Opinion Dynamics

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Group behaviors

Asch experiment
(Social Pressure)

Group behaviors

Reservoir Dogs, Quentin Tarantino
(Persuasive Arguments)

+ Group behavior

Imitation

Why can people change their opinions or behaviors? Social influence and Persuasion

From www.sociologyencyclopedia.com

social influence

Lisa Rashotte

Social influence is defined as change in an individual's thoughts, feelings, attitudes, or behaviors that results from interaction with another individual or a group. Social influence is distinct from conformity, power, and authority. Confor-

Social influence, however, is the process by which individuals make *real* changes to their feelings and behaviors as a result of interaction with others who are perceived to be similar, desirable, or expert. People adjust their beliefs with respect to others to whom they feel similar in accordance with psychological principles

Persuasion →

Current research on persuasion, broadly defined as change in attitudes or beliefs based on information received from others, focuses on written or spoken messages sent from source

Framing the problem

- When a group of inter-related individuals discuss around a given item, they are prone to change their initial opinions in order to get like or dissimilar from other subjects in the group.
- This interpersonal dynamics leads to different consequences which can be categorized either by consensus or coexistence of opinions.
- What are the mechanisms leading to the formation of these collective states?

Sociological Theories

- Imitation (Akers et al, 1979): In situations of high uncertainty, it can be rational for individuals to imitate the behavior and opinions of others (Bikhchandani, Hirshleifer and Welch 1992).
- Social Pressure: In some situations, interactions partners may exert social pressure to conform with each other (Festinger Schachter & Black 1950, Homans 1951).
- Cognition theories (Festinger 1957, Heider 1967): imply that we want to be like people we like to interact with. To achieve this, we can convince the others or change ourselves
- Persuasive Arguments Theory (PAT): Interactions partners exchange arguments and persuade each other that certain opinions are more adequate (Myers 1982, Wood 2000).

Empirical Results also demonstrate the importance of social influence.

ATTITUDE CHANGE: Persuasion and Social Influence

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Key Words influence, motives, fear appeals, social identity

■ **Abstract** This chapter reviews empirical and theoretical developments in research on social influence and message-based persuasion. The review emphasizes research published during the period from 1996–1998. Across these literatures, three central motives have been identified that generate attitude change and resistance. These involve concerns with the self, with others and the rewards/punishments they can provide, and with a valid understanding of reality. The motives have implications for information processing and for attitude change in public and private contexts. Motives in persuasion also have been investigated in research on attitude functions and cognitive dissonance theory. In addition, the chapter reviews the relatively unique aspects of each literature: In persuasion, it considers the cognitive and affective mechanisms underlying attitude change, especially dual-mode processing models, recipients' affective reactions, and biased processing. In social influence, the chapter considers how attitudes are embedded in social relations, including social identity theory and majority/minority group influence.

CONTENTS

- Psychological experiments consistently show that subjects adjust their opinions after being informed about the opinions of another person (Wood 2000)
- When subject share some attribute with that person, they tend to decrease their opinion distance (Berscheid 1966, van Knioоernberg and Wilke 1988).

What would we like to know?

- Under what conditions does a group reach consensus or a given opinion becomes predominant?
- Can we predict the final collective outcomes when different mechanisms compete among each other, as for instance, when some individuals tend to agree and others to disagree?

Theoretical & numerical models

REVIEWS OF MODERN PHYSICS, VOLUME 81, APRIL–JUNE 2009

Statistical physics of social dynamics

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Statistical physics has proven to be a fruitful framework to describe phenomena outside the realm of traditional physics. Recent years have witnessed an attempt by physicists to study collective phenomena emerging from the interactions of individuals as elementary units in social structures. A wide list of topics are reviewed ranging from opinion and cultural and language dynamics to crowd behavior, hierarchy formation, human dynamics, and social spreading. The connections between these problems and other, more traditional, topics of statistical physics are highlighted. Comparison of model results with empirical data from social systems are also emphasized.

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CONTENTS

From Physics perspective

- How do the interactions between social agents create order out of an initial disordered situation?
- Order is a translation in the language of physics of what is denoted in social sciences as consensus, agreement, uniformity, while disorder stands for fragmentation or disagreement.
- It is reasonable to assume that without interactions, heterogeneity dominates left alone, each agent would choose a personal response to a political question.
- Still, it is common experience that shared opinions, cultures, and languages do exist.
- **The focus of the statistical physics approach to social dynamics is to understand how this comes about.**
- The key factor is that agents interact, and this generally tends to make people more similar (or not!)
- Repeated interactions in time can lead (or not) to higher degrees of homogeneity, which can be partial or complete depending on the temporal or spatial scales.

Representing opinions

The first decision to make in order to build opinion formation models is to choose how to represent opinions.
If we have a single topic of discusion, opinion will be represented by a single variable that could be discrete or continuous.
If we model opinions in several topics, we represent them as vectors.

The Classical discrete models

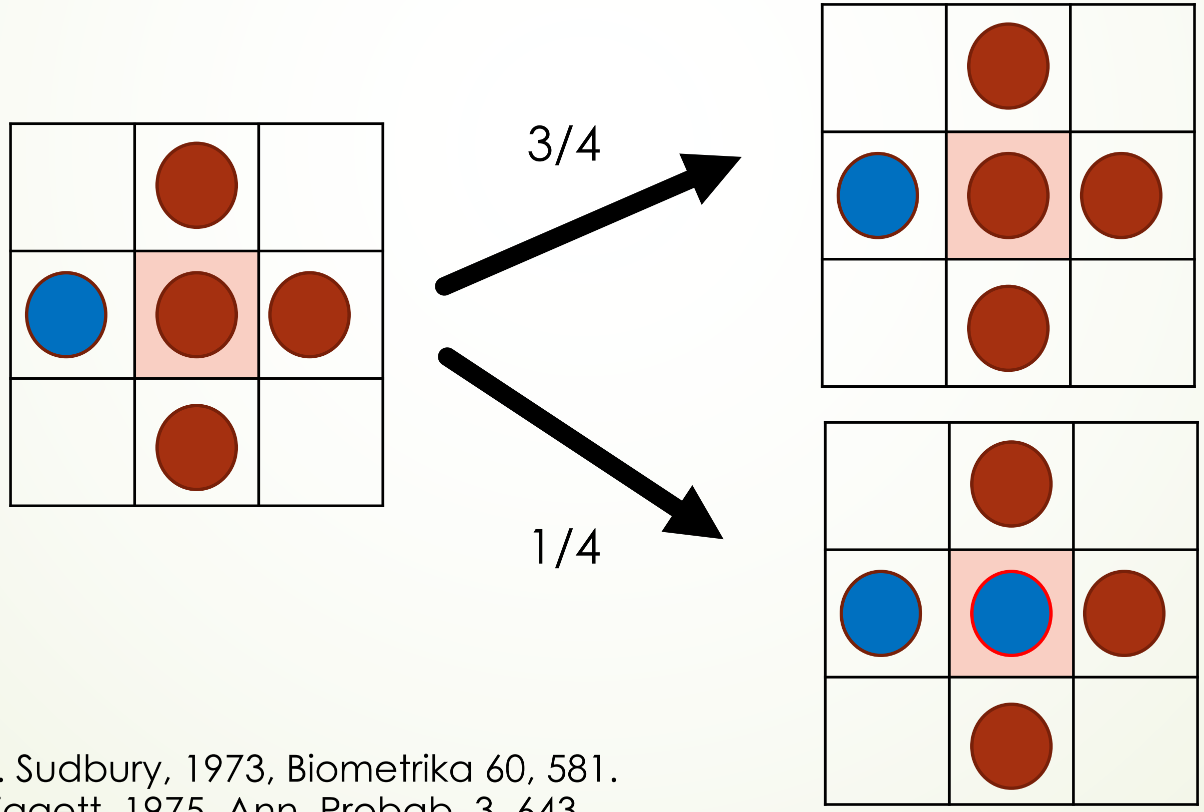
Voter Model

- Each agent can be in one of two states (pro, con) (black, white) (up, down)
- Initially solved in a 2D grid
- Follow an imitation dynamics (Akers 1979) which is the behavior followed in low information environments.
- Agents can be considered as non-confidence in their own opinion.
- Bulk noise is absent so consensus (all agents in the same state) is an absorbing state (at least in finite systems)
- Starting from a disordered initial condition, voter dynamics tends to increase the order of the system, as in usual coarsening processes (Scheucher and Spohn, 1988).
- The question is whether full consensus is reached in a system of infinite size.

– Clifford, P., and A. Sudbury, 1973, Biometrika 60, 581.
- Holley, R., and T. Liggett, 1975, Ann. Probab. 3, 643.

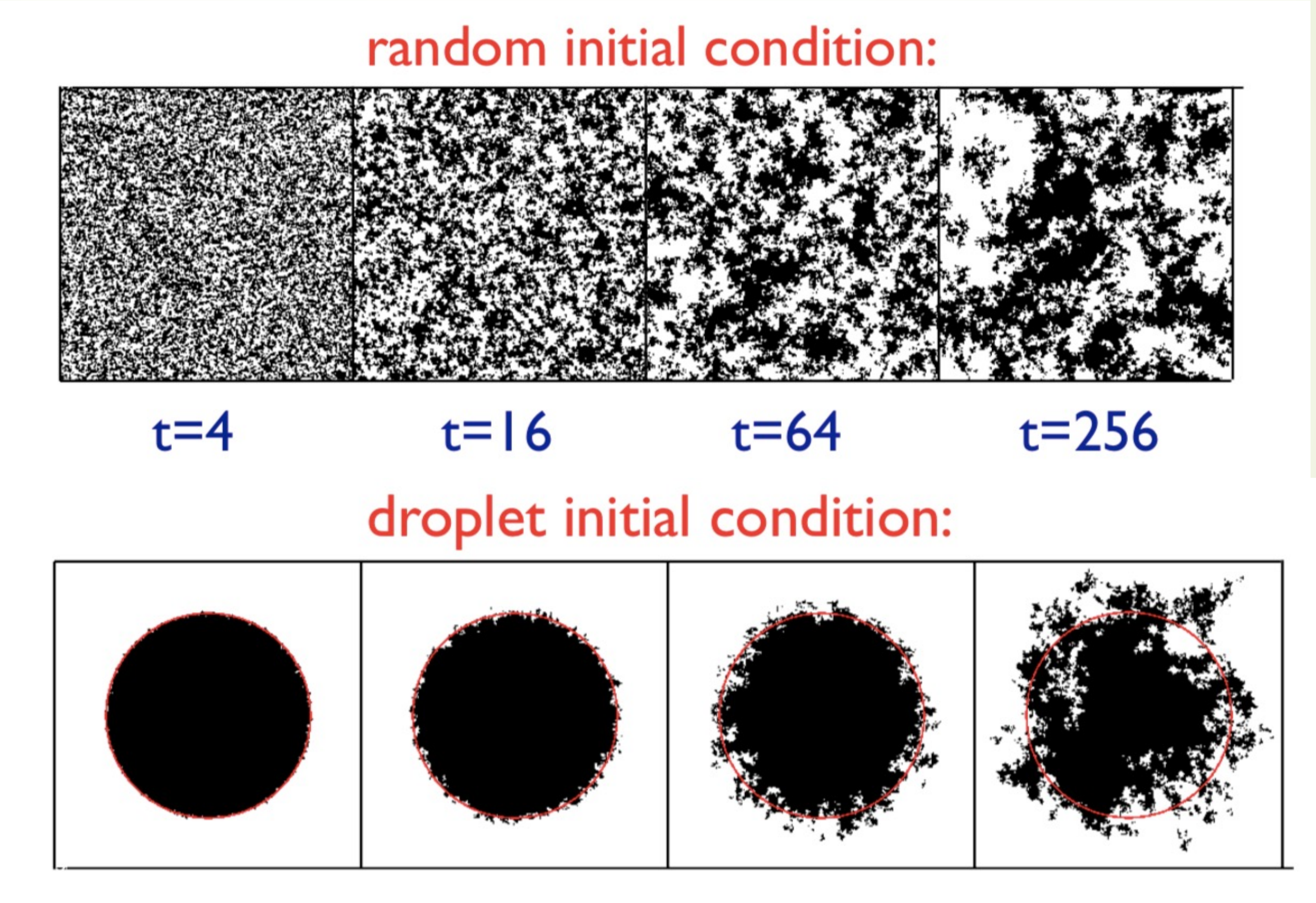
Voter Model: Dynamical rules

1 – Pick a random agent
2 – Assume the state of a randomly selected neighbor
3 - Repeat 1 & 2 until consensus necessarily occurs in a finite system.



– Clifford, P., and A. Sudbury, 1973, Biometrika 60, 581.
- Holley, R., and T. Liggett, 1975, Ann. Probab. 3, 643.

Voter Model evolution



Voter model main properties

- A very useful quantity to characterize the dynamics of the voter model is the density of active links ρ:
- An active links is a link between two agents with different state (and therefore able to change)
- In d-dimensional- grid, the density of active links behaves as:

$$\rho = \begin{cases} t^{-1/2} & d = 1 \\ \dfrac{1}{\ln(t)} & d = 2 \\ a - bt^{-\frac{d}{2}} & d > 2 \end{cases}$$

- Its mean that, in the thermodynamic limit, the system converge to consensus if d ≤ 2.
- For d>2 instead, it exhibits asymptotically a finite density of interfaces, i.e., no consensus is reached (in an infinite system) and domains of opposite opinions coexist indefinitely in time
- In finite systems, the time to reach consensus T_N, depends on size N:
- $T_N \sim N^2$ for d=1, $T_N \sim N.\ln(N)$ for d=2 and $T_N \sim N$ for d>2

Voter model in complex networks

PHYSICAL REVIEW E **72**, 036132 (2005)

Voter model dynamics in complex networks: Role of dimensionality, disorder, and degree distribution

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We analyze the ordering dynamics of the voter model in different classes of complex networks. We observe that whether the voter dynamics orders the system depends on the effective dimensionality of the interaction networks. We also find that when there is no ordering in the system, the average survival time of metastable states in finite networks decreases with network disorder and degree heterogeneity. The existence of hubs, i.e., highly connected nodes, in the network modifies the linear system size scaling law of the survival time. The size of an ordered domain is sensitive to the network disorder and the average degree, decreasing with both; however, it seems not to depend on network size and on the heterogeneity of the degree distribution.

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Here, the authors analyze the dynamics of the voter model when the connectivity is given by different complex networks

It is possible to derive master equations for voter model in networks?

The Voter model in uncorrelated networks

Analytical solution of the voter model on uncorrelated networks

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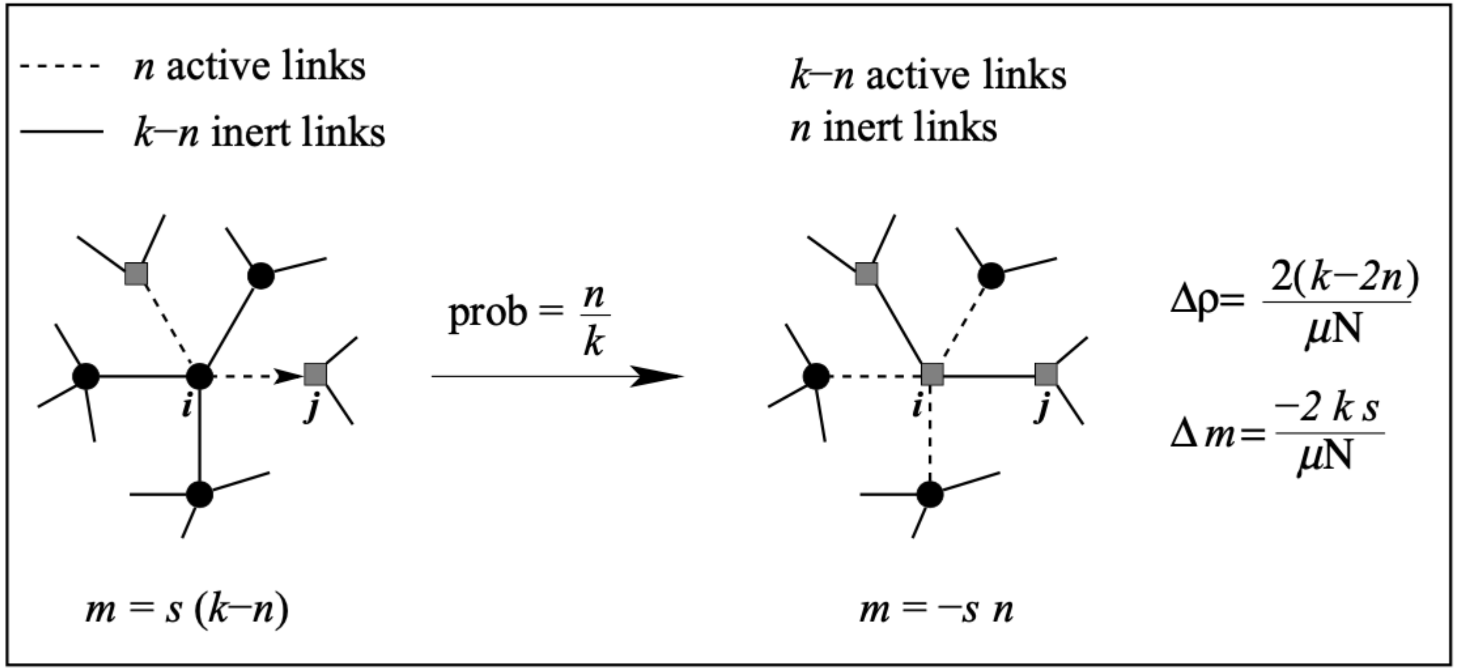
The Voter Model in uncorrelated networks

We will follow the developing of master equations for the evolution of the voter model in uncorrelated networks as V&E perform in the paper.

This can be done in terms of two macroscopic variables:

ρ: density of active links
m: magnetization

What happen to ρ and ***m*** when a node of degree **k** changes is state from **s** to **–s**?



μ : <k>
# of links: $\frac{\mu N}{2}$

The Voter Model in uncorrelated networks

Let's formulate a Master Equation (ME) for the density of active links:

$$\frac{d\rho}{dt} = \sum_k P_k \left.\frac{d\rho}{dt}\right|_k = \sum_k \frac{P_k}{1/N} \left. d\rho \right|_k$$

Where we use that dt=1/N and **dρ)$_k$** is the change in the density of active links when a node of degree k is chosen.

$$\left. d\rho \right|_k = \sum_{n=0}^{k} B(n,k) . \frac{n}{k} \frac{2(k-2n)}{\mu N}$$

Probability that n active links are connected to a node of degree **k**

Probability of randomly choose active link

Change in ρ due to a spin flip

The Voter Model in uncorrelated networks

The probability of having **n** active links among **k** ones will depend on the state **s** of the node.

-During the evolution the density of opinions +/-, σ_+ and σ_- will change, we can write:

$$B(n,k) = \sum_{s=\pm} \sigma_s \, B(n,k|s)$$

Then:

$$\frac{d\rho}{dt} = \frac{2}{\mu} \sum_k P_k \sum_{s=\pm} \sigma_s \sum_{n=0}^{k} B(n,k|s) \frac{n}{k} (k-2n)$$

Which can be written as:

$$\frac{d\rho}{dt} = \frac{2}{\mu} \sum_k P_k \sum_{s=\pm} \sigma_s \left[\langle n \rangle_{k,s} - \frac{2}{k} \langle n^2 \rangle_{k,s} \right]$$

The Voter Model in uncorrelated networks

Where we have used that:

$$\langle n\rangle_{k,s} = \sum_{n=0}^{k} n\, B(n,k|s) \qquad \langle n^2\rangle_{k,s} = \sum_{n=0}^{k} n^2\, B(n,k|s)$$

Here, we use the hypotesis of uncorrelated networks: We assume that the probability of having an active link depends only of the states of the two connected nodes and is independent of the others neighboors.

Therefore, if we call $p = P(-s|s)$ the probability that a neighboor of node "i" be in a state "-s" given that "i" is in state "s", then B$(n,k|s)$ becomes a binomial distribution and:

$$B(n,k|s) = \binom{k}{n} p^n (1-p)^{k-n} \quad \text{and} \quad \begin{aligned} \langle n\rangle_{k,s} &= kp \\ \langle n^2\rangle_{k,s} &= kp + k(k-1)p^2 \end{aligned}$$

$$\text{Where } p = P(-s|s) = \frac{\rho\left(\mu N/_2\right)}{\mu N \sigma_s} = \frac{\rho}{2\sigma_s}$$

→ # of active links conecting "s" wirh "–s"

→ # of links connecting to a node with state "s"

The Voter Model in uncorrelated networks

Replacing p:

$$\langle n \rangle_{k,s} = \frac{k\rho}{2\sigma_s}$$

$$\langle n^2 \rangle_{k,s} = \frac{k\rho}{2\sigma_s} + \frac{k(k-1)\rho^2}{4\sigma_s^2}$$

$$\frac{d\rho}{dt} = \frac{2}{\mu} \sum_k P_k \sum_{s=\pm} \sigma_s \left[\langle n \rangle_{k,s} - \frac{2}{k} \langle n^2 \rangle_{k,s} \right]$$

$\langle k \rangle = \mu = \sum_k k P_k$ and $\sum_k P_k = 1$

....

$$\frac{d\rho}{dt} = \frac{2\rho}{\mu} \left[(\mu - 1) \left(1 - \frac{\rho}{2\sigma_+(1-\sigma_+)} \right) - 1 \right] \quad (1)$$

Master equation for density of active links

$$\frac{d\rho}{dt} = \frac{2\rho}{\mu}\left[(\mu - 1)\left(1 - \frac{\rho}{2\sigma_+(1-\sigma_+)}\right) - 1\right] \quad (1)$$

This equation has two stationary solutions:

$\rho^* = 0$ or $\rho^* = \frac{(\mu-2)}{(\mu-1)} 2\sigma_+(1-\sigma_+) = 4\xi(\mu)\sigma_+(1-\sigma_+)$

$\rho^* = 0$ → Stable if $\mu \leq 2$

$\rho^* = 4\xi(\mu)\sigma_+(1-\sigma_+)$ → Stable if $\mu > 2$

with $\xi(\mu) = \frac{(\mu - 2)}{2(\mu - 1)}$

Stationary state

The stationary solution of the voter model is a complete or a partially ordered state depending on the value of the mean degree μ = <k>

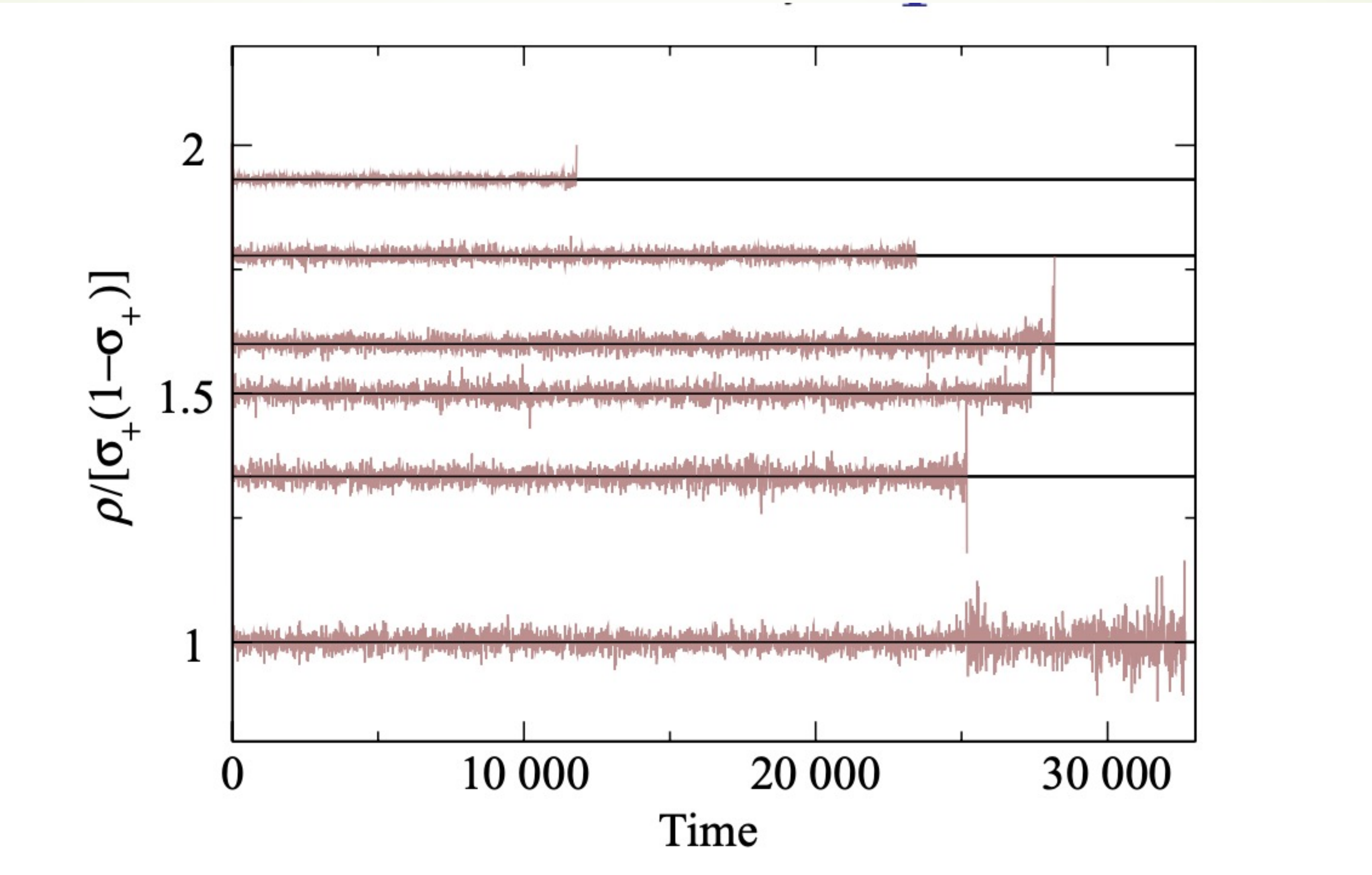
If $\mu > 2$, the solution is a partially ordered state with a fraction of active links different from zero as long as σ_+ or σ_- were also different from zero.

If $N \to \infty$, fluctuations goes to zero (in ρ and σ_s) and $\sigma_+(t) \to \sigma_+(0)$ and:

$$\rho^* = 4\xi\sigma_+(0)\big(1 - \sigma_+(0)\big) \equiv cte$$

The system does not reach complete order in the thermodinamic limit. However, in finite systems always exist a fluctuation that leads to all nodes have the same opinion and therefore $\rho = 0$.

Master equation for density of active links



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Let's introduce link magnetization

Although $\boldsymbol{\rho}$ is useful to find the absorbing state, it does not provide which of the two states is reached: all + or all -. Let's define:

ρ_{++} : density of links connecting two nodes with opinions +1
ρ_{--} : density of links connecting two nodes with opinions -1

Then, m = ρ_{++} - ρ_{++} . Let's write the relation between m and ρ:

First, we will write ρ_{ss} : in terms of σ_s :

1. # of total links from a node with opinion "s": $\boldsymbol{\sigma_s \mu N}$
2. # of total links from "s" to "-s": $\boldsymbol{\rho(\frac{\mu N}{2})}$
3. # of total links from "s" to "s" : $\boldsymbol{\rho_{ss} \mu N}$

Then, (1) = (2) + (3) and $\boldsymbol{\rho_{ss} = \sigma_s - {}^{\rho}/_{2}}$, leading to: $\boldsymbol{m = \rho_{++} - \rho_{--} = 2\sigma_+ - 1}$

Using that $\boldsymbol{\rho = 4\xi\sigma_+(1 - \sigma_+)}$ we obtain:

$$\boldsymbol{\rho = \xi(1 - m^2)}$$

Link magnetization and density of active links

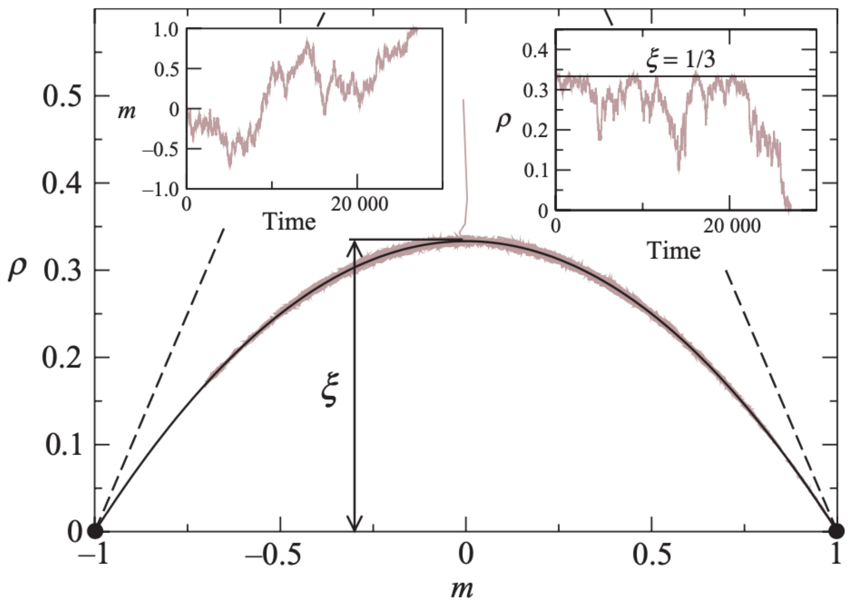

Figure 3. Trajectory of the system in a single realization plotted on the active links density-link magnetization ($\rho - m$) plane, for a DR random graph of size $N = 10^4$ and degree $\mu = 4$. Insets: time evolution of m (left) and ρ (right) for the same realization. We note that ρ and m are not independent but fluctuate in coupled manner, following a parabolic trajectory described by $\rho = \frac{1}{3}(1 - m^2)$ from equation (9) (solid line).

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Master equation for the link magnetization

$$P(m,t+\delta t)=\sum_k P_k\{W_{m+\delta k\to m}\,P(m+\delta_k,t)+W_{m-\delta k\to m}\,P(m-\delta_k,t)+W_{m\to m}\,P(m,t)\}$$

Probability of going from **m+δk** to **m**.

Probability of going from **m-δk** to **m**.

Probability of not changing **m**.

Where **P(m,t)** is the probability of having magnetization **m** at time **t.**

We can write down the probabilities of the possible changes in **m**, due to the selection of a node of degree **k** in :

- A node with spin **s** and degree **k**, change its state with probability: $\sigma_s P(-s|s) = {}^{\rho}/_{2}$, producing a change in magnetiation $\Delta m = s\delta_k$ with $\delta_k = \frac{2k}{\mu N}$.
- The same node remains in the same state with probability: $\sigma_s[1-P(-s|s)] = \sigma_s\left(1-{}^{\rho}/_{2\sigma_s}\right)$.

With $P(-s|s)=\frac{\rho}{2\sigma_s}$, as before

Master equation for the link magnetization

Using that $\boldsymbol{\rho = \xi(1 - m^2)}$ we can write the transition probabilities as:

$$W_{m \to m \pm \delta k} = \frac{\xi}{2}(1 - m^2) \quad \text{and} \quad W_{m \to m} = (1 - \rho) = [1 - \xi(1 - m^2)]$$

$$\boldsymbol{P(m, t + \delta t) = \sum_k P_k \left\{ \frac{\xi}{2}[1 - (m + \delta_k)^2]\ P(m + \delta_k, t) + \frac{\xi}{2}[1 - (m - \delta_k)^2] P(m - \delta_k, t) + [1 - \xi(1 - m^2)] P(m, t) \right\}}$$

Now, we go to the continuous limit by expanding **P** at first order in **t** and second order in **m**:

$$P(m, t + \delta t) = P(m, t) + \frac{\partial P}{\partial t}\delta t$$

$$P(m + \Delta, t) = P(m, t) + \frac{\partial P}{\partial t}\Delta + \frac{1}{2}\frac{\partial^2 P}{\partial m^2}\Delta^2$$

with $\Delta = \pm\delta_k = \pm\frac{2k}{\mu N}$

.....

Master equation for the link magnetization

After a while:

$$N\delta t\frac{\partial P}{\partial t}=\frac{2\xi}{\mu^2}\sum_k k^2P_k\left\{-2P-4m\frac{\partial P}{\partial m}+(1-m^2)\frac{\partial^2 P}{\partial m^2}\right\}$$

Now, if we identify $\mu_2=\sum_k k^2P_k$ as the second moment of the degree distribution and we define a caracteristic time $\tau=\frac{\mu^2 N}{2\xi\mu_2}$, we take $t'={}^t/_\tau$ and the limit $\delta t={}^1/_N\to 0$ as $N\to\infty$:

$$\boxed{\frac{\partial P(m,t')}{\partial t'}=\frac{\partial^2}{\partial m^2}[(1-m^2)P(m,t')]} \qquad (2)$$

The general solution of (2) is:

$$P(m,t')=\sum_{l=0}^{\infty}A_lC_l^{3/2}(m)e^{-(l+1)(l+2)t'} \qquad (3)$$

$C_l^{3/2}(x)$: Gegembauer polynomials

Average dynamics of density of active links

Having **P(m,t')** we can calculate the time evolution of **<ρ>(t):**

$$\langle\rho(t')\rangle = \xi\langle 1 - m^2(t')\rangle = \int_{-1}^{+1} dm(1 - m^2)P(m,t')$$

Replacing P(m,t') from (3) and finding the coefficients with an initial condition $m_0 = \xi(1 - m_0^2)$ then:

$$\langle\rho(t')\rangle = \xi(1 - m_0^2)e^{-2t'}$$

And replacing back t' and ξ we have:

$$\langle\rho(t)\rangle = \frac{(\mu - 2)}{2(\mu - 1)}(1 - m_0^2)e^{-2t/\tau}$$

Temporal evolution & different networks

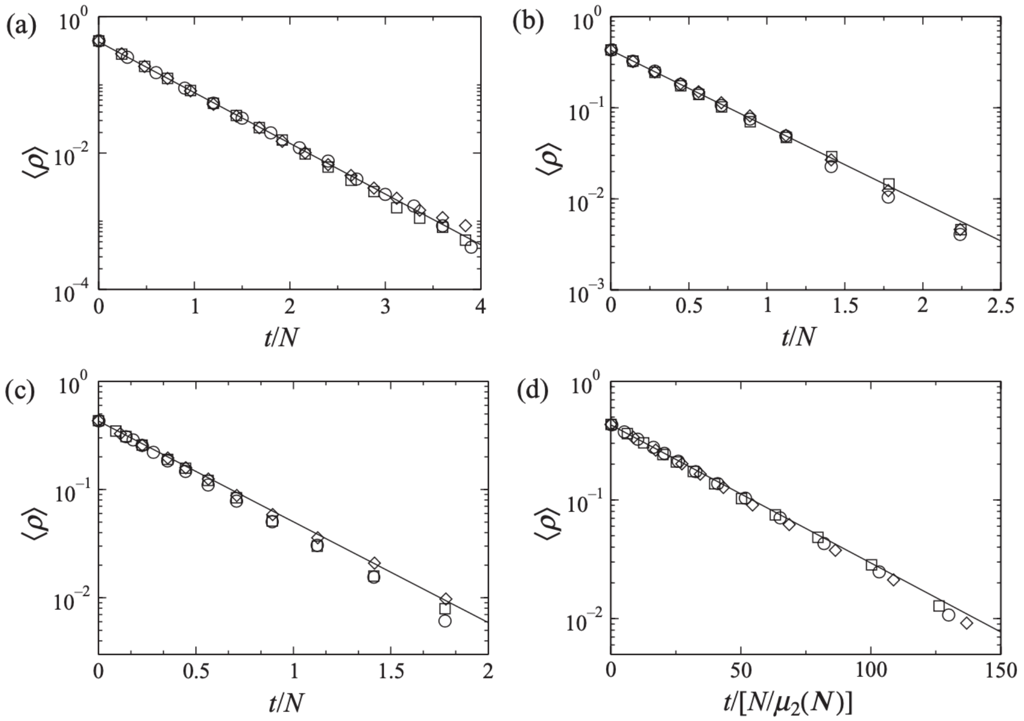

Figure 4. Time evolution of the average density of active links $\langle \rho(t) \rangle$ for (a) DR, (b) ER, (c) EN and (d) BA networks with average degree $\mu = 8$. The open symbols correspond to networks of different sizes: $N = 1000$ (circles), $N = 5000$ (squares) and $N = 10\,000$ (diamonds). Solid lines are the analytical predictions from equation (17). The average was taken over 1000 independent realizations, starting from a uniform distribution with magnetization $m_0 = 0$.

Table 1. Node degree distribution P_k, its second moment μ_2 and the decay time constant of the average density of active links τ, for different networks.

Network	P_k	μ_2	$\tau(\mu, N)$
DR	$\delta_{k,\mu}$	μ^2	$\frac{(\mu-1)}{(\mu-2)}N$
ER	$e^{-\mu}\frac{\mu^k}{k!}$	$\mu(\mu+1)$	$\frac{\mu(\mu-1)}{(\mu+1)(\mu-2)}N$
EN	$\frac{2\,e}{\mu}\exp\left(-\frac{2k}{\mu}\right)$	$\frac{5}{4}\mu^2$	$\frac{4(\mu-1)}{5(\mu-2)}N$
BA	$\frac{\mu(\mu+2)}{2k(k+1)(k+2)}$	$\frac{\mu(\mu+2)}{4}\ln\left(\frac{\mu(\mu+2)^3 N}{(\mu+4)^4}\right)$	$\frac{4\mu(\mu-1)N/(\mu^2-4)}{\ln\left(\frac{\mu(\mu+2)^3}{(\mu+4)^4}N\right)}$
CG	$\delta_{k,N-1}$	$(N-1)^2$	N

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Beyond the classical voter model

Not only the node'states can change in time. Also the contact network can evolve and the final states depend on the interplay between both dynamics

Co-evolutionary voter model in complex networks

PRL **100,** 108702 (2008) PHYSICAL REVIEW LETTERS week ending 14 MARCH 2008

Generic Absorbing Transition in Coevolution Dynamics

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(Received 22 October 2007; published 14 March 2008)

We study a coevolution voter model on a complex network. A mean-field approximation reveals a absorbing transition from an active to a frozen phase at a critical value $p_c = \frac{\mu-2}{\mu-1}$ that only depends on th average degree μ of the network. In finite-size systems, the active and frozen phases correspond to connected and a fragmented network, respectively. The transition can be seen as the sudden change in th trajectory of an equivalent random walk at the critical point, resulting in an approach to the final froze state whose time scale diverges as $\tau \sim |p_c - p|^{-1}$ near p_c.

DOI: 10.1103/PhysRevLett.100.108702 PACS numbers: 89.75.Fb, 05.40.−a, 05.65.+b, 89.75.H

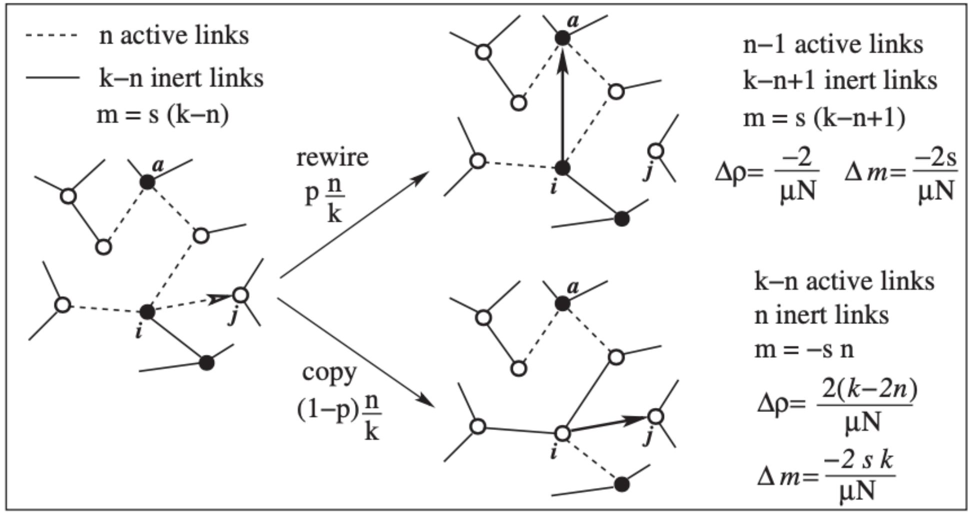

FIG. 1. Update events and the associated changes in the density of active links ρ and the link magnetization $m = \rho_{++} - \rho_{--}$ when two neighbors i and j with states $S_i = s$ and $S_j = -s$ are chosen ($s = \pm 1$).

Co-evolving voter model in complex networks

How they look like?

$$\frac{d\rho}{dt} = \sum_k \frac{P_k}{1/N} \sum_{n=0}^{k} B_{n,k} \frac{n}{k}\left[(1-p)\frac{2(k-2n)}{\mu N} - p\frac{2}{\mu N}\right]$$
$$= \sum_k P_k \frac{2}{\mu k}[(1-p)(k\langle n\rangle_k - 2\langle n^2\rangle_k) - p\langle n\rangle_k], \qquad (1)$$

$$\frac{d\rho}{dt} = \frac{2\rho}{\mu}[(1-p)(\mu-1)(1-2\rho)-1]. \qquad (2)$$

Summary and conclusions.—In summary, the coevolution mechanism on the voter model induces a fragmentation transition that is a consequence of the competition between the copying and the rewiring dynamics. In the connected active phase, the system falls in a dynamical steady state with a finite fraction of active links. The slow and permanent rewiring of these links keeps the network evolving and connected until by a finite-size fluctuation the system reaches the fully ordered state (all nodes in the same state) and freezes in a single component. In the frozen phase, the fast rewiring dynamics quickly leads to the fragmentation of the network into two components, before the system becomes fully ordered.

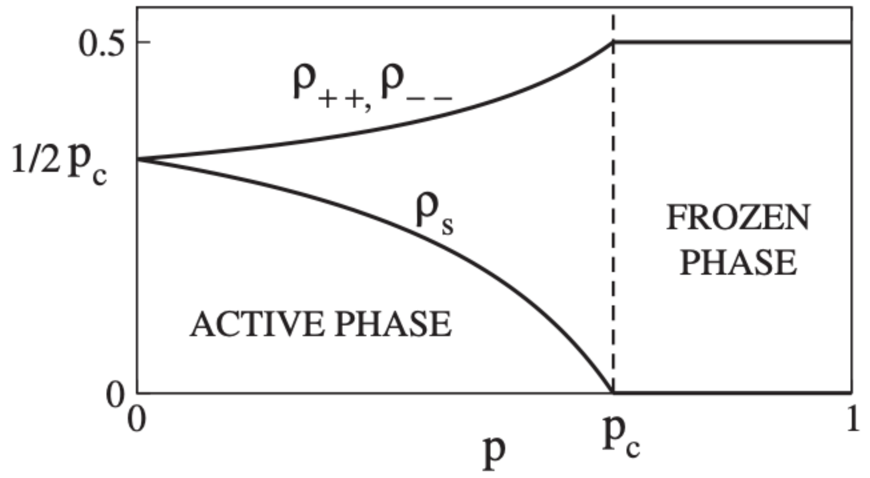

FIG. 2. Stationary density of active links ρ_s and the two types of inert links ρ_{++} and ρ_{--} vs the rewiring probability p as described by the mean-field theory for a network with average degree $\mu = 4$. The critical point p_c separates an active from a frozen phase.

Co-evolving nonlinear voter model

Fragmentation transitions in a coevolving nonlinear voter model

Byungjoon Min & Maxi San Miguel

We study a coevolving nonlinear voter model describing the coupled evolution of the states of the nodes and the network topology. Nonlinearity of the interaction is measured by a parameter *q*. The network topology changes by rewiring links at a rate *p*. By analytical and numerical analysis we obtain a phase diagram in *p*,*q* parameter space with three different phases: Dynamically active coexistence phase in a single component network, absorbing consensus phase in a single component network, and absorbing phase in a fragmented network. For finite systems the active phase has a lifetime that grows exponentially with system size, at variance with the similar phase for the linear voter model that has a lifetime proportional to system size. We find three transition lines that meet at the point of the fragmentation transition of the linear voter model. A first transition line corresponds to a continuous absorbing transition between the active and fragmented phases. The other two transition lines are discontinuous transitions fundamentally different from the transition of the linear voter model. One is a fragmentation transition between the consensus and fragmented phases, and the other is an absorbing transition in a single component network between the active and consensus phases.

Co-evolving nonlinear voter model

Dynamics

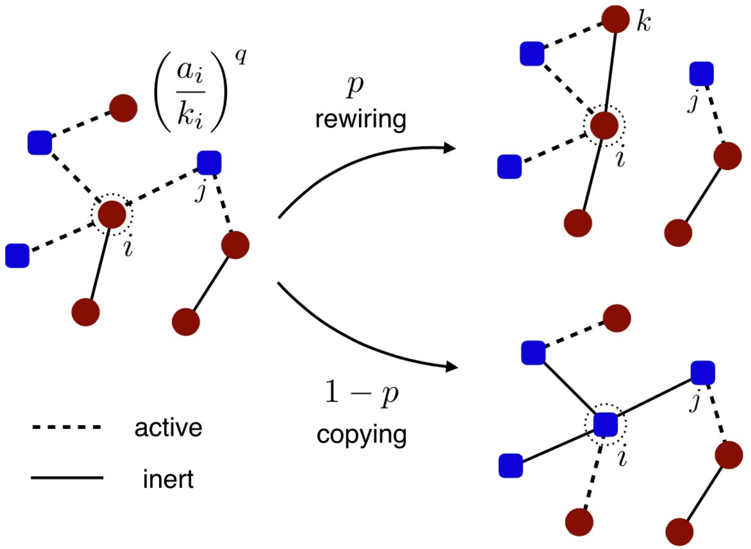

Figure 1. Schematic illustration of update rule of the coevolving nonlinear voter model. Each node is in either up (red circle) or down (blue rounded square) state. Solid and dashed lines indicate respectively inert and active links. At each step, we randomly choose a node *i*. And we choose one of its neighbors *j* connected by an active link with a probability $((a_i)/(k_i))^q$. Then, we rewire an active link with a probability *p* and copy the state of the neighbor with a probability $1-p$.

Phase Diagram

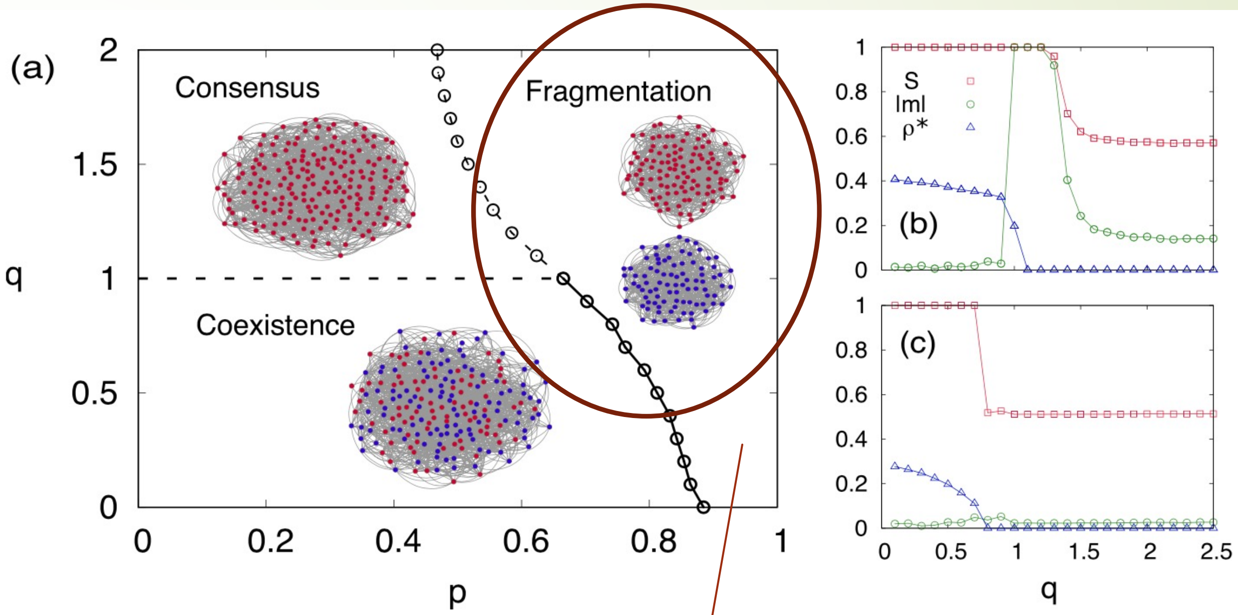

Figure 5. (**a**) Phase diagram with respect to *p* and *q* shows consensus, coexistence, and fragmented phases, obtained numerically on degree regular networks with $\langle k\rangle=8$, $N=10^4$ and initial condition $m=0$, averaged over 10^3 realizations. Examples of network configuration at the steady-state of the coevolution model are also shown with $N=200$ and $(p, q)=(0.2, 0.5)$ for coexistence, (0.2, 2) for consensus, and (0.8,0.5) for fragmentation. Size of giant component, magnetization, and density of active links at (**b**) $p=0.55$ and (**c**) $p=0.75$ are also shown.

Let's remember this!!

Multi-state voter model

Local and global ordering dynamics in multi-state voter models

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We investigate the time evolution of the density of active links and of the entropy of the distribution of agents among opinions in multi-state voter models with all-to-all interaction and on uncorrelated networks. Individual realisations undergo a sequence of eliminations of opinions until consensus is reached. After each elimination the population remains in a meta-stable state. The density of active links and the entropy in these states varies from realisation to realisation. Making some simple assumptions we are able to analytically calculate the average density of active links and the average entropy in each of these states. We also show that, averaged over realisations, the density of active links decays exponentially, with a time scale set by the size and geometry of the graph, but independent of the initial number of opinion states. The decay of the average entropy is exponential only at long times when there are at most two opinions left in the population. Finally, we show how meta-stable states comprised of only a subset of opinions can be artificially engineered by introducing precisely one zealot in each of the prevailing opinions.

Voter model & Data

Can be used the voter model to reproduce real data?

Voter model & Data

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Is the Voter Model a Model for Voters?

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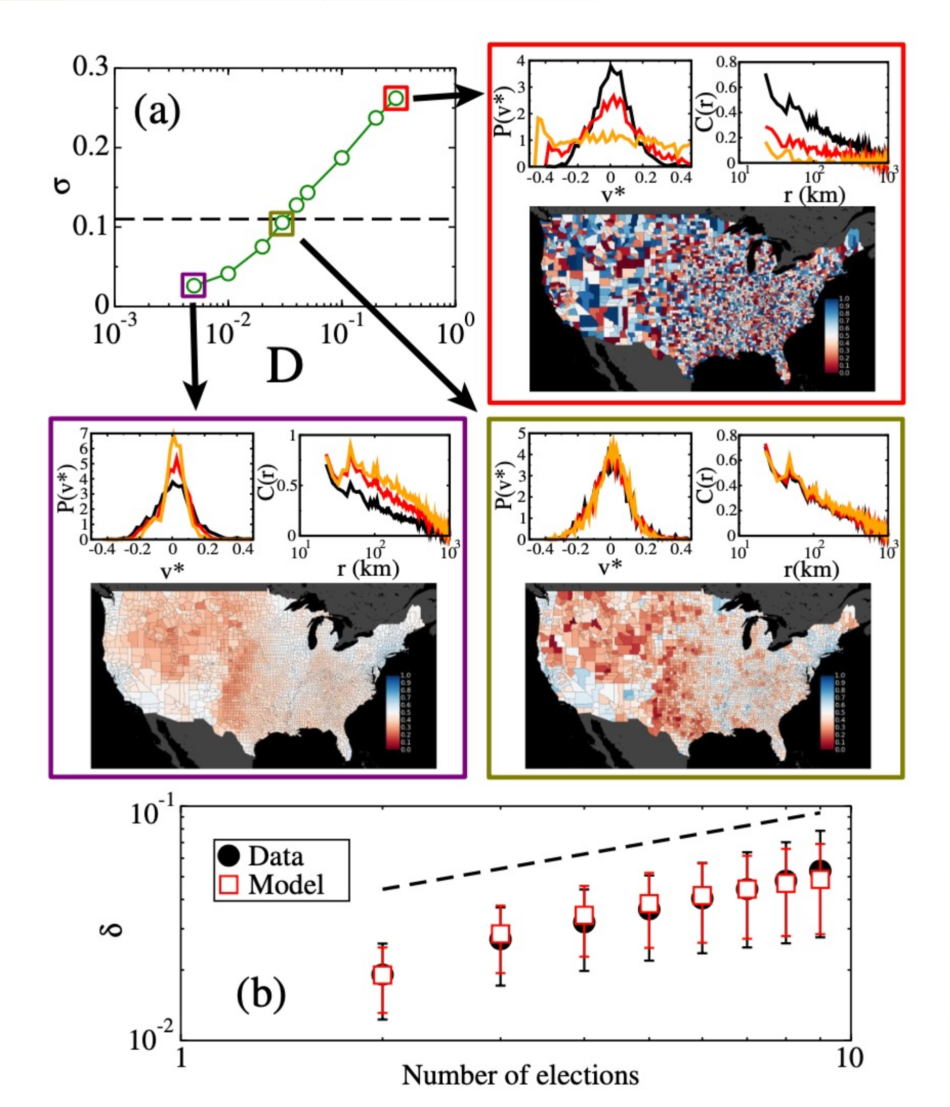
The voter model has been studied extensively as a paradigmatic opinion dynamics model. However, its ability to model real opinion dynamics has not been addressed. We introduce a noisy voter model (accounting for social influence) with recurrent mobility of agents (as a proxy for social context), where the spatial and population diversity are taken as inputs to the model. We show that the dynamics can be described as a noisy diffusive process that contains the proper anisotropic coupling topology given by population and mobility heterogeneity. The model captures statistical features of U.S. presidential elections as the stationary vote-share fluctuations across counties and the long-range spatial correlations that decay logarithmically with the distance. Furthermore, it recovers the behavior of these properties when the geographical space is coarse grained at different scales—from the county level through congressional districts, and up to states. Finally, we analyze the role of the mobility range and the randomness in decision making, which are consistent with the empirical observations.

Voter model & Data



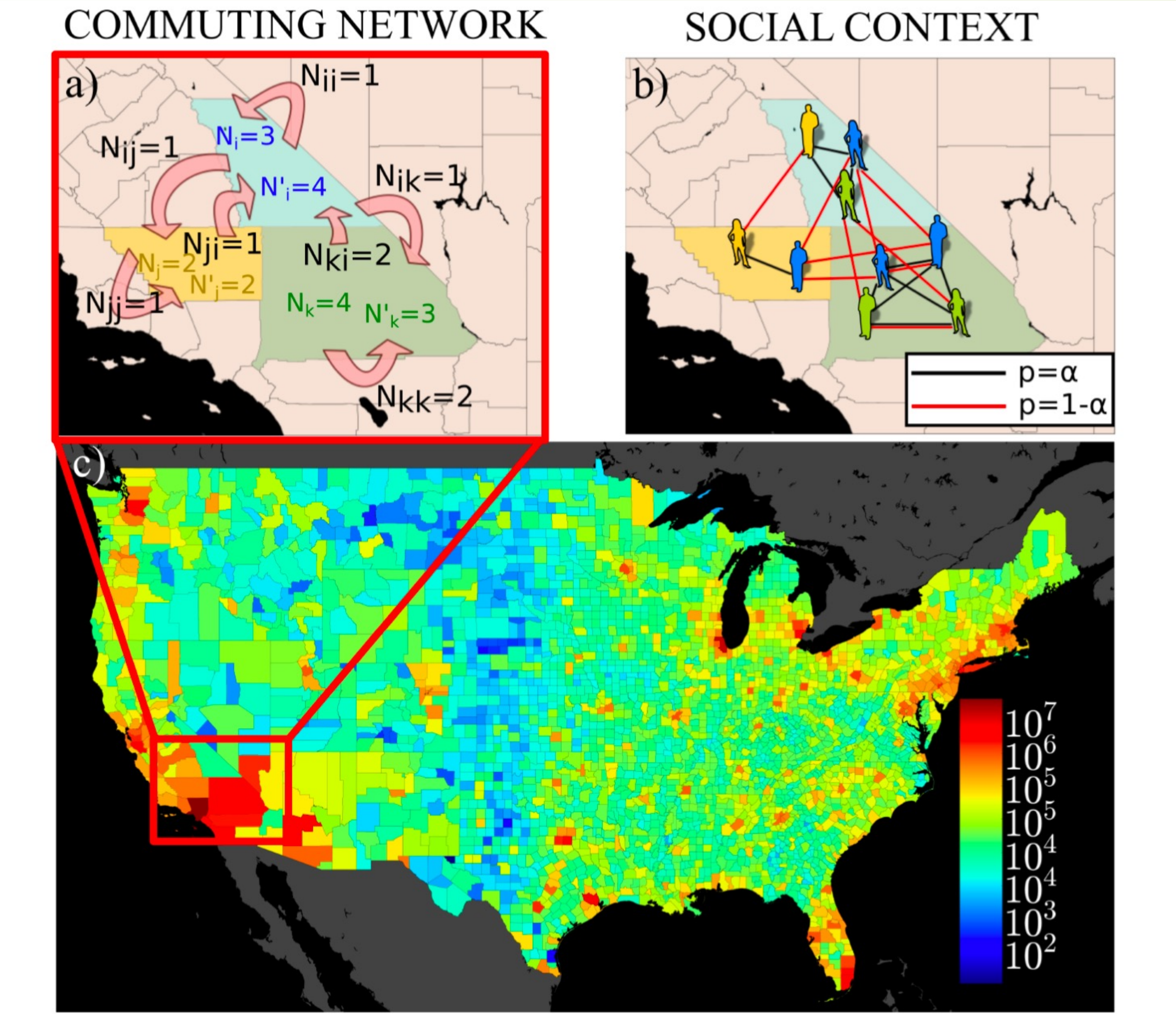


FIG. S2: **Recurrent mobility and population heterogeneities. a)** Schematic representation of the commuting network obtained from census data. **b)** Schematic representation of the different agent interactions. The home county interactions (black edges) and work county interactions (red edges) occur with different probabilities (α and $1-\alpha$ respectively). The agents

Summary

- We have seen sociological theories of social influence
- We focus today on discrete opinion model, like different variations of the classical voter model
- We derived the master equations of the voter model in uncorrelated networks
- We show variations of voter model with co-evolving networks. It appears echo-chambers? What is this?
- Can voter model be applied to understand voting data?

See you next class!!