Magnetic screening mass for neutral pions
Luis Alberto Hernández Rosas


In collaboration with: A. Ayala, R. Farias, A. Mizher, C. Villavicencio and R. Zamora
Outline

1. Debye mass
2. Magnetic Debye mass
3. Magnetic screening mass in the LSMq
4. Unifying our understanding. NJL $\leftrightarrow$ LSMq
5. Results
On Tuesday ...

William, Norberto and Ricardo showed results for the magnetic modification to the pole mass for different hadrons.


Carlomagno, Gómez Dumm, Noguera and Scoccola, Physics. Rev. D106 (2022), 074002
Many other results

LQCD and effective models results


Magnetic screening mass


B. Sheng, Y. Wang, X. Wang and L. Yu, Phys. Rev. D103 (2021) 9, 094001
The Coulomb potential is modified by collective effects as

$$V(r) = Q \int \frac{d^3 p}{(2\pi)^3} \frac{e^{i\vec{p} \cdot \vec{r}}}{\vec{p}^2 + \Pi(p_0 = 0, \vec{p})}$$

The position of the pole is called the Debye mass or the screening mass. Also the potential can be written as

$$V(r) = e^{-m_D r} \frac{Q}{r},$$

where $m_D = (r_D)^{-1}$. Then, if we want to compute the screening mass at finite $T$, we need to solve the equation

$$[p_0^2 - \vec{p}^2 - \Pi(p_0, \vec{p}, T)]|_{p_0=0} = 0$$
Now, if we want to compute in general the screening mass at finite $|eB|$, we need to solve the equation

$$[p_0^2 - p_\perp^2 - p_3^2 - m^2 - \Pi(p_0, p_\perp, p_3, |eB|)]|_{p_0=0} = 0,$$

where $p^2 \rightarrow p_\perp^2 + p_3^2$ and $\Pi(p_0, p_\perp, p_3, |eB|)$ should be computed according the Lagrangian that we use.
Linear Sigma Model with quarks

Renormalizable effective model to describe dynamics at low energies.

\[
\mathcal{L} = \frac{1}{2} (\partial_\mu \sigma)^2 + \frac{1}{2} (D_\mu \vec{\pi})^2 + \frac{a^2}{2} (\sigma^2 + \vec{\pi}^2) - \frac{\lambda}{4} (\sigma^2 + \vec{\pi}^2)^2
\]

\[+ \ i \bar{\psi} \gamma^\mu D_\mu \psi - g A_{\mu} (\sigma + i \gamma_5 \vec{\tau} \cdot \vec{\pi}) \psi ,\]

where \( \vec{\pi} = (\pi^+, \pi^-, \pi^0) \), the model has two species of quarks represented by an \( SU(2) \) isospin doublet \( \psi \), and \( \sigma \) meson is a scalar included by means of an isospin singlet.

\[D_\mu = \partial_\mu + i q_{f,b} A_\mu ,\]

with

\[A^\mu = \frac{B}{2}(0, -y, x, 0) .\]

To allow for spontaneous symmetry breaking

\[\sigma \rightarrow \sigma + \nu .\]

As a consequence of SSB

\[m_\sigma^2 = 3 \lambda \nu^2 - a^2 , \quad m_\pi^2 = \lambda \nu^2 - a^2 , \quad m_f = g \nu .\]
Feynman rules for the LSMq

Meson interactions in the LSMq. Dashed lines are used to represent the neutral and charged pions, whereas double lines represent the $\sigma$. Solid lines represent the quarks. Thin solid lines represent the $d$ quark, and thick solid lines represent the $u$ quark.

Quark-meson interactions in the LSMq. Dashed lines represent the neutral and charged pions, whereas the double lines represent the $\sigma$. Solid lines represent the quarks. Thin solid lines represent the $d$ quark, and thick solid lines represent the $u$ quark.
Come back with the main topic. In order to obtain the screening mass for the NEUTRAL PION, we need to solve the equation

\[
[p_0^2 - p_\perp^2 - p_3^2 - m_\pi^2 - \Pi(p_0, p_\perp, p_3, |eB|)]|_{p_0=0} = 0
\]

\[\downarrow\]

dynamical mass
Come back with the main topic. In order to obtain the screening mass for the NEUTRAL PION, we need to solve the equation

\[
[p_0^2 - p_\perp^2 - p_3^2 - m_\pi^2 - \Pi(p_0, p_\perp, p_3, |eB|)]|_{p_0=0} = 0
\]

\[
[p_0^2 - p_\perp^2 - p_3^2 - (\lambda v_0^2 - a^2) - \Pi(p_0, p_\perp, p_3, |eB|)]|_{p_0=0} = 0
\]

\(V_0 = \text{vacuum spectator value (changes as a function of } |eB|)\)
Screening mass within the LSMq

Come back with the main topic. In order to obtain the screening mass for the NEUTRAL PION, we need to solve the equation

\[
[p_0^2 - p_\perp^2 - p_3^2 - m_\pi^2 - \Pi(p_0, p_\perp, p_3, |eB|)]|_{p_0=0} = 0
\]

\[
[p_0^2 - p_\perp^2 - p_3^2 - (\lambda v_0^2 - a^2) - \Pi(p_0, p_\perp, p_3, |eB|)]|_{p_0=0} = 0
\]
In order to obtain the vev, we compute the effective potential up to 1-loop order.

\[
V_{\text{eff}} = V_{\text{tree}} + V_{\pi^+}^1 + V_{\pi^-}^1 + V_{\pi^0}^1 + V_\sigma^1 + \sum_f V_f^1 .
\]

where

\[
V_b^1 = -\frac{i}{2} \int \frac{d^4k}{(2\pi)^4} \ln \left[ -D_b^{-1}(k) \right], \quad V_f^1 = iN_c \int \frac{d^4k}{(2\pi)^4} \text{Tr} \ln \left[ S_f^{-1}(k) \right]
\]

with the propagators given by

\[
S_f(p) = \int_0^\infty \frac{ds}{\cos(|q_f B|s)} e^{is\left(p_\parallel^2 - p_\perp^2 \frac{\tan(|q_f B|s)}{|q_f B|s} - m_f^2 + i\epsilon\right)} \\
\times \left( \cos(|q_f B|s) + \gamma_1 \gamma_2 \sin(|q_f B|s) \text{sign}(q_f B) \right) \times \left( m_f + p_\parallel \right) \frac{p_\perp}{\cos(|q_f B|s)}
\]

\[
D_i(p) = \int_0^\infty \frac{ds}{\cos(|q_b B|s)} e^{is\left(p_\parallel^2 - p_\perp^2 \frac{\tan(|q_b B|s)}{|q_b B|s} - m_b^2 + i\epsilon\right)}.
\]
Vacuum expectation value and the Magnetic Catalysis

In order to obtain the vev, we compute the effective potential up to 1-loop order.

\[ V^{\text{eff}} = V^{\text{tree}} + V^{1}_{\pi^+} + V^{1}_{\pi^-} + V^{1}_{\pi^0} + V^{1}_{\sigma} + \sum_{f} V^{1}_{f}. \]

where

\[ V^{1}_{b} = -\frac{i}{2} \int \frac{d^4 k}{(2\pi)^4} \ln \left[ -D^{-1}_b(k) \right], \quad V^{1}_{f} = iN_c \int \frac{d^4 k}{(2\pi)^4} \text{Tr} \ln \left[ S^{-1}_f(k) \right] \]

Introducing the vacuum stability conditions

\[ \frac{1}{2v} \frac{dV^{\text{vac}}}{dv} \bigg|_{v=v_0} = 0, \quad \frac{d^2 V^{\text{vac}}}{dv^2} \bigg|_{v=v_0} = 2a^2 + 2m_0^2. \]

\[ V^{\text{vac}} = -\frac{(a^2 + m_0^2 + \delta a^2)}{2} v^2 + \frac{(\lambda + \delta \lambda)}{4} v^4 - 3 \frac{m_0^4}{64\pi^2} \left[ \frac{3}{2} + \ln \left( \frac{\mu^2}{m_0^2} \right) \right] \]

\[ - \frac{m_\sigma^4}{64\pi^2} \left[ \frac{3}{2} + \ln \left( \frac{\mu^2}{m_\sigma^2} \right) \right] + 2N_c \frac{m_f^4}{16\pi^2} \left[ \frac{3}{2} + \ln \left( \frac{\mu^2}{m_f^2} \right) \right]. \]
Then, the effective potential is

\[
V^{\text{eff}}(B) = -\frac{(a^2 + m_0^2)}{2}v^2 - \frac{\delta a^2}{2}v_0^2 + \frac{\lambda}{4}v^4 + \frac{\delta \lambda}{4}v_0^4 - 3\frac{m_0^4(v_0)}{64\pi^2} \left[ \frac{3}{2} + \ln \left( \frac{\mu^2}{m_0^2(v_0)} \right) \right] \\
- \frac{m_0^4(v_0)}{64\pi^2} \left[ \frac{3}{2} + \ln \left( \frac{\mu^2}{m_0^2(v_0)} \right) \right] + 2N_c \sum_f \frac{m_f^4(v_0)}{16\pi^2} \left[ \frac{3}{2} + \ln \left( \frac{\mu^2}{m_f^2(v_0)} \right) \right] \\
+ \frac{2}{16\pi^2} \left[ 2|eB|^2 \psi^{-2} \left( \frac{1}{2} + \frac{m_0^2(v)}{2|eB|} \right) + \frac{3m_0^4(v)}{8} - \frac{1}{2}|eB| m_0^2(v) \ln(2\pi) \right] \\
- \frac{m_0^4(v)}{4} \ln \left( \frac{m_0^2(v)}{2|eB|} \right) - \frac{N_c}{8\pi^2} \sum_f \left[ 4|q_f B|^2 \psi^{-2} \left( \frac{m_f^2(v)}{2|q_f B|} \right) + \frac{3}{4} m_f^4(v) \right] \\
- \frac{m_f^4(v)}{2} \ln \left( \frac{m_f^2(v)}{2|q_f B|} \right) - m_f^2(v)|q_f B| + m_f^2(v)|q_f B| \ln \left( \frac{m_f^2(v)}{4\pi|q_f B|} \right). 
\]
Magnetic catalysis
Neutral pion self-energy

\[ \Pi(B, q) = \sum_f \Pi_{f\bar{f}}(B, q) + \Pi_{\pi^-}(B) + \Pi_{\pi^+}(B) + \Pi_{\pi^0} + \Pi_{\sigma}. \]

with
Neutral pion self-energy

\[-i\Pi_{f\bar{f}}(B, q) = -g^2 \int \frac{d^4 k}{(2\pi)^4} \text{Tr}[\gamma_5 iS_f(k)\gamma_5 iS_f(k + q)] + CC,\]

\[-i\Pi_{\pi\pm} = \int \frac{d^4 k}{(2\pi)^4} (-2i\lambda) iD_{\pi\pm}(k).\]

where the propagators are

\[S_f(p) = \int_0^\infty \frac{ds}{\cos(|q_f B|s)} e^{is(p^2 - p^2 \frac{\tan(|q_f B|s)}{|q_f B|s} - m_f^2 + i\epsilon)} \times \left( \cos(|q_f B|s) + \gamma_1\gamma_2 \sin(|q_f B|s)\text{sign}(q_f B) \right) \times \left( m_f + \frac{\not{p}_\parallel}{\cos(|q_f B|s)} \right),\]

\[D_i(p) = \int_0^\infty \frac{ds}{\cos(|q_b B|s)} e^{is(p^2 - p^2 \frac{\tan(|q_b B|s)}{|q_b B|s} - m_b^2 + i\epsilon)}.\]
Neutral pion self-energy

Boson contribution

\[ \Pi_{\pi \pm} = \Pi_{\pi \pm}^{\text{vac}} + \Pi_{\pi \pm}^{B} \]
\[ \quad = \frac{\lambda}{4\pi^2} \left[ \frac{m_{\pi}^2}{2} \ln \left( \frac{\mu^2}{m_{\pi}^2} \right) + \frac{m_{\pi}^2}{2} \ln \left( \frac{m_{\pi}^2}{2|q_{b}B|} \right) \right. \]
\[ \quad \quad - |q_{b}B| \left( \ln \left( \Gamma \left( \frac{1}{2} + \frac{m_{\pi}^2}{2|q_{b}B|} \right) \right) + \ln(\sqrt{2\pi}) - \frac{m_{\pi}^2}{2} \right) \]

Fermion contribution

\[ \Pi_{f \bar{f}} = \Pi_{f \bar{f}}^{\text{vac}} + \Pi_{f \bar{f}}^{B} \]

\[ \text{Computed without any approximation} \]
\[ \Rightarrow \text{numerically.} \]
We are ready to find the magnetic screening mass for the neutral pion by joining all the results showed

\[
[p_0^2 - p_\perp^2 - p_3^2 - m_\pi^2 - \Pi(p_0, p_\perp, p_3, |eB|)]|_{p_0=0} = 0
\]

... Hold your horses! We can include one more ingredient in the recipe. \(\rightarrow\) Effective coupling constants.

\[
\lambda_{\text{eff}} = \lambda(1 + \Gamma^B_\lambda),
\]

\[
ge_{\text{eff}} = g(1 + \Gamma^B_g).
\]
Effective coupling constants

Magnetic corrections to the boson self-coupling

\[-i6\lambda \Gamma^B = \int \frac{d^4k}{(2\pi)^4} (-2i\lambda) iD_{\pi -}(k)(-2i\lambda) \times iD_{\pi -}(k + p + r) + \text{CC},\]

\[\Gamma^B = -\frac{\lambda}{12\pi^2} \left[ \ln \left( \frac{\mu^2}{2|q_bB|} \right) - \psi^0 \left( \frac{|q_bB| + m^2_{\pi}}{2|q_bB|} \right) \right].\]

Magnetic corrections to the boson-fermion coupling

\[\Gamma^{LLL}_g = \Gamma^B_{1,g} + \Gamma^B_{2,g} + \Gamma^B_{3,g}.\]

\[g\gamma^5 \Gamma^B_{1,g} = \int \frac{d^2 s_\perp d^2 t_\perp}{\pi^2 |eB|^2} \frac{d^4k}{(2\pi)^4} \left( \sqrt{2g\gamma^5} \right) iS_d(k_{\parallel} + p_{\parallel}, s_\perp) \left( -g\gamma^5 \right) iS_d(k_{\parallel} + r_{\parallel}, t_\perp) \]
\[\quad \times \left( \sqrt{2g\gamma^5} \right) iD_{\pi -}(k_{\parallel}, k_\perp) e^{i\frac{2}{|eB|} \epsilon_{ij}(s-q-t)_i(s-p-k)_j} + \text{CC},\]

\[g\gamma^5 \Gamma^B_{2,g} = \int \frac{d^4k}{(2\pi)^4} \left( g\gamma^5 \right) iS_u(k + p) \left( g\gamma^5 \right) iS_u(k + r) \left( g\gamma^5 \right) iD_{\pi 0}(k) + \text{CC},\]

\[g\gamma^5 \Gamma^B_{3,g} = \int \frac{d^4k}{(2\pi)^4} (-ig) iS_u(k + p) \left( g\gamma^5 \right) iS_u(k + r) (-ig) iD_\sigma(k) + \text{CC}.\]
Effective coupling constants behaviour
Where only the quark-antiquark pair fluctuation is considered, we have

\[ [-p_\perp^2 - p_3^2 - m_\pi^2 - \Pi(0, p_\perp, p_3, |eB|)] = 0 \]

which can be rewritten as follows

\[ (-p_\perp^2 - p_3^2 - m_\pi^2) \left( 1 - \frac{\Pi(0, p_\perp, p_3, |eB|)}{-p_\perp^2 - p_3^2 - m_\pi^2} \right) = 0 \]

\[ (-p_\perp^2 - p_3^2 - m_\pi^2) \left( 1 - \frac{g^2\tilde{\Pi}(0, p_\perp, p_3, |eB|)}{-p_\perp^2 - p_3^2 - m_\pi^2} \right) = 0 \]

Using random phase approximation

\[ \frac{2iG}{1 - 2G\tilde{\Pi}(p_0, p_\perp, p_3, |eB|)} \]

It is interpreted as an effective meson propagator where the pole mass is obtained when \( p_\perp \) and \( p_3 \) go to zero, and the screening mass is obtained when \( p_0 \) goes to zero and \( p_\perp \) or \( p_3 \) is finite. Then, the equation to solve is

\[ 1 - 2G\tilde{\Pi}(0, p_\perp, p_3, |eB|) = 0 \]
magnetic screening mass for $\rho_3 = 0$
magnetic screening mass for $\rho_\perp = 0$
Both magnetic screening masses

\[ m_{\mu c}/m_{\tau} \] vs. \( eB \) [GeV²]

- \( \lambda=15, g=3.4, m_{bc,\perp} \)
- \( \lambda=15, g=3.4, m_{bc,\parallel} \)

\[ m_{bc}/m_{\tau} \] vs. \( eB \) [GeV²]

- \( \lambda=14, g=3.4, m_{bc,\perp} \)
- \( \lambda=14, g=3.4, m_{bc,\parallel} \)
¡Gracias!

lherandez.rosas@izt.uam.mx
luis.hr@xanum.uam.mx