Strange dwarfs
and the question of their dynamical stability

José C. Jiménez

Institute of Physics, University of São Paulo (IF-USP)

in collaboration V. Gonçalves and L. Lazzari (UFPel)

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Outline

1. Motivation

2. Classic and state-of-the-art results

3. Our results for the strange-dwarf stability

4. Conclusions
1. Motivation

OPEN
Evidence for quark-matter cores in massive neutron stars

Eemeli Annala©1, Tyler Gorda©2 □, Aleksi Kurkela©3,4 □, Joonas Näätäliä©5,6,7 □ and Aleksi Vuorinen©1 □

[Nature Phys. 16 (2020) 9, 907-910]
Motivation

Equations of state for QCD matter
[Annala et al., Nature Phys. 2020]
Motivation

Quark-matter core masses in NSs

[Annala et al., Nature Phys. 2020]
Motivation

Mass-Radius diagram (cartoon) for compact and non-compact stellar objects [Alford et al., 2017]
2. **Seminal work:**

FROM STRANGE STARS TO STRANGE DWARFS

**N. K. GLENDENNING, CH. KETTNER,² AND F. WEBER**

Nuclear Science Division and Institute for Nuclear and Particle Astrophysics, Lawrence Berkeley Laboratory, MS: 70A-3307, University of California, Berkeley, California 94720

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**ABSTRACT**

We determine all possible equilibrium sequences of compact strange-matter stars with nuclear crusts, which range from massive strange stars to strange white dwarf–like objects (strange dwarfs). The properties of such stars are compared with those of their nonstrange counterparts—neutron stars and ordinary white dwarfs. The main emphasis of this paper is on strange dwarfs, which we divide into two distinct categories. The first one consists of a core of strange matter enveloped within ordinary white dwarf matter. Such stars are hydrostatically stable with or without the strange core and are therefore referred to as “trivial” strange dwarfs. This

Equation of state (left) and corresponding MR diagram (right) [Glendenning et al., 1995]
Dynamical stability of their strange dwarfs (SD)

They perform the dynamical stability analysis by solving the Sturm-Liouville problem [Chandrasekhar, 1964] for the amplitudes, $u_n$, i.e.

$$\frac{d}{dr} \left( \prod \frac{du_n}{dr} \right) + (Q + \omega_n^2 W) u_n = 0.$$ 

Oscillation frequencies in terms of $\Phi(\omega_n^2) \equiv \text{sign}(\omega_n^2) \log[1 + |\omega_n^2|]$.
The analysis of Alford, Harris, Sachdeva (2017)

They modeled the transition as a crossover with width $\delta P$ which must recover the discontinuity when $\delta P \to 0$. Also they adjusted the BPS EoS into a continuous function. Their SD EoS is given by [Alford et al., 2017]

$$\epsilon(P) = \frac{1}{2} (1 - f(P)) \epsilon_{\text{BPS}}(P) + \frac{1}{2} (1 + f(P)) (3P + 4B).$$

where $f(P) = \tanh \left( \frac{(P - P_{\text{crit}})}{\delta P} \right)$. 

![Graph showing phase transition and energy density vs pressure](image)
Dynamical stability of Alford, Harris, Sachdeva (2017)

They also perform the dynamical stability analysis by solving the Sturm-Liouville problem [Chandrasekhar, 1964] for the amplitudes, $u_n$, i.e.

$$
\frac{d}{dr} \left( \prod \frac{du_n}{dr} \right) + (Q + \omega_n^2 W) u_n = 0.
$$
Phase Transition Effects on the Dynamical Stability of Hybrid Neutron Stars

Jonas P. Pereira\textsuperscript{1,2}, César V. Flores\textsuperscript{1,3}, and Germán Lugones\textsuperscript{1}\textsuperscript{©}

\textsuperscript{1} Universidade Federal do ABC, Centro de Ciências Naturais e Humanas, Avenida dos Estados, 5001- Bangú, CEP 09210-170, Santo André, SP, Brazil
jonas.pereira@ufabc.edu.br, cesarovfsky@gmail.com, german.lugones@ufabc.edu.br

\textsuperscript{2} Mathematical Sciences and STAG Research Centre, University of Southampton, Southampton, SO17 1BJ, UK

\textsuperscript{3} Universidade Federal do Maranhão, Departamento de Física, Campus Universitário do Bacanga, CEP 65080-805, São Luís, Maranhão, Brazil

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Abstract

We study radial oscillations of hybrid nonrotating neutron stars composed by a quark matter core and hadronic external layers. At first, we physically deduce the junction conditions that should be imposed between the two phases in these systems when perturbations take place. Then we compute the oscillation spectrum focusing on the effects of slow and rapid phase transitions at the quark-hadron interface. We use a generic MIT-bag model for
Boundary conditions in 1st-order phase transitions

\[ \tau_{\text{reactions}} \ll \omega_0^{-1} \sim 1 \text{ ms} \]

- rapid conversions
  - \( t_1 \): quarks \( \rightarrow \) hadrons
  - \( t_2 \): quarks \( \rightarrow \) hadrons
  - \( t_3 \): quarks \( \rightarrow \) hadrons

\[ \Delta p^+ = \Delta p^- \]
\[ \left[ \xi - \frac{\Delta p}{r p_0} \right]^+ = \left[ \xi - \frac{\Delta p}{r p_0} \right]^ - \]

\[ \tau_{\text{reactions}} \gg \omega_0^{-1} \sim 1 \text{ ms} \]

- slow conversions
  - \( t_1 \): quarks \( \rightarrow \) hadrons
  - \( t_2 \): quarks \( \rightarrow \) hadrons
  - \( t_3 \): quarks \( \rightarrow \) hadrons

\[ \Delta p^+ = \Delta p^- \]
[\[ \xi^+ = \xi^- \]
Recent result of
Di Clemente, Drago, Char, Pagliara (2022)

Stability and instability of strange dwarfs

Francesco Di Clemente\textsuperscript{1,2}, Alessandro Drago\textsuperscript{1,2}, Prasanta Char\textsuperscript{3} and Giuseppe Pagliara\textsuperscript{1,2}

\textsuperscript{1} Dipartimento di Fisica e Scienze della Terra, Università di Ferrara, Via Saragat 1, I-44122 Ferrara, Italy,
\textsuperscript{2} INFN Sezione di Ferrara, Via Saragat 1, I-44122 Ferrara, Italy,
\textsuperscript{3} Space Sciences, Technologies and Astrophysics Research (STAR) Institute, Université de Liège, Bât. B5a, 4000 Liège, Belgium

More than 20 years ago, Glendenning, Kettner and Weber proposed the existence of stable white dwarfs with a core of strange quark matter. More recently, by studying radial modes, Alford, Harris and Sachdeva concluded instead that those objects are unstable. We investigate the stability of these objects by looking again at their radial oscillations, while incorporating boundary conditions at the quark-hadron interface which correspond either to a rapid or to a slow conversion of hadrons into quarks. Our analysis shows that objects of this type are stable if the star is not strongly perturbed and ordinary matter cannot transform into strange quark matter because of the Coulomb barrier separating the two components. On the other hand, ordinary matter can be transformed into strange quark matter if the star undergoes a violent process, as in the preliminary stages of a type Ia supernova, and this causes the system to become unstable and to collapse into a strange quark star. In this way, accretion induced collapse of strange dwarfs can be facilitated and km-sized objects with subsolar masses can be produced.

arXiv:2207.08704
Recent result of
Di Clemente, Drago, Char, Pagliara (2022)

arXiv:2207.08704
4. **OUR FRAMEWORK:**

**First-order) Radial oscillation equations**

Instead of solving the Sturm-Liouville problem, we use the first-order formalism developed by [Gondek et al., 1997] which is given by

\[
\frac{d\xi}{dr} = -\frac{1}{r} \left( 3\xi + \frac{\Delta P}{\Gamma P} \right) - \frac{dP}{dr} \frac{\xi}{(P + \epsilon)} ,
\]

and

\[
\frac{d\Delta P}{dr} = \xi \left\{ \omega^2 e^{\lambda - \nu} (P + \epsilon)r - 4 \frac{dP}{dr} \right\} + \xi \left\{ \left( \frac{dP}{dr} \right)^2 \frac{r}{(P + \epsilon)} - 8\pi e^\lambda (P + \epsilon)Pr \right\} + \Delta P \left\{ \frac{dP}{dr} \frac{1}{P + \epsilon} - 4\pi (P + \epsilon)re^\lambda \right\} ,
\]

where \(\omega\) is the oscillation frequency.
OUR FRAMEWORK:
Cold-quark matter equations of state

Pressures for the MIT bag model (B) and perturbative QCD (FKV)
[arXiv:1311.5154,1906.11189]
OUR RESULTS:
Strange-dwarf families with the MIT bag

Using the MIT bag model (B) with different $\epsilon_{\text{crust}}$
OUR RESULTS:
Strange-dwarf families with the MIT bag

Using the MIT bag model (B) with different $\epsilon_{\text{crust}}$
OUR RESULTS:
Strange-dwarf spectrum with the MIT bag

Using the MIT bag model (B)
OUR RESULTS:
Strange-dwarf families with pQCD

Using pQCD (FKV[X]) and fixed $\epsilon_{\text{crust}} = 4 \times 10^{11}$ g/cm$^3$
OUR RESULTS:
Strange-dwarf families with pQCD

Using pQCD (FKV[X]) and fixed $\epsilon_{\text{crust}} = 4 \times 10^{11} \text{g/cm}^3$
OUR RESULTS:
Strange-dwarf spectrum with pQCD

Using pQCD (FKV[X]) and fixed $\epsilon_{\text{crust}} = 4 \times 10^{11} \text{g/cm}^3$
4. Conclusions

- We revisit the dynamical-stability question of strange-dwarf hybrid stars solving the radial-oscillation equations analyzing properly the associated eigenfrequencies.

- In contrast to past results, our calculations indicate that the strange-dwarf family is really stable when rapid and slow conversions occur. The so-called reaction mode plays a fundamental role since in most situations one finds that $R_{\text{core}} \ll R_{\text{crust}}$.

- In order to prove the robustness our findings we performed all our radial-oscillation calculations also using the FKV pocket formula containing state-of-the-art results from cold and dense pQCD.
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