Exploring the effects of Delta Baryons in magnetars

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with the collaboration of
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25/10/2022


Quantum Field Theory

Equations of Motion

Chemical Equilibrium & Charge Neutrality

Matter Composition

Equations of State

Phase Transition

Hydrostatic Equilibrium

Compact Star

Magnetic Fields
Relativistic effective models in compact star description

\[ \mathcal{L} = \sum_b \bar{\psi}_b \left[ \gamma^\mu \left( i \partial^\mu - g_{\omega b} \omega^\mu - g_{\phi b} \phi^\mu - \frac{g_{\rho b}}{2} \vec{\tau} \cdot \vec{\rho}^\mu \right) - M \right] \psi_b \]

\[ + \frac{1}{2} \left( \partial_\mu \sigma \partial^\mu \sigma - m^2 \sigma^2 \right) - \frac{\lambda_1}{3} \sigma^3 - \frac{\lambda_2}{4} \sigma^4 \]

\[ - \frac{1}{4} \Omega_{\mu\nu} \Omega^{\mu\nu} + \frac{1}{2} m^2 \omega_\mu \omega^\mu - \frac{1}{4} \Phi_{\mu\nu} \Phi^{\mu\nu} + \frac{1}{2} m^2 \phi_\mu \phi^\mu \]

\[ - \frac{1}{4} \vec{R}_{\mu\nu} \cdot \vec{R}^{\mu\nu} + \frac{1}{2} m^2 \vec{\rho}_\mu \cdot \vec{\rho}^\mu + g_{\omega \rho} \omega_\mu \omega^\mu \vec{\rho}_\mu \cdot \vec{\rho}^\mu, \quad (1) \]
\[
\mathcal{L} = \sum_b \bar{\psi}_b \left[ \gamma_{\mu} \left( i \partial^\mu - g_{\omega b} \omega^\mu - g_{\phi b} \phi^\mu - \frac{g_{\rho b}}{2} \vec{\tau} \cdot \vec{\rho}^\mu \right) - M \right] \psi_b
\]

\[
\frac{1}{2} \left( \partial_{\mu} \sigma \partial^\mu \sigma - m^2 \sigma^2 \right) - \frac{\lambda_1}{3} \sigma^3 - \frac{\lambda_2}{4} \sigma^4
\]

\[
- \frac{1}{4} \Omega_{\mu\nu} \Omega^{\mu\nu} + \frac{1}{2} m^2 \omega_{\mu} \omega^\mu - \frac{1}{4} \Phi_{\mu\nu} \Phi^{\mu\nu} + \frac{1}{2} m^2 \phi_{\mu} \phi^\mu
\]

\[
- \frac{1}{4} \vec{R}_{\mu\nu} \cdot \vec{R}^{\mu\nu} + \frac{1}{2} m^2 \vec{\rho}_{\mu} \cdot \vec{\rho}^\mu + g_{\omega \rho} \omega_{\mu} \omega^\mu \vec{\rho}_{\mu} \cdot \vec{\rho}^\mu
\]  

\[ (1) \]

<table>
<thead>
<tr>
<th>Model</th>
<th>(n_0)</th>
<th>(B/A)</th>
<th>(K)</th>
<th>(S)</th>
<th>(L)</th>
<th>(M/m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>GM1</td>
<td>0.153</td>
<td>16.33</td>
<td>300.5</td>
<td>32.5</td>
<td>94</td>
<td>0.70</td>
</tr>
<tr>
<td>L3(\omega\rho)</td>
<td>0.156</td>
<td>16.20</td>
<td>256</td>
<td>31.2</td>
<td>74</td>
<td>0.69</td>
</tr>
<tr>
<td>DDME2</td>
<td>0.152</td>
<td>16.14</td>
<td>251</td>
<td>32.3</td>
<td>51</td>
<td>0.57</td>
</tr>
<tr>
<td>Constr.</td>
<td>0.148–0.170</td>
<td>15.8–16.5</td>
<td>220–260</td>
<td>28.6–34.4</td>
<td>36.0–86.8</td>
<td>0.6–0.8</td>
</tr>
</tbody>
</table>

**Table:** symmetric nuclear matter properties at saturation density for the models employed in this work.
Relativistic effective models in compact star description

\[ \mu_b = \mu_n - q_b \mu_e \]  \hspace{1cm} (2)

\[ \sum_{i=b,l} q_i n_i = 0 \]  \hspace{1cm} (3)
\begin{align}
\mu_b &= \mu_n - q_b \mu_e \\
\sum_{i=b,l} q_i n_i &= 0
\end{align}

\begin{align}
\varepsilon &= \sum_b \frac{1}{\pi^2} \int_0^{p_{fb}} dp \, p^2 \sqrt{p^2 + M_b^2} + \frac{1}{2} m_\sigma^2 \sigma_0^2 + \frac{\lambda_1}{3} \sigma_0^3 + \frac{\lambda_2}{4} \sigma_0^4 \\
&\quad - \frac{1}{2} m_\omega^2 \omega_0^2 - \frac{1}{2} m_\phi^2 \phi_0^2 - \frac{1}{2} m_\rho^2 \rho_0^2 - g_{\omega \rho} \omega_0^2 \rho_0^2 + \varepsilon_{\text{leptons}}, \\
P &= -\varepsilon + \sum_b \mu_b n_b
\end{align}
\[ \mu_b = \mu_n - q_b \mu_e \] (2)

\[ \sum_{i=b,l} q_i n_i = 0 \] (3)

\[ \varepsilon = \sum_b \frac{1}{\pi^2} \int_0^{p_F_b} dp \, p^2 \sqrt{p^2 + M_b^2} + \frac{1}{2} m_\sigma^2 \sigma_0^2 + \frac{\lambda_1}{3} \sigma_0^3 + \frac{\lambda_2}{4} \sigma_0^4 
- \frac{1}{2} m_\omega^2 \omega_0^2 - \frac{1}{2} m_\phi^2 \phi_0^2 - \frac{1}{2} m_\rho^2 \rho_0^2 - g_\omega \rho_0^2 \rho_0^2 + \varepsilon_{\text{leptons}}, \] (4)

\[ P = -\varepsilon + \sum_b \mu_b n_b \] (5)

\[ \frac{dP}{dr} = - \frac{[\varepsilon(r) + P(r)] [m(r) + 4\pi r^3 P(r)]}{r [r - 2m(r)]}, \] (6)

\[ m(r) = \int_0^r dr' 4\pi r'^2 \varepsilon(r') \] (7)
<table>
<thead>
<tr>
<th></th>
<th>$M_b$ (MeV)</th>
<th>$q_b(e)$</th>
<th>$I_{3b}$</th>
<th>$S_b$</th>
<th>$\mu_b/\mu_N$</th>
<th>$\kappa_b/\mu_N$</th>
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</thead>
<tbody>
<tr>
<td>$p$</td>
<td>939</td>
<td>+1</td>
<td>+1/2</td>
<td>1/2</td>
<td>2.79</td>
<td>1.79</td>
</tr>
<tr>
<td>$n$</td>
<td>939</td>
<td>0</td>
<td>−1/2</td>
<td>1/2</td>
<td>−1.91</td>
<td>−1.91</td>
</tr>
<tr>
<td>$\Lambda$</td>
<td>1116</td>
<td>0</td>
<td>0</td>
<td>1/2</td>
<td>−0.61</td>
<td>−0.61</td>
</tr>
<tr>
<td>$\Sigma^+$</td>
<td>1193</td>
<td>+1</td>
<td>1</td>
<td>1/2</td>
<td>2.46</td>
<td>1.67</td>
</tr>
<tr>
<td>$\Sigma^0$</td>
<td>1193</td>
<td>0</td>
<td>0</td>
<td>1/2</td>
<td>1.61</td>
<td>1.61</td>
</tr>
<tr>
<td>$\Sigma^-$</td>
<td>1193</td>
<td>−1</td>
<td>−1</td>
<td>1/2</td>
<td>−1.16</td>
<td>−0.37</td>
</tr>
<tr>
<td>$\Xi^0$</td>
<td>1315</td>
<td>0</td>
<td>+1/2</td>
<td>1/2</td>
<td>−1.25</td>
<td>−1.25</td>
</tr>
<tr>
<td>$\Xi^-$</td>
<td>1315</td>
<td>−1</td>
<td>−1/2</td>
<td>1/2</td>
<td>−0.65</td>
<td>0.06</td>
</tr>
<tr>
<td>$\Delta^{++}$</td>
<td>1232</td>
<td>+2</td>
<td>+3/2</td>
<td>3/2</td>
<td>4.99</td>
<td>3.47</td>
</tr>
<tr>
<td>$\Delta^+$</td>
<td>1232</td>
<td>+1</td>
<td>+1/2</td>
<td>3/2</td>
<td>2.49</td>
<td>1.73</td>
</tr>
<tr>
<td>$\Delta^0$</td>
<td>1232</td>
<td>0</td>
<td>−1/2</td>
<td>3/2</td>
<td>0.06</td>
<td>0.06</td>
</tr>
<tr>
<td>$\Delta^-$</td>
<td>1232</td>
<td>−1</td>
<td>−3/2</td>
<td>3/2</td>
<td>−2.45</td>
<td>−1.69</td>
</tr>
</tbody>
</table>

$$g_{ib} = x_{ib}g_i$$ (8)
Figure 2 The $P$–$\dot{P}$ diagram illustrating the placement of the different isolated neutron star classes. The blue dots mark pulsars detected both in the radio and X-ray bands, the red ones those observed only at X-ray energies. The lines of constant age and magnetic field are also shown (courtesy R.P. Mignani).
\[
\int d^3 k \rightarrow \frac{|q| B}{(2\pi)^2} \sum \nu \int dk_z, \quad \text{where} \quad \nu = n + \frac{1}{2} - \frac{s}{2} \frac{q_b}{|q_b|}
\]

\[
\nu_{\text{max}}(s) = \left[ \frac{(E_{Fb}^* + s \kappa_b B)^2 - M_{b}^*}{2|q_b|B} \right]^{\frac{1}{2}}
\]
Matter composition under extreme magnetic fields

\[ \int d^3 k \rightarrow \frac{|q|B}{(2\pi)^2} \sum_{\nu} \int dk_z, \quad \text{where} \quad \nu = n + \frac{1}{2} - \frac{s}{2} \frac{q_b}{|q_b|} \]

(9)

\[ \nu_{\text{max},b}(s) = \left[ \frac{(E_{F,b}^* + s\kappa_b B)^2 - M_b^*^2}{2|q_b|B} \right] \]

(10)

\[ q_b = 0: \]

\[ k_{F,b}^2(s) = E_{F,b}^*^2 - (M_b^* - s\kappa_b B)^2 \]

(11)

\[ n_b = \frac{1}{2\pi^2} \sum_s \left\{ \frac{k_{F,b}^2(s)}{3} - \frac{s\kappa_b B}{2} \left[ (M_b^* - s\kappa_b B) k_{F,b} E_{F,b}^*^2 \left( \arcsin \left( \frac{M_b^* - s\kappa_b B}{E_{F,b}^*} \right) - \frac{\pi}{2} \right) \right] \right\} \]

(12)

\[ n_{s,b} = \frac{M_b^*}{4\pi^2} \sum_s \left[ E_{F,b}^* k_{F,b}(s) - (M_b^* - s\kappa_b B)^2 \ln \left| \frac{k_{F,b}(s) + E_{F,b}^*}{M_b^* - s\kappa_b B} \right| \right] \]

(13)

\[ q_b \neq 0: \]

\[ k_{F,b}^2(\nu, s) = E_{F,b}^*^2 - \left( \sqrt{M_b^*^2 + 2\nu |q_b|B - s\kappa_b B} \right)^2 \]

(14)

\[ n_b = \frac{|q_b|B}{2\pi^2} \sum_{\nu, s} k_{F,b}(\nu, s) \]

(15)

\[ n_{s,b} = \frac{|q_b|B M_b^*}{2\pi^2} \sum_{s, \nu} \sqrt{M_b^*^2 + 2\nu |q_b|B - s\kappa_b B} \ln \left| \frac{k_{F,b}(\nu, s) + E_{F,b}^*}{\sqrt{M_b^*^2 + 2\nu |q_b|B - s\kappa_b B}} \right| \]

(16)
Figure: Particle composition of neutron-star matter with \( \Delta s \), with \( B = 0 \) (top panels) and magnetic field \( B = 3 \times 10^{18} \) G (bottom panels), when considering (solid lines) or disregarding (dashed lines) the effects of the AMMs.
\[ Y_{\text{spin}} = \frac{\sum_{b,s} s n_b(s)}{\sum_{b,s} n_b(s)} , \]  

\textbf{Figure:} Spin polarization fraction as a function of baryon number density for neutron-star matter with magnetic field \( B = 3 \times 10^{18} \) G, when considering (solid lines) or disregarding (dashed lines) the effects of the AMMs.
Macroscopic structure effects of magnetic fields

Figure: Stellar mass as a function of equatorial radius for different compositions and interaction strengths, for central magnetic fields $B = 0$ (solid lines), $B = 5 \times 10^{17}$ G (dashed lines), and $B = 10^{18}$ G (dotted lines).

Table: Central baryon ($n_c$) and energy ($\varepsilon_c$) densities as a function of magnetic field strength for neutron stars of radius 12 km with L3$\omega rho$ model for $x_{\sigma\Delta} = x_{\omega\Delta} = 1.0(1.2)$ in the top panel and CMF model in the bottom panel.
Figure: Magnetic field distribution inside a neutron star of mass $1.8M_\odot$ and central magnetic field of $B = 5 \times 10^{17} \text{ G}$. Solid, dashed, dashed-dotted and dotted are, respectively, the first four even multipoles of the magnetic field norm ($l = 0, 2, 4, 6$).
Magnetic field effects on the deconfinement transition

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Magnetic field effects on the deconfinement transition

**Figure**: Mass-radius diagram for hybrid EoS with chemical equilibrium in both phases, showing results without magnetic field effects.

**Figure**: Example of equations of state of parameter choices that allow the hadron-quark phase transition to occur at $B = 3 \times 10^{18} \text{ G}$.

\[
m_i = m_{i0} + \frac{D}{n_b^{1/3}} + C n_b^{1/3} = m_{i0} + m_l, \tag{18}
\]
### Magnetic field effects on the deconfinement transition

<table>
<thead>
<tr>
<th>$C = 0$</th>
<th>$\sqrt{D} = 155$ MeV</th>
<th>B = 0</th>
<th>B = 3 × 10$^{18}$ G</th>
<th>B-W</th>
</tr>
</thead>
<tbody>
<tr>
<td>no crossing</td>
<td>no crossing</td>
<td>yes</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$C = 0$</td>
<td>$\sqrt{D} = 158.5$ MeV</td>
<td>$\mu_0 = 960$</td>
<td>$\mu_0 = 958$</td>
<td>yes</td>
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<tr>
<td>$p_0 = 1.55$</td>
<td>$p_0 = 1.80$</td>
<td></td>
<td></td>
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<tr>
<td>$C = 0$</td>
<td>$\sqrt{D} = 165$ MeV</td>
<td>$\mu_0 = 1062$</td>
<td>$\mu_0 = 1066$</td>
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<tr>
<td>$p_0 = 21.98$</td>
<td>$p_0 = 24.70$</td>
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<td></td>
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<tr>
<td>$C = 0.23$</td>
<td>$\sqrt{D} = 155$ MeV</td>
<td>$\mu_0 = 1130$</td>
<td>$\mu_0 = 1145$</td>
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<tr>
<td>$p_0 = 43.62$</td>
<td>$p_0 = 51.32$</td>
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<tr>
<td>$C = 0.365$</td>
<td>$\sqrt{D} = 142$ MeV</td>
<td>$\mu_0 = 1105$</td>
<td>$\mu_0 = 1109$</td>
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<tr>
<td>$p_0 = 34.98$</td>
<td>$p_0 = 38.30$</td>
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<tr>
<td>$C = 0.5$</td>
<td>$\sqrt{D} = 135.75$ MeV</td>
<td>$\mu_0 = 1202$</td>
<td>$\mu_0 = 1242$</td>
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<tr>
<td>$p_0 = 72.66$</td>
<td>$p_0 = 94.93$</td>
<td></td>
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<tr>
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<td>$\mu_0 = 1440$</td>
<td>$\mu_0 = 1475$</td>
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</tr>
<tr>
<td>$p_0 = 215.50$</td>
<td>$p_0 = 247.53$</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Table:** Values for $\mu_0$ (in MeV) and $p_0$ (in MeV/fm$^3$) for which the conditions of phase coexistence are satisfied at $T = 0$. The latter column specifies whether or not the Bodmer-Witten conjecture is satisfied.
- The understanding of the meson-delta coupling parameters can be refined by symmetry group considerations, as it is made for the hyperon coupling schemes.

- The magnetic field effects on $\Delta$-admixed matter can be more robustly understood by having the complete solution of the spin-3/2 Rarita-Schwinger equation under a magnetic field.
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Muito Obrigado!

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