Strange dwarfs
and the question of their dynamical stability

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Outline

1. Motivation

nature physics

LETTERS

https://doi.org/10.1038/s41567-020-0914-9

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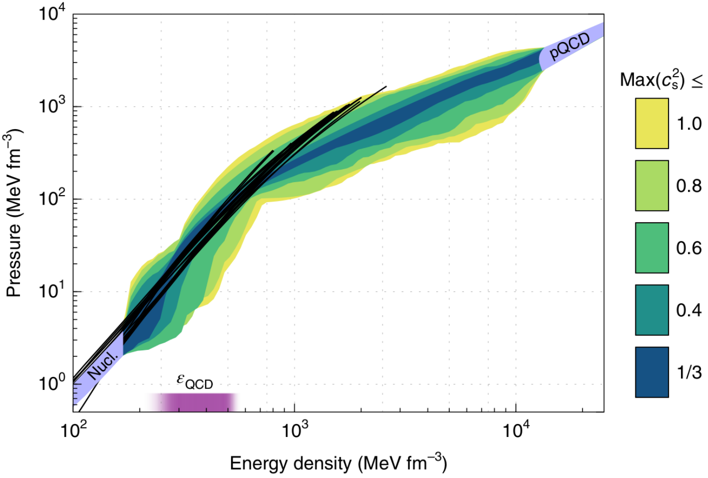
Evidence for quark-matter cores in massive neutron stars

Eemeli Annala[1], Tyler Gorda[2] ✉, Aleksi Kurkela[3,4] ✉, Joonas Nättilä[5,6,7] and Aleksi Vuorinen[1] ✉

[Nature Phys. 16 (2020) 9, 907-910]

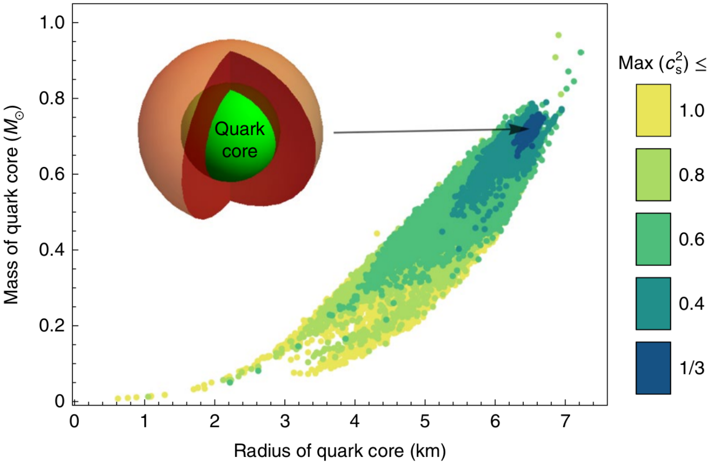
Motivation



Equations of state for QCD matter

[Annala *et al.*, Nature Phys. 2020]

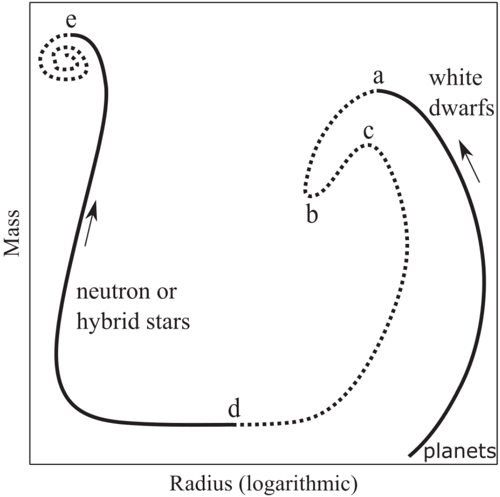
Motivation



Quark-matter core masses in NSs

[Annala *et al.*, Nature Phys. 2020]

Motivation



Mass-Radius diagram (cartoon) for compact and non-compact stellar objects [Alford *et al.*, 2017]

2. Seminal work:
Glendenning, Kettner, Weber (1995)

THE ASTROPHYSICAL JOURNAL, 450:253–261, 1995 September 1

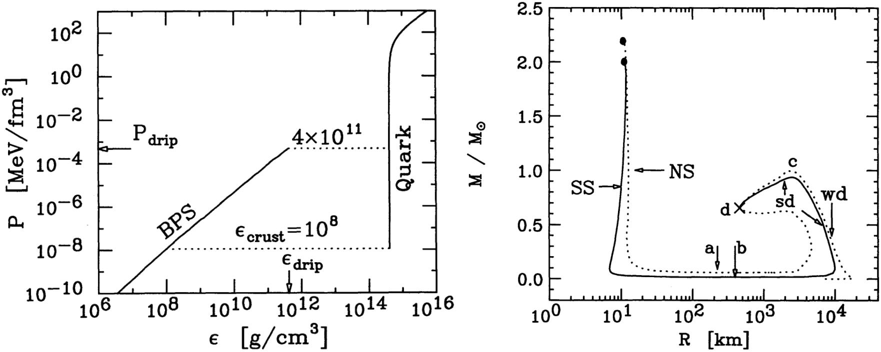
FROM STRANGE STARS TO STRANGE DWARFS[1]

N. K. GLENDENNING, CH. KETTNER,[2] AND F. WEBER
Nuclear Science Division and Institute for Nuclear and Particle Astrophysics, Lawrence Berkeley Laboratory, MS: 70A-3307, University of California, Berkeley, California 94720
Received 1994 December 7; accepted 1995 March 10

ABSTRACT

We determine all possible equilibrium sequences of compact strange-matter stars with nuclear crusts, which range from massive strange stars to strange white dwarf–like objects (strange dwarfs). The properties of such stars are compared with those of their nonstrange counterparts—neutron stars and ordinary white dwarfs. The main emphasis of this paper is on strange dwarfs, which we divide into two distinct categories. The first one consists of a core of strange matter enveloped within ordinary white dwarf matter. Such stars are hydrostatically stable with or without the strange core and are therefore referred to as "trivial" strange dwarfs. This

2. Results of Glendenning, Kettner, Weber (1995)

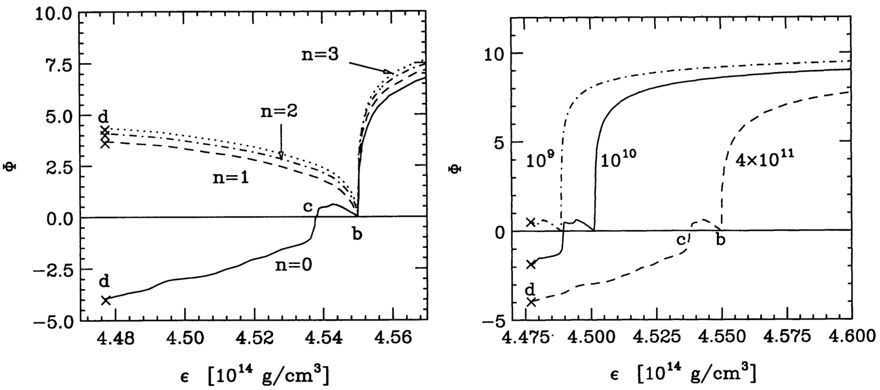


Equation of state (left) and corresponding MR diagram (right)
[Glendenning *et al.*, 1995]

Dynamical stability of their strange dwarfs (SD)

They perform the dynamical stability analysis by solving the Sturm-Liouville problem [Chandrasekhar, 1964] for the amplitudes, u_n, i.e.

$$\frac{d}{dr}\left(\Pi\frac{du_n}{dr}\right) + (Q + \omega_n^2 W)u_n = 0.$$



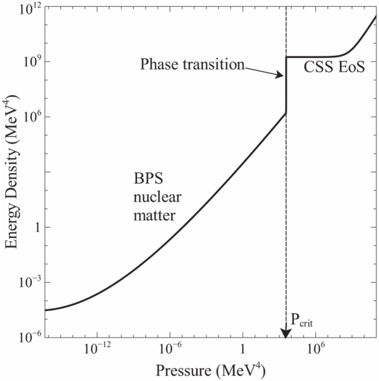
Oscillation frequencies in terms of $\Phi(\omega_n^2) \equiv \mathrm{sign}(\omega_n^2)\log[1 + |\omega_n^2|]$

The analysis of Alford, Harris, Sachdeva (2017)

They modeled the transition as a crossover with width δP which must recover the discontinuity when $\delta P \to 0$. Also they adjusted the BPS EoS into a continuous function. Their SD EoS is given by [Alford *et al.*, 2017]

$$\epsilon(P) = \frac{1}{2}\left(1 - f(P)\right)\epsilon_{\mathrm{BPS}}(P) + \frac{1}{2}\left(1 + f(P)\right)(3P + 4B).$$

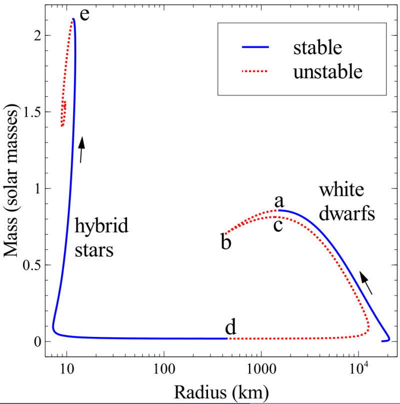
where $f(P) = \tanh\left((P - P_{\mathrm{crit}})/\delta P\right)$.

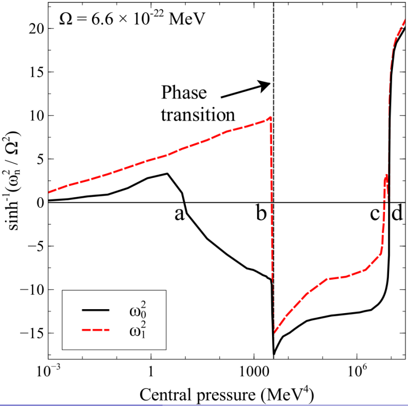


Dynamical stability of Alford, Harris, Sachdeva (2017)

They **also** perform the dynamical stability analysis by solving the Sturm-Liouville problem [Chandrasekhar, 1964] for the amplitudes, u_n, i.e.

$$\frac{d}{dr}\left(\Pi\frac{du_n}{dr}\right)+(Q+\omega_n^2 W)u_n=0.$$





Radial oscillations with interior discontinuities

THE ASTROPHYSICAL JOURNAL, 860:12 (14pp), 2018 June 10 https://doi.org/10.3847/1538-4357/aabfbf

CrossMark

Phase Transition Effects on the Dynamical Stability of Hybrid Neutron Stars

Jonas P. Pereira[1,2], César V. Flores[1,3], and Germán Lugones[1]
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[2] Mathematical Sciences and STAG Research Centre, University of Southampton, Southampton, SO17 1BJ, UK
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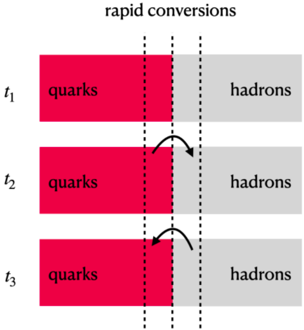
Abstract

We study radial oscillations of hybrid nonrotating neutron stars composed by a quark matter core and hadronic external layers. At first, we physically deduce the junction conditions that should be imposed between the two phases in these systems when perturbations take place. Then we compute the oscillation spectrum focusing on the effects of slow and rapid phase transitions at the quark-hadron interface. We use a generic MIT-bag model for

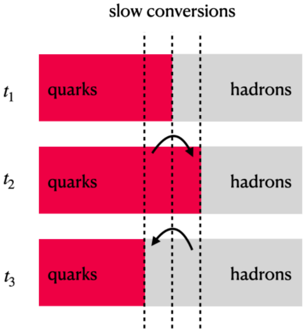
Boundary conditions in 1st-order phase transitions

$\tau_{reactions} \ll \omega_0^{-1} \sim 1$ ms



$$\begin{cases} \Delta p^+ = \Delta p^- \\ \left[\xi - \dfrac{\Delta p}{r p_0'}\right]^+ = \left[\xi - \dfrac{\Delta p}{r p_0'}\right]^- \end{cases}$$

$\tau_{reactions} \gg \omega_0^{-1} \sim 1$ ms



$$\begin{cases} \Delta p^+ = \Delta p^- \\ \xi^+ = \xi^- \end{cases}$$

Recent result of
Di Clemente, Drago, Char, Pagliara (2022)

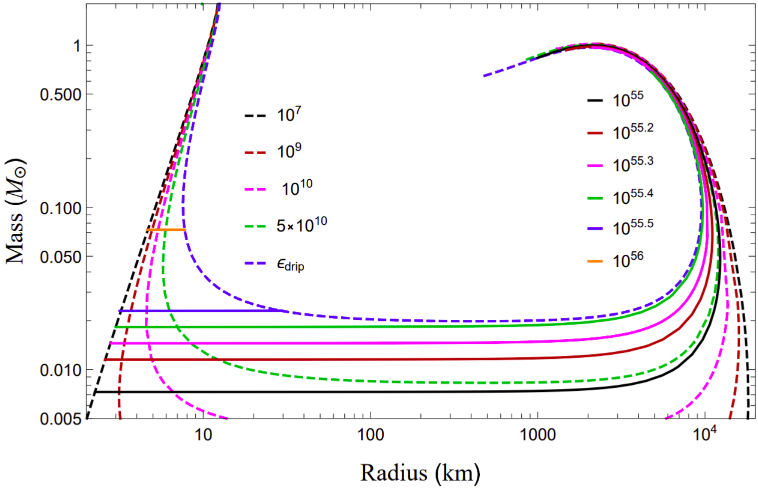
Stability and instability of strange dwarfs

Francesco Di Clemente[1,2], Alessandro Drago[1,2], Prasanta Char[3] and Giuseppe Pagliara[1,2]
[1] *Dipartimento di Fisica e Scienze della Terra, Università di Ferrara, Via Saragat 1, I-44122 Ferrara, Italy,*
[2] *INFN Sezione di Ferrara, Via Saragat 1, I-44122 Ferrara, Italy,*
[3] *Space Sciences, Technologies and Astrophysics Research (STAR) Institute, Université de Liège, Bât. B5a, 4000 Liège, Belgium*

More than 20 years ago, Glendenning, Kettner and Weber proposed the existence of stable white dwarfs with a core of strange quark matter. More recently, by studying radial modes, Alford, Harris and Sachdeva concluded instead that those objects are unstable. We investigate the stability of these objects by looking again at their radial oscillations, while incorporating boundary conditions at the quark-hadron interface which correspond either to a rapid or to a slow conversion of hadrons into quarks. Our analysis shows that objects of this type are stable if the star is not strongly perturbed and ordinary matter cannot transform into strange quark matter because of the Coulomb barrier separating the two components. On the other hand, ordinary matter can be transformed into strange quark matter if the star undergoes a violent process, as in the preliminary stages of a type Ia supernova, and this causes the system to become unstable and to collapse into a strange quark star. In this way, accretion induced collapse of strange dwarfs can be facilitated and km-sized objects with subsolar masses can be produced.

arXiv:2207.08704

Recent result of Di Clemente, Drago, Char, Pagliara (2022)



arXiv:2207.08704

4. OUR FRAMEWORK:

(First-order) Radial oscillation equations

Instead of solving the Sturm-Lioville problem, we use the first-order formalism developed by [Gondek *et al.*, 1997] which is given by

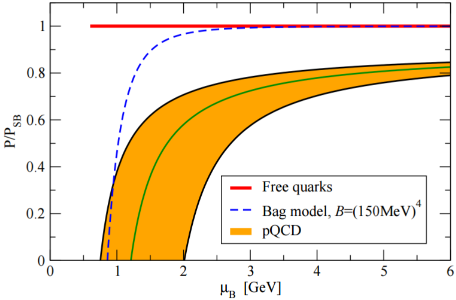
$$\frac{d\xi}{dr} = -\frac{1}{r}\left(3\xi + \frac{\Delta P}{\Gamma P}\right) - \frac{dP}{dr}\frac{\xi}{(P+\epsilon)} ,$$

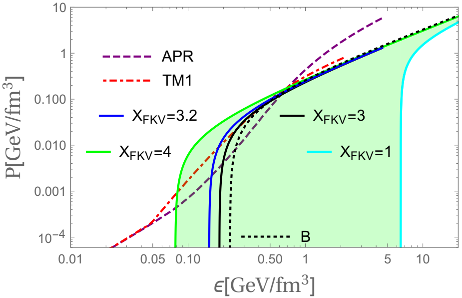
and

$$\frac{d\Delta P}{dr} = \xi\left\{\omega^2 e^{\lambda-\nu}(P+\epsilon)r - 4\frac{dP}{dr}\right\} + \xi\left\{\left(\frac{dP}{dr}\right)^2\frac{r}{(P+\epsilon)} - 8\pi e^{\lambda}(P+\epsilon)Pr\right\} + \Delta P\left\{\frac{dP}{dr}\frac{1}{P+\epsilon} - 4\pi(P+\epsilon)re^{\lambda}\right\} ,$$

where ω is the oscillation frequency.

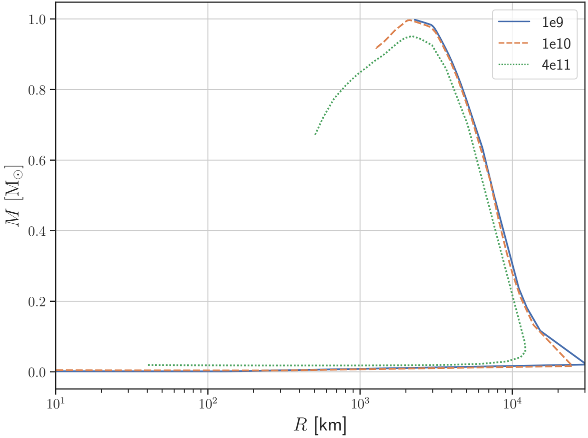
OUR FRAMEWORK:
Cold-quark matter equations of state





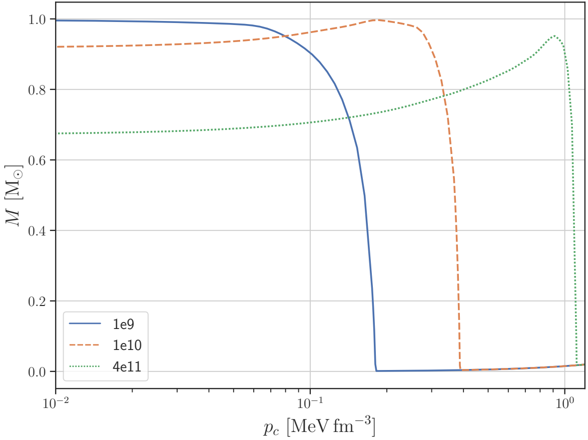
Pressures for the MIT bag model (B) and perturbative QCD (FKV)
[arXiv:1311.5154,1906.11189]

OUR RESULTS:
Strange-dwarf families with the MIT bag



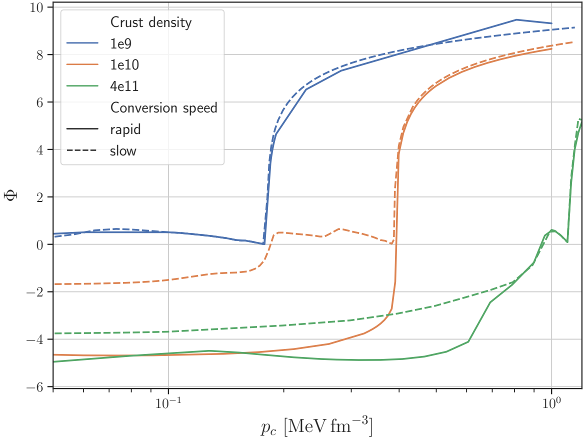
Using the MIT bag model (B) with different ϵ_{crust}

OUR RESULTS:
Strange-dwarf families with the MIT bag



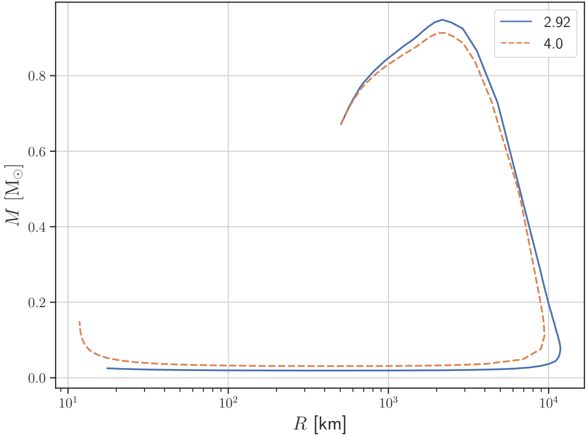
Using the MIT bag model (B) with different ϵ_{crust}

OUR RESULTS:
Strange-dwarf spectrum with the MIT bag



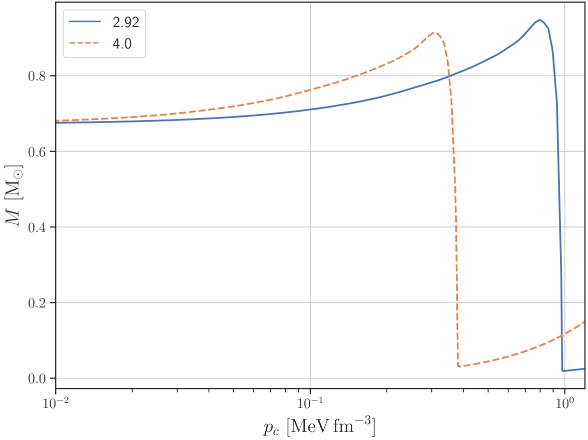
Using the MIT bag model (B)

OUR RESULTS:
Strange-dwarf families with pQCD



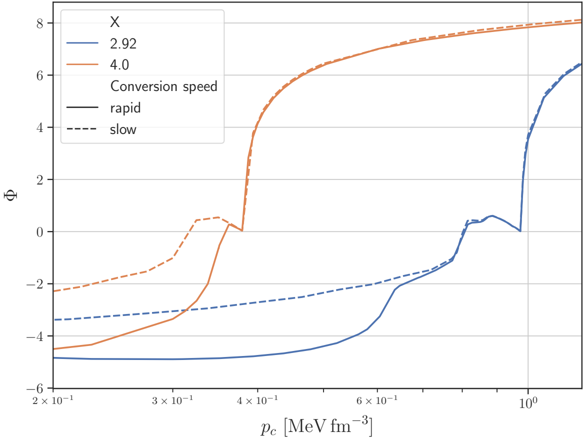
Using pQCD (FKV[X]) and fixed $\epsilon_{\rm crust} = 4 \times 10^{11} {\rm g/cm}^3$

OUR RESULTS:
Strange-dwarf families with pQCD



Using pQCD (FKV[X]) and fixed $\epsilon_{\rm crust} = 4 \times 10^{11} \mathrm{g/cm^3}$

OUR RESULTS:
Strange-dwarf spectrum with pQCD



Using pQCD (FKV[X]) and fixed $\epsilon_{\text{crust}} = 4 \times 10^{11}\text{g/cm}^3$

4. Conclusions

- We revisit the dynamical-stability question of strange-dwarf hybrid stars solving the radial-oscillation equations analyzing properly the associated eigenfrequencies.

- In contrast to past results, our calculations indicate that the strange-dwarf family is really **stable** when rapid and slow conversions occur. The so-called *reaction mode* plays a fundamental role since in most situations one finds that $R_{\rm core} \ll R_{\rm crust}$.

- In order to prove the robustness our findings we performed all our radial-oscillation calculations also using the FKV pocket formula containing state-of-the-art results from cold and dense pQCD.

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