Chiral plasma instability in the magnetosphere of magnetars

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October 27, 2022
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Pulsars

- **Neutron stars** are laboratories of matter under extreme conditions

- **Prediction**
  

- **Observation**
  

- **Pulsars** are neutron stars that are
  
  - rapidly rotating \((P \sim 1 \text{ ms to } 10 \text{ s})\)
  
  - strongly magnetized \((B \sim 10^8 \text{ to } 10^{15} \text{ G})\)

- Pulsar radiation is beamed along the magnetic field direction (the “lighthouse” effect)
Pulsars in $P - \dot{P}$ plane

- **Characteristic age**
  \[
  \tau \simeq \frac{P}{2\dot{P}}
  \]

- **Spin-down luminosity**
  \[
  -\dot{E} \simeq 4\pi^2 I \frac{\dot{P}}{P^3}
  \]

- **Characteristic magnetic field**
  \[
  B \simeq 3 \times 10^{19} \left( \frac{P \dot{P}}{s} \right)^{1/2} G
  \]

MAGNETOSPHERES

Image credit: Aurore Simonnet, Sonoma State University
Pulsar electrodynamics (VDM)

- Vacuum dipole model (VDM) \((\rho = 0 \text{ and } J = 0 \text{ outside the star})\)
- Stellar interior (good conductor):
  \[\vec{E}''_{in} = \vec{E}_{in} + \frac{\vec{\Omega} \times \vec{r}}{c} \times \vec{B}_{in} = 0\]
  \[\vec{E} = \ldots\] [see Deutsch, Ann. Astrophys. 18, 1 (1955)]

- Fields outside the pulsar are
  \[\vec{B} = \frac{B_0 R^3}{2r^3} (3(\hat{m} \cdot \hat{r})\hat{r} - \hat{m})\]
  \[\vec{E} = \ldots\]

where \(\vec{m}\) is the magnetic moment and \(\Omega\) is the angular frequency

- There is a nonzero charge density and a strong electric field on the surface \((E_{surf} \sim \Omega R B_0 \sim 10^{12} \text{ to } 10^{15} \text{ V/m})\)
Pulsar electrodynamics (VDM)

- Charged particles
  
  i. pulled up from the surface ($\vec{E} \neq 0$)
  
  ii. move along curved trajectories ($\vec{B} \neq 0$)
  
  iii. produce curvature radiation
  
  iv. $\gamma$-quanta produce $e^+ e^-$ pairs
      
      \[ l_\gamma \approx \frac{2R_c B_c m_e}{15 B \varepsilon_\gamma} \]
      
  v. Secondary particles produce synchrotron & curvature radiation
  
- **End result:** (I) magnetized vacuum is nontransparent for photons with $\varepsilon_\gamma \gtrsim 2m_e$; (II) vacuum turns into plasma
Pulsar electrodynamics (RMM)

- Rotating magnetosphere model (RMM) (assuming a highly conducting plasma outside the star)

\[ \mathbf{E}' = \mathbf{E} + \frac{\mathbf{\Omega} \times \mathbf{r}}{c} \times \mathbf{B} = 0 \]

i.e., \( E_\parallel = 0 \)

- Plasma motion is determined by

\[ \mathbf{v}_{\text{drift}} = c \frac{\mathbf{E} \times \mathbf{B}}{B^2} = \mathbf{\Omega} \times \mathbf{r} + j_\parallel \mathbf{B} \]

- Corotating plasma is charged

\[ \rho_{\text{GJ}} = \nabla \cdot \mathbf{E} = -\frac{2}{c} \mathbf{\Omega} \cdot \mathbf{B} \]

Gaps in magnetosphere

• If one assumes that \( E_\parallel = 0 \) everywhere, the magnetic field lines are equipotential (\( V = \text{const} \))

• Then,

\[
0 = \oint \vec{E} \cdot d\vec{l} = \int (\nabla \times \vec{E}) \cdot d\vec{s} = -\frac{\partial}{\partial t} \int \vec{B} \cdot d\vec{s}
\]

• Thus, \( E_\parallel = 0 \) cannot be enforced everywhere if \( \vec{B} \) changes in time

• Regions ("gaps") with unscreened \( E_\parallel \) will necessarily develop (they result from dynamical charge/current starvation)

[Ruderman & Sutherland, Astrophys. J. 196, 51 (1975)]
Gaps in magnetosphere

• Gaps can develop at various locations

• Intermittent gaps are caused by rapid outflow of charge

• The gap size $h$ grows at a speed close to the speed of light

• Electric potential difference grows like $\Delta V = E_{\parallel} h \propto h^2$

• $\Delta V$ & photon flux cause an avalanche production of electron-positron pairs

• Since $B \propto 1/r^3$, anomalous effects are strongest near polar caps

[Ruderman & Sutherland, Astrophys. J. 196, 51 (1975)]
Pulsar gaps

- Estimates for the **electric field** and the **gap size**

\[ \vec{E}_\parallel \approx Bh/R_{LC} \]

\[ h \approx 3.6 \text{ m} \left( \frac{R}{10 \text{ km}} \right)^{2/7} \left( \frac{\Omega}{1 \text{ s}^{-1}} \right)^{-3/7} \left( \frac{B}{10^{14} \text{ G}} \right)^{-4/7} \]

where \( R_{LC} = c/\Omega \) is the radius of light cylinder.

The field scales with pulsar parameters as follows

\[ \vec{E}_\parallel \approx 2.7 \times 10^{-8} E_c \left( \frac{R}{10 \text{ km}} \right)^{2/7} \left( \frac{\Omega}{1 \text{ s}^{-1}} \right)^{4/7} \left( \frac{B}{10^{14} \text{ G}} \right)^{3/7} \]

where \( E_c = m_e^2/e = 1.3 \times 10^{18} \text{ V/m} \).

[Ruderman & Sutherland, Astrophys. J. 196, 51 (1975)]
Charge in the gap

formation of the gap

relaxation

Gap parameters

- Quantitative estimate of the gap size and fields

<table>
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where

$$E_c = \frac{m_e^2}{e} = 1.3 \times 10^{18} \text{ V/m}$$

$$B_c = \frac{m_e^2}{e} = 4.4 \times 10^{13} \text{ G}$$

October 27, 2022
Workshop on Electromagnetic Effects in Strongly Interacting Matter, São Paulo, Brazil
Chiral charge production

- The evolution of the chiral charge is determined by
  \[
  \frac{\partial n_5}{\partial t} + \nabla \cdot \mathbf{j}_5 = \frac{e^2 \mathbf{E} \cdot \mathbf{B}}{2\pi^2} - \Gamma_m n_5
  \]

- While the chiral anomaly produces \( n_5 \), the chirality flipping tries to wash it away.

- The chiral charge \( n_5 \) approaches the following steady-state value:
  \[
  n_5 = \frac{e^2}{2\pi^2 \Gamma_m} \mathbf{E} \cdot \mathbf{B}
  \]

- The estimates for the chirality flip rate in a hot plasma:
  \[
  \Gamma_m \approx \frac{\alpha^2 m_e^2}{T} \quad (T \lesssim m_e/\sqrt{\alpha}) \quad \text{and} \quad \Gamma_m \approx \frac{\alpha m_e^2}{T} \quad (T \gg m_e/\sqrt{\alpha})
  \]

[Boyarsky, Cheianov, Ruchayskiy, Sobol, Phys. Rev. Lett. 126, 021801 (2021)]
Time scales

- The gap formation time
  \[ t_h \sim \frac{h}{c} \sim 10^{-8} \text{ s} \]

- Timescale for chiral charge production
  \[ t^* \sim \frac{1}{\Gamma_m} \sim 10^{-17} \text{ s} \]

- Note that
  \[ t_h \gg t^* \]

- Thus, the chirality production is nearly instantaneous
Estimate for $n_5$ in magnetars

- The estimate for the chiral charge is given by

\[
  n_5 \approx \frac{e^2 E \parallel B}{2\pi^2 \Gamma_m} \approx 1.5 \times 10^{-5} \text{ MeV}^3 \left(\frac{T}{1 \text{ MeV}}\right)
\]
\[
  \times \left(\frac{R}{10 \text{ km}}\right)^{2/7} \left(\frac{\Omega}{1 \text{ s}^{-1}}\right)^{4/7} \left(\frac{B}{10^{14} \text{ G}}\right)^{10/7}
\]

- The corresponding chiral chemical potential is

\[
  \mu_5 \approx \frac{3n_5}{T^2} \approx 4.6 \times 10^{-5} \text{ MeV} \left(\frac{T}{1 \text{ MeV}}\right)^{-1}
\]
\[
  \times \left(\frac{R}{10 \text{ km}}\right)^{2/7} \left(\frac{\Omega}{1 \text{ s}^{-1}}\right)^{4/7} \left(\frac{B}{10^{14} \text{ G}}\right)^{10/7}
\]
Values of $n_5$ and $\mu_5$

- The corresponding numerical values for chiral charge and chiral chemical potential are

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CHIRAL PLASMA INSTABILITY

Image credit: European Southern Observatory
Plasma with $\mu_5 \neq 0$

- Nonzero $\mu_5$ and $B$ drive the chiral magnetic effect (CME)
  \[ \vec{j} = \frac{e^2 B}{2\pi^2 \mu_5} \]

- The effect comes from the spin-polarized LLL ($s=\downarrow$)
  - L-handed states ($p_3 < 0 \& |E| < \mu_5$) are empty (holes with $p_3 > 0$)
  - R-handed states ($p_3 < 0 \& E < \mu_5$) are occupied

- However, plasma at $\mu_5 \neq 0$ is unstable
Maxwell equations at $\mu_5 \neq 0$

- The total current (CME + Ohm)
  \[ j = \frac{2\alpha}{\pi} \mu_5 B + \sigma E \]

- By substituting $j$ into Ampere’s law
  \[ \nabla \times B = j + \frac{\partial E}{\partial t} \]
  and solving for the electric field, one derives
  \[ E = \frac{1}{\sigma} \left( \nabla \times B - k_* B - \frac{\partial E}{\partial t} \right) \]
  where $k_* = \frac{2\alpha \mu_5}{\pi}$

- Finally, by calculating the curl and using Faraday’s law,
  \[ \frac{\partial B}{\partial t} = -\frac{1}{\sigma} \left( \nabla \times (\nabla \times B) - k_* \nabla \times B + \frac{\partial^2 B}{\partial t^2} \right) \]
Helical modes at $\mu_5 \neq 0$

- Search for a solution as a superposition of helical eigenstates

$$\nabla \times \mathbf{B}_{\lambda,k} = \lambda k \mathbf{B}_{\lambda,k}$$

e.g.,

$$\mathbf{B}_{\lambda,k} = B_0 (\hat{x} + i\lambda \hat{y}) e^{-i\omega t + ikz}$$

Then, for a fixed eigenmode, the evolution equation reads

$$\frac{d\mathbf{B}_{\lambda,k}}{dt} = \frac{1}{\sigma} \left( \lambda k_\star k - k^2 - \frac{\partial^2}{\partial t^2} \right) \mathbf{B}_{\lambda,k}$$

- The two solutions for the frequency are

$$\omega_{1,2} = -\frac{i}{2} \left( \sigma \pm \sqrt{\sigma^2 + 4k(\lambda k_\star - k)} \right)$$
Long-wavelength modes

- For a plasma with high conductivity

\[
\omega_{1,2} \simeq \begin{cases} 
-\frac{i}{\sigma} \left( \sigma + \frac{k(\lambda k_* - k)}{\sigma} \right) \\
\frac{i}{\sigma} \frac{k(\lambda k_* - k)}{\sigma} 
\end{cases}
\]

- The 1\textsuperscript{st} mode is damped by charge screening:

\[
B_{k,1} \propto B_0 e^{-\sigma t}
\]

- The 2\textsuperscript{nd} mode is unstable when \( k < \lambda k_* \):

\[
B_{k,2} \propto B_0 e^{+tk(\lambda k_* - k)/\sigma}
\]

- The momentum of the fastest growing mode \( B_{k,2} \) is

\[
\frac{1}{2} k_*
\]

[Joyce & Shaposhnikov, PRL 79, 1193 (1997)]
[Boyarsky, Frohlich, Ruchayskiy, PRL 108, 031301 (2012)]
[Tashiro, Vachaspati, Vilenkin, PRD 86, 105033 (2012)]
[Akamatsu & Yamamoto, PRL 111, 052002 (2013)]
[Tuchin, PRC 91, 064902 (2015)]
[Manuel & Torres-Rincon, PRD 92, 074018 (2015)]
[Hirono, Kharzeev, Yin, PRD 92, 125031 (2015)]
[Sigl & Leite, JCAP 01, 025 (2016)]
Instability in pulsars

- The estimate for $k_*$

$$k_* \approx 2.2 \times 10^{-7} \text{ MeV} \left(\frac{T}{1 \text{ MeV}}\right)^{-1} \left(\frac{R}{10 \text{ km}}\right)^{2/7} \left(\frac{\Omega}{1 \text{ s}^{-1}}\right)^{4/7} \left(\frac{B}{10^{14} \text{ G}}\right)^{10/7}$$

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Observational consequences

• Unstable plasma in the gaps produces helical (circularly polarized) modes in the frequency range

\[ 0 \lesssim \omega \lesssim k_\star \]

• For magnetars, these span radio frequencies and may reach into the near-infrared range

• Available energy is of the order of \( \Delta \mathcal{E} \sim \mu^2 T^2 h^3 \), i.e.,

\[
\Delta \mathcal{E} \simeq 2.1 \times 10^{25} \text{ erg} \left( \frac{T}{1 \text{ MeV}} \right) \left( \frac{R}{10 \text{ km}} \right)^{6/7} \\
\times \left( \frac{\Omega}{1 \text{ s}^{-1}} \right)^{-9/7} \left( \frac{B}{10^{14} \text{ G}} \right)^{2/7}
\]

• The energy is sufficient to feed the fast radio bursts (FRB)
Outstanding problems

- Interplay of chiral charge and electron-positron pair production induced by energetic photons should be studied in detail.

- The modification of the chiral flip rate $\Gamma_m \approx \frac{\alpha^2 m_e^2}{T}$ by the strong magnetic field (extra suppression?)

- The role of the inverse magnetic cascade and the chiral-magnetic turbulence should be quantified.

- Self-consistent dynamics of chiral plasma in the gap regions should be simulated in detail.

- Detailed mechanism of the energy transfer from unstable helical modes to radio emission in FRBs.
Summary

- Chiral anomaly can have *macroscopic* implications in pulsars
- It leads to a *significant* chiral charge production (up to $10^{34}$ m$^{-3}$) in strongly magnetized magnetospheres
- The chiral chemical potential $\mu_5$ can be up to $10^{-3}$ MeV
- This is sufficient to trigger emission of helical waves with frequencies up to about $k_* \approx \frac{2}{\pi} \alpha \mu_5$ (radio to infrared range)
- Helical waves can affect the pulsar jets and observable features of the fast radio bursts
- For quantitative effects, further detailed studies are needed