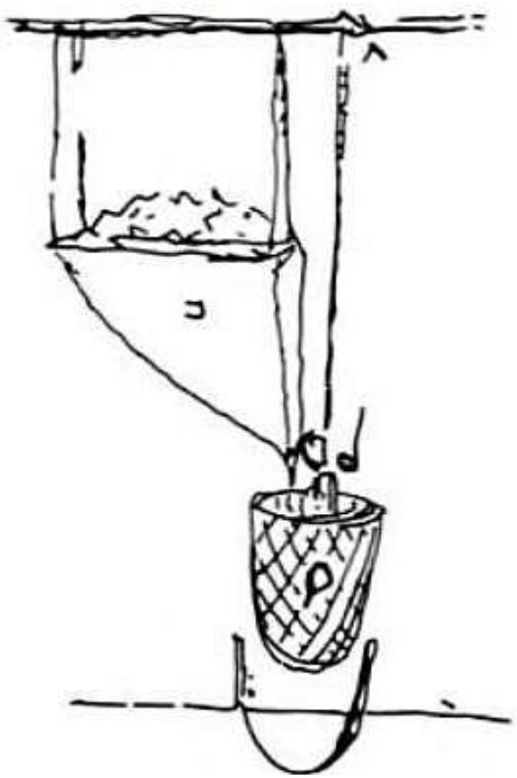
Example: Crackling Fracture

Power-law fracture precursors



Leonardo DaVinci

Larger is weaker:

- longer wires fail at smaller force
- thicker wires fail at smaller stress
- weakest portion dominates
- infinite system breaks at zero stress
- no precursor avalanches

Precursors are a finite size effect:

finite size criticality

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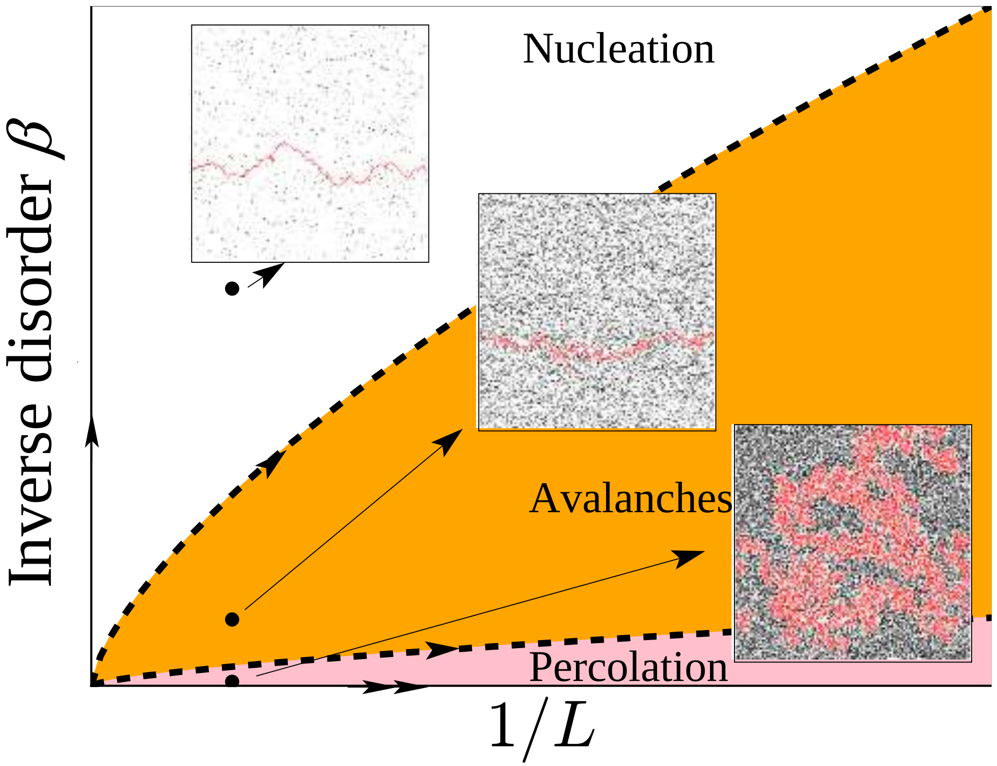
Crackling Fracture and irrelevant variables*

Shekhawat: Power-law fracture precursors



Large disorder: 'avalanches' sizes S=1

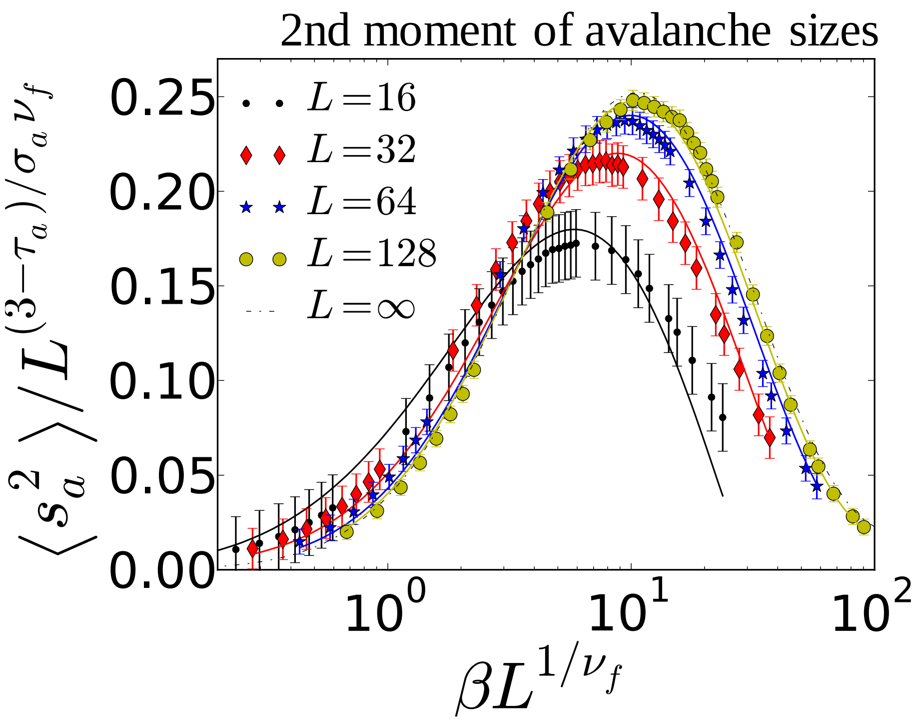
Small disorder: one big avalanche

Universal crossover

$P(S) \sim S^{-\tau} \mathcal{F}(\beta L^{1/\nu}, S L^{-1/\sigma\nu}, u L^{-\Delta/\nu})$

Percolation controls scaling

New relevant perturbation β



These curves don't overlap! They depend on $U = u L^{-\Delta/\nu}$.

They converge to $\mathcal{F}(\beta L^{1/\nu}, S L^{-1/\sigma\nu}, 0)$

as L diverges, with corrections (lines through data)

$u\ S^{-\tau} L^{-\Delta/\nu} \mathcal{F}^{[0,0,1]}(\beta L^{1/\nu}, S L^{-1/\sigma\nu}, 0)$.

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Dangerous Irrelevant Variables*

Suppose we have $z(x, y, u) = x^\beta \mathcal{Z}(Y, U) = x^\beta \mathcal{Z}(y/x^{\beta\delta}, u\, x^\omega)$, with x and y relevant. u is irrelevant, but dangerous: $\mathcal{Z}(Y, U)$ is singular as $U \to 0$.

- **Ising model ($D > 4$).** The magnetization $M(t, h, g)$ minimizes a quartic potential
$$V(m, t) = (t/2)m^2 + (g/4!)m^4.$$
g is irrelevant, but dangerous: setting it to zero for $t = T - T_c < 0$ would force $M \to \pm\infty$. Thus $M(t, h, g) \sim t^\beta \mathcal{M}(H, G) = t^\beta \mathcal{M}(h/t^{\beta\delta}, g t^\omega)$ is singular as $G \to 0$.

- **Zero temperature fixed point.** Glasses and disordered systems will have exponentially diverging time scales as they freeze if temperature is irrelevant: as one coarse grains, the effective barriers stay constant while the effective temperature decreases, leading to barrier crossing times which diverge as $\exp(A/(T - T_c)^{\theta\nu})$.

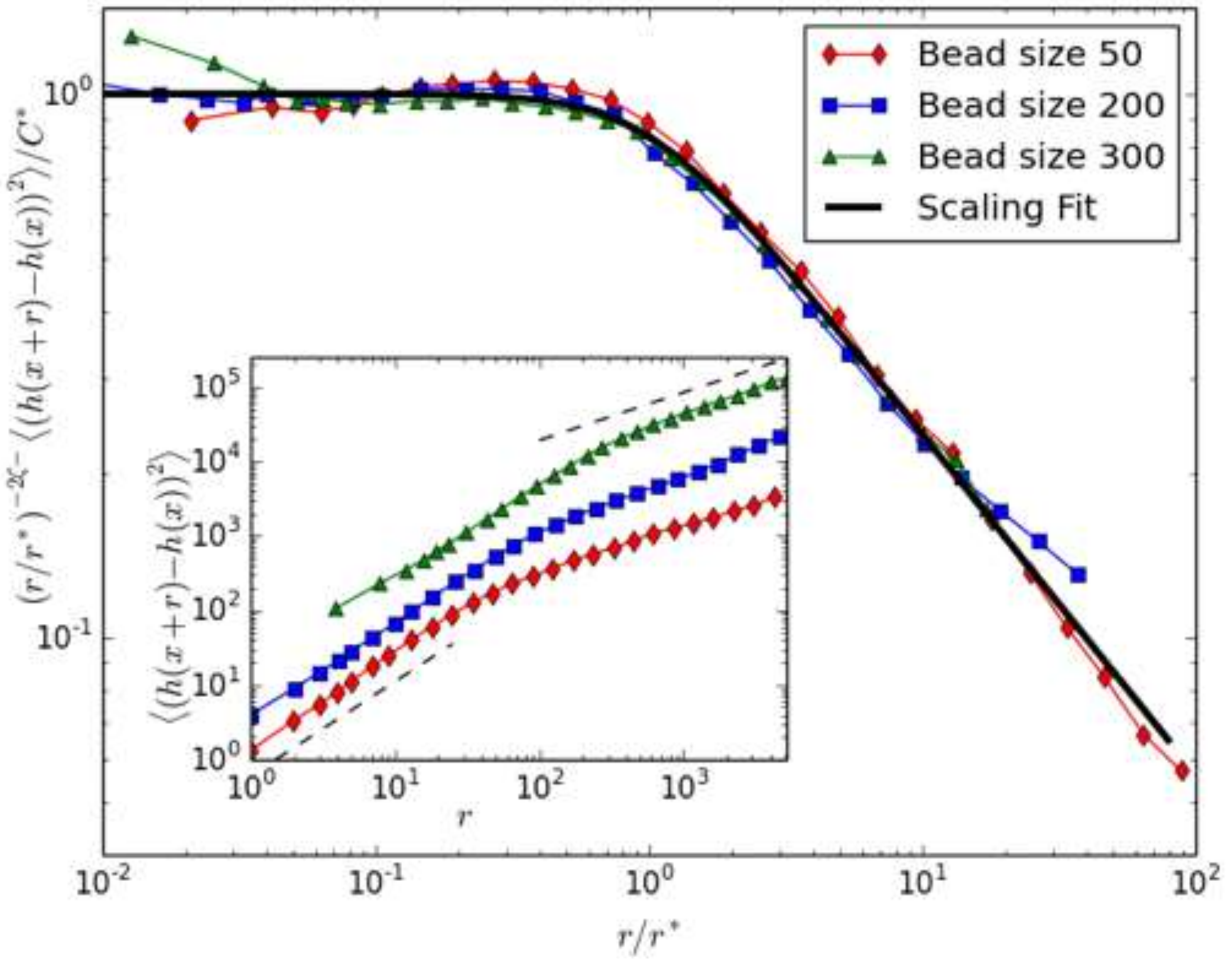
- **Jamming**. We shall see on Friday that jamming and rigidity percolation has an elastic lifetime at low frequencies that diverges when an irrelevant variable vanishes.

Fracture Roughness and Crossover Scaling*

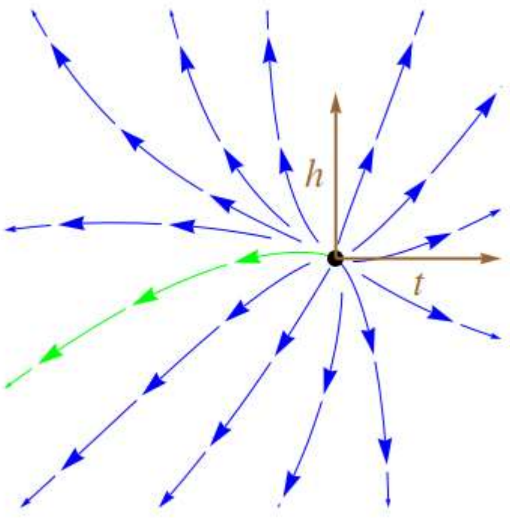
The height-height correlation function that measures the roughness of a crack surface has two distinct universality classes, each with their own power-law growth $r^{2\zeta}$ with distance. The short-distance Larkin class is unstable to the long-distance depinning class, relevant variable λ:

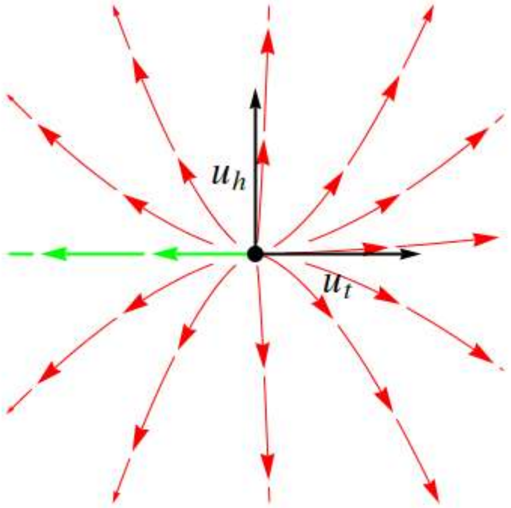
$$C(r,\lambda) \sim r^{2\zeta_{lark}}\, \mathcal{C}_{lark}(r/\lambda^{-\phi})$$

The long-distance depinning behavior is recapitulated as part of the universal function $\mathcal{C}_{lark}$ of the unstable fixed point: $\mathcal{C}_{lark}(X) \sim X^{2(\zeta_{depin} - \zeta_{lark})}$



Chen, Zapperi

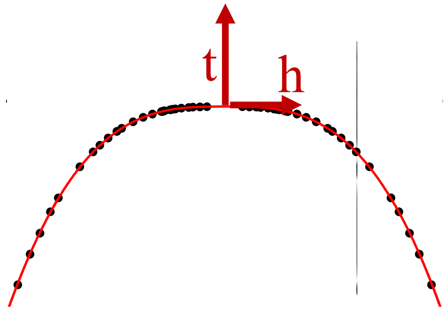


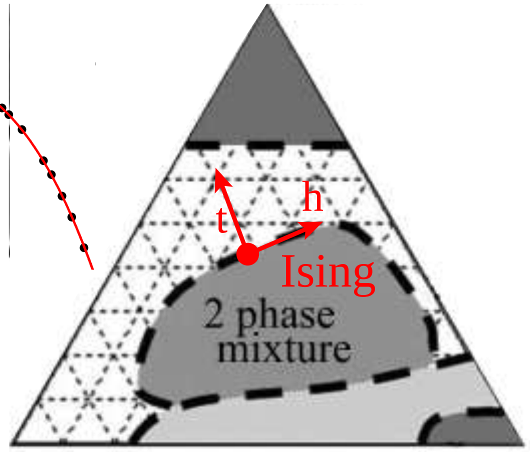


Physical System

Normal Form (hyperbolic)

Normal forms, universality families, and ***corrections to scaling***





Nature uses different coordinates than your model does.

Analytic corrections to scaling: changing variables gives rapid convergence

Normal form theory (dynamical systems applied to RG flow)

Depends on which bifurcation (hyperbolic, transcritical, pitchfork, …)

Traditional power law scaling = hyperbolic bifurcation (includes ***singular corrections to scaling***)

Lower critical dimensions, upper critical dimensions, 2D Ising, 2D RFIM, …

Raju, Hayden, Liarte

Normal forms explain logs in UCD, exponentials in LCD, …

Universality family	Systems	Normal form	Invariant scaling combinations
Hyperbolic	**3D Ising Model** (t) 3D RFIM (w)	$dt/dl = (1/\nu)t$	Lt^{ν}
Pitchfork	**2D RFIM** (w) 6D Potts model (q)	$dw/dl = w^3 + Bw^5$	$Le^{1/(2w^2)}(1/w^2+B)^{-B/2}$
Transcritical	**4D Ising model** (u, t) 2D NERFIM $(-w, S)$ 1D Ising model $(-t, h)$	$du/dl = -u^2 + Du^3$ $dt/dl = 2t - Atu$	$Le^{1/u-D}[1/(Du) - 1]^D = Ly^D$ $tL^2[W(yL^{1/D})/(1/(Du) - 1)]^{-A}$
Resonance	**2D Ising model**	$df/dl = 2f - t^2 - L^{-2}$ $dt/dl = t + AL^{-1}$	$tL + A \log L$
Higher Codimension	**2D XY model**	$dx/dl = -y^2[1 + xf(x^2)]$ $dy/dl = -xy$	$y^2 - 2\int_0^x s/[1 + sf(s^2)]\, ds$ $= y^2 - x^2 + [2f(0)/3]x^3 - [f(0)^2/2]x^4 + \mathcal{O}(x^5)$

Hayden, Raju

Normal form prediction for 2D RFIM avalanches

2D RFIM: disorder $w = (R - R_c)/R)$ flow has no linear term:

$$dw/d\ell = w^2 + B_1 w^3 + B_2 w^4 + \dots$$

We can change variables $w = \widetilde{w} + b_1 \widetilde{w}_2 + b_2 \widetilde{w}_3 + b_3 \widetilde{w}_4 + \dots$ to make

$$d\widetilde{w}/d\ell = \widetilde{w}^2 + B_1 \widetilde{w}^3 + (B_1 b_1 + b_1^2 + B_2 - b_2)\widetilde{w}^4 + \dots$$

We can make $\widetilde{w}^n$ vanish for $n > 3$, but $B_1 \equiv B$ is fixed and universal. The avalanche size S has a linear term, but one nonlinear term involving w cannot be removed, so our transcritical normal form is

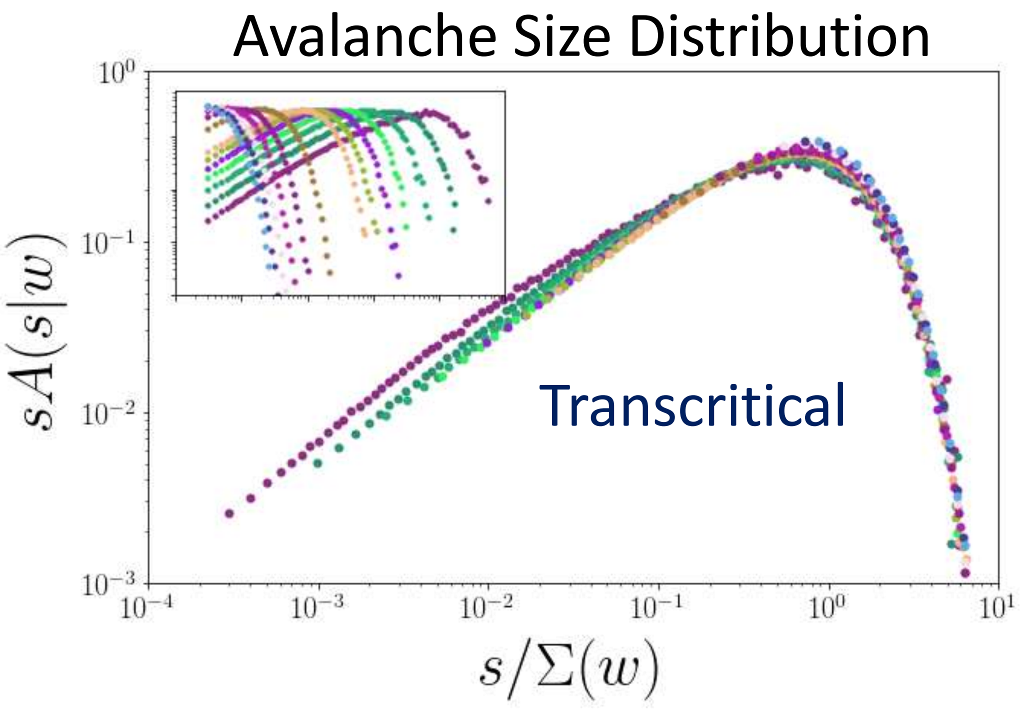
$$dw/d\ell = w^2 + Bw^3 \qquad dS/d\ell = -d_f S - CSw.$$

The invariant scaling combination $\mathcal{S}(S, w) \neq S w^{1/\sigma}$ is no longer a ratio of power laws!

$$\mathcal{S}(S, w) = S/\Sigma(w) = S\left((B + 1/w)^{Bd_f - C} \exp(-d_f/w)\right).$$

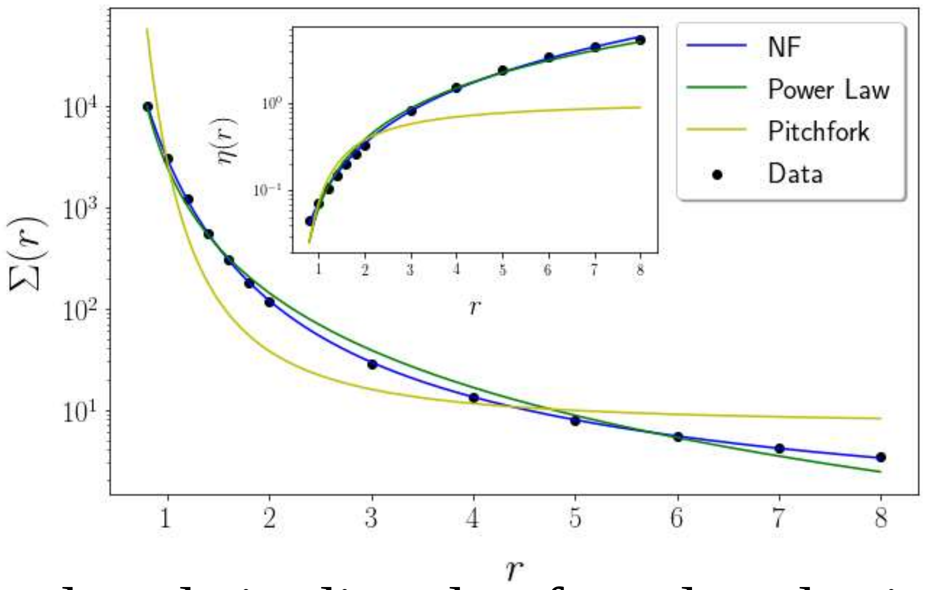
Hayden, Raju

Avalanches and normal forms in 2D
Invariant scaling variable for avalanche size cutoff *S(w)*



Avalanche size cutoff in lower critical dimension

$$\Sigma(w) \neq w^{-1/\sigma}$$
$$= \left((B + 1/w)^{-Bd_f + C} \exp(d_f/w)\right)$$



One decade in disorder, four decades in size!

Hayden, Raju