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Multidimensional social models

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ICTP SAIFR

The beginnings: A model of cultural dissemination

A discrete multidimensional social model

Homogeneity and diversity in culture

- Understanding how a culture can get established, spread, and be sustained has growing importance in today's world.
- We are in favor of the spread of a common culture when it favors efficient communication, prevents unnecessary conflict and foster action for global needs as sustainable growth.
- On the other hand, we hate the harm done to peoples whose cultures are destroyed, the lost to the rest of us of the wisdom embodies in these vanishing cultures.

The Axelrod's puzzle.

- If people tend to become more alike in their beliefs, attitudes, and behavior when they interact, why do not all such differences eventually disappear?

Robert Axelrod

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Robert Axelrod is the Walgreen Professor for the Study of Human Understanding at the University of Michigan. He has appointments in the Department of Political Science and the Gerald R. Ford School of Public Policy. Prior to coming to Michigan he taught at the University of California, Berkeley (1968-74). He holds a BA in mathematics from the University of Chicago (1964), and a PhD in political science from Yale (1969).

He is best known for his interdisciplinary work on the evolution of cooperation which has been cited more than 30,000 times. His current research interests include international security and sense-making. Among his honors and awards are membership in the National Academy of Sciences, a five year MacArthur Prize Fellowship, the Newcomb Cleveland Prize of the American Association for the Advancement of Sciences for an outstanding contribution to science, and the National Academy of Sciences Award for Behavioral Research Relevant to the Prevention of Nuclear War. He served as President of the American Political Science Association (2006-07).

Axelrod has consulted and lectured on promoting cooperation and harnessing complexity for the United Nations, the World Bank, the U.S. Department of Defense, and various organizations serving health care professionals, business leaders, and K-12 educators.

Characterizing the individuals state in terms of culture

- ✓ The term *culture* will be used to indicate the set of individual attributes that are subject to social influence.
- ✓ The meaning of these attributes is not specified in the model.
- ✓ The question being investigated is how people influence each other on a given set of features and why this influence does not lead, necessarily to homogeneity.

However, non of previous consider this fundamental principle:

"the transfer of ideas occurs most frequently between individuals . . . who are similar in certain attributes such as beliefs, education, social status, and the like" (Rogers 1983, 274; see also Homans 1950)

"The model of social influence offered here abstracts this fundamental principle to say that communication is most effective between similar people.

- The likelihood that a given cultural feature will spread from one individual (or group) to another depends on how many other features they may already have in common.
- Similarity leads to interaction, and interaction leads to still more similarity

R. Axelrod, The Journal of Conflict Resolution 41, 203 (1997).

Axelrod and cultural dissemination

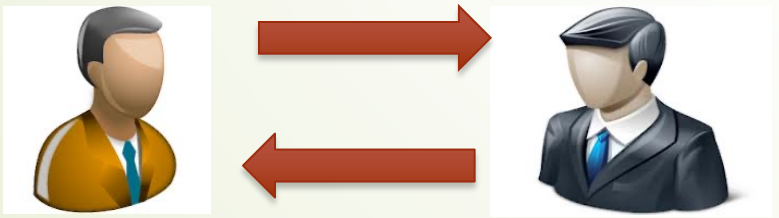
Research questions

Under what conditions can a diversity of cultures be created and / or maintained? What mechanisms would lead to a culturally unique state and under what conditions would it occur?

Axelrod's proposal

Let's develop a theoretical model in which agents interacts according to known mechanisms and analyze the emergent behavior.

Pairwise Interaction



?

The ingredients of the Axelrod's model

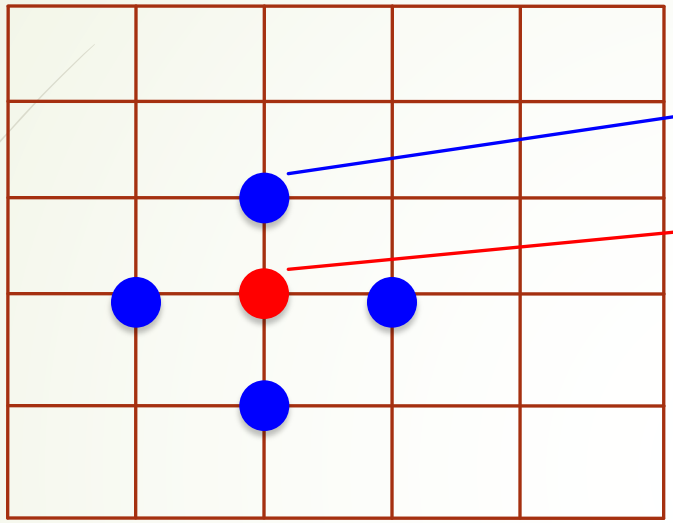
- Agent Based Modeling: Local interactions produces global behavior
- No central authority.
- Adaptative rather than rational agents. The individuals follow the simple rules sketched bellow:
- The interaction between people is more frequent and intense if they are more similar (i.e. they share a series of cultural attributes: language - race - religion) → Homophily
- The interaction between people with a certain degree of similarity makes them more similar Social Influence → Social Influence
- Can these two mechanisms maintain a state of cultural diversity?

  Or do we have to explicitly include mechanisms of differentiation?

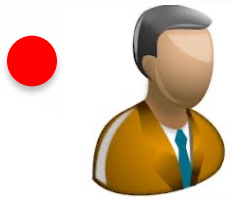
A discrete multidimensional model

R. Axelrod, The Journal of Conflict Resolution 41, 203 (1997).

Axelrod's model



neighbor



agent

Cultural state of each agent: Vector of dimension F (=3) and Q (=2) traits

0	0	0	0	1	1	1	1
0	0	1	1	0	0	1	1
0	1	0	1	0	1	0	1

Language: English (1)/Spanish (0)

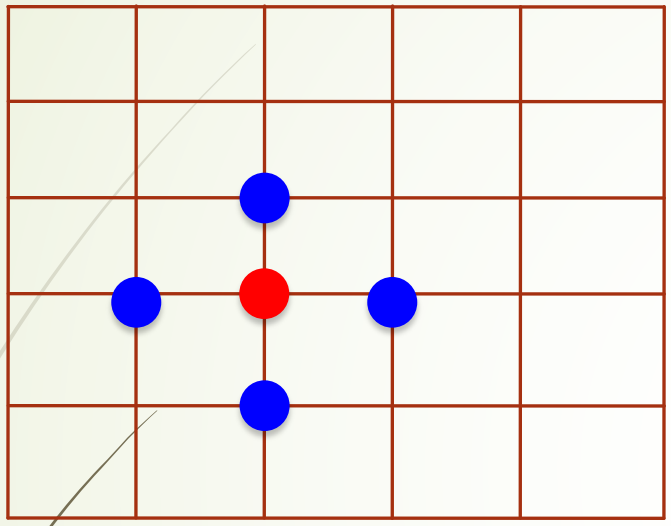
Polítics: Democrat (1)/Republican (0)

Civil State: Single (1) / Married (0)

→ e/color defines a given state

Other possible features: Sports, Music, etc

Dynamics on Axelrod's model



Active Agent

Elected Neighbour

They become similar

The more similar, the more able to interact.

Ej. 1

0	1		0	*
0	0	P=2/3 →	0	
0	0		0	

Ej. 2

0	0		0	
0	1	P=1/3 →	0	*
0	1		1	

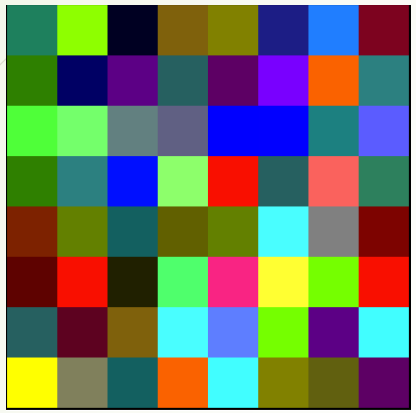
For fixed features(F): larger values of Q →More cultures co-exist

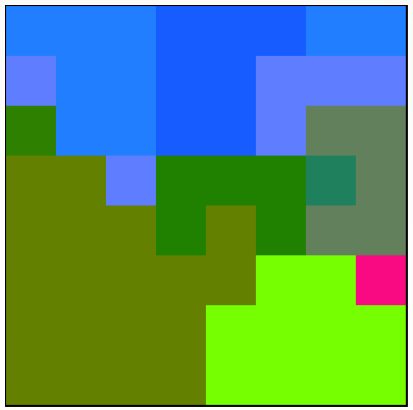
t=0

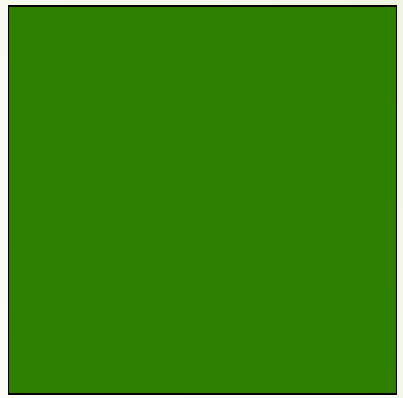
Intermediate times

Asymptotic

Q=5







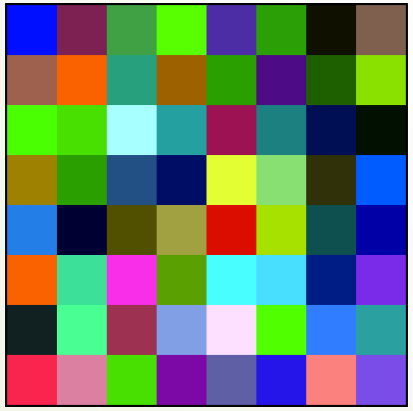
(F_1,F_2,F_3,F_4,F_5)

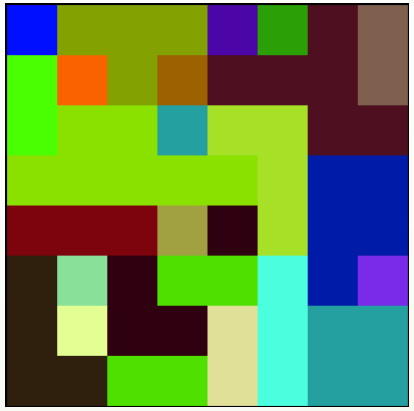
F=5

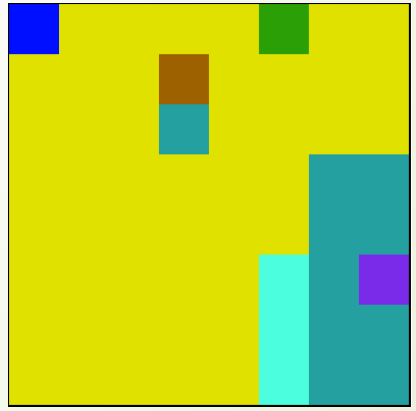
Mono-cultural state

Flecha del tiempo

Q=10





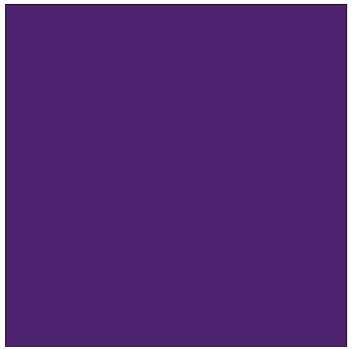


(F_1,F_2,F_3,F_4,F_5)

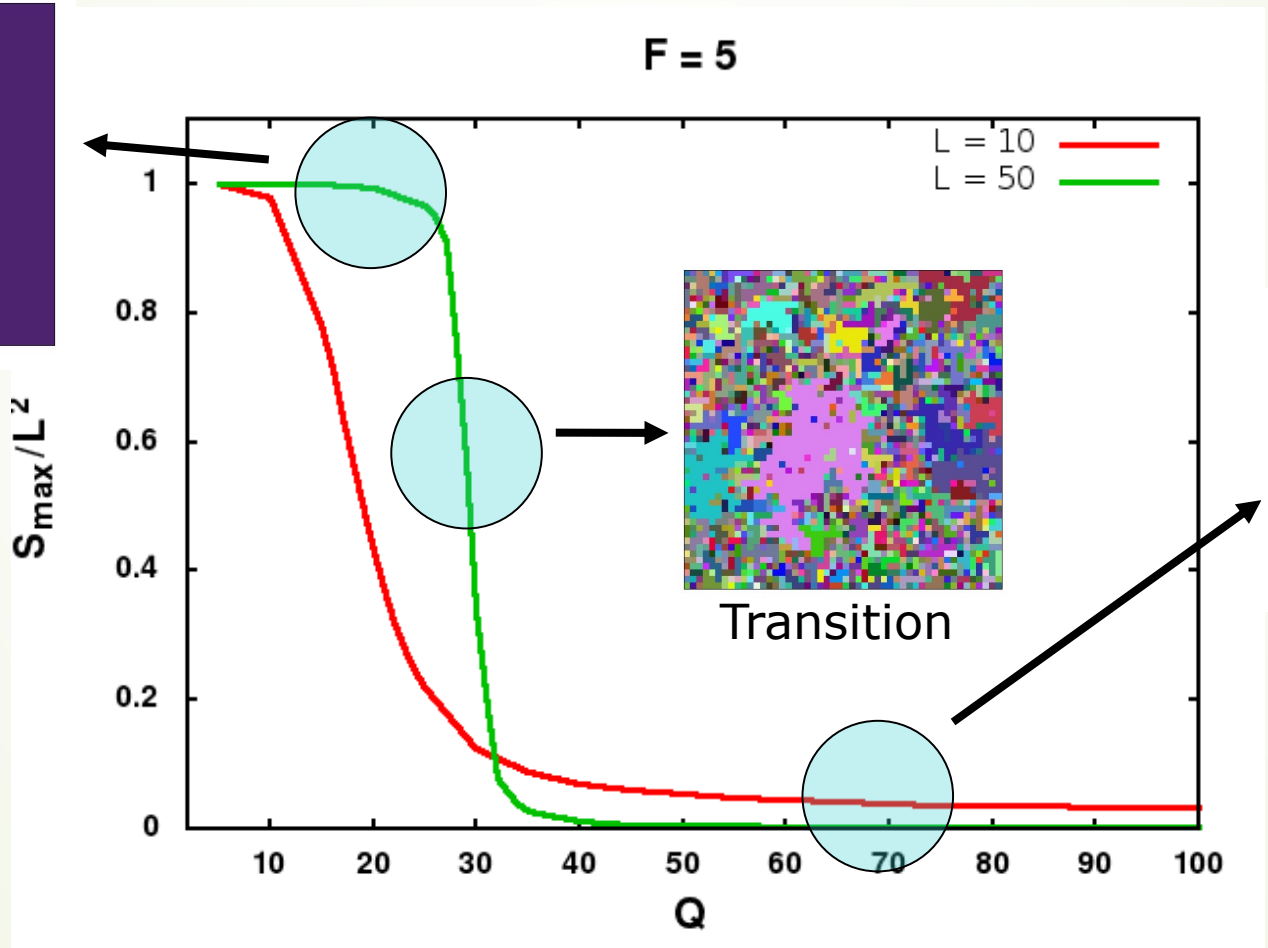
Diversity

Order-disorder transition in a 2D grid

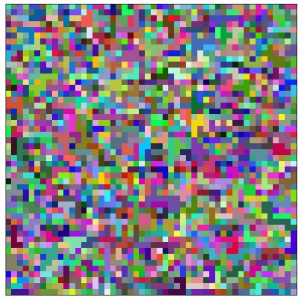
Order Parameter: Normalized size of the biggest region



Monocultural



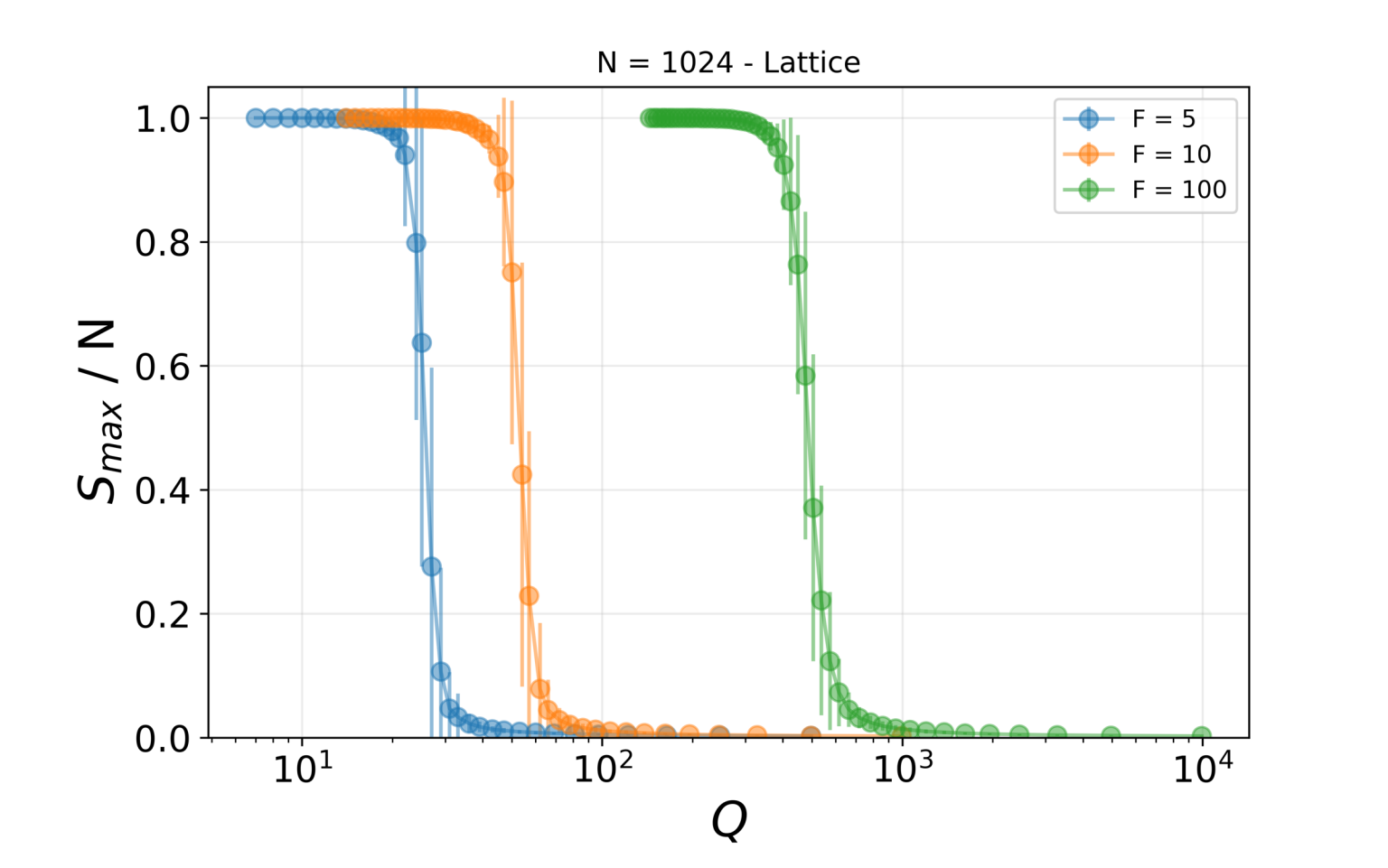
Transition



Multicultural

Order-disorder transition in a 2D grid

Dependence of the transition with F



Q_c (F=5) = 28
Q_c (F=10) = 55
Q_c (F=100) = 490

The larger F, the larger Q_c

The nature of the transition

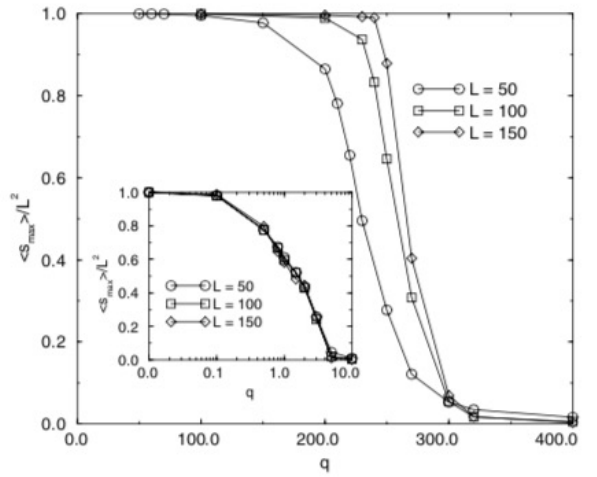


FIG. 1. Behavior of s_{max}/L^2 vs q for three different system sizes and $F = 10$. In the inset the same quantity is reported for $F = 2$.

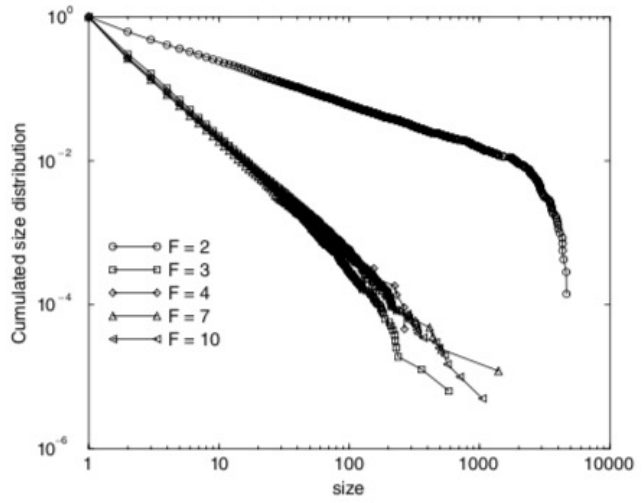


FIG. 2. Cumulated distribution $U_L(s,q)$ of region sizes for $q \approx q_c$, $L = 100$, and several values of F.

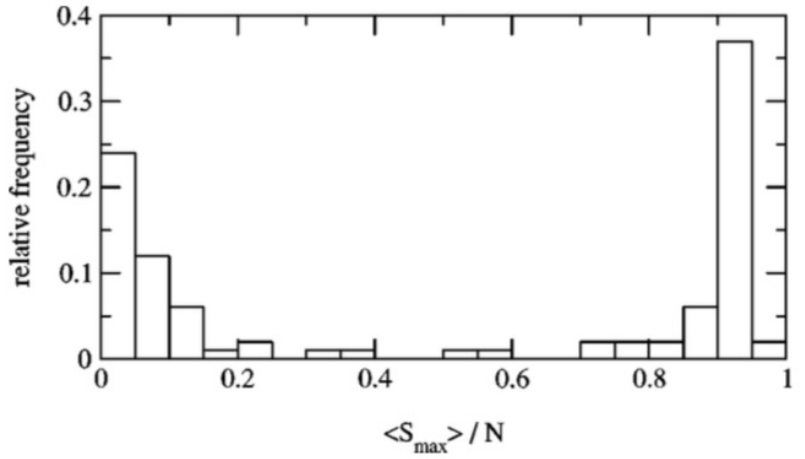


FIG. 2. Distribution of the order parameter at $q = 55$, $F = 10$ in a square lattice of size $N = 100^2$. The distribution is based on 100 independent realizations.

Phase Transition: 2nd Order for F=2
1st Order for F > 2 (3-10)

- Nonequilibrium Phase Transition in a Model for Social Influence,
C. Castellano, M. Marsili, and A. Vespignani. Phys. Rev. Lett. 85, 3536 (2000)

- Nonequilibrium transitions in complex networks: A model of social interaction
K. Klemm, V. M. Eguíluz, R. Toral, and M. San Miguel Phys. Rev. E 67, 026120 (2003)

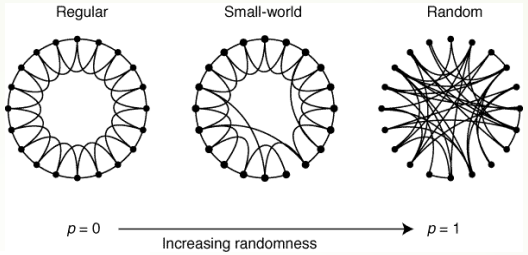
Dependence with underlying topology

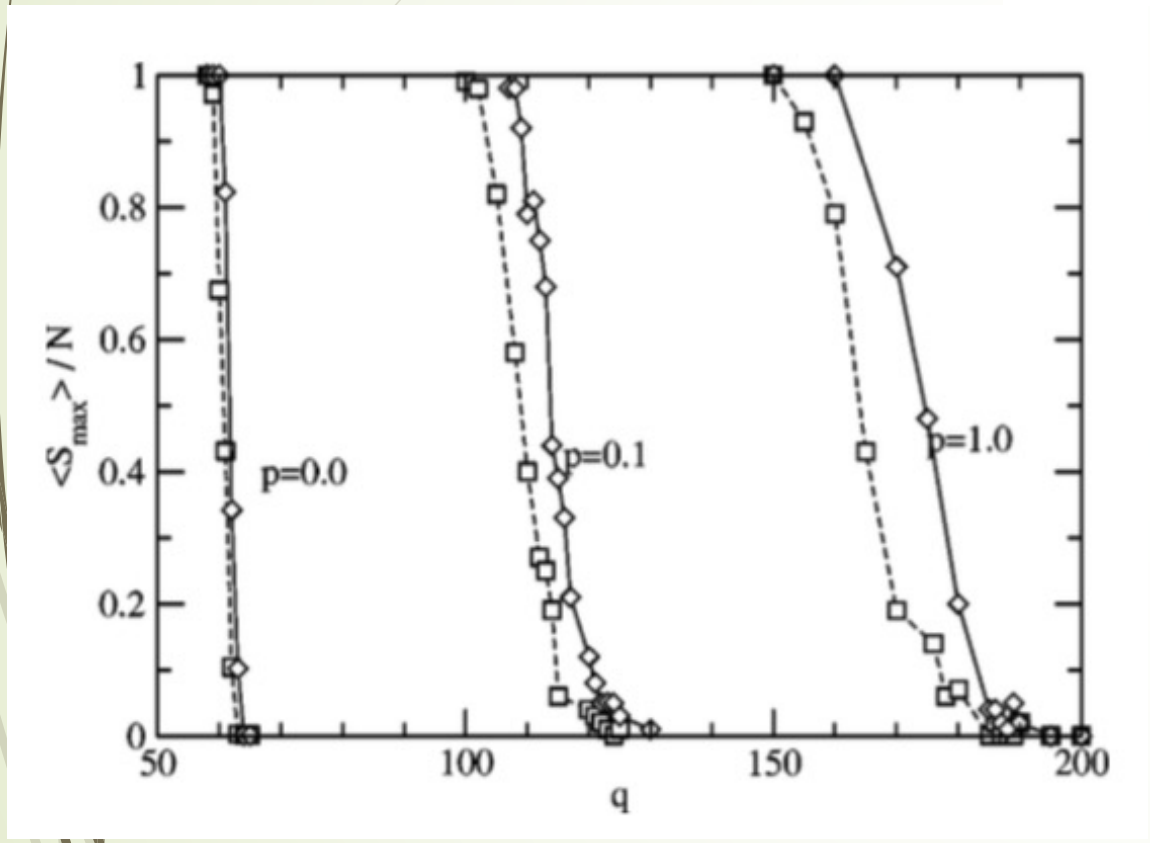
- Nonequilibrium transitions in complex networks: A model of social interaction
K. Klemm, V. M. Eguíluz, R. Toral, and M. San Miguel Phys. Rev. E 67, 026120 (2003)

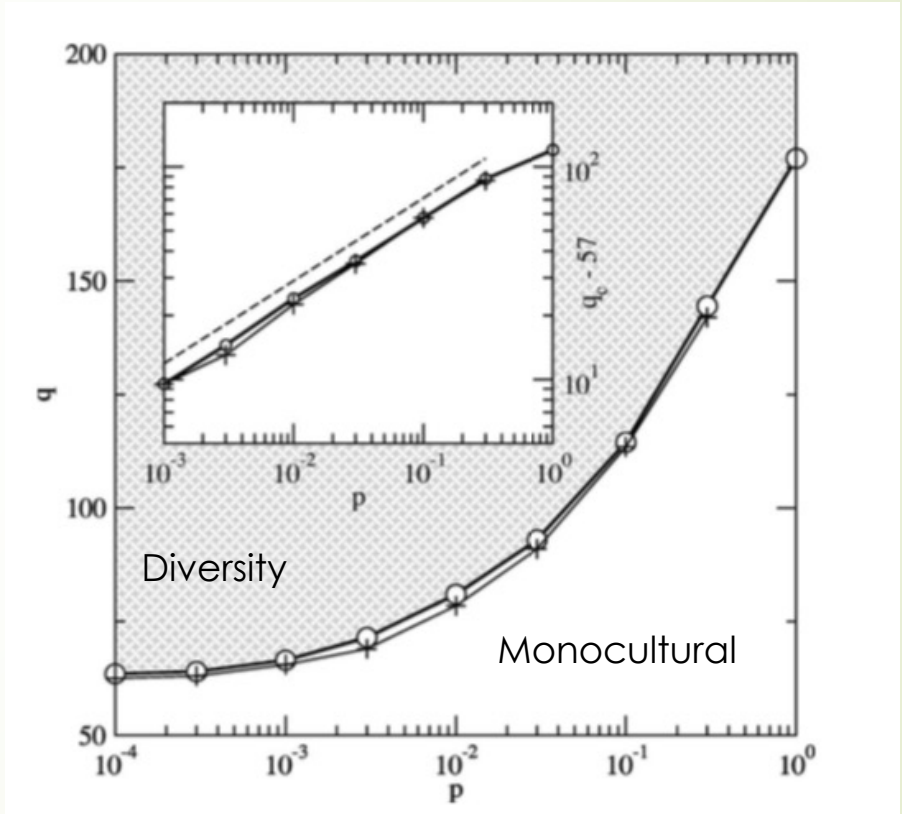
- Dynamical organization towards consensus in the Axelrod model on complex networks,
B. Guerra, J. Poncela, J. Gómez-Gardeñes, V. Latora, and Y. Moreno Phys. Rev. E 81, 056105 (2010)

Phase transition in a small world network (F=10)

q_c increase with the amount of spatial disorder



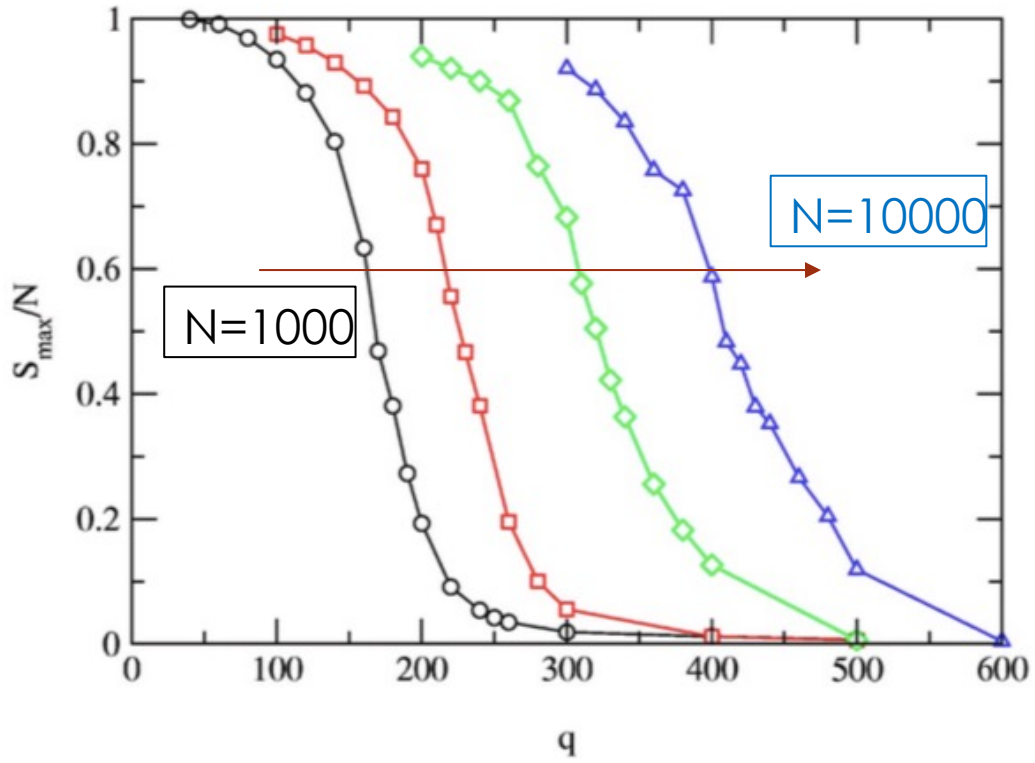


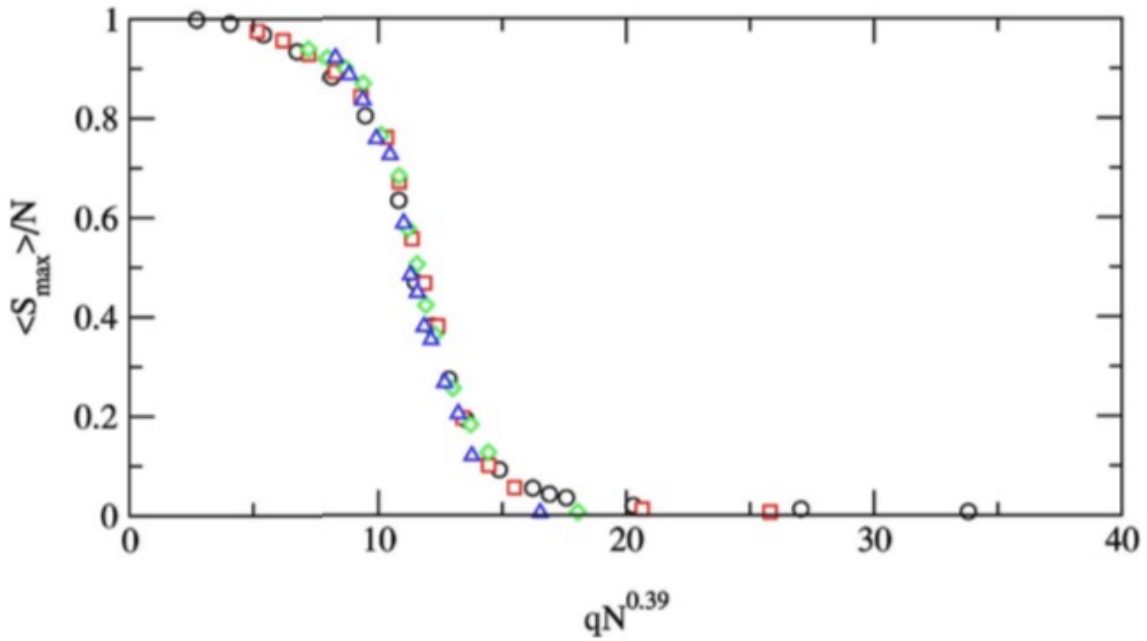


- Nonequilibrium transitions in complex networks: A model of social interaction K. Klemm, V. M. Eguíluz, R. Toral, and M. San Miguel Phys. Rev. E 67, 026120 (2003)

Phase transition in a scale free network

Size dependence of the transition for F=10

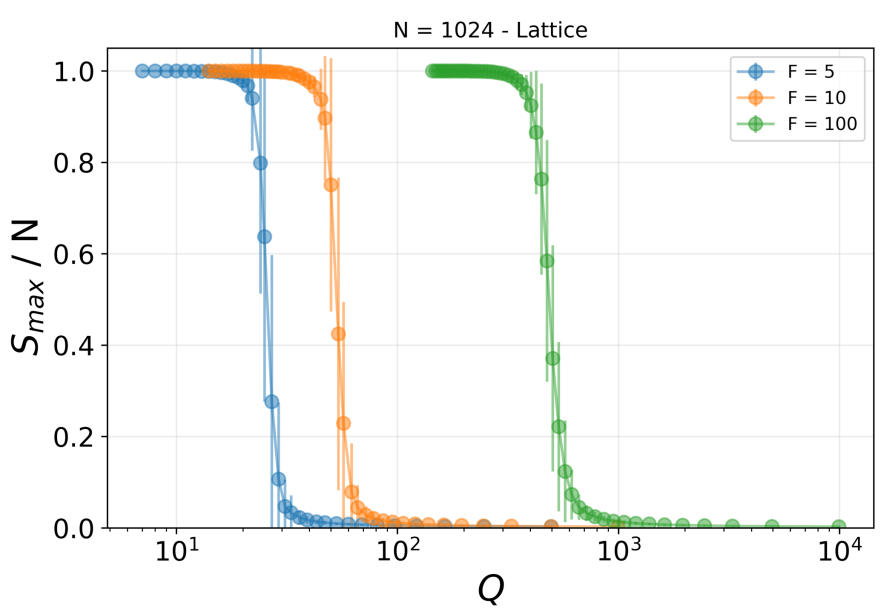


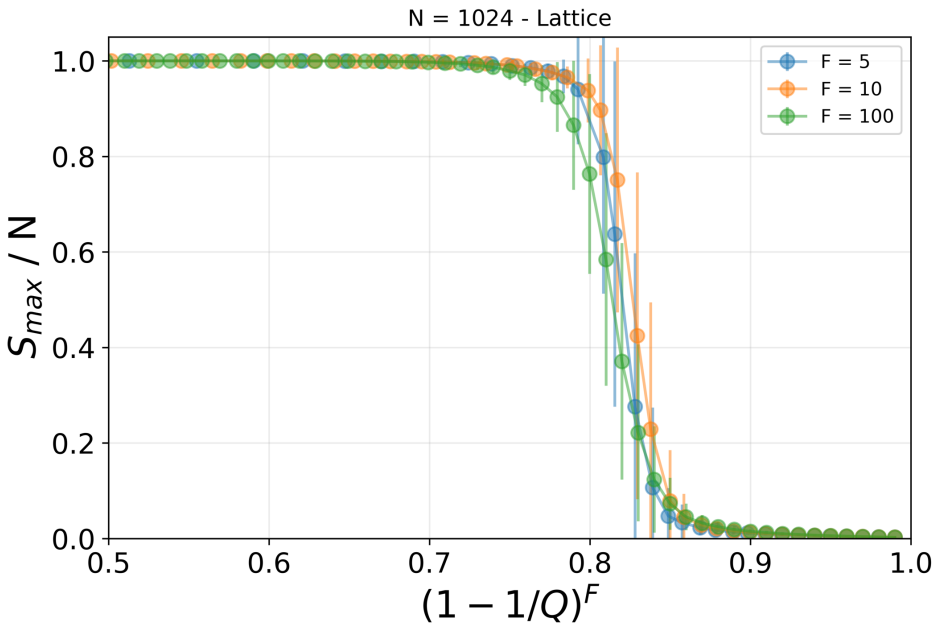


$q_c \sim N^{0.39}$ - The transitions disappear in the thermodynamics limit .

- Nonequilibrium transitions in complex networks: A model of social interaction
K. Klemm, V. M. Eguíluz, R. Toral, and M. San Miguel Phys. Rev. E 67, 026120 (2003)

There are scaling relation in 2D Grid





- The relevant quantity is homophily (h: how similar they are)
- $\rho = (1 - {}^{1}/_{Q})^{F}$ is the probability of having a link with h=0 in the initial state.
- If two agents have h=0 they will not interact.

In the limit $F \to \infty$:

If we use same scaling in complete networks, phase transition is fully determined by initial conditions in this limit

PHYSICAL REVIEW E **101**, 052319 (2020)

Erdós-Rényi phase transition in the Axelrod model on complete graphs

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and Instituto de Física de Buenos Aires (IFIBA), CONICET. Av.Cantilo s/n, Pabellón 1, Ciudad Universitaria, 1428 Buenos Aires, Argentina

(Received 20 December 2019; revised manuscript received 31 March 2020; accepted 5 May 2020; published 28 May 2020)

The Axelrod model has been widely studied since its proposal for social influence and cultural dissemination. In particular, the community of statistical physics focused on the presence of a phase transition as a function of its two main parameters, F and Q. In this work, we show that the Axelrod model undergoes a second-order phase transition in the limit of $F \to \infty$ on a complete graph. This transition is equivalent to the Erdős-Rényi phase transition in random networks when it is described in terms of the probability of interaction at the initial state, which depends on a scaling relation between F and Q. We also found that this probability plays a key role in sparse topologies by collapsing the transition curves for different values of the parameter F.

DOI: 10.1103/PhysRevE.101.052319

Axelrod transition is of 2nd order and from the same class of random network (ER) phase transition

Can we obtain master equations in Axelrod's Model?

PHYSICAL REVIEW E **103**, 012307 (2021)

Analytical approach to the Axelrod model based on similarity vectors

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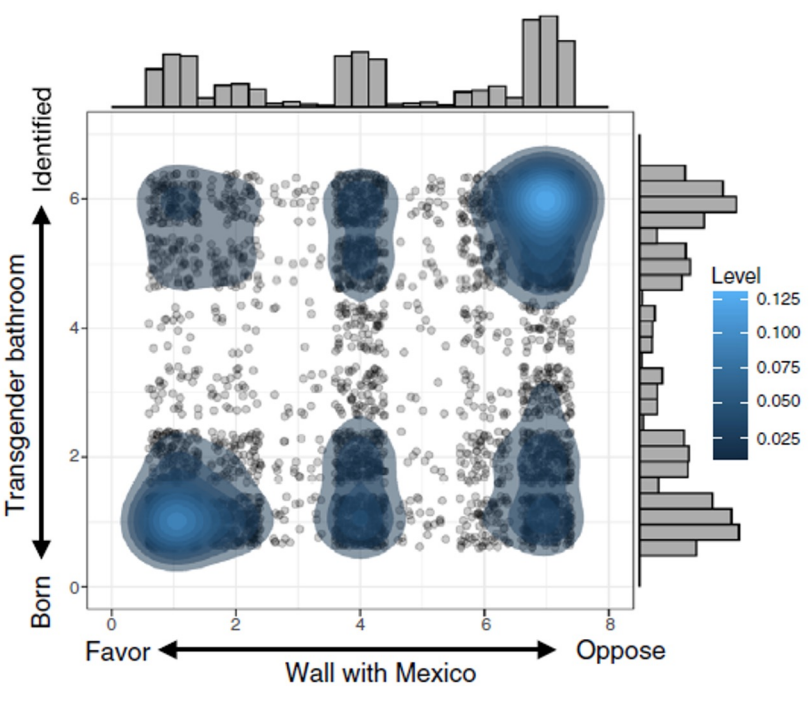
Complex problems of social interaction are usually studied within the framework of agent-based models. Some of these problems, such as issue alignment and opinion polarization, are better suited in the framework of n-dimensional opinion space. Although this kind of complex problem may be explored by numerical simulations, these simulations can hinder our ability to obtain general results. In this work, we show how, under certain conditions, a classical multidimensional opinion model such as the Axelrod model can give rise to a closed set of master equations in terms of vector similarities between agents. The analytical results fully agree with the simulations on complete networks, accurately predict the similarity distribution of the whole system in sparse topologies, and provide a good approximation of the similarity of physical links that improves when the mean degree of the system increases.

DOI: 10.1103/PhysRevE.103.012307

Yes for for complete networks in F=2 & F=3

Multidimentional opinion models: there are more complexity to be considered



- Topics are rarely discussed in isolation
- individuals are much more likely to have a certain combination of opinions than others.
- It can be modelled by assuming that topics are not independent of each other.
- A non-orthogonal basis of topics should be used.
- This idea is grounded in topic decomposition when analyze corpus of data using NLP (Natural Language Processing)

Data from ANES surveys

ANES
American National Election Studies

A multidimensional continuous model with non-orthogonal topics

F. Baumann, P. Lorenz-Spreen, I. M. Sokolov, and M. Starnini

Phys. Rev. X 11, 011012 (2021)

Opinion model in non orthogonal N dimensions

PHYSICAL REVIEW X **11,** 011012 (2021)

Emergence of Polarized Ideological Opinions in Multidimensional Topic Spaces

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Agents are represented by opinion vectors of opinion x_i:

$$x_i = (x_i^1, x_i^2, \ldots, x_i^{T-1}, x_i^T) \qquad x_i^v \in [-\infty, \infty]$$

For each topic v, agent "i" have an opinion x_i^v :

- $\mathrm{sgn}(x_i^v)$: describes the qualitative stance towards the topic (in favor or against)
- $|x_i^v|$: quantifies the strength of his/her opinion about topic v.

Opinion model in non orthogonal 2 dimensions

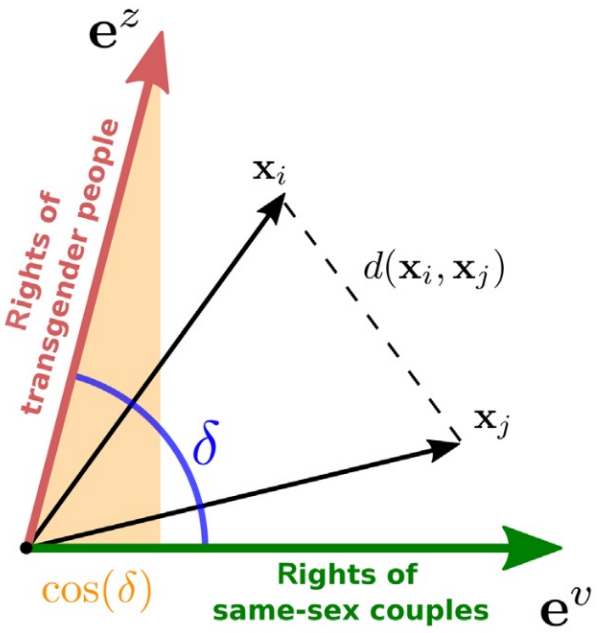


FIG. 1. Illustration of two nonorthogonal topics as a basis for the topic space $\mathcal{T}$. For $T = 2$, the nonorthogonal, normalized basis is uniquely defined by the angle δ. Geometrically, $\cos(\delta)$ quantifies the overlap between basis vectors, interpreted as a topical overlap, here the rights of same-sex couples ($\mathbf{e}^{(u)}$) and transgender people ($\mathbf{e}^{(v)}$). The opinion distance between two agents i and j, $d(\mathbf{x}_i, \mathbf{x}_j)$, is computed by the scalar product defined in Eq. (2).

In 2D dynamical equations for opinion vectors x_i can be written as:

$$\dot{x}_i^1 = -x_i^1 + K \sum_j A_{ij}(t)\, tanh(\alpha(x_j^1 + cos(\delta)x_j^2))$$

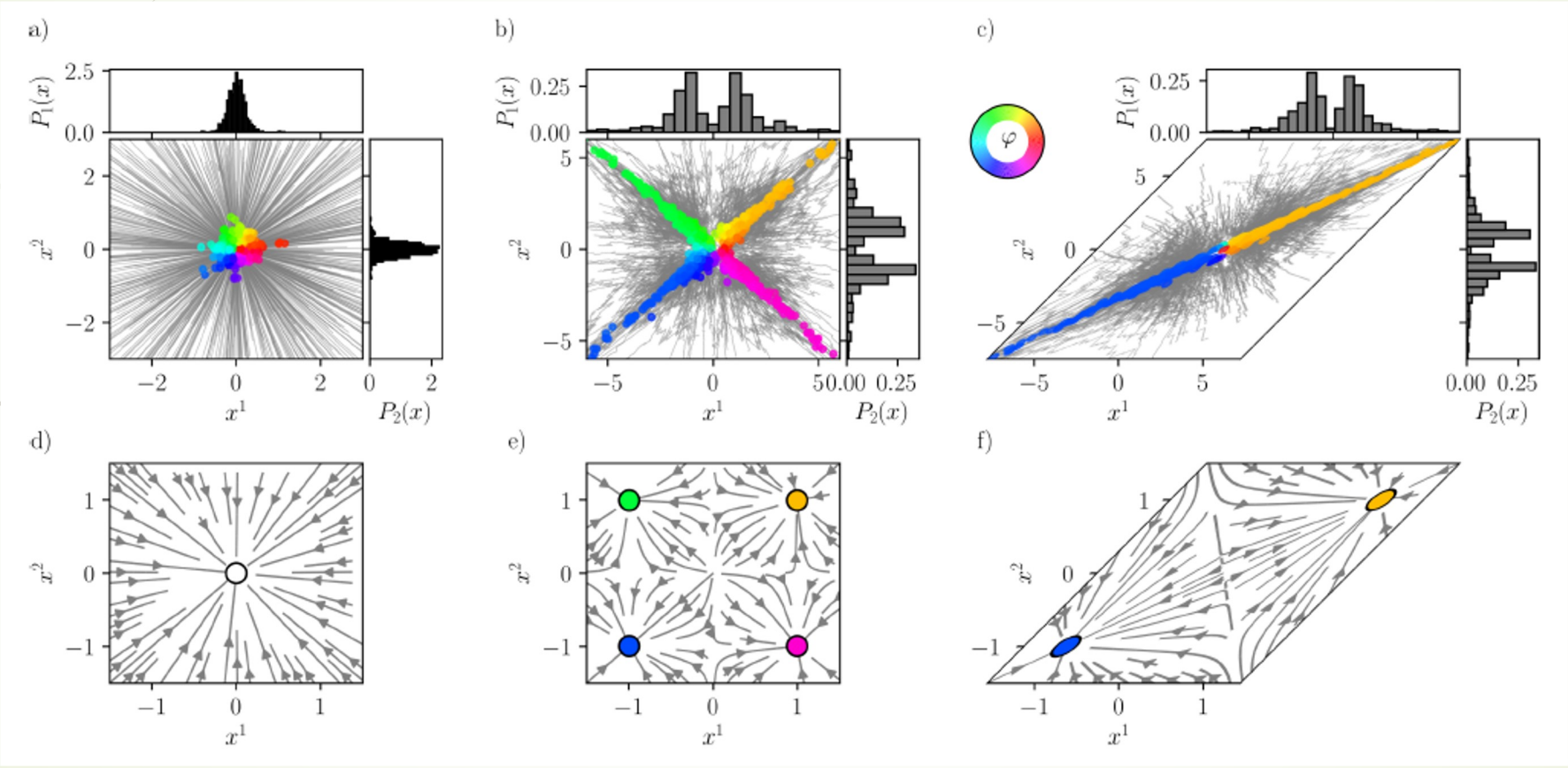
$$\dot{x}_i^2 = -x_i^2 + K \sum_j A_{ij}(t)\, tanh(\alpha(cos(\delta)x_j^1 + x_j^2))$$

Where:

K is the strength of the social influence (a global parameter)
$A_{ij}(t)$ is the temporal adjacency matrix.
$\boldsymbol{\alpha}$ is the controversialness of a topic
Φ is the topic overlap matrix

$$\begin{pmatrix} 1 & cos(\delta) \\ cos(\delta) & 1 \end{pmatrix}$$

Collective states (depends on δ y α)

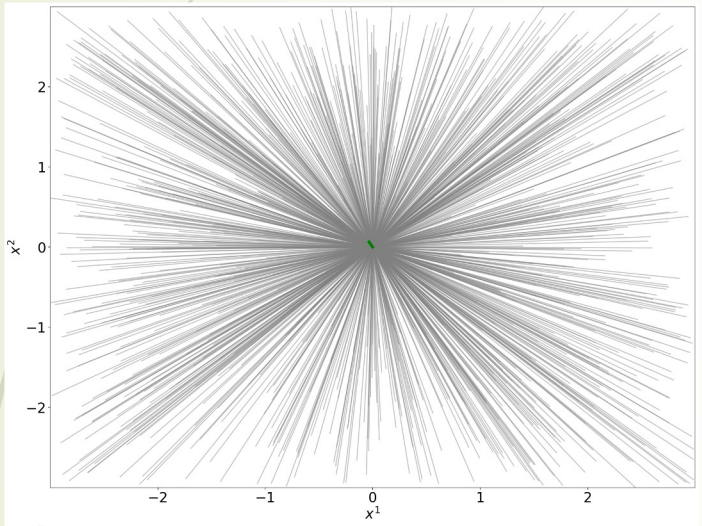


Consensus
(low α)

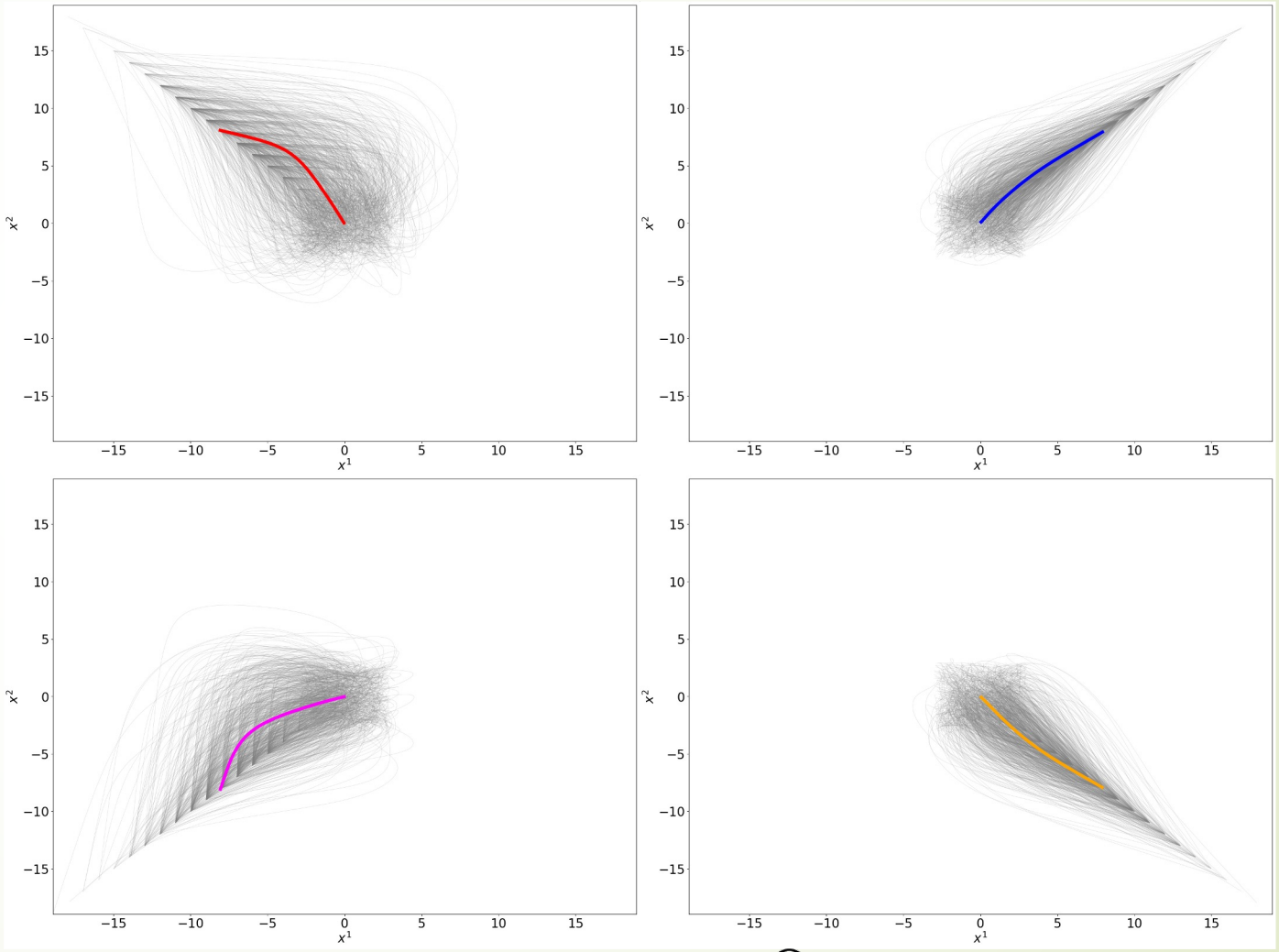
Uncorrelated polarization
(low α and δ)

Ideological states (correlated polarization)
(low α and high δ)

Dynamics of the network



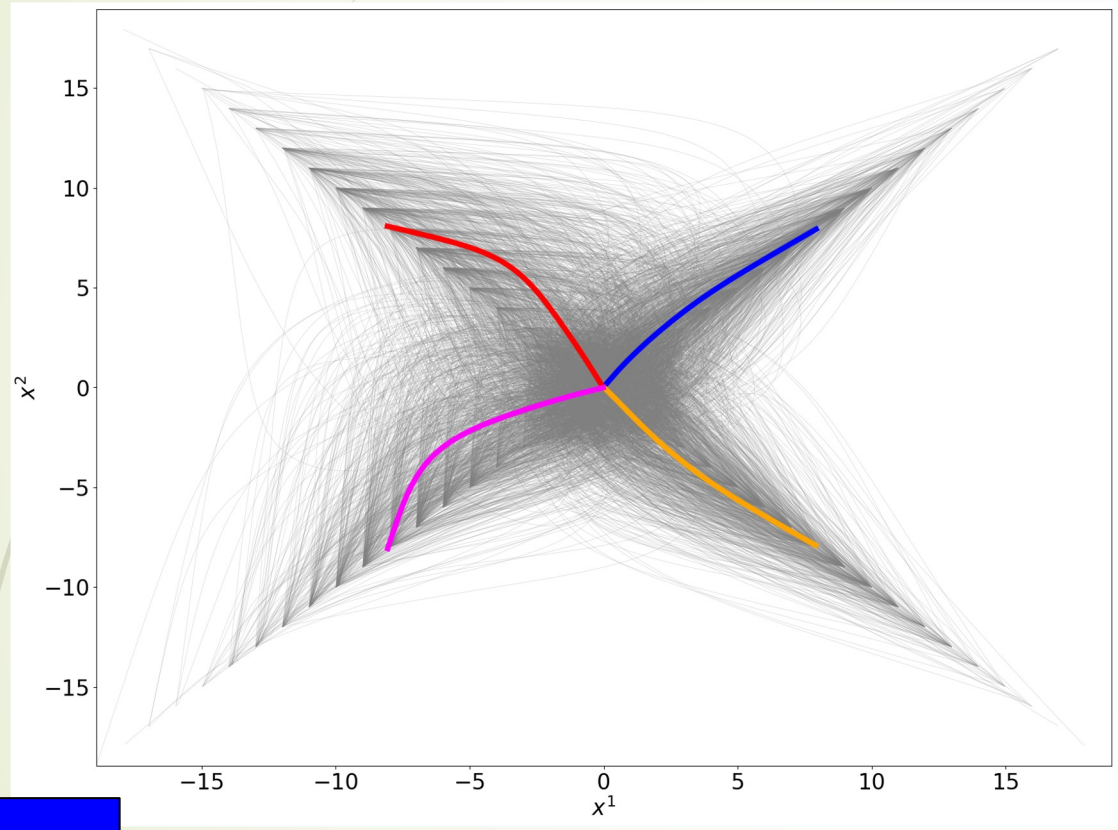
$$\alpha \approx 0$$

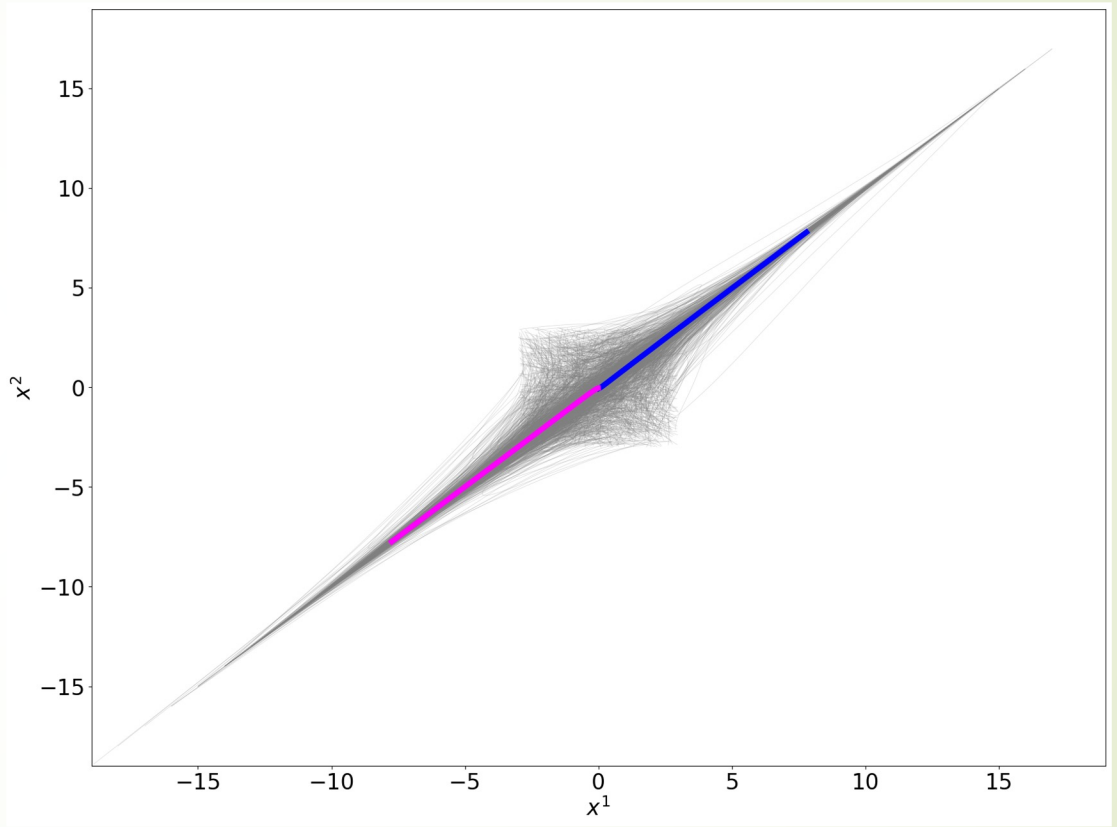


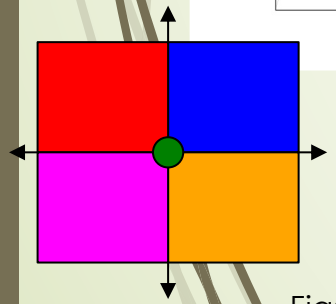
$$\alpha \gg 0$$

Figs. From F. Di Ciocco Msc. Thesis

Dynamics of the network







$$\alpha \gg 0 \quad cos(\delta) \approx 0$$

$$\alpha \gg 0 \quad |cos(\delta)| \gg 0$$

Figs. From F. Di Ciocco Msc. Thesis

Network dynamics

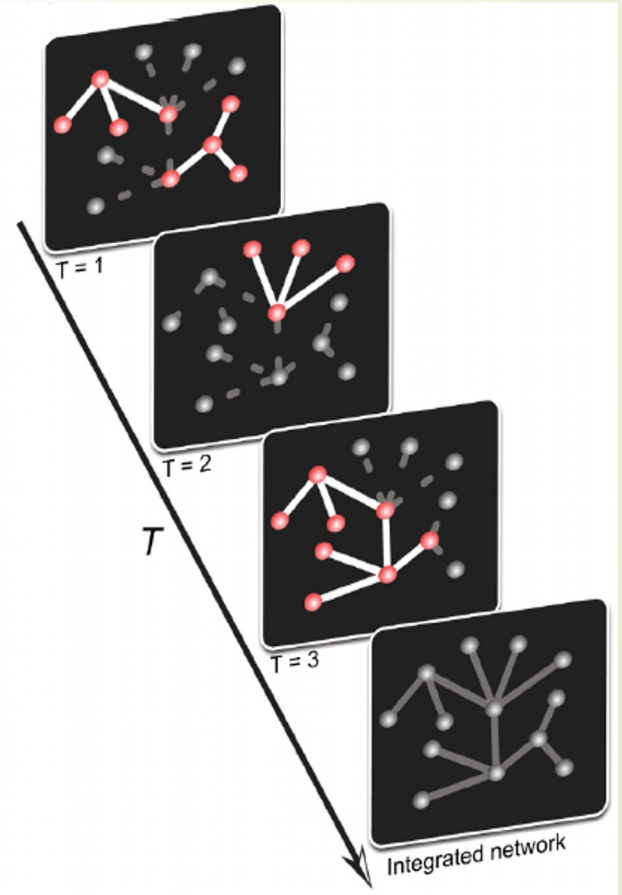
- Contacts between individuals (A_{ij} (t)) evolve according to activity driven model .
- This model produces a temporal network which changes at discrete time intervals, and can be described like this:

- Each node is characterized by an activity $a_i \in [\varepsilon, 1]$. a_i is randomly chosen from a power law distribution $F(a) \sim a^{-\gamma}$.
- Upon activation, agent i contacts ***m*** distinct other agents chosen at random with probability $\boldsymbol{p_{ij}}$ given by:

$$p_{ij} = \frac{d(\boldsymbol{x_i}, \boldsymbol{x_j})^{-\beta}}{\sum_j d(\boldsymbol{x_i}, \boldsymbol{x_j})^{-\beta}}$$

β is the exponent that control homophily.

Active agents are activated with higher probability and, the higher the β, the most similar "m" agents are connected.



(*) R. Pastor-Santorras y A. Vespignani N. Perra B. Gonçalves. "Activity driven modeling of time". En: Scientific Reports 2.1 (2012),

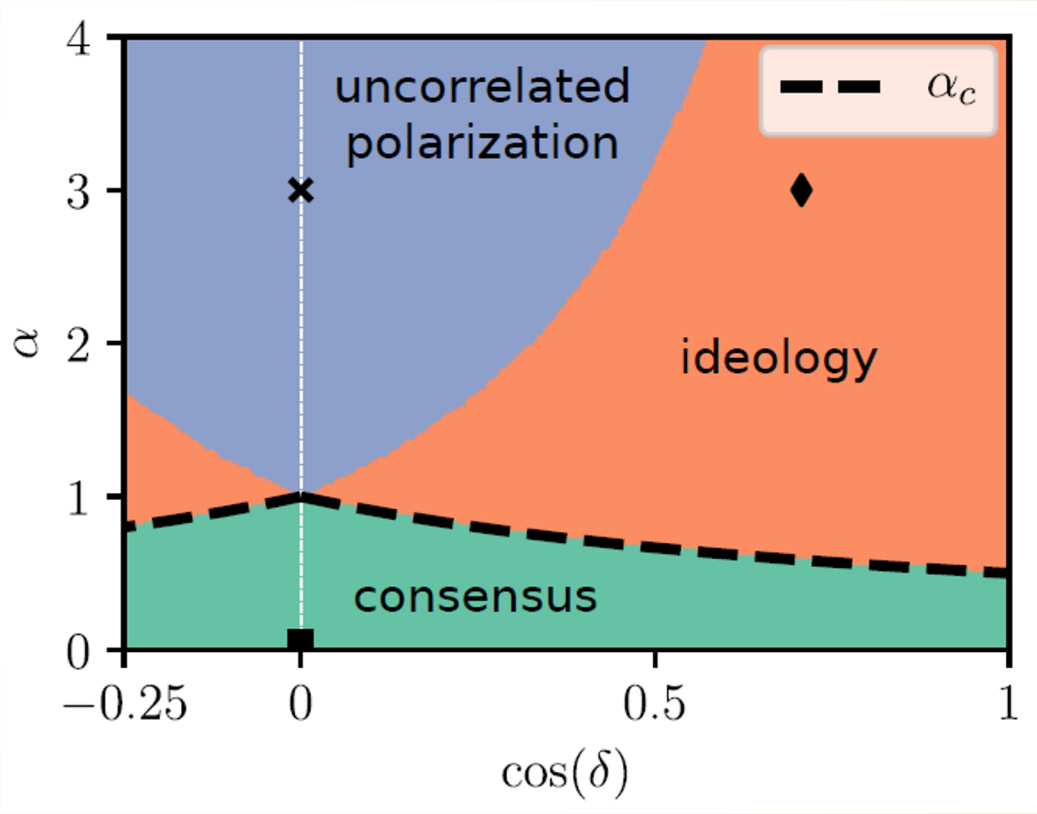
Mean field approximation

In the limit $N \to \infty$ and considering high values of homophily $\beta >> 1$, the dynamics of the model can be captured by a mean field approach.

- They analyze the stability of the stable fixed point at **x**=0 (global consensus.
- This stability depends on α and δ. The consensus is stable if:

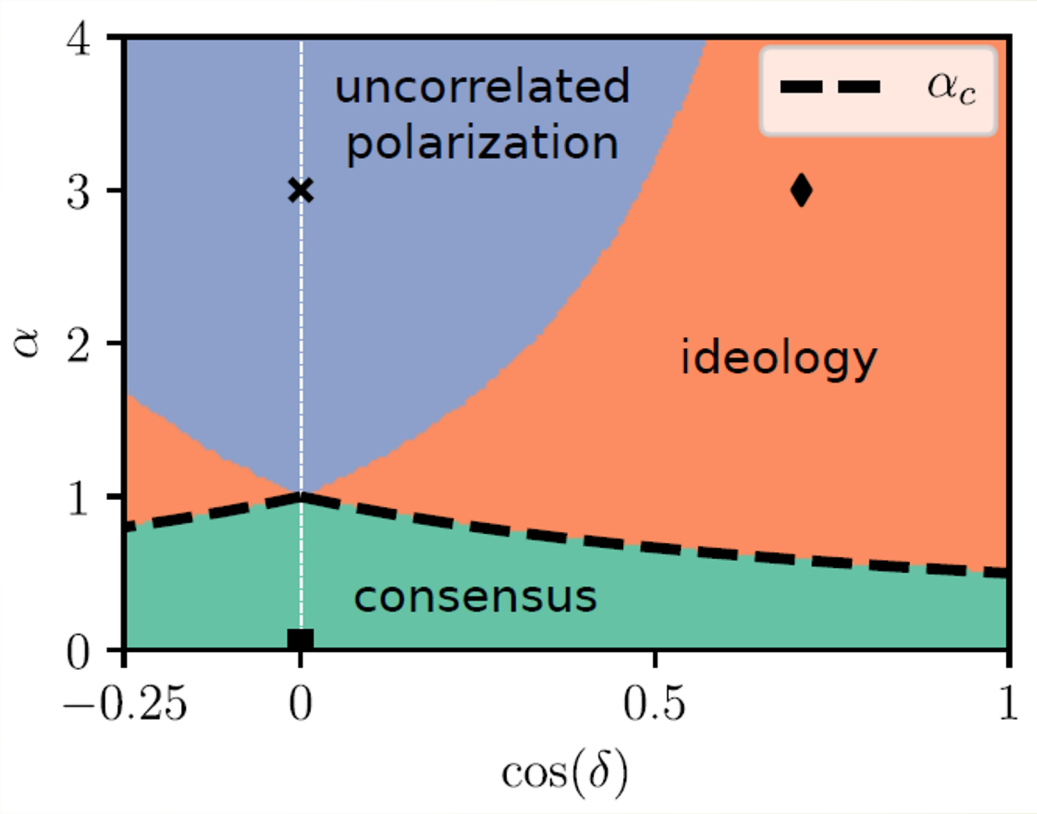
$$\alpha < \frac{1}{K\langle k[1 + cos(\delta)]\rangle} = \alpha_c$$

Model Phase Diagram



Model Phase Diagram

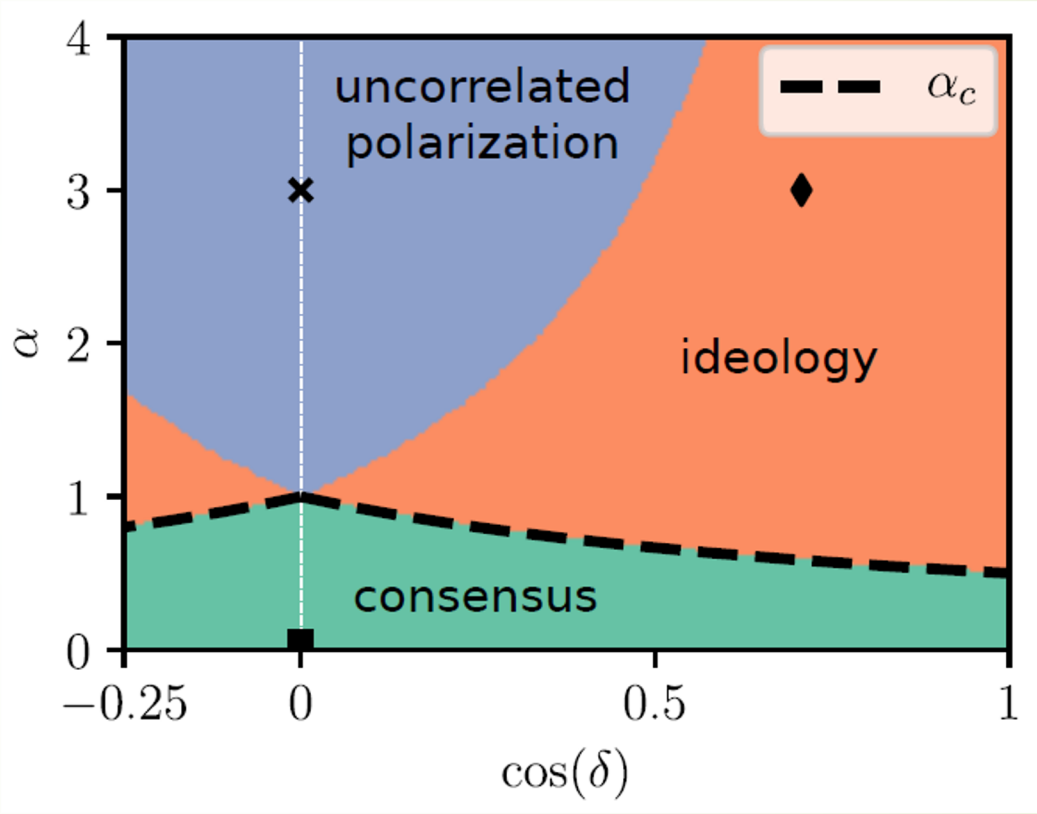
$$\alpha_c = \frac{1}{Km\langle a\rangle[1 + cos(\delta)]}$$



Model Phase Diagram

$$\alpha_c = \frac{1}{Km\langle a\rangle[1+cos(\delta)]}$$

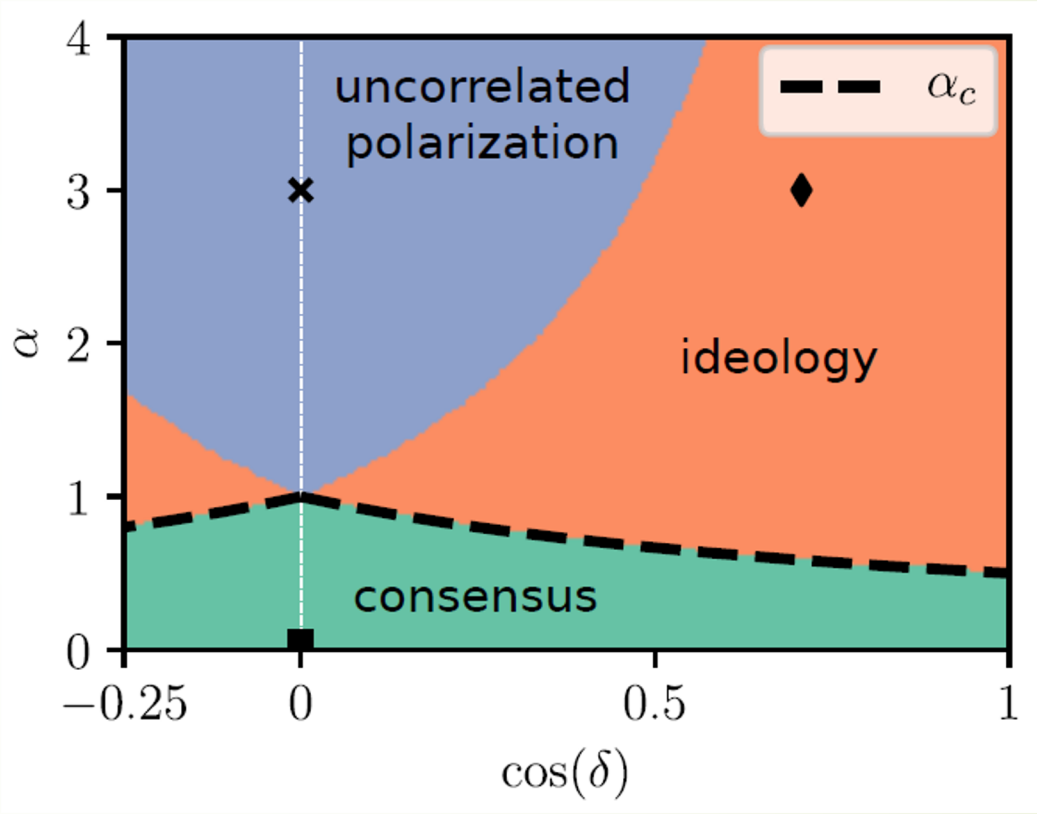
$$\dot{x}_i^1 = -x_i^1 + K\sum_j A_{ij}(t)\, tanh(\alpha(x_j^1 + cos(\delta)x_j^2))$$



Model Phase Diagram

$$\alpha_c = \frac{1}{K\langle m\rangle\langle a\rangle[1 + cos(\delta)]}$$

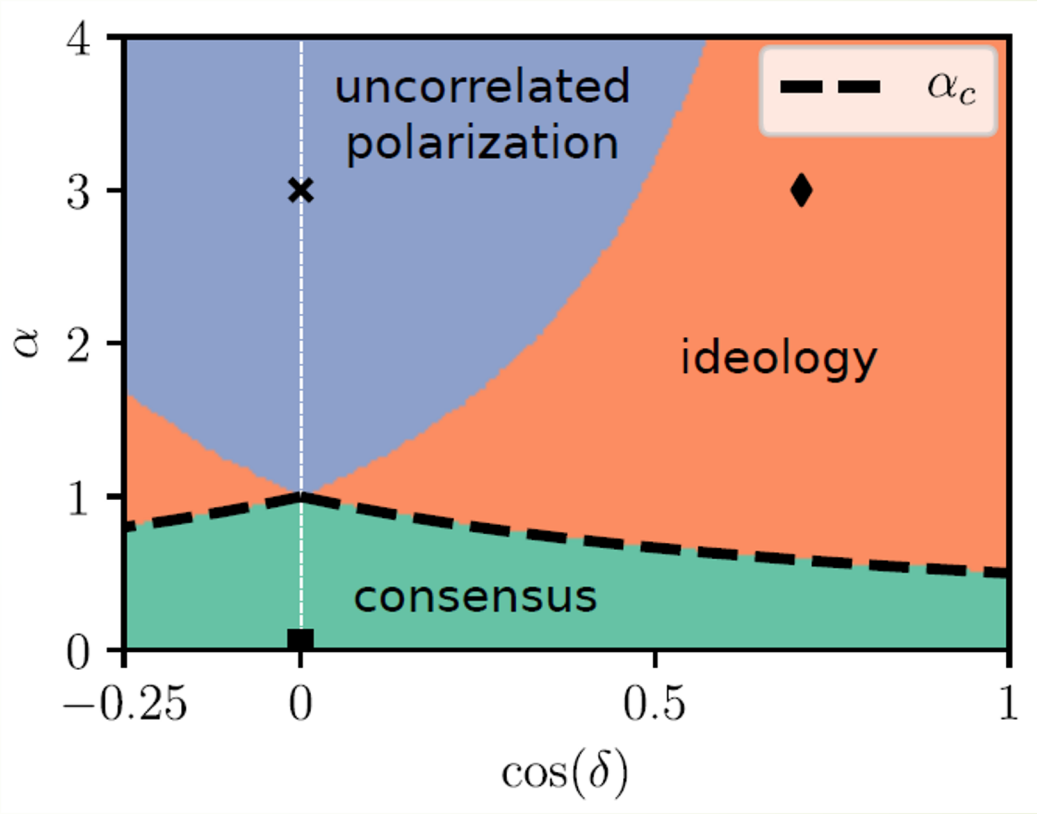
Number of nodes that an agent activates on connection



Model Phase Diagram

$$\alpha_c = \frac{1}{Km\langle a \rangle[1 + cos(\delta)]}$$

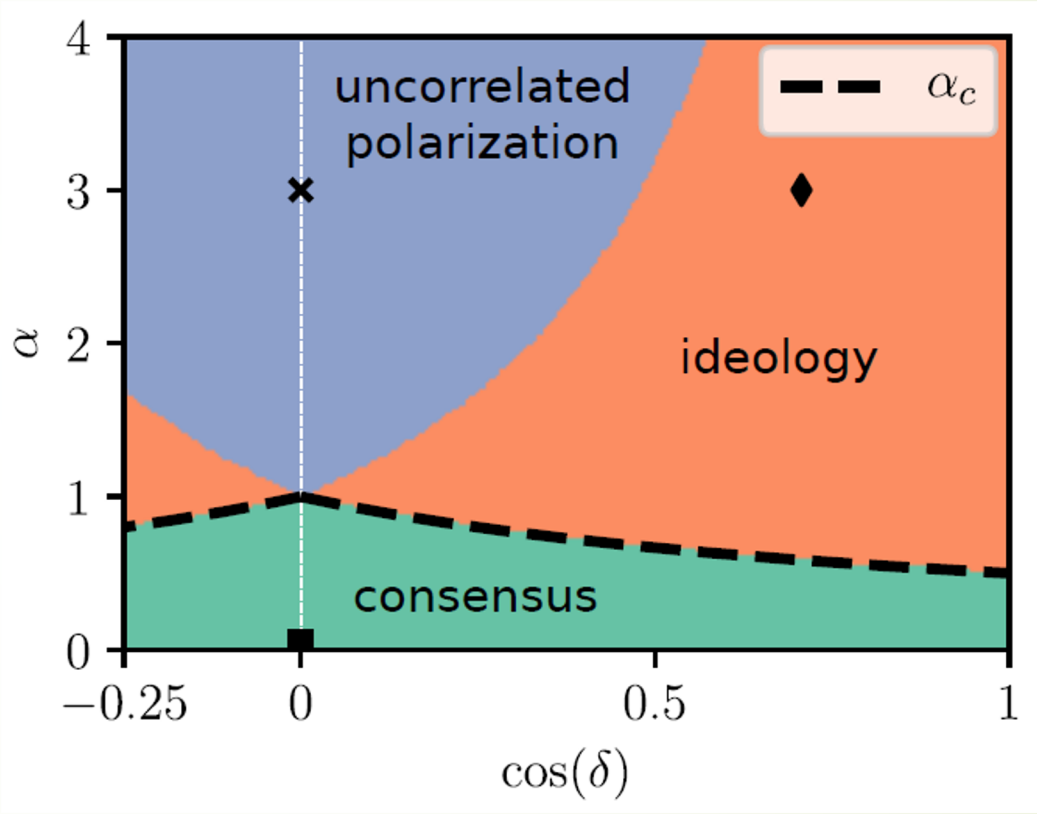
Mean activity of the network: m<a> = <k>



Model Phase Diagram

$$\alpha_c = \frac{1}{Km\langle a\rangle[1+cos(\delta)]}$$

$$\dot{x}_i^1 = -x_i^1 + K\sum_j A_{ij}(t)\, tanh(\alpha(x_j^1 + cos(\delta)x_j^2))$$



Comparison with echo-chambers

The join evolution of opinion and network dynamics can given rise to the formation of echo-chambers. The emerging topology is shaped by opinion dynamics

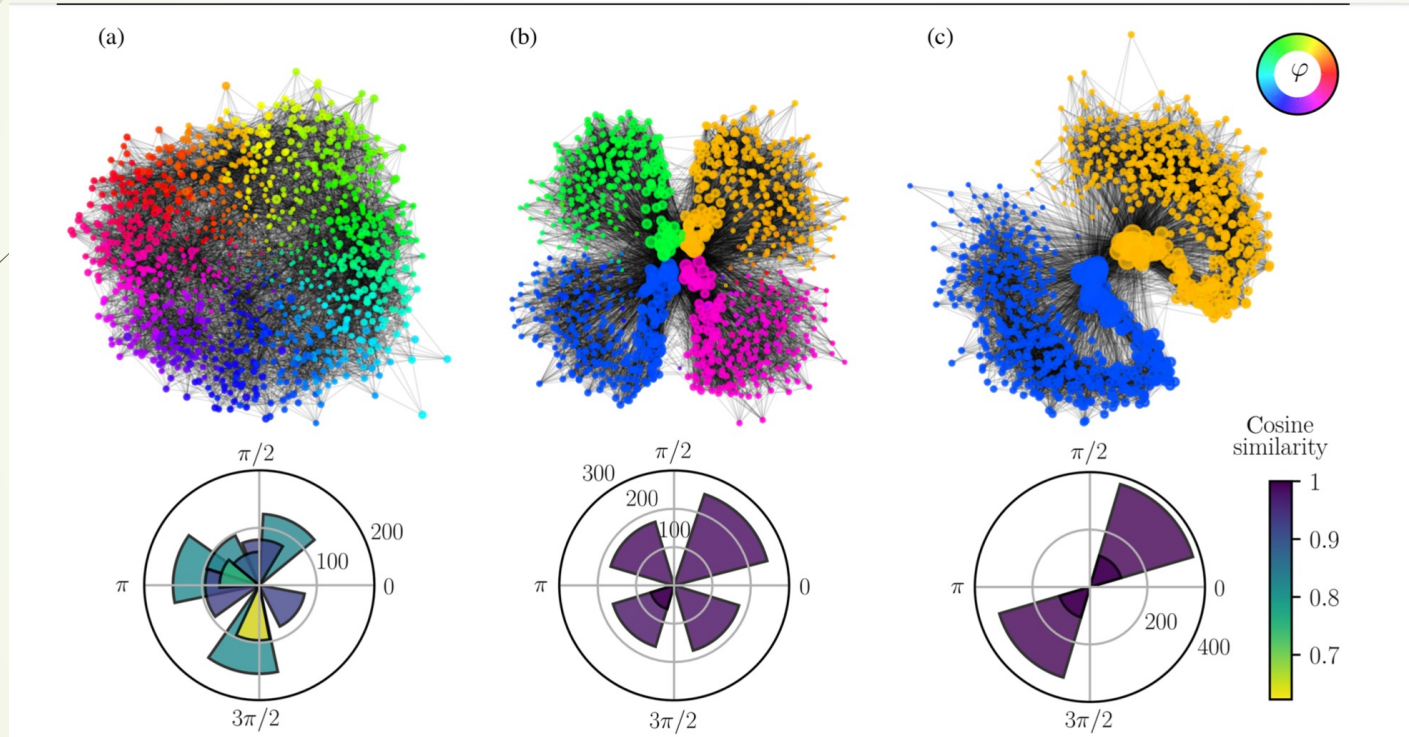


FIG. 5. Community structure of the social networks. Visualization of the social networks aggregated over the last 70 time steps (top) and corresponding community detection (bottom) for three different dynamical regimes: (approaching) consensus (a), uncorrelated polarized state (b), and ideological state (c). The model parameters are set as in Figs. 2(a)–2(c), i.e., $\alpha = 0.05$, $\delta = \pi/2$ (a), $\alpha = 3$, $\delta = \pi/2$ (b), and $\alpha = 3$, $\delta = \pi/4$ (c). Top: In the network illustrations, each node is colored according to its opinion angle φ, and its size is proportional to its conviction r. Bottom: Communities are represented in the polar bar plot below each network. Each community is represented by a bar: The radius represents the size, and color and width correspond to the average cosine similarity between all pairs of agents within the community. The orientation represents the average opinion angle $\langle\varphi\rangle$ of all agents within the community. Communities containing less than 5% of the total number of nodes are not shown.

ANES surveys

They have been conducted since 1948 to analyze public opinion and the electoral behavior of society. Those of this study correspond to 2016. They choose 67 questions answered by 253984 people.

Question label	Question/Issue	ANES ID
Obamacare	Summary: favor/oppose 2010 health care law **V161114a**: IF R FAVORS THE 2010 HEALTH CARE LAW: Do you favor that [a great deal, moderately, or a little / a little, moderately, or a great deal]? **V161114b**: IF R OPPOSES THE 2010 HEALTH CARE LAW: Do you oppose that [a great deal, moderately, or a little / a little, moderately, or a great deal]?	V161114x
voting: duty or choice	Summary: Voting as duty or choice **V161151a**: IF R CONSIDERS VOTING A DUTY: How strongly do you feel that voting is a duty? [Very strongly, moderately strongly, or a little strongly / A little strongly, moderately strongly, or very strongly]? **V161151b**: IF R CONSIDERS VOTING A CHOICE: How strongly do you feel that voting is a choice? [Very strongly, moderately strongly, or a little strongly / A little strongly, moderately strongly, Or very strongly]?	V161151x
use of military	How willing should the United States be to use military force to solve international problems? [Extremely willing, very willing, moderately willing, a little willing, or not at all willing / Not at all willing, a little willing, moderately willing, very willing, or extremely willing]?	V161154
insurance plan	Where would you place yourself on this scale, or haven't you thought much about this? 1 (Govt insurance plan) – 7 (Private insurance plan)	V161184

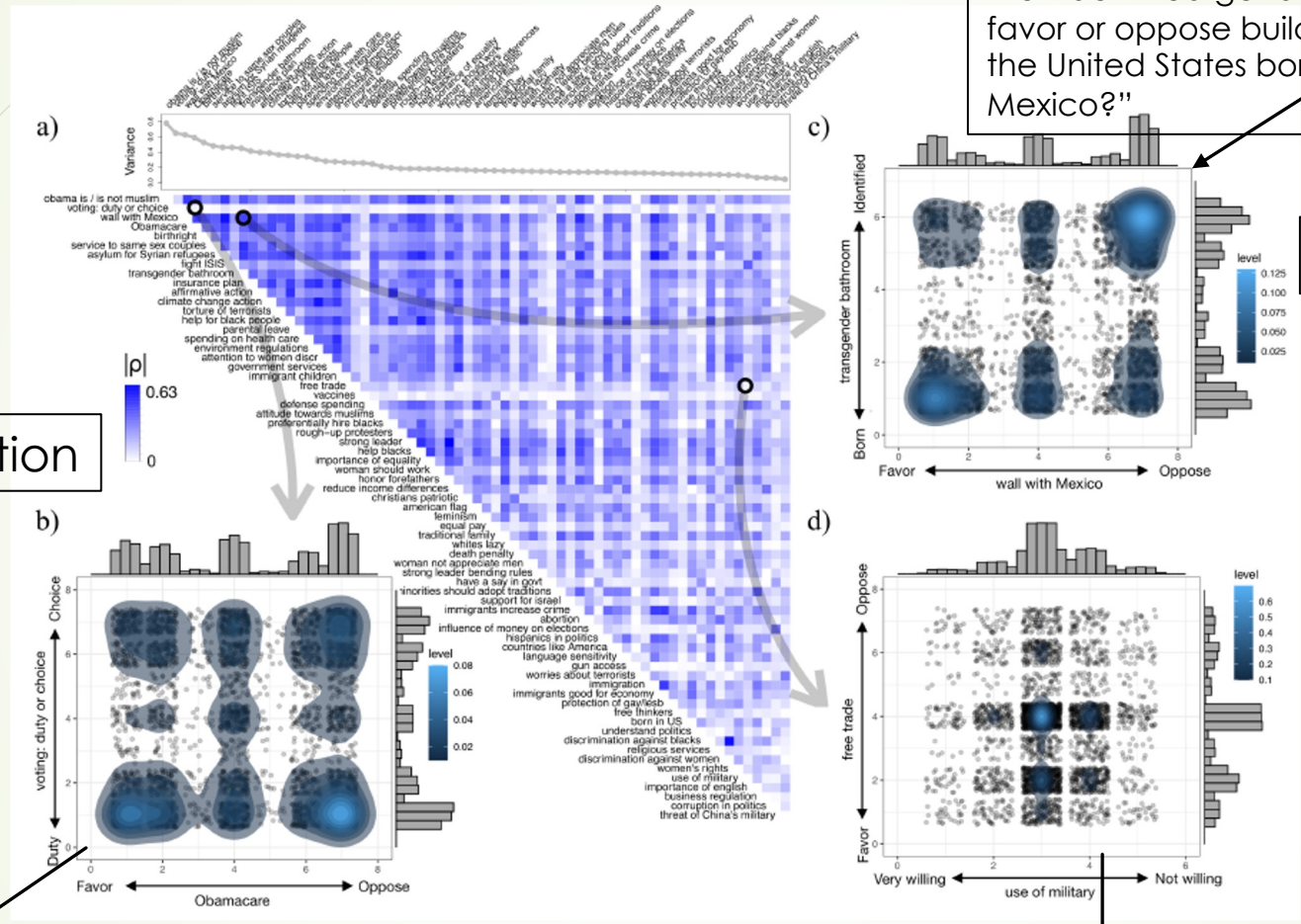
wall with Mexico	Summary: Build wall with Mexico **V161196**: Do you favor, oppose, or neither favor nor oppose building a wall on the U.S. border with Mexico? **V161196a**: IF R FAVORS BUILDING A WALL ON THE U.S. BORDER WITH MEXICO / IF R OPPOSES BUILDING A WALL ON THE U.S. BORDER WITH MEXICO: Do you favor that [a great deal, a moderate amount, or a little / a little, a moderate amount, or a great deal]? / Do you oppose that[a great deal, a moderate amount, or a little / a little, a moderate amount, or a great deal]?	V161196x
help for black people	Where would you place yourself on this scale, or haven't you thought much about this? 1 (Govt should help Blacks) – 7 (Blacks should help themselves)	V161198
environment regulations	Where would you place yourself on this scale, or haven't you thought much about this? 1 (Regulate business to protect the environment and create jobs) – 7 (No regulation because it will not work and will cost jobs)	V161201
free trade	Summary: Favor/oppose free trade agreements **V162176**: Do you favor, oppose, or neither favor nor oppose the U.S. making free trade agreements with other countries? **V162176a**: How strongly do you [favor/oppose] it?	V162176x

ANES survey's results

"Should transgender people have to use the bathrooms of the gender they were born as, or should they be allowed to use the bathrooms of their identified gender?" vs "Do you favor or oppose building a wall on the United States border with Mexico?"

Ideological state

Polarization



Consensus

"Do you consider voting a choice or duty" vs
"Do you favor or oppose the health care reform law passed in 2010?"

"Do you favor or oppose the United States making free trade agreements with other countries?" vs "How willing should the United States be to use military force to solve international problems?"

Thanks!

http://sophy.df.uba.ar/ , @SoPhyLabDF in

SoPhy Lab

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Murchison F.

Cicchini T.
Albanese F.
Espejo D.

Favio Di Ciocco

Sebastián Pinto

Conclusions & Perspectives

- We developed a framework in order to study quantitatively Public and Media Agendas
- We observed significative changes in the diversity of agendas on specific dates indicating how attention collapses on single important issues
- We also observe significative changes in the distance between agendas on specific days indicating temporal decoupling of public attention from Media offers
- This methodology allows to perform quantitatively analysis of framing and coverage bias
- The observed agenda dynamics should guide the production of future data driven models of media and social influence.