

This course: *Focus on shape-changing metamaterials*

1. Intuitive examples and tool to characterize metamaterials
(Tuesday)
- 2. Designing metamaterial using combinatorics (today)**
3. Beyond the Unit cell (Thursday)
4. Active metamaterials (Friday)

Exercise (30 min)

$$\frac{d(d+1)}{2} + N_0 - N_S = dN - N_B$$



1. Use Maxwell-Calladine theorem to estimate $N_0 - N_S$ in these two structures
2. Determine the zero modes and states of self-stress by intuition
3. Code up the compatibility matrix
4. Compute its null-space to determine the number of zero modes and their shape
5. Compute the null-space of the equilibrium matrix (transpose of the compatibility matrix)

Reminder:

The compatibility matrix relates the displacement of the nodes to the elongation of the bonds

$$e_\beta = \hat{\mathbf{b}}_\beta \cdot (\mathbf{u}(s'_\beta) - \mathbf{u}(s_\beta)),$$

Hint:

- *Define a matrix of node positions (in 2d, Nx2 matrix)
- *Define an adjacency matrix (N^*N) that has ones when node i and node j share a bond
- *Write a function that calculate the bond vector $\hat{\mathbf{b}}_\beta$.
- *populate the compatibility matrix with the bond vectors
- *See Gitlab for help

<https://uva-hva.gitlab.host/published-projects/CourseMechanicalMetamaterials>

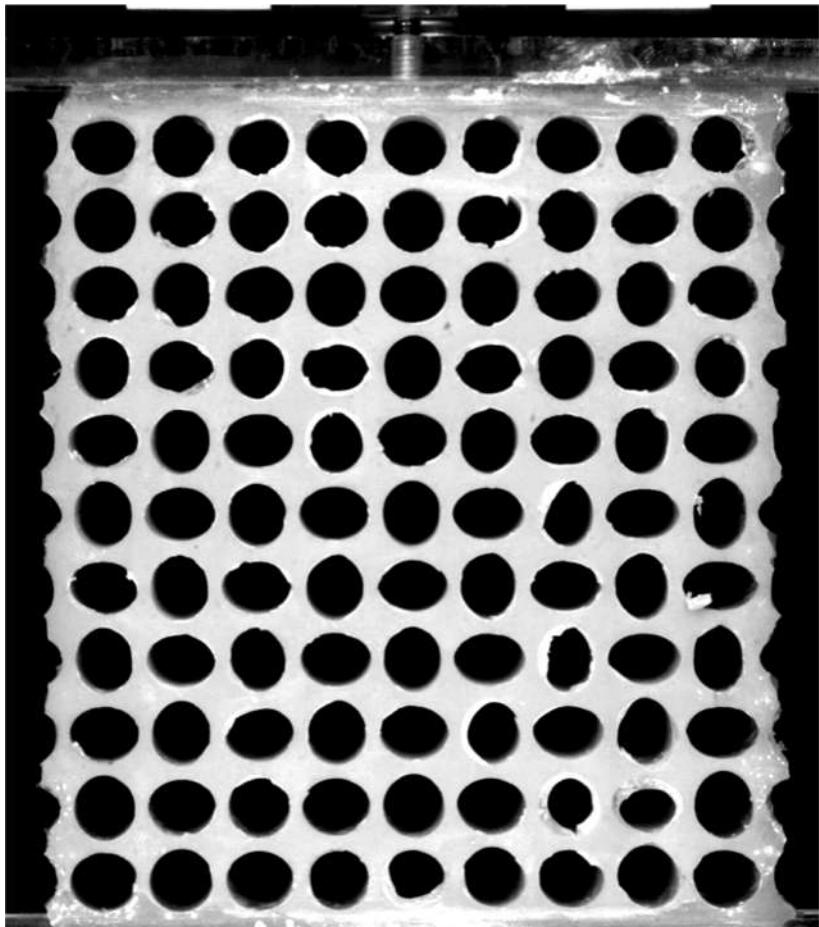


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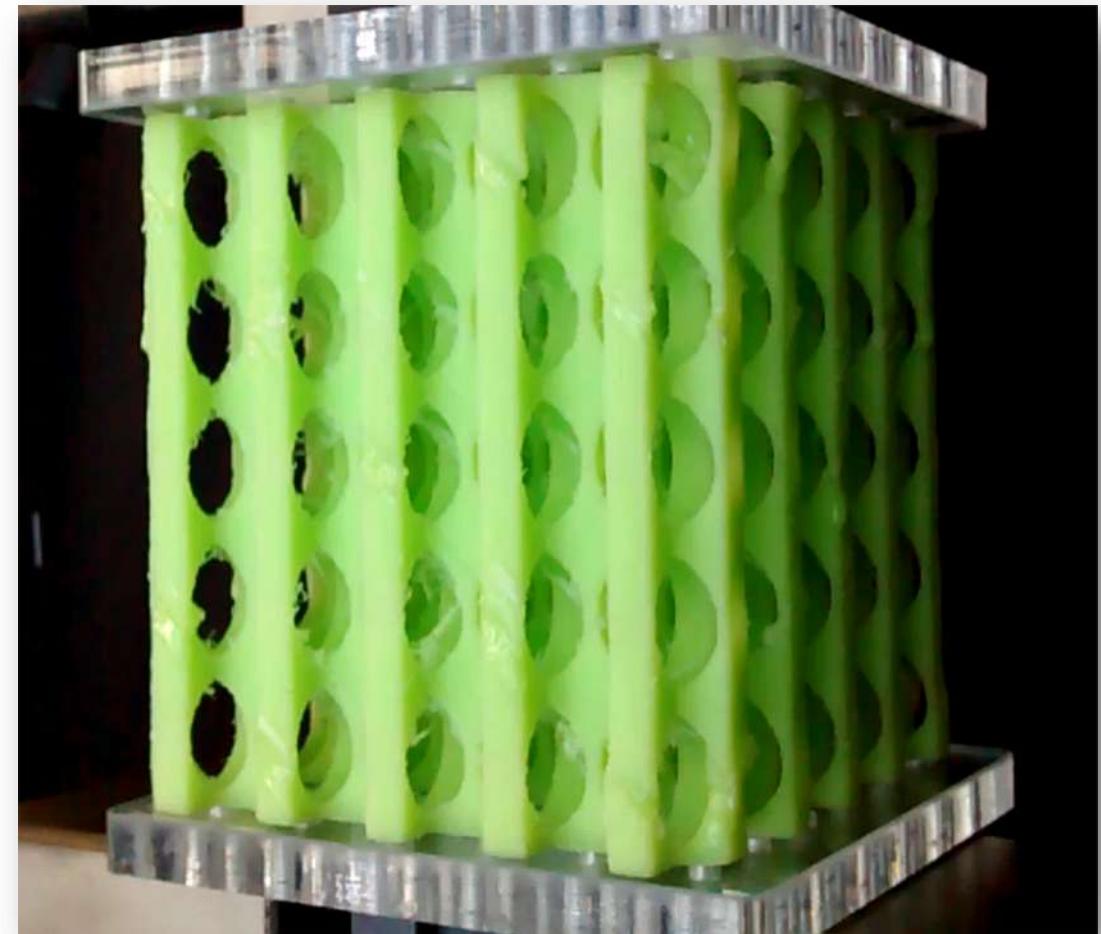
Basic shape changes

2D

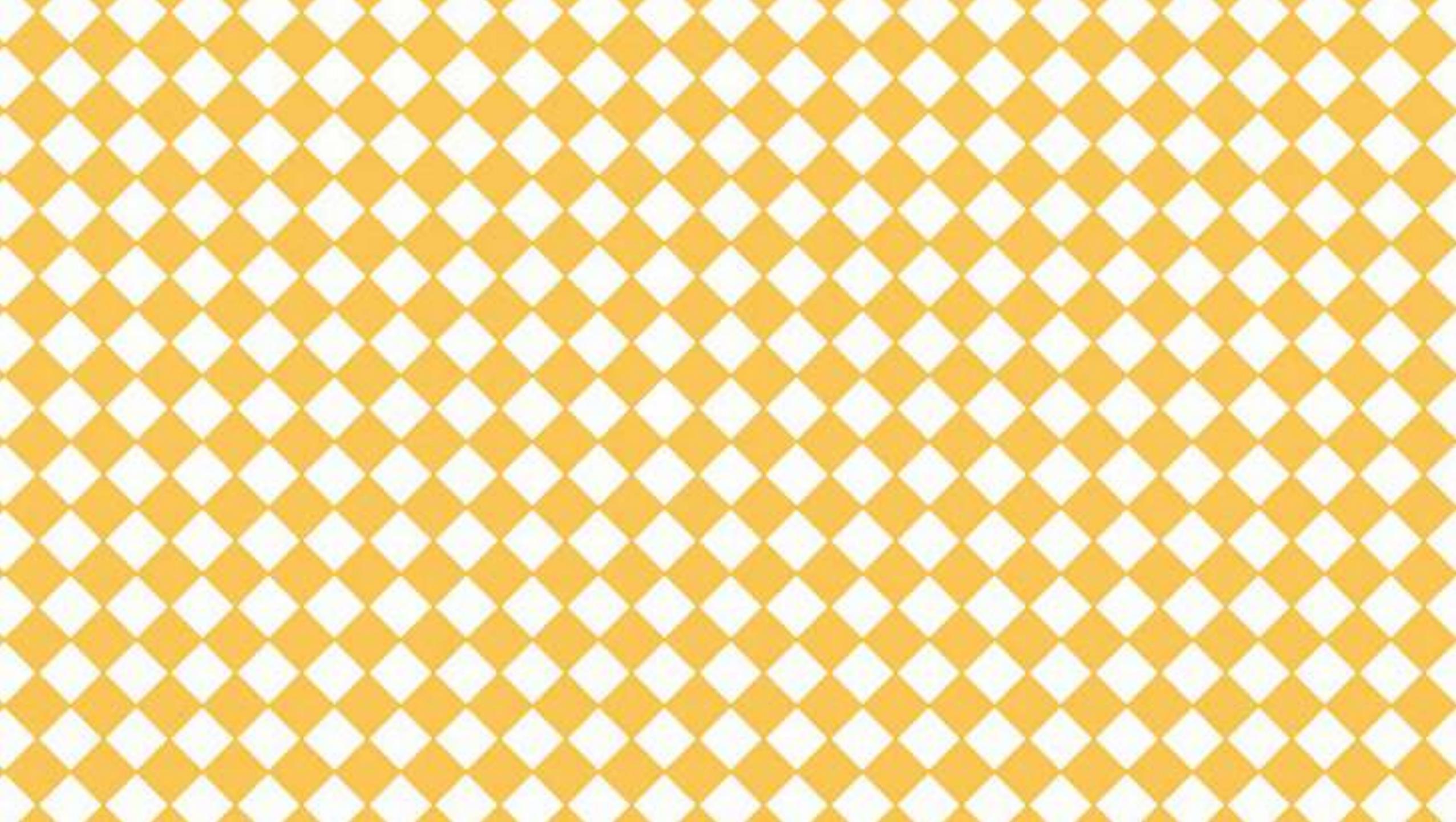


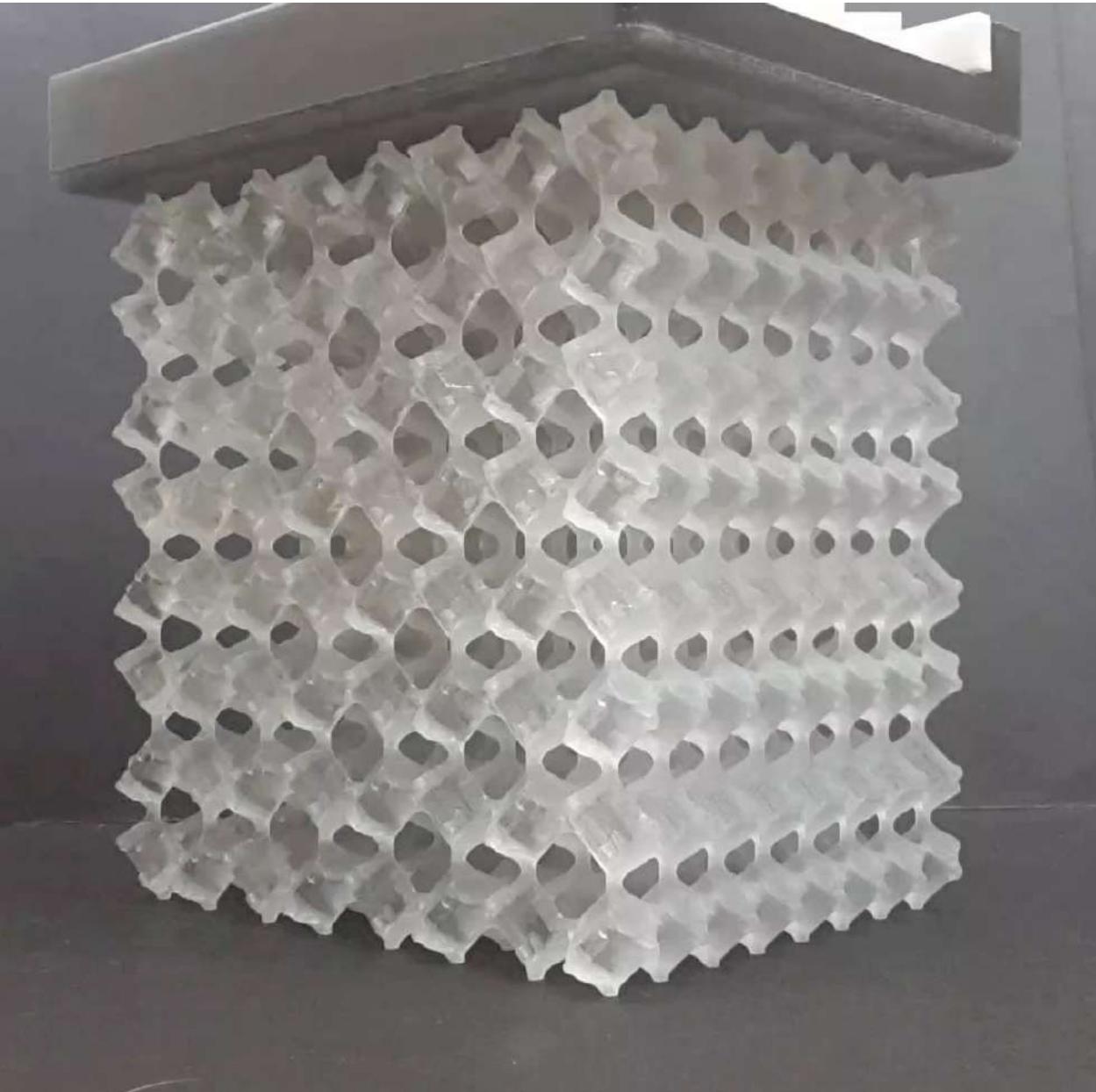
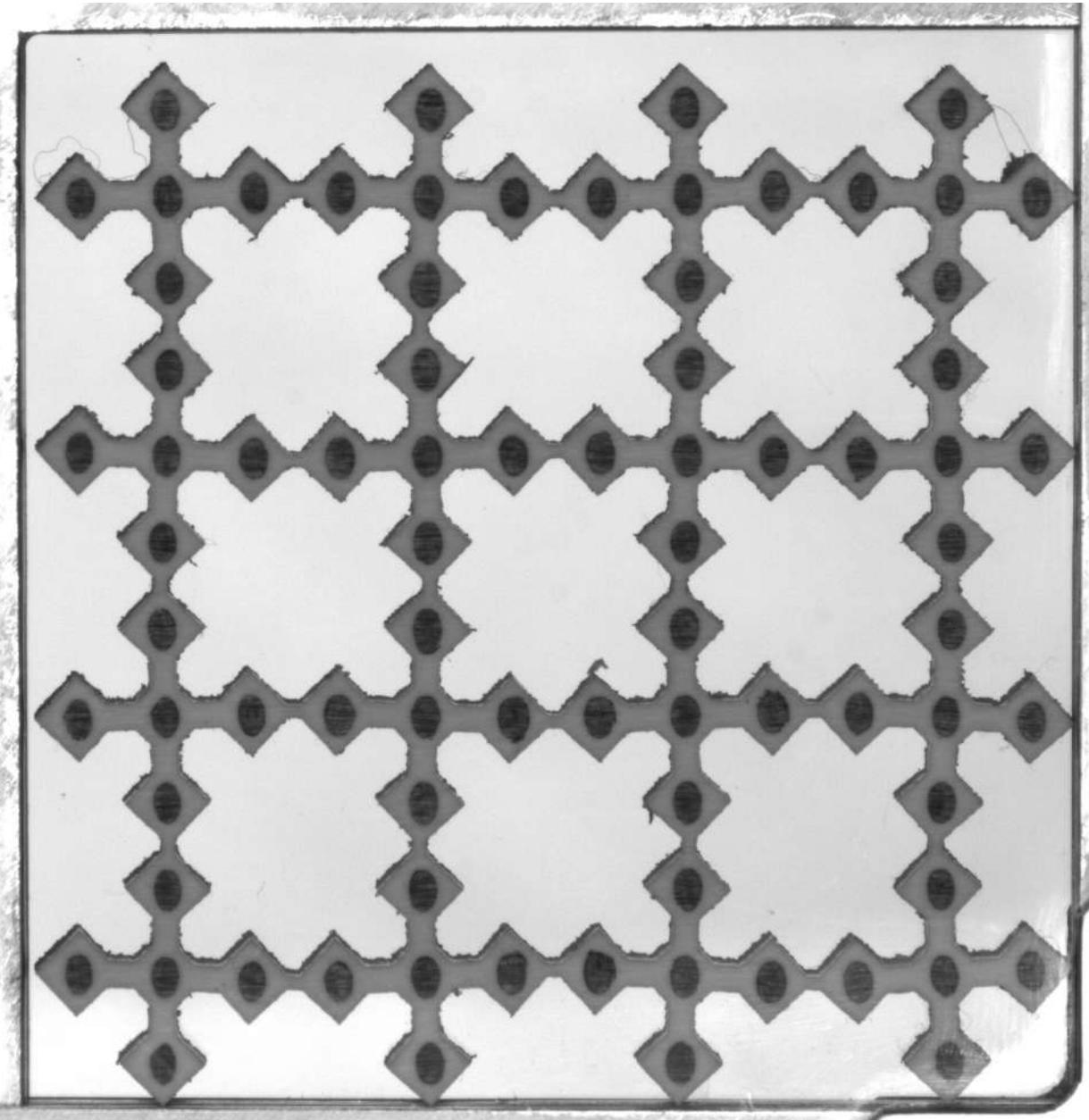
Mullin et al. PRL 2007

3D



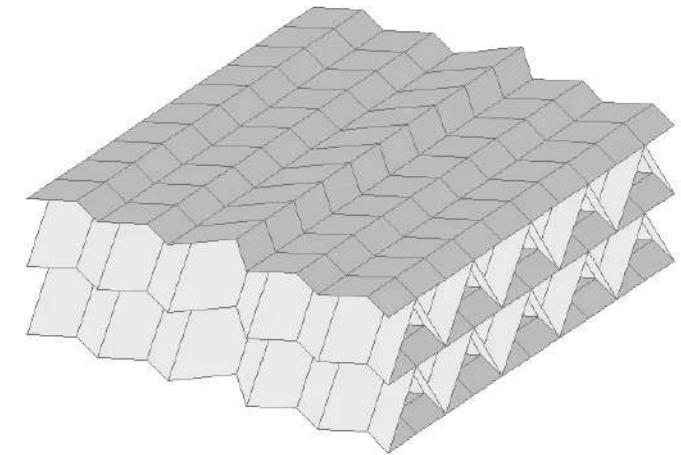
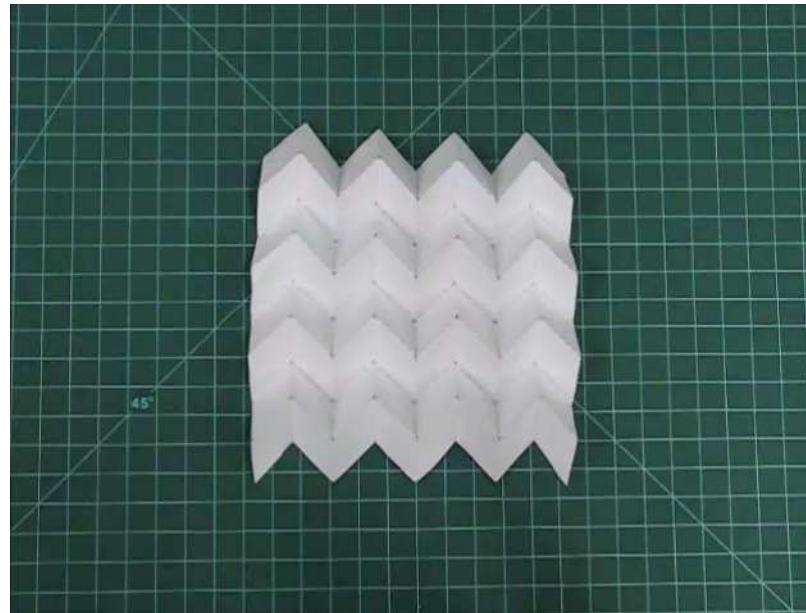
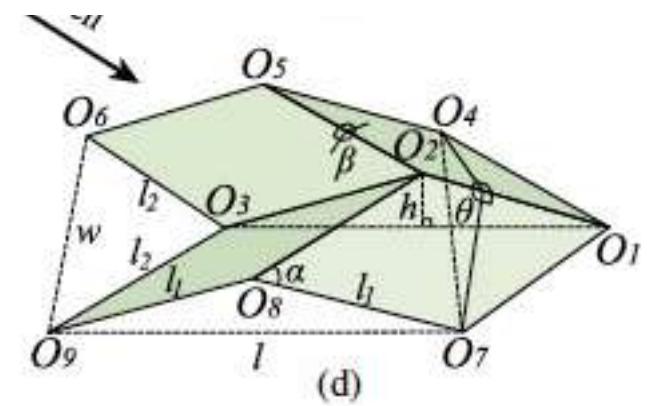
Coulais et al. Nature 2016





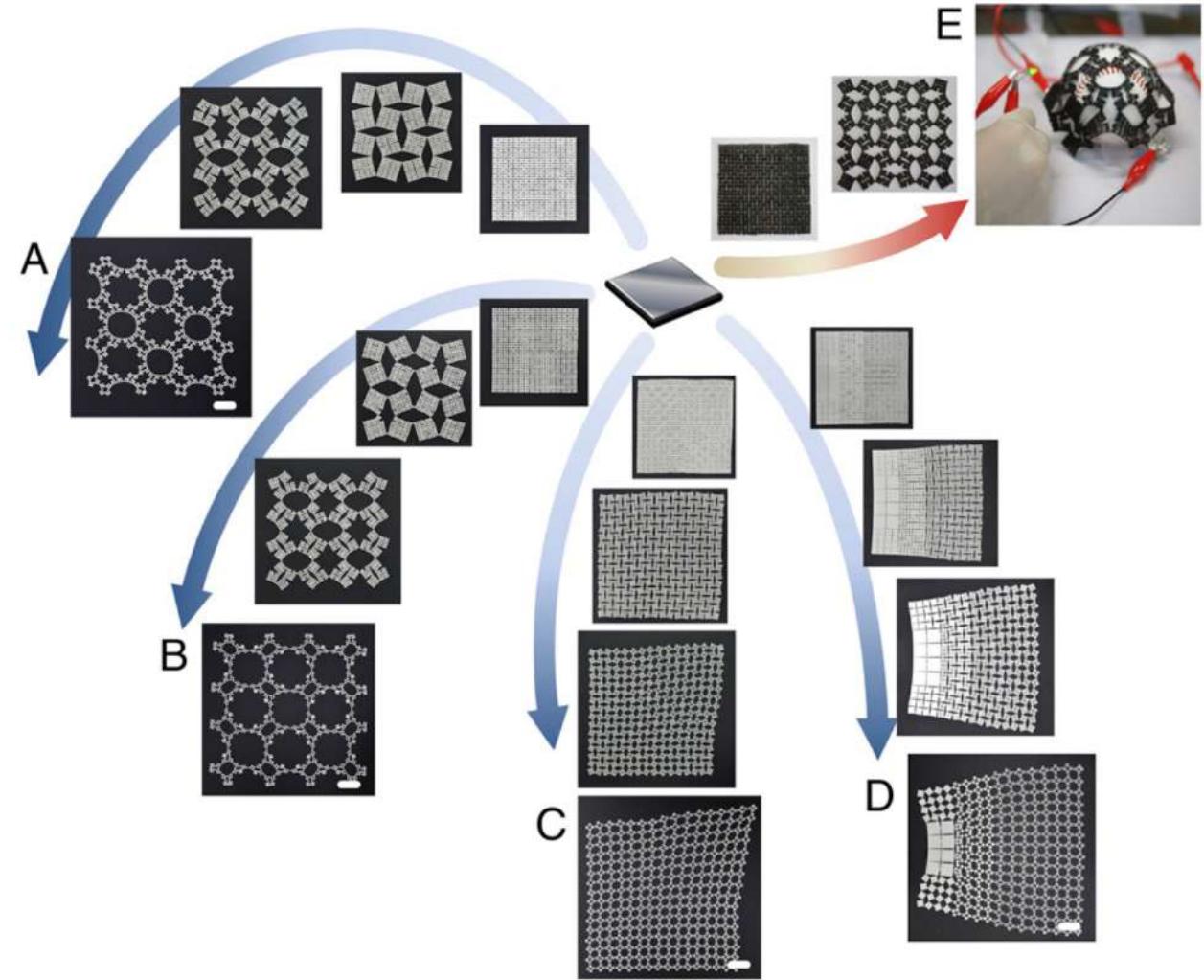
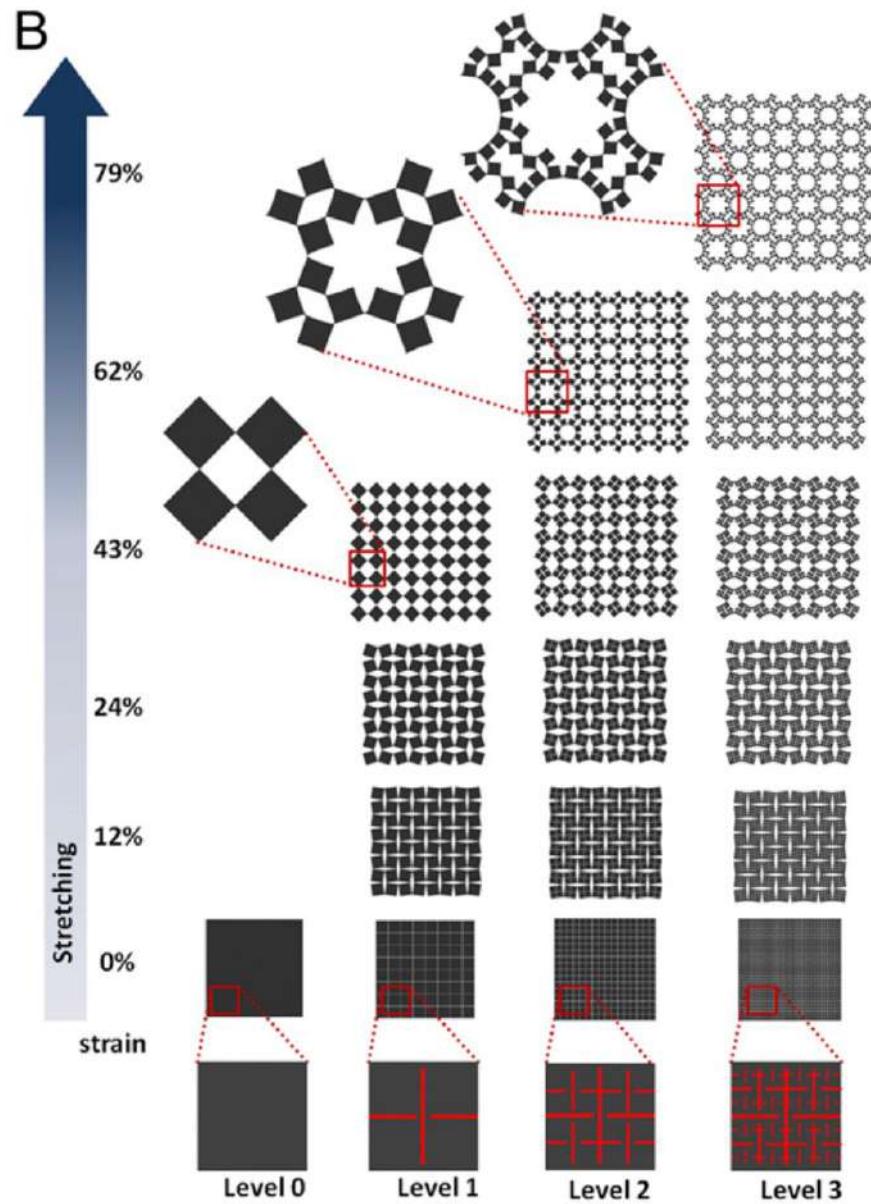
Coulais, Sabbadini, Vink, van Hecke (2018) *Nature*

Origami shape-changers



Miura (1985) *Rep. Inst. of Space Astronautical Sc.*
Wei et al. (2013) *PRL*
Silverberg et al. (2014) *Science*
Schenk & Guest (2013) *PNAS*

Kirigami

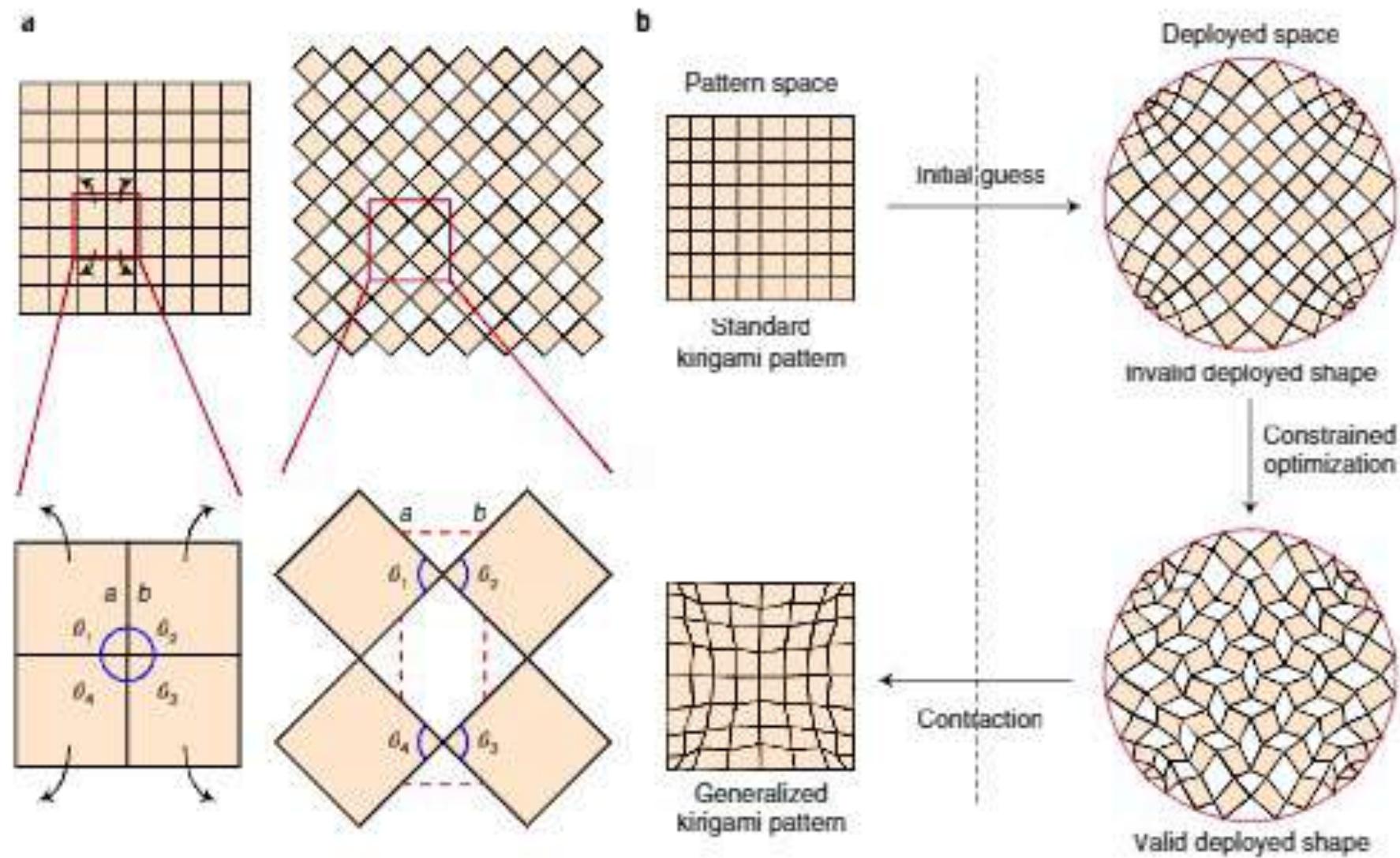


...But how can we design for
shape-changes?

(1) Optimization

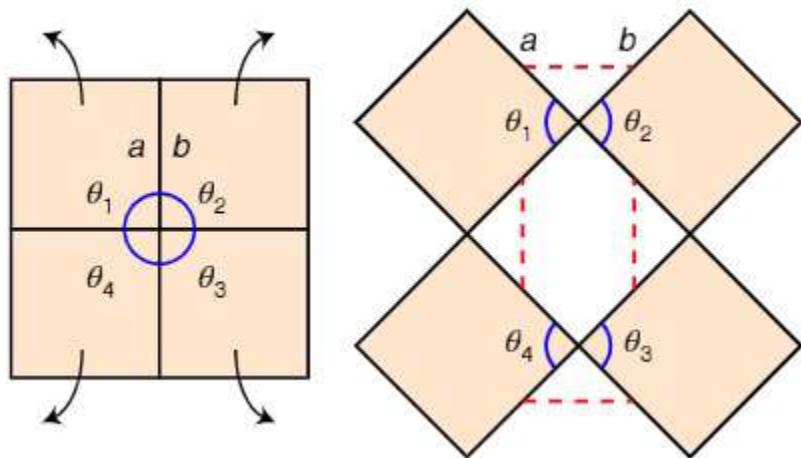
(2) Combinatorial design

Example I: Optimization with constraints



Example I: Optimization with constraints

Constraints to enforce the mechanism



$$a^2 - b^2 = 0$$

$$\theta_1 + \theta_2 + \theta_3 + \theta_4 = 2\pi$$

$$\|\mathbf{p}_i - \tilde{\mathbf{p}}_i\|^2 = 0$$

$$\langle (\mathbf{b} - \mathbf{a}) \times (\mathbf{c} - \mathbf{a}), \hat{\mathbf{n}} \rangle \geq 0$$

(1) Adjacent edges have same length

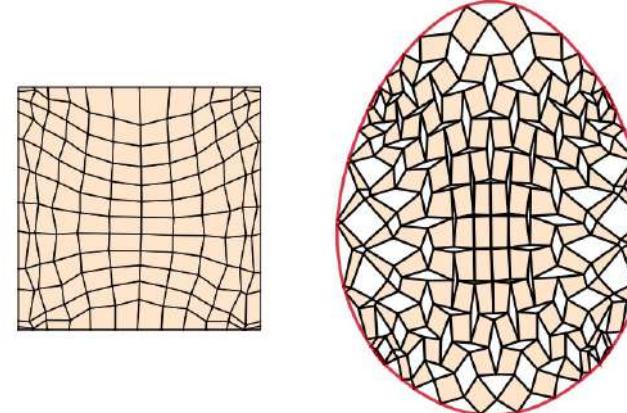
(2) Sum of vertices angle is 2π

(3) Distance to target shape is zero

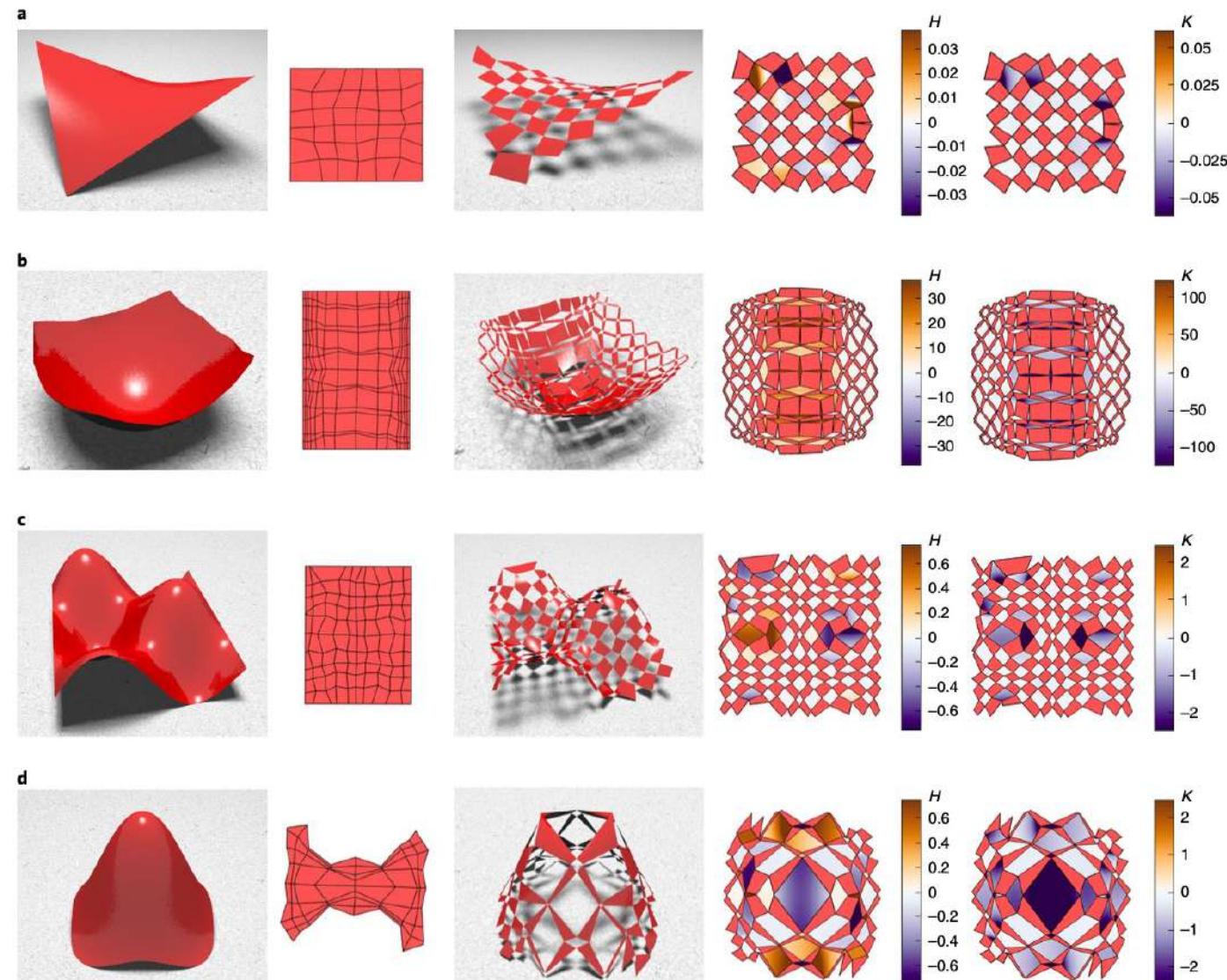
(4) Quads don't overlap

Minimize the objective function
to keep a smooth minimization gradient

$$\frac{1}{M} \sum_{i=1}^M \left(\sum_j (\alpha_{ij} - \beta_{ij})^2 + \sum_k (\alpha_{ik} - \beta_{ik})^2 \right) \quad (5)$$



Example I: Optimization with constraints

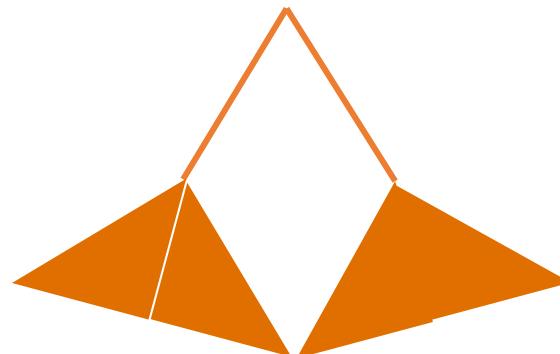
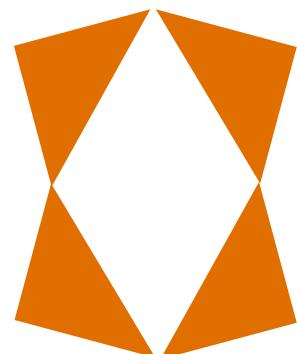
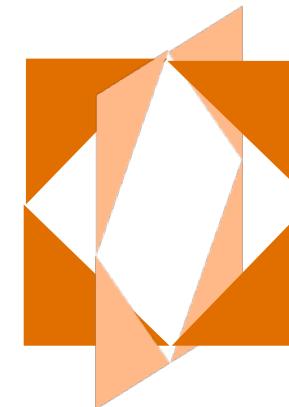
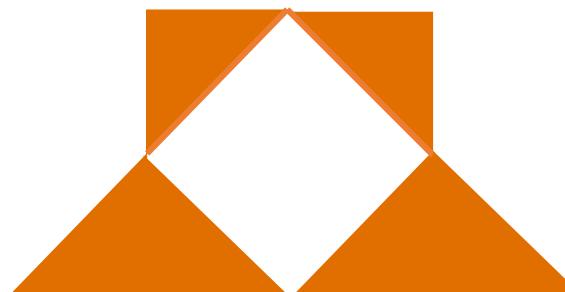
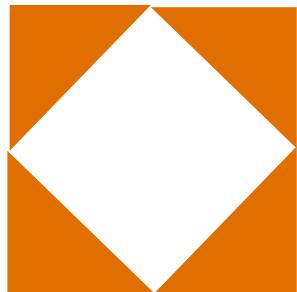


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(1) Optimization

(2) Combinatorial design

How to gain design freedom?

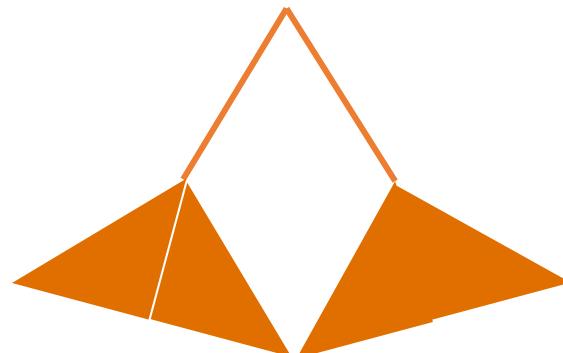
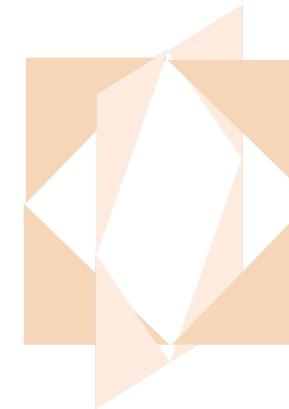
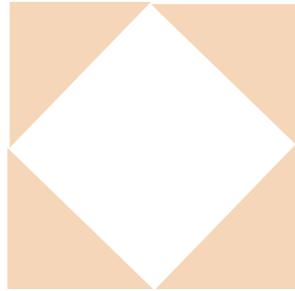


Meeussen et al. Nat. Phys. 2020

Coulais et al. Nature 2016
Van Mastrigt et al., PRL 2022

Bossart, Dykstra et al. PNAS 2021
Van Mastrigt et al., PRL 2022

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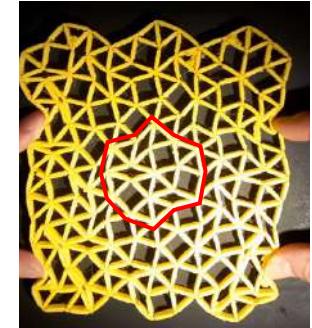
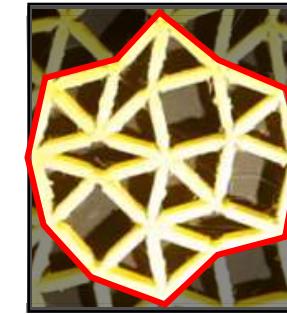
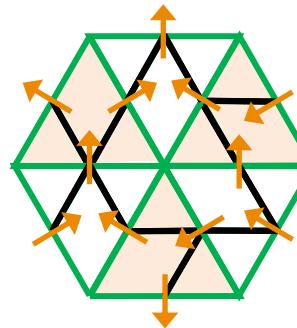
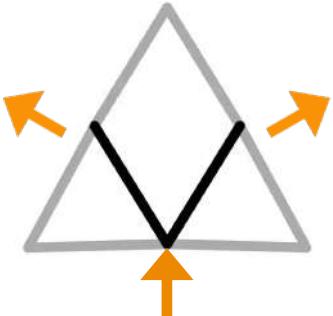
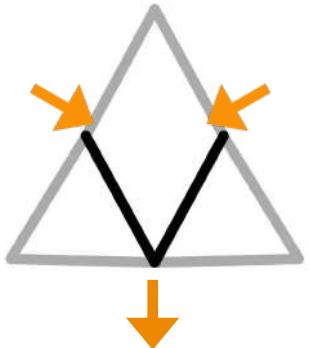
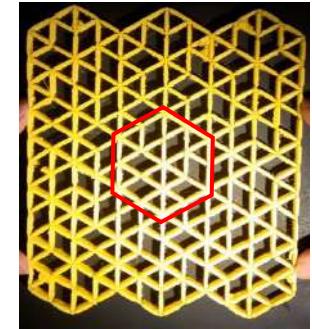
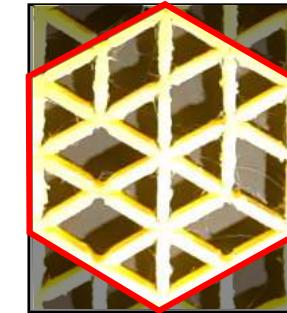
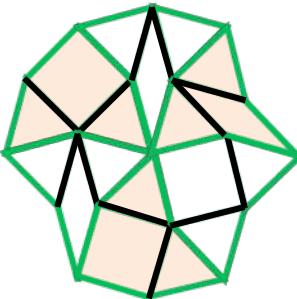
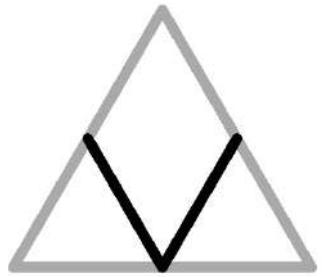


Meeussen et al. Nat. Phys. 2020

Coulais et al. Nature 2021
Van Mastrigt et al., PRL 2022

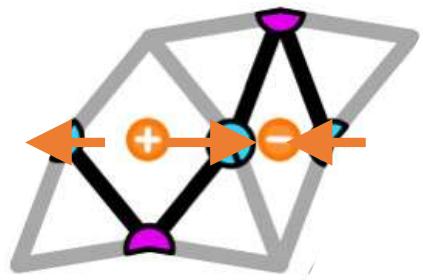
Bossart, Dykstra et al. PNAS 2021
Van Mastrigt et al., PRL 2022

Kinematic compatibility = spin problem

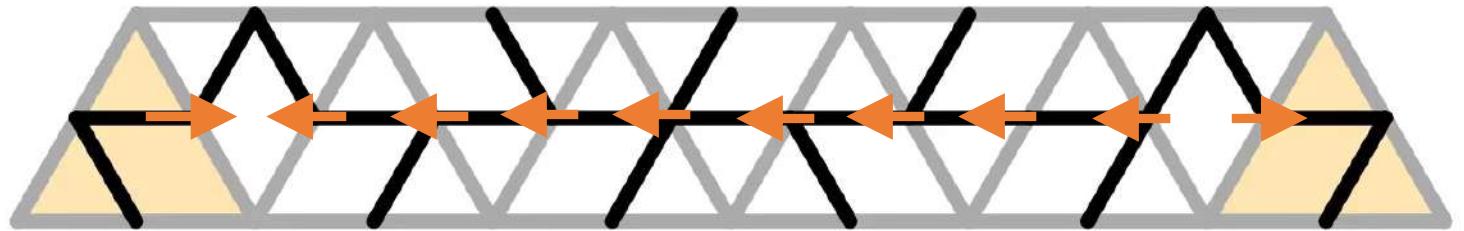


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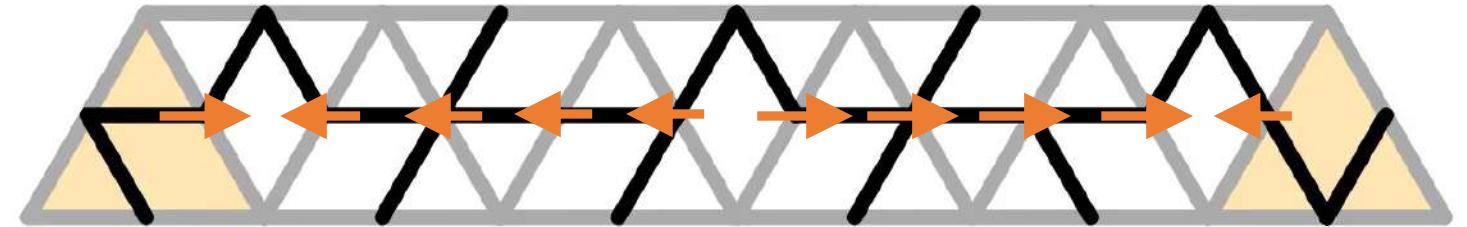
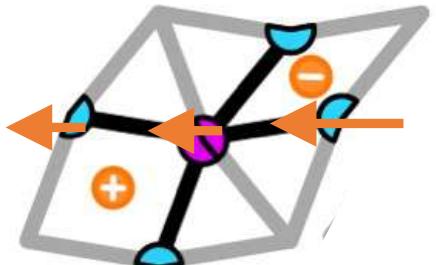
Antiferromagnetic order



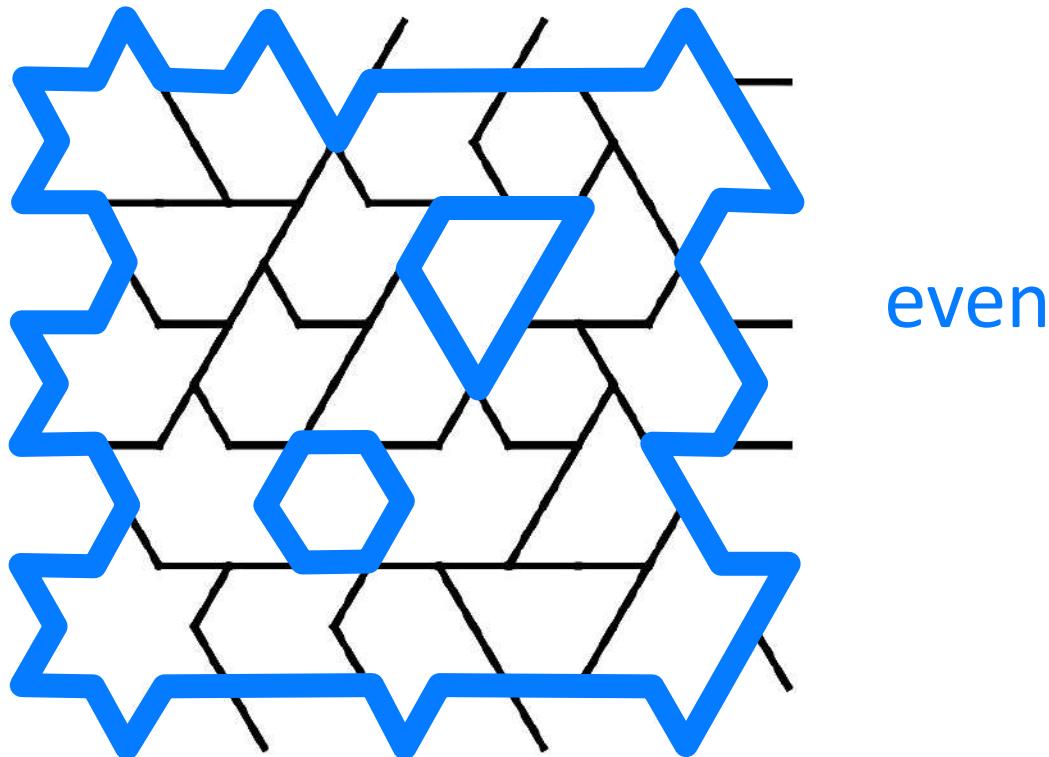
Parity matters!



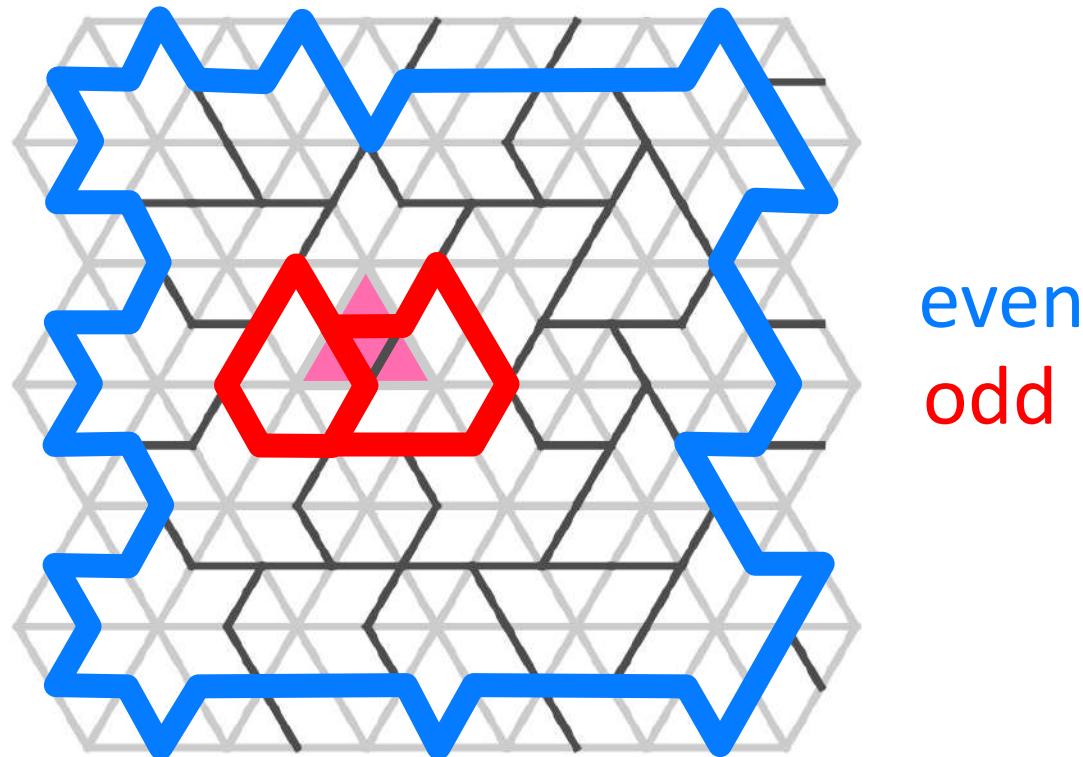
Ferromagnetic order



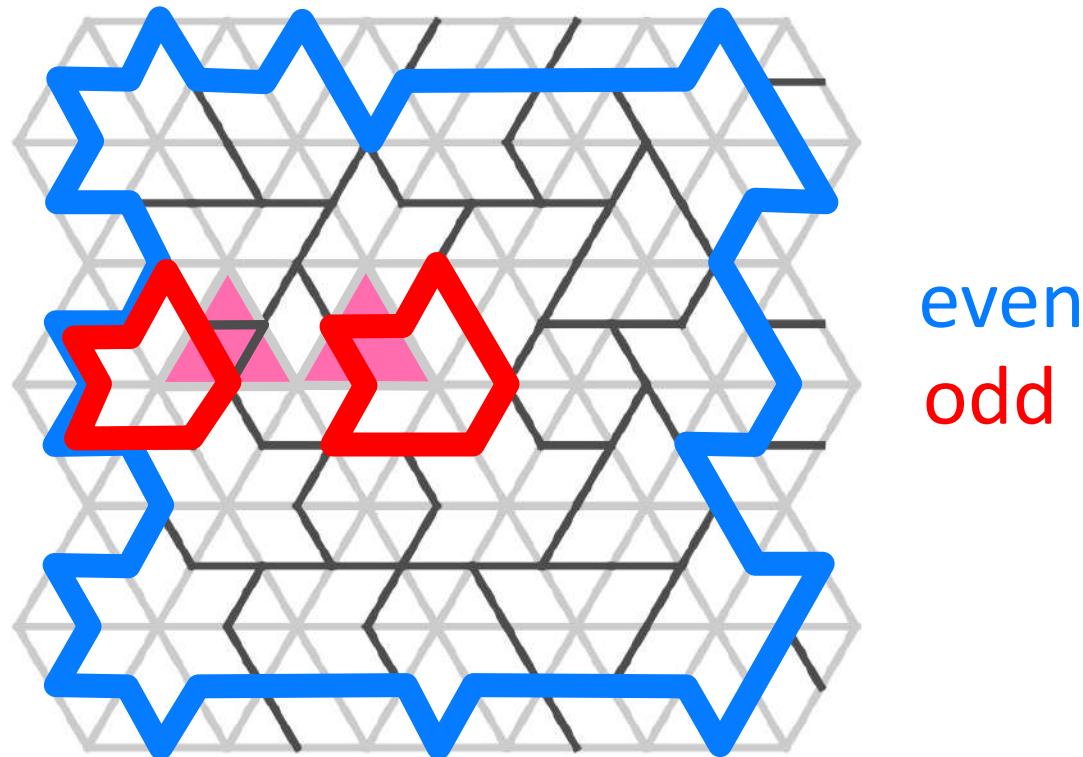
Design via parity and loops



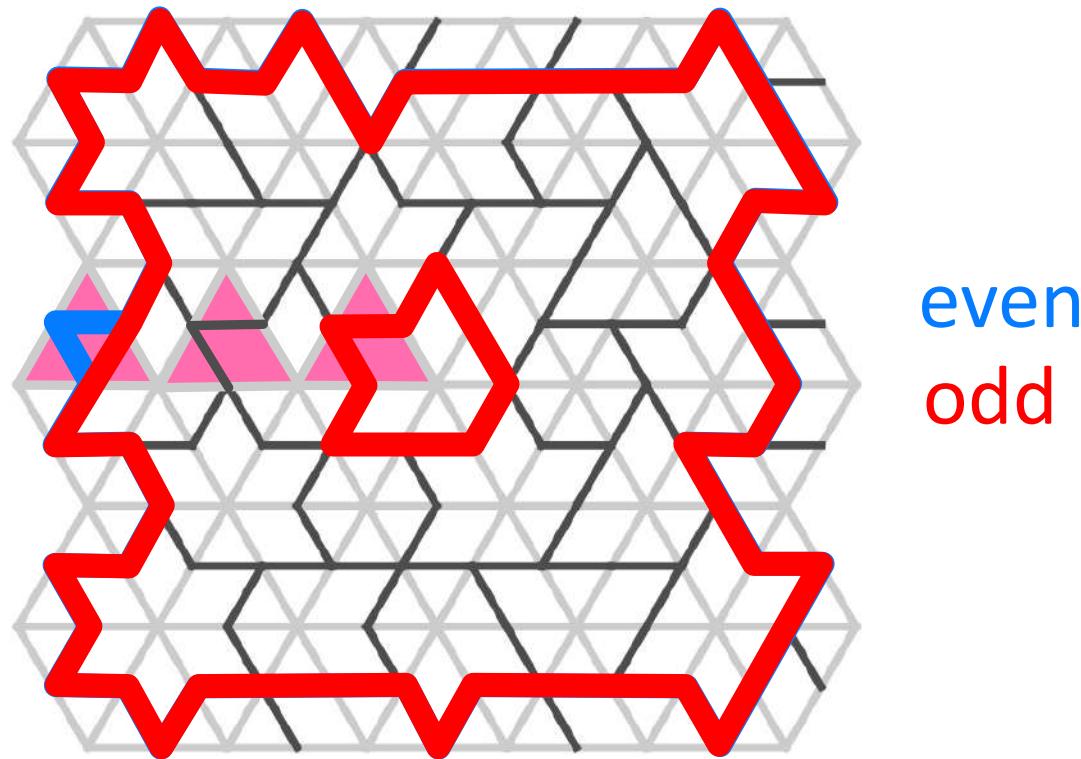
Design via parity



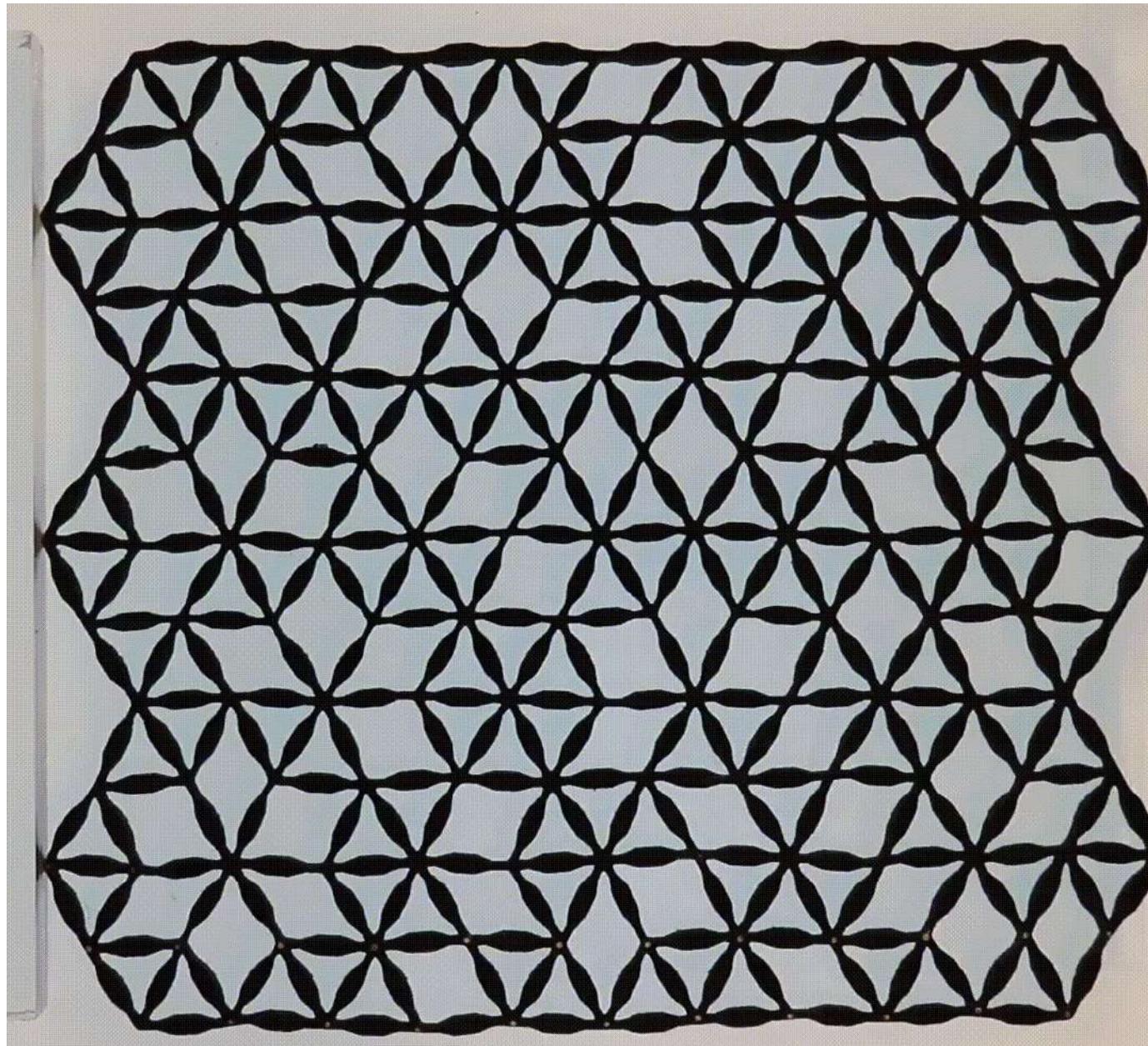
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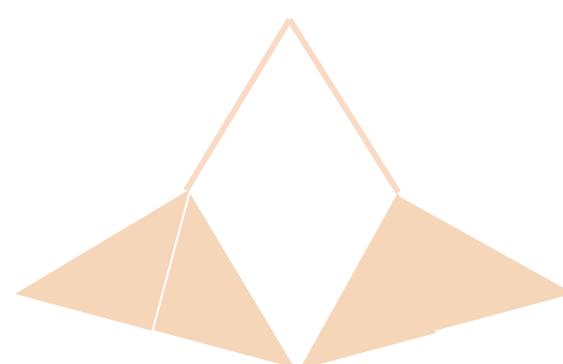
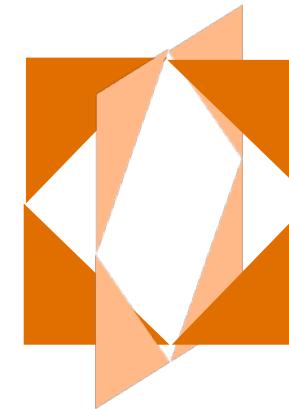
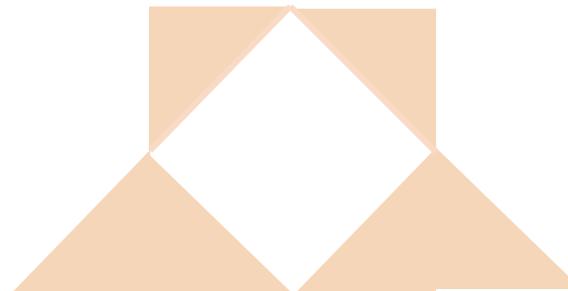
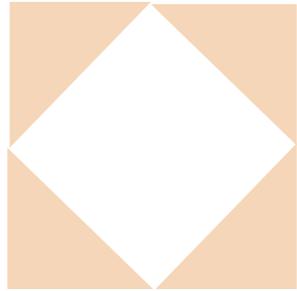
Design via parity and loops



Design via parity and loops



How to gain design freedom?

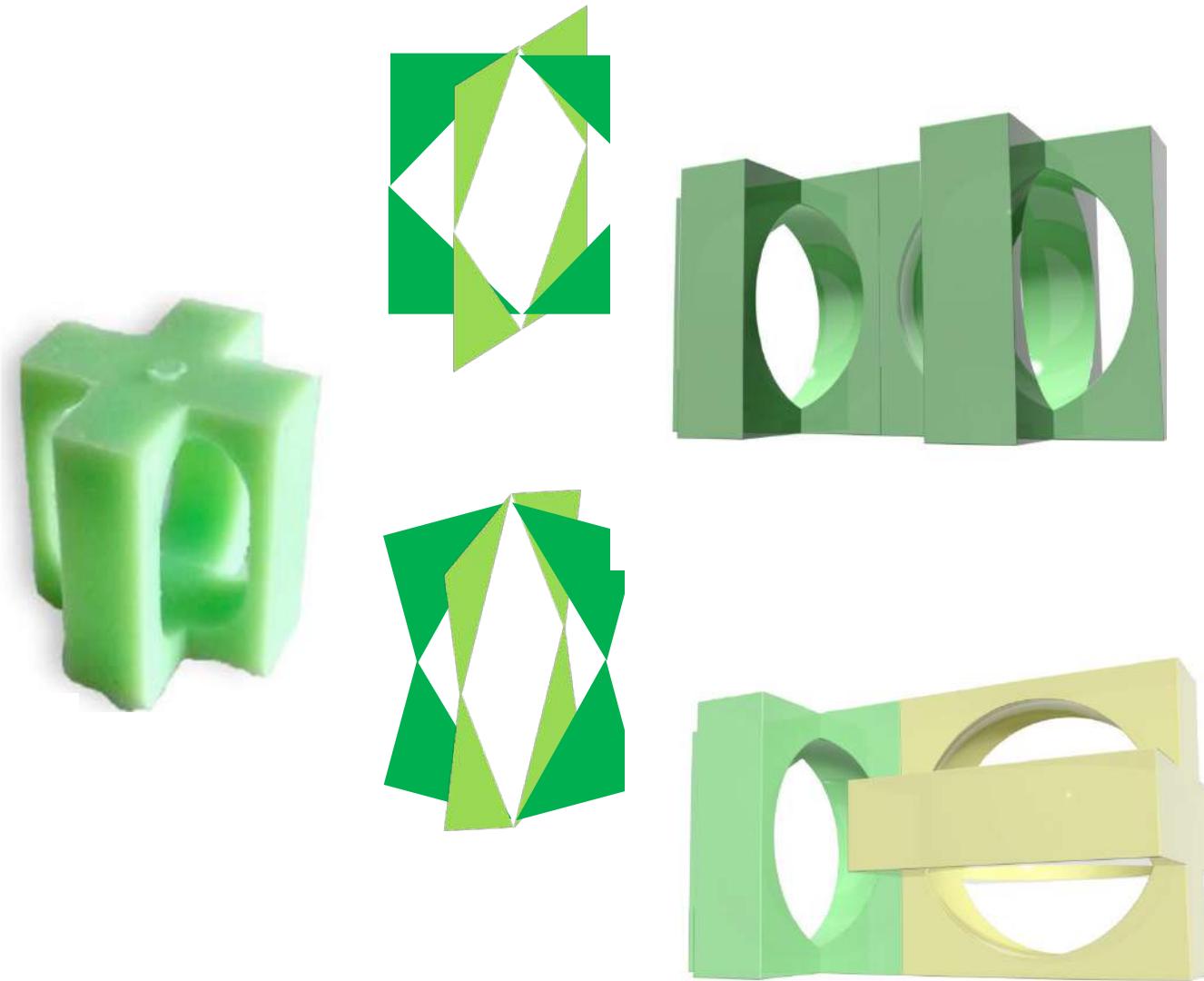


Meeussen et al. Nat. Phys. 2020

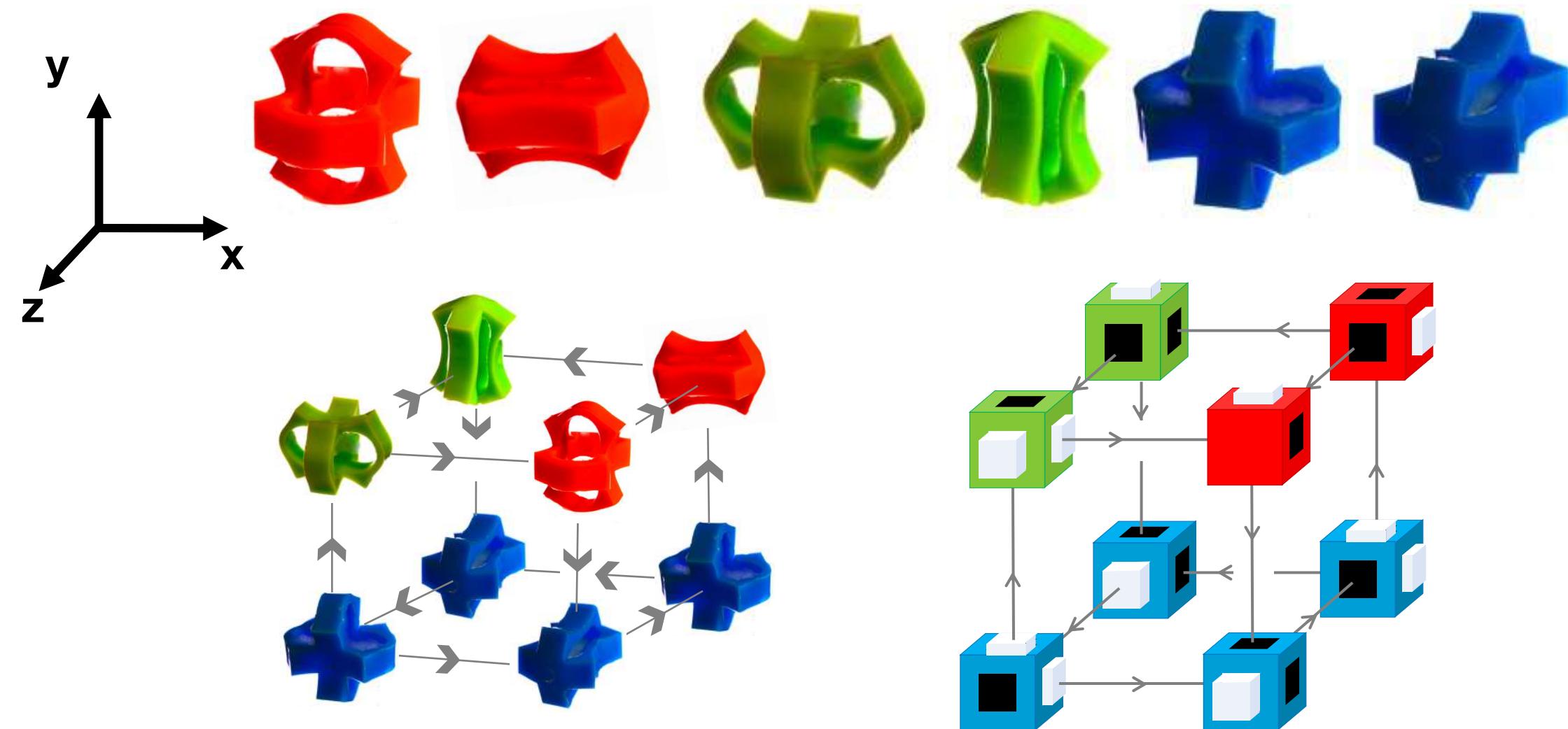
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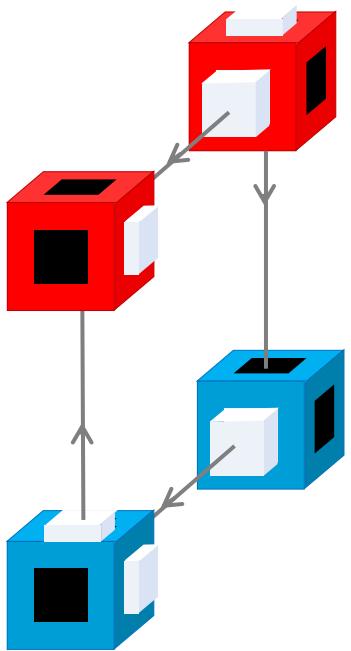
3D Anisotropic building block



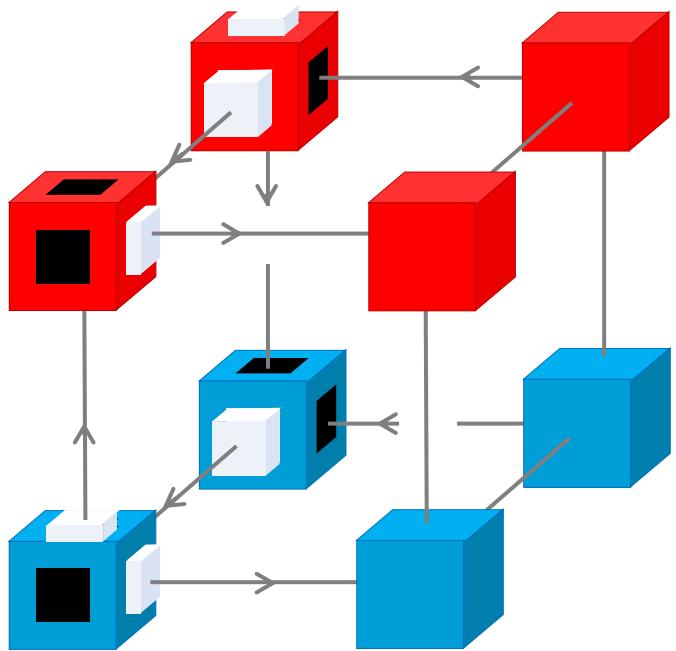
3 building blocks for the price of 1



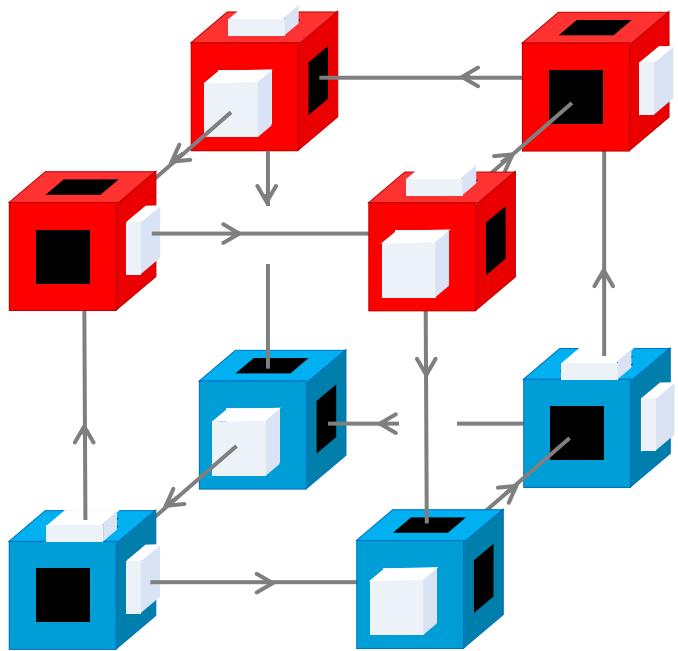
Anti-ferromagnetic order



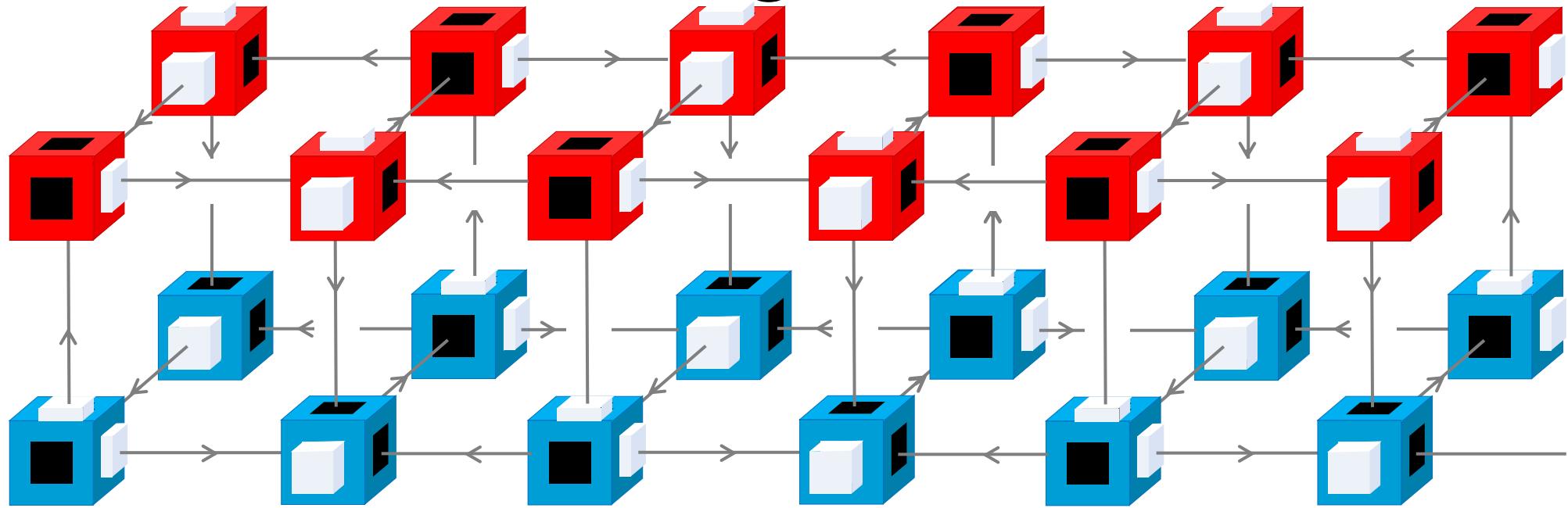
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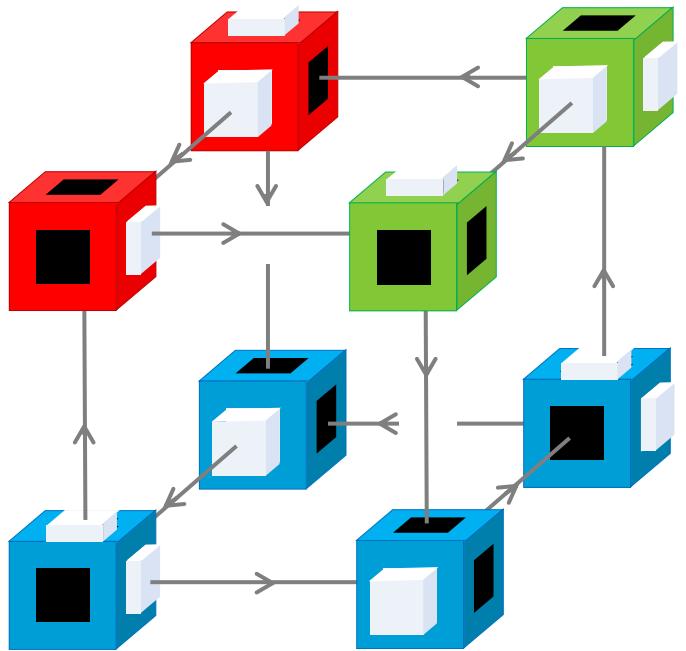
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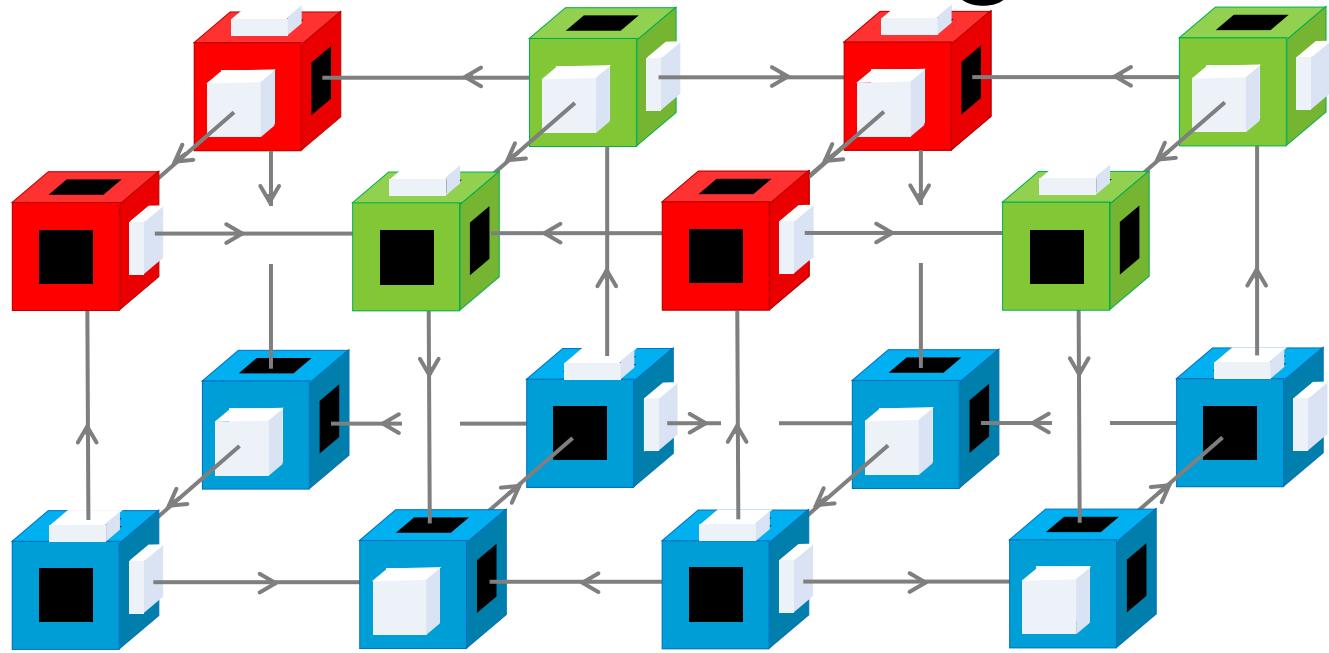
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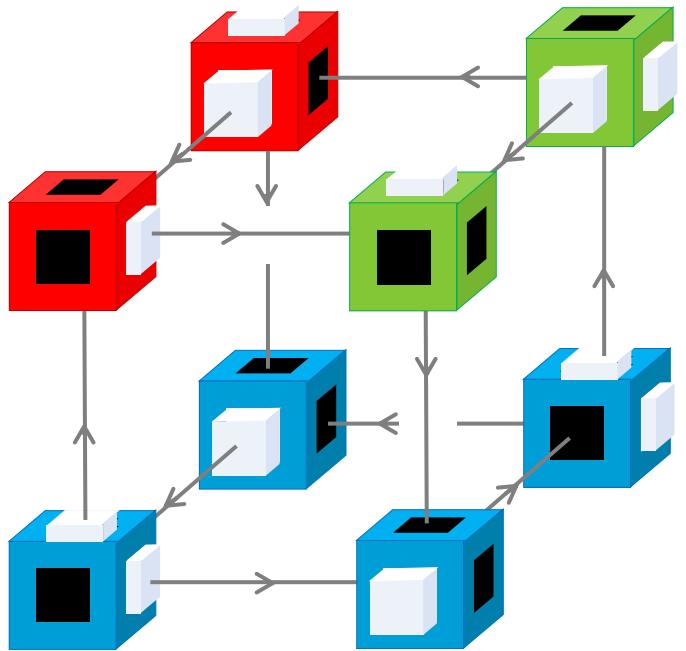
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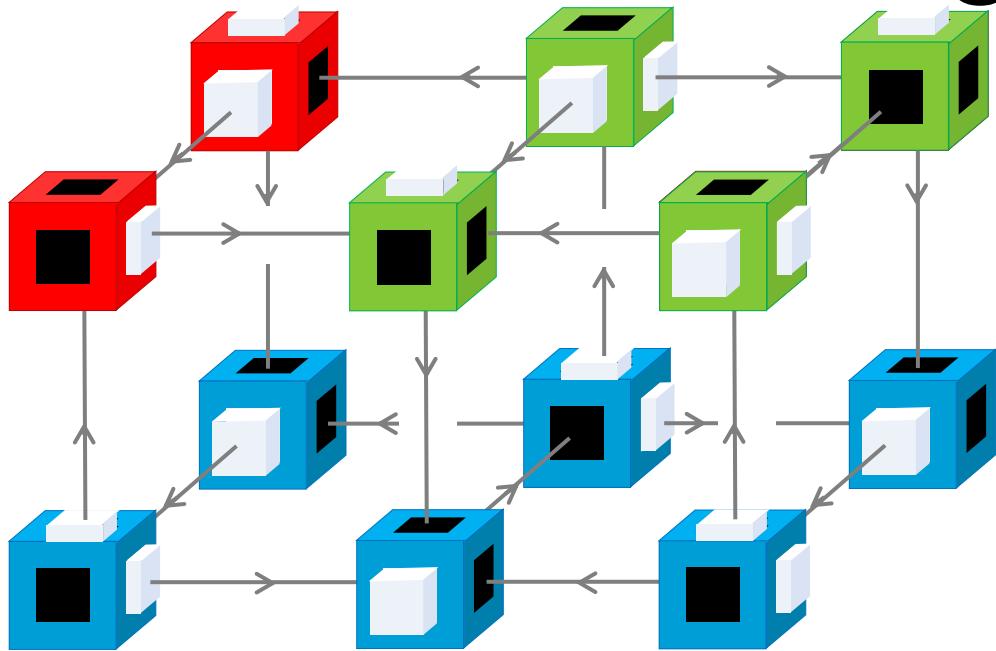
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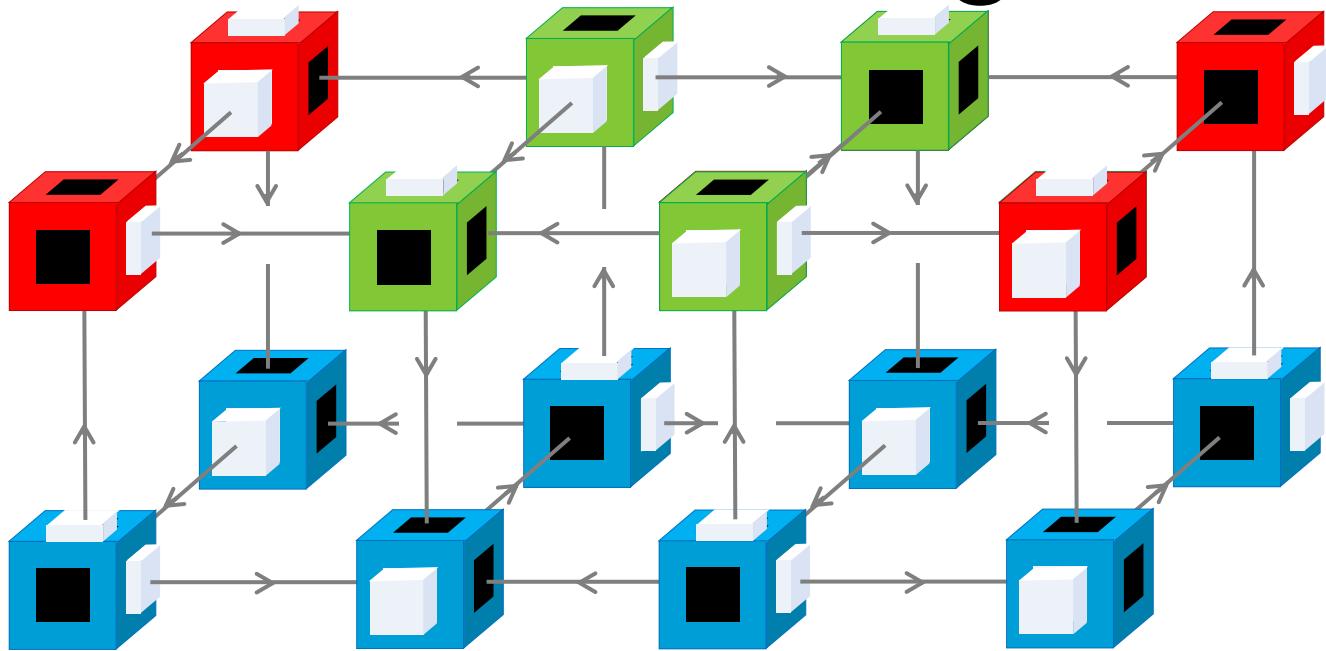
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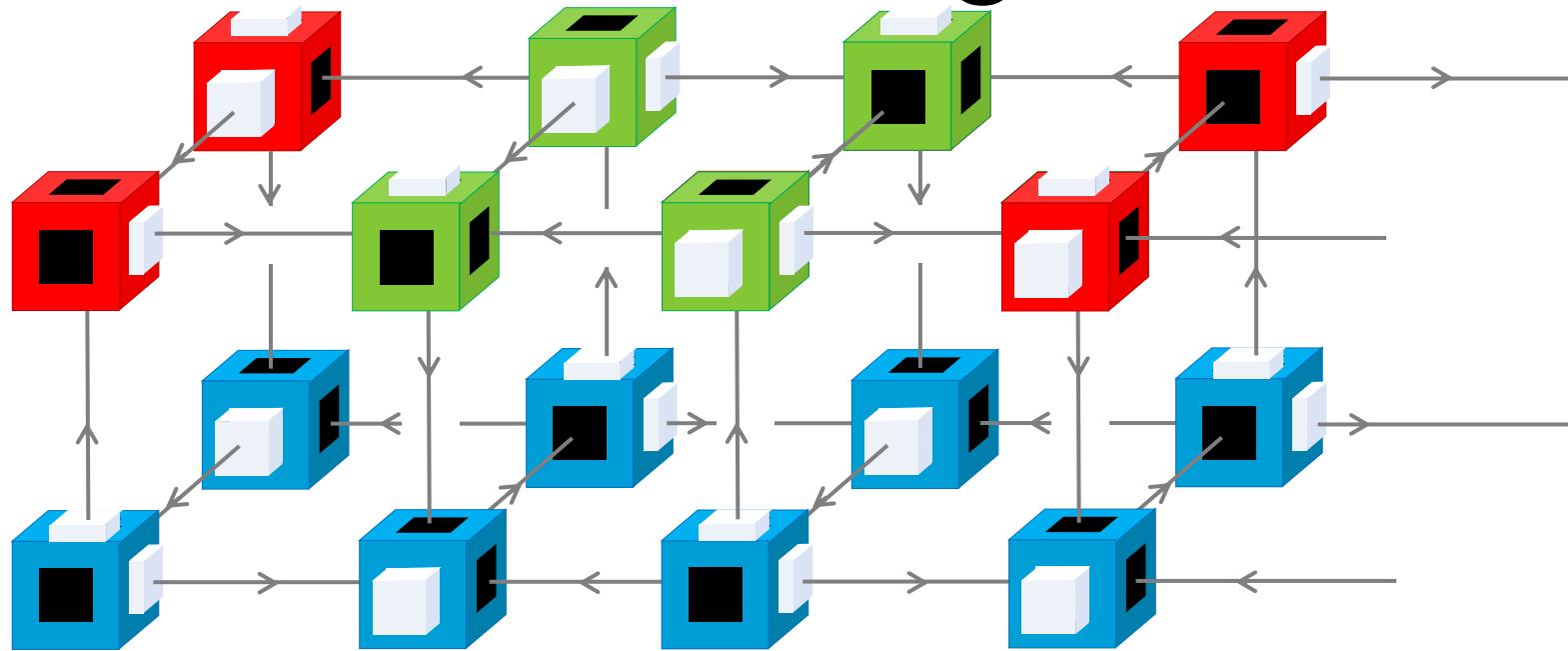
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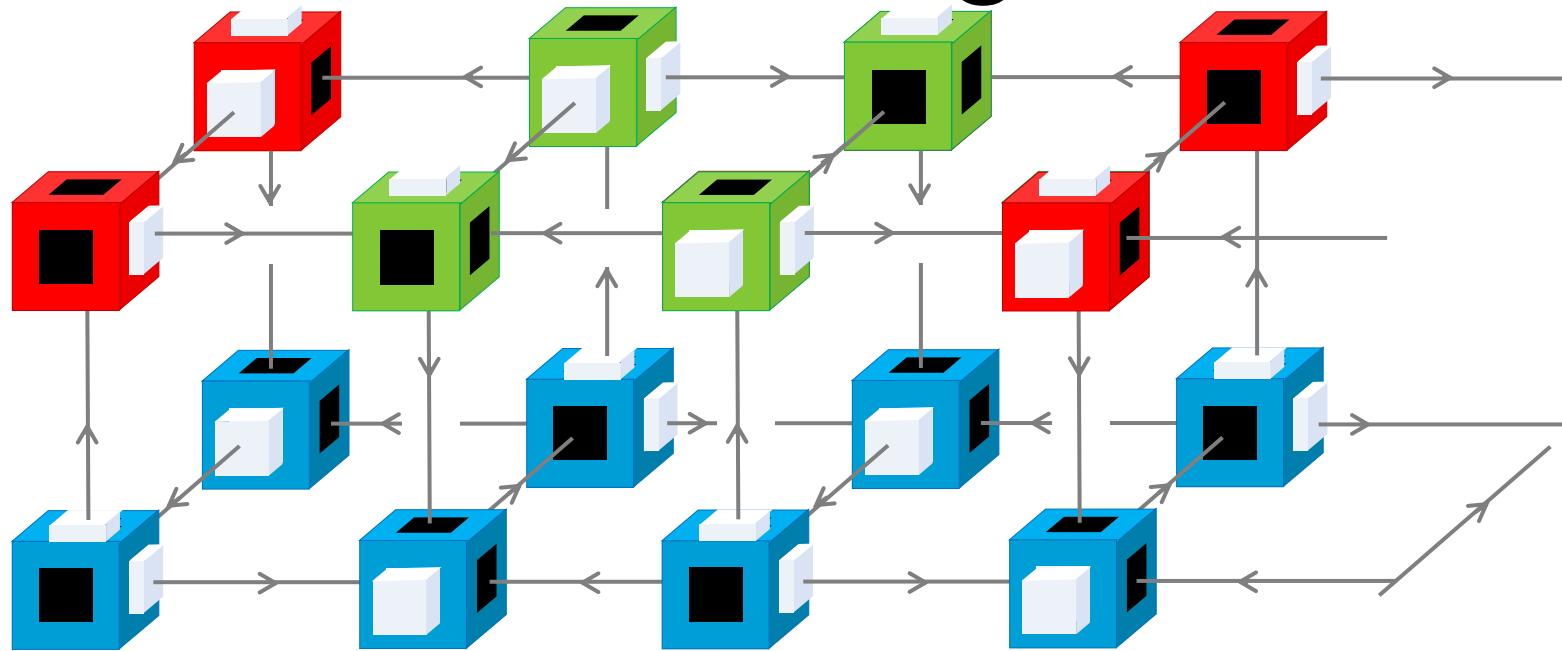
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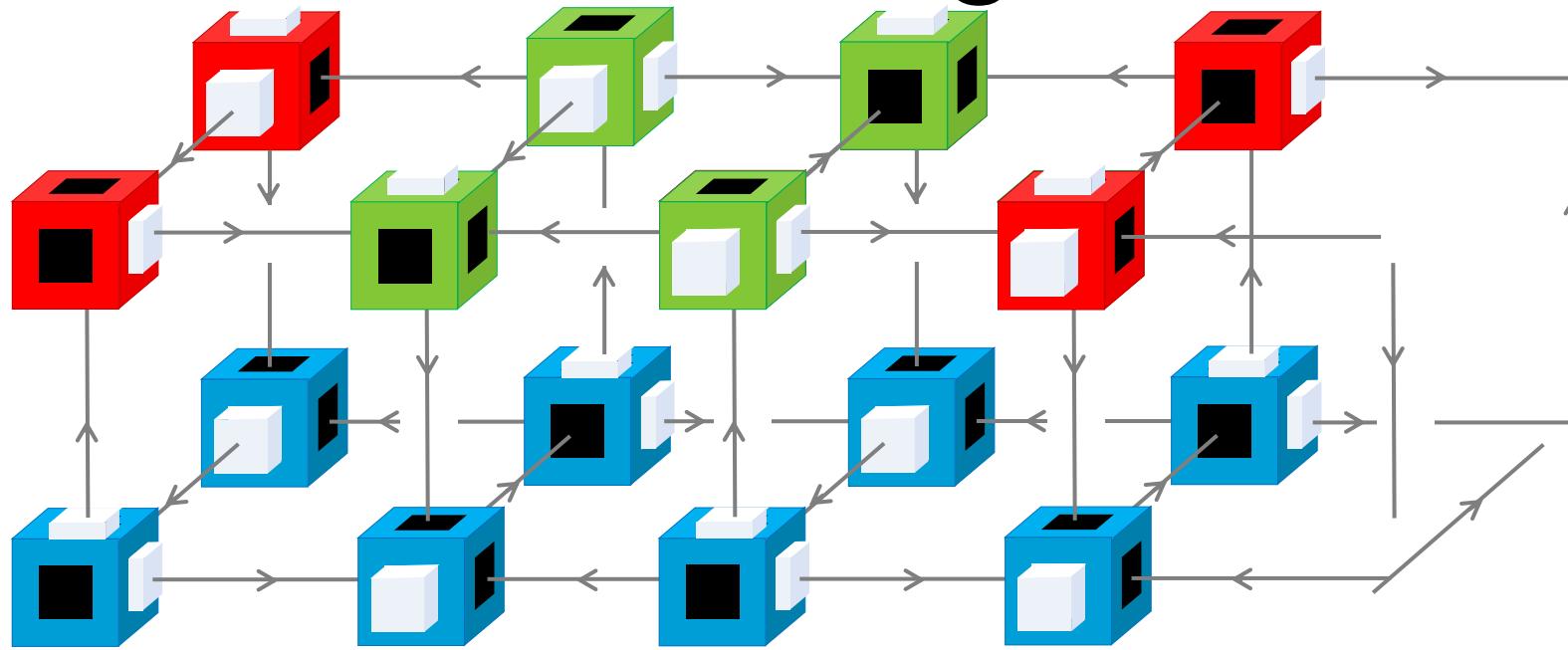
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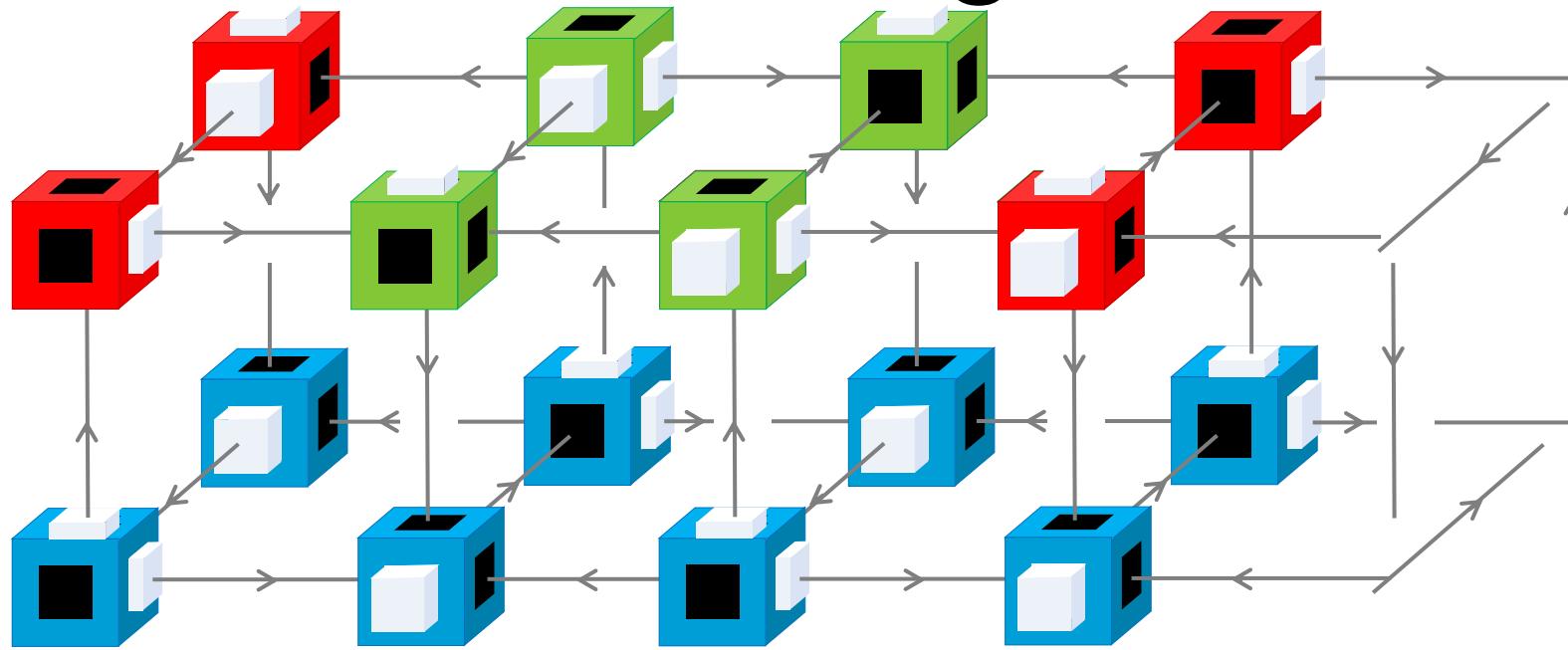
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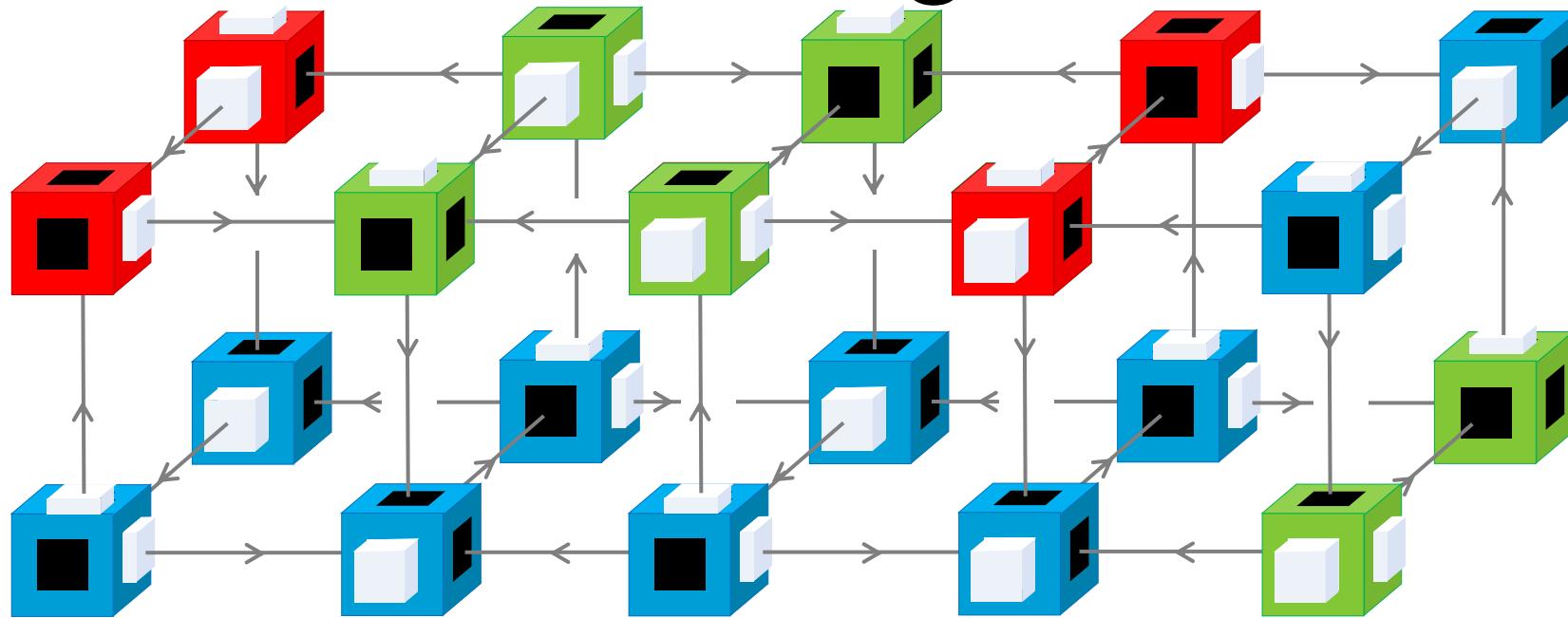
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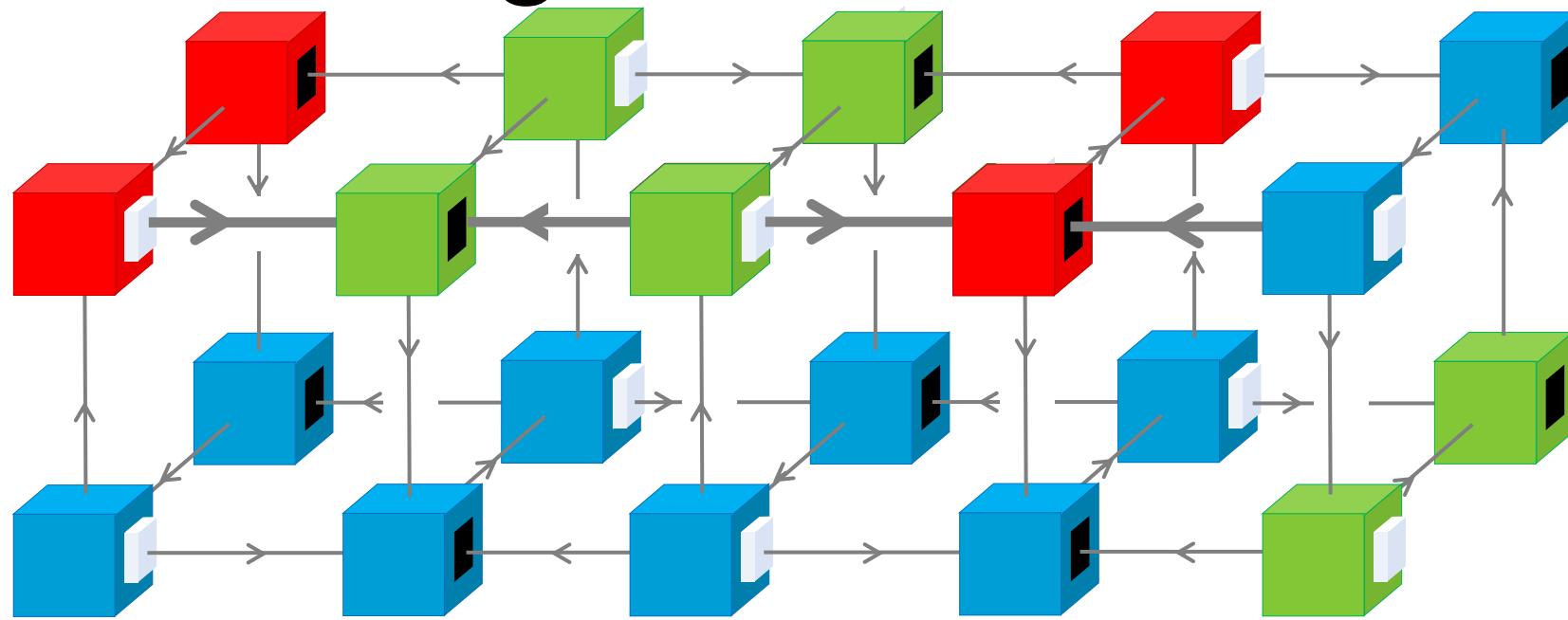
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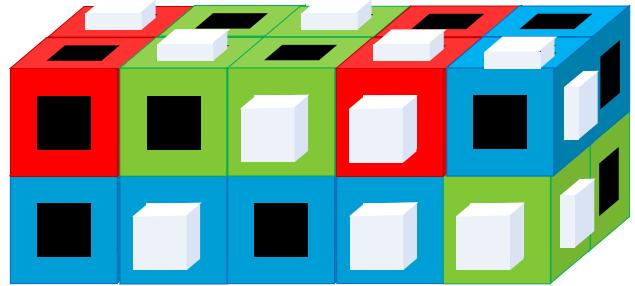
Anti-ferromagnetic order



Anti-ferromagnetic order -> Holography



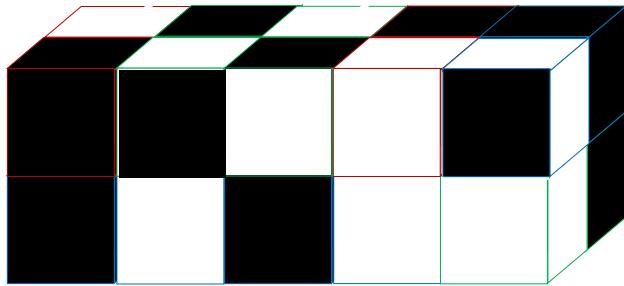
Anti-ferromagnetic order -> Holography



LxLxL Configs?

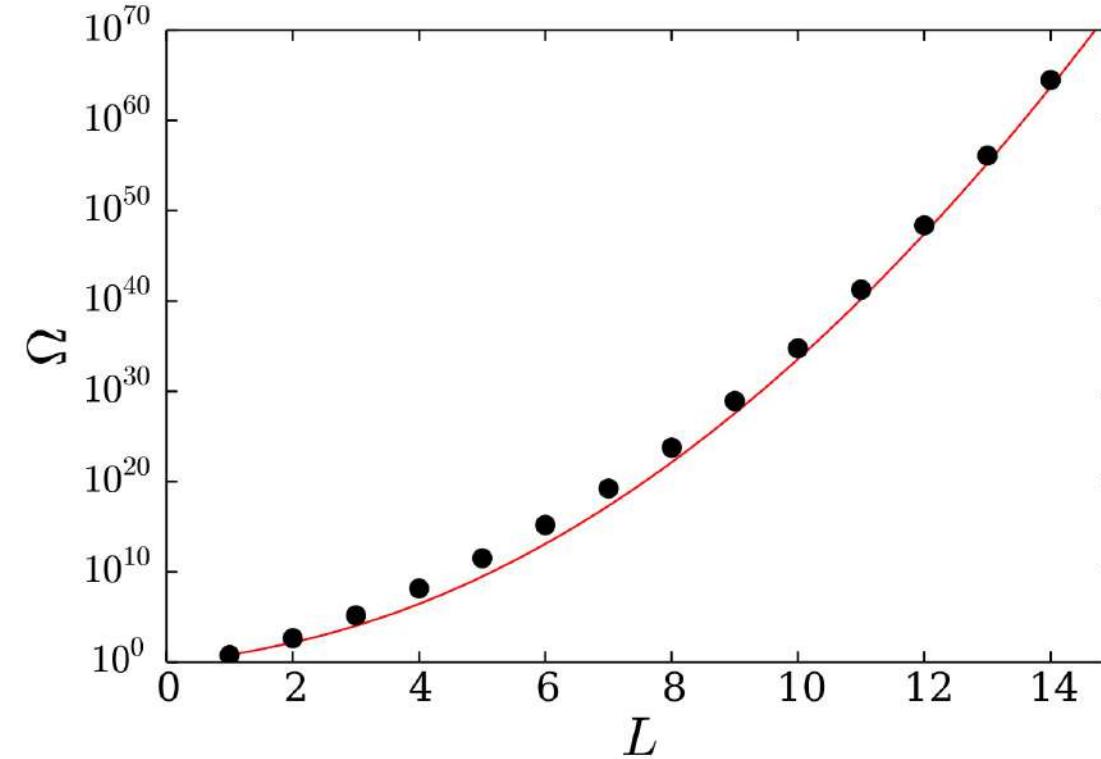
$$3^{L^3} \rightarrow 2^{3L^2}$$

Anti-ferromagnetic order -> Holography



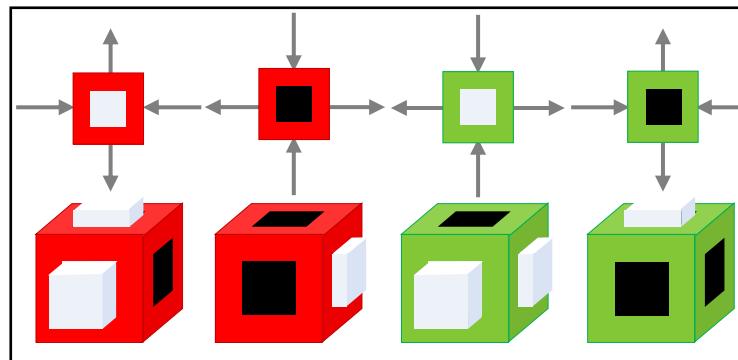
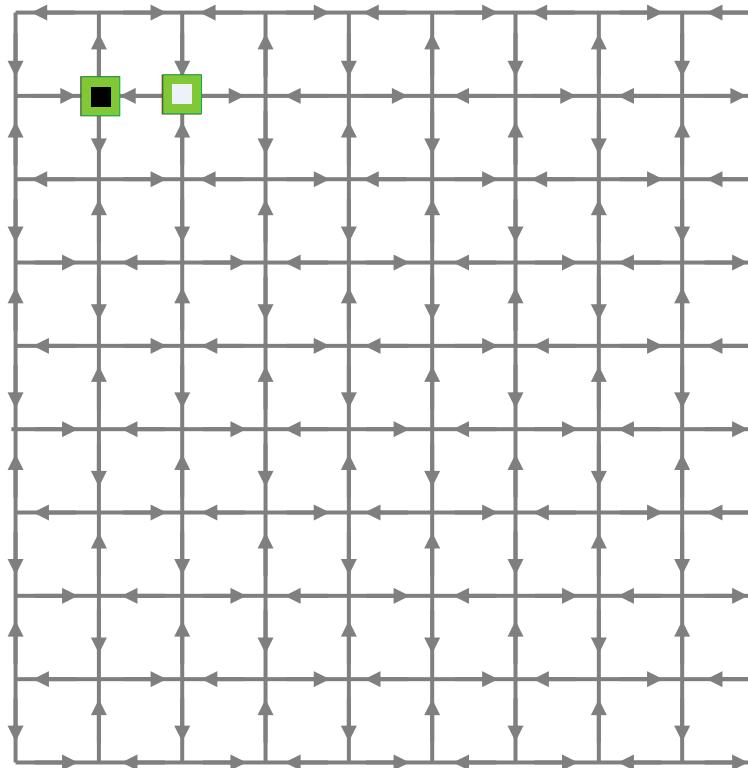
LxLxL Configs?

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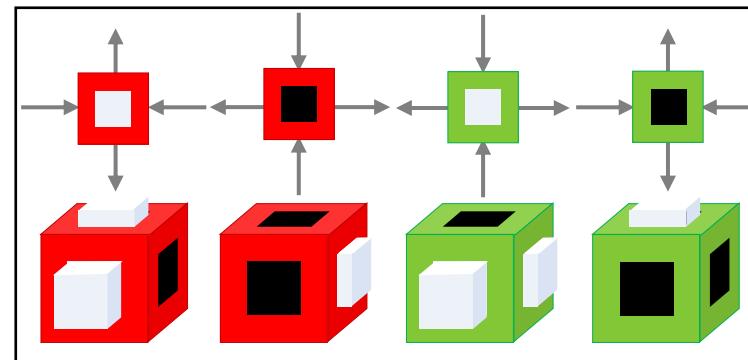
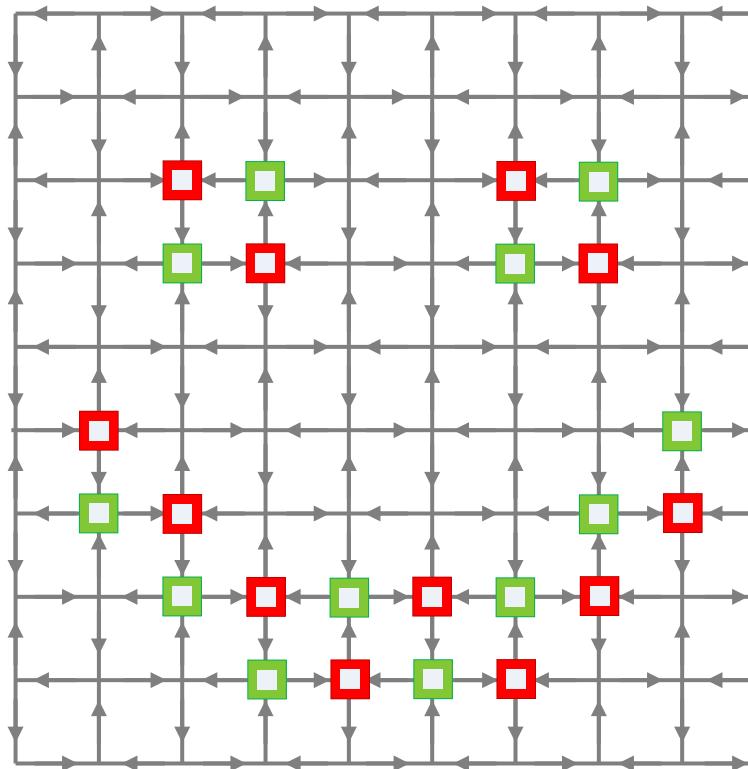


Lower Bound: $2^{L^2} \times 2^L = 2^{L+L^2}$

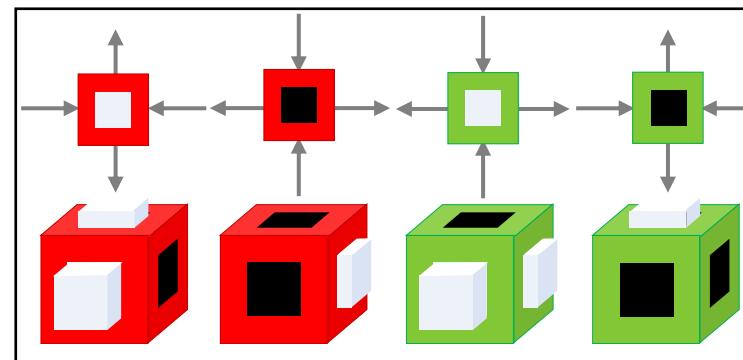
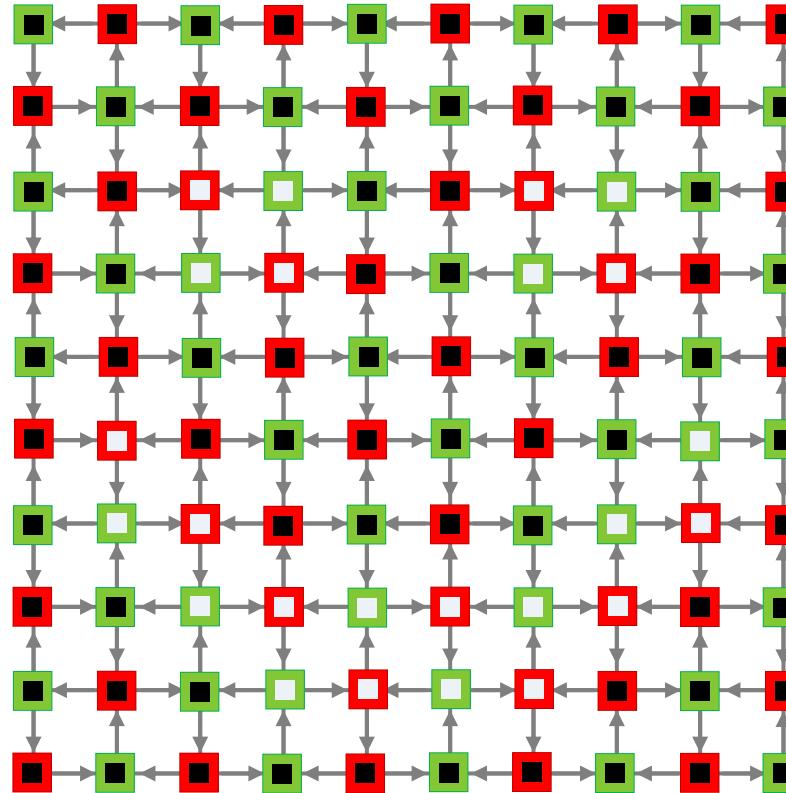
Finally, some design



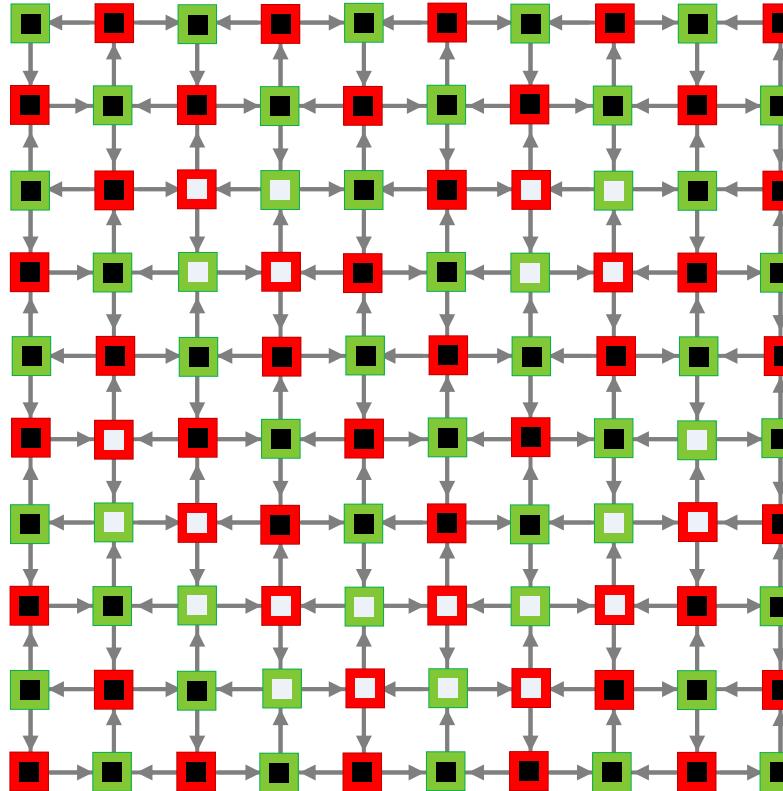
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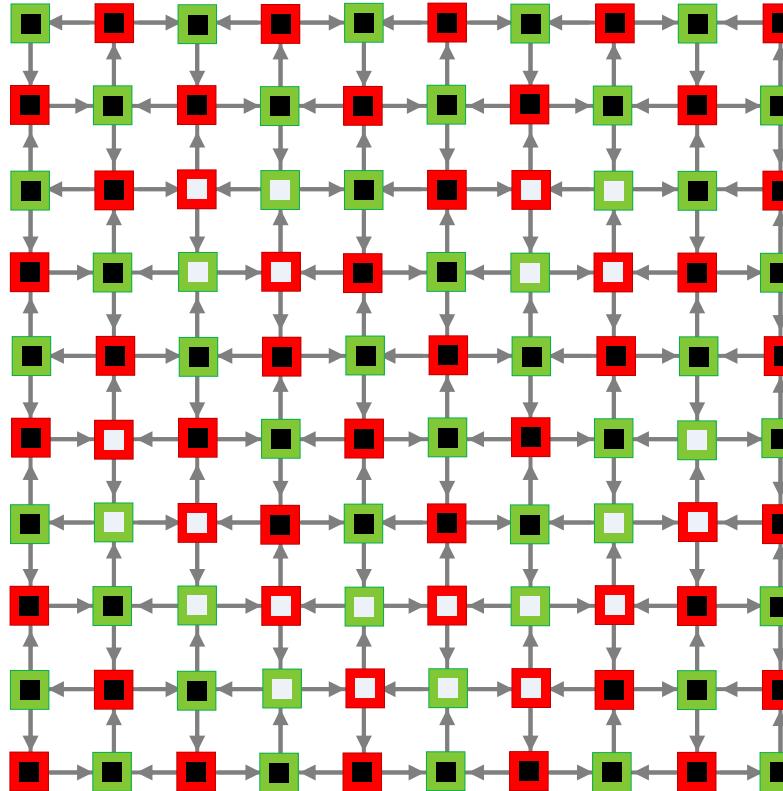
Finally, some design



LxLx1 Motifs

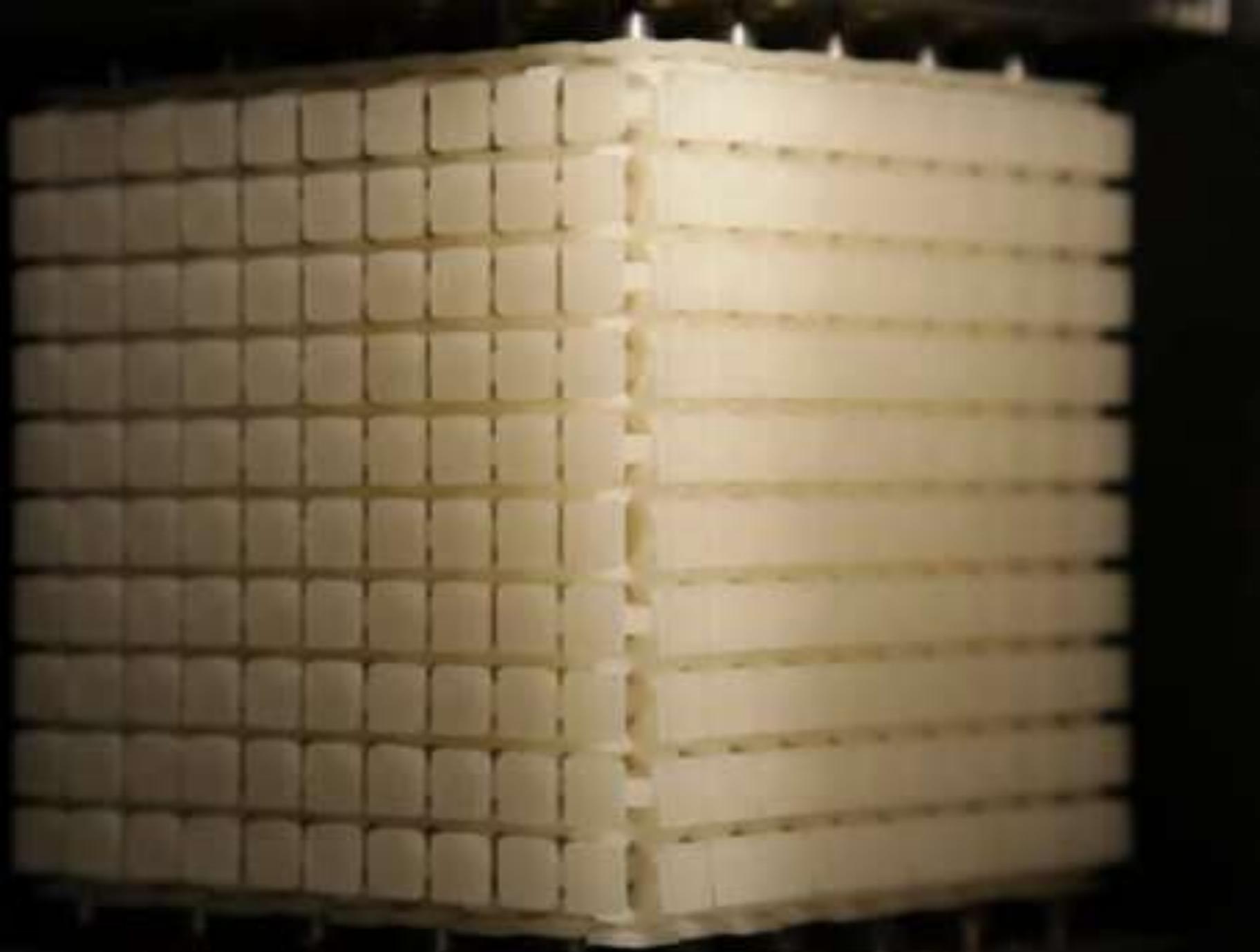
$$2^{L^2} \times 2$$

Finally, some design



LxLxL Motifs

$$2^{L^2} \times 2^L = 2^{L+L^2}$$

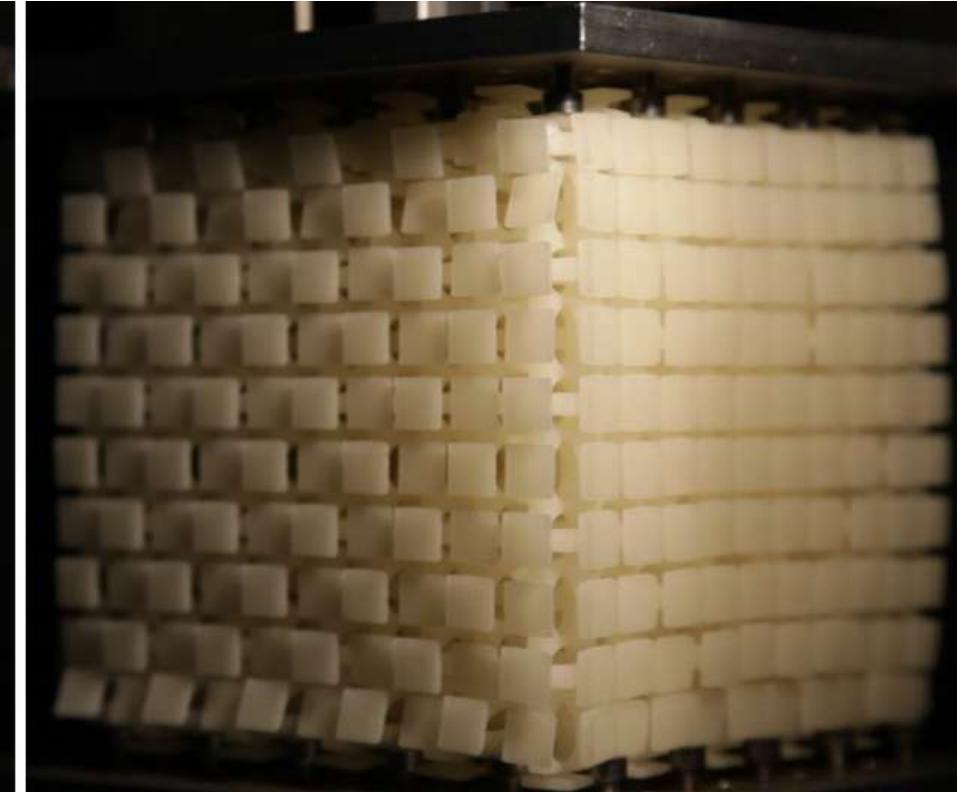


Coulais, Teomy, de Reus, Shokef and van Hecke (2016) **Nature**

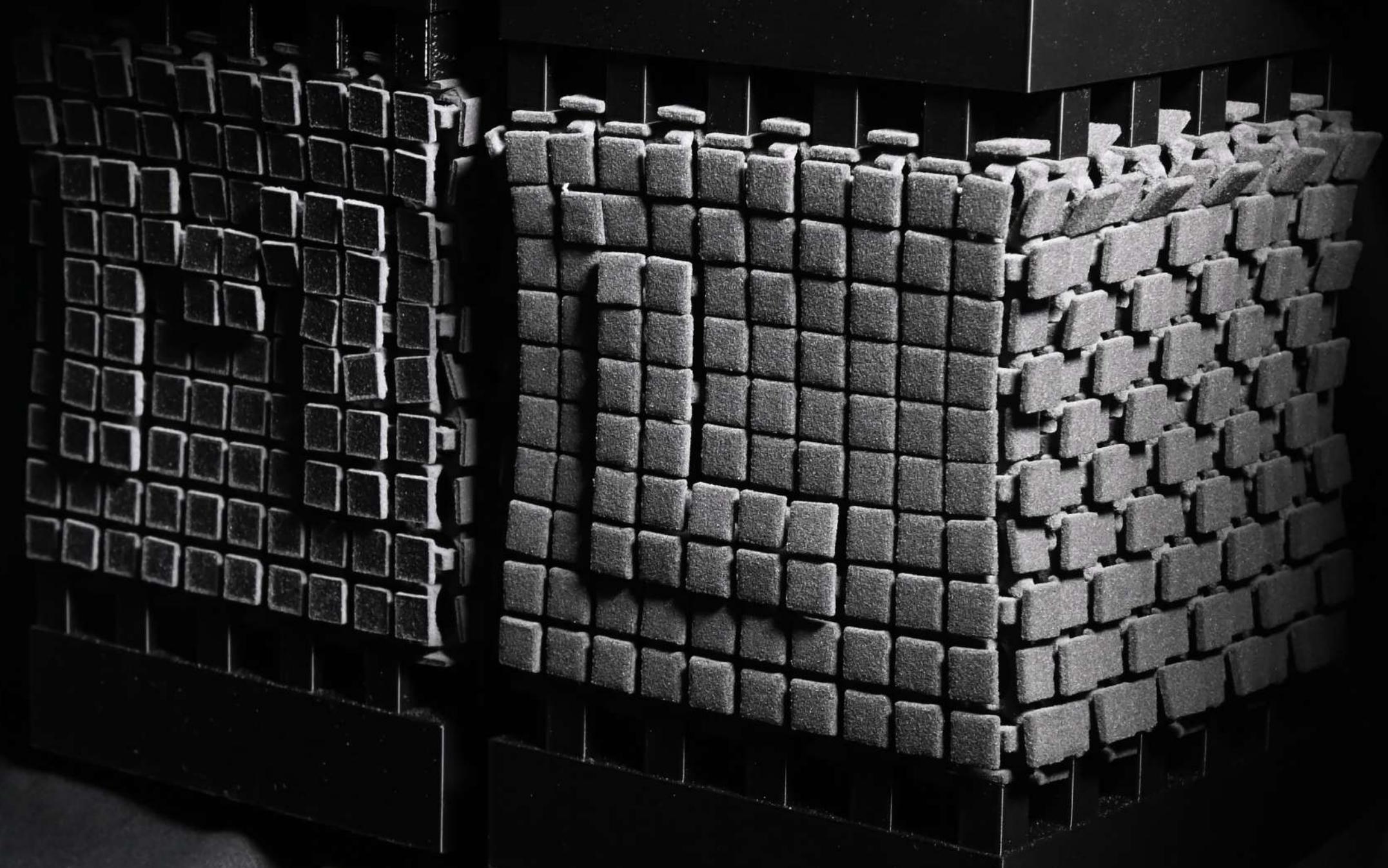
Finally, some design



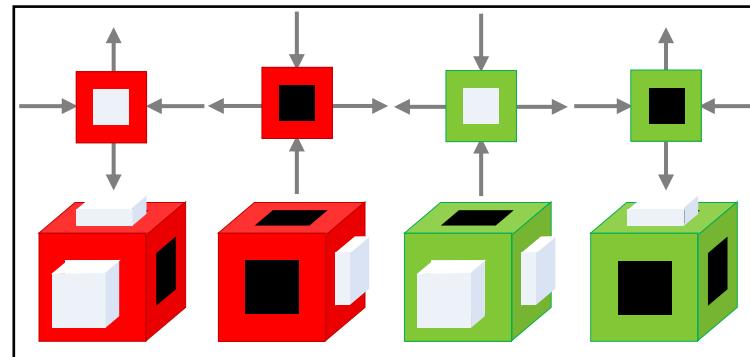
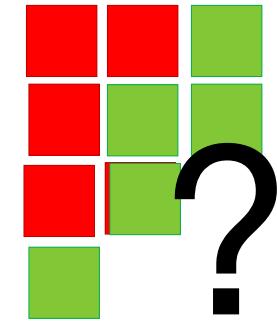
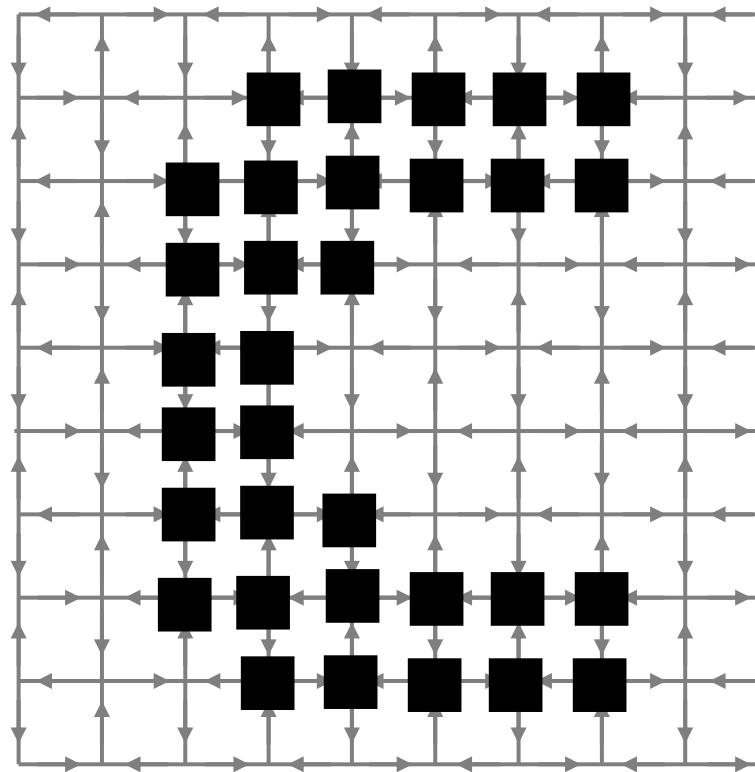
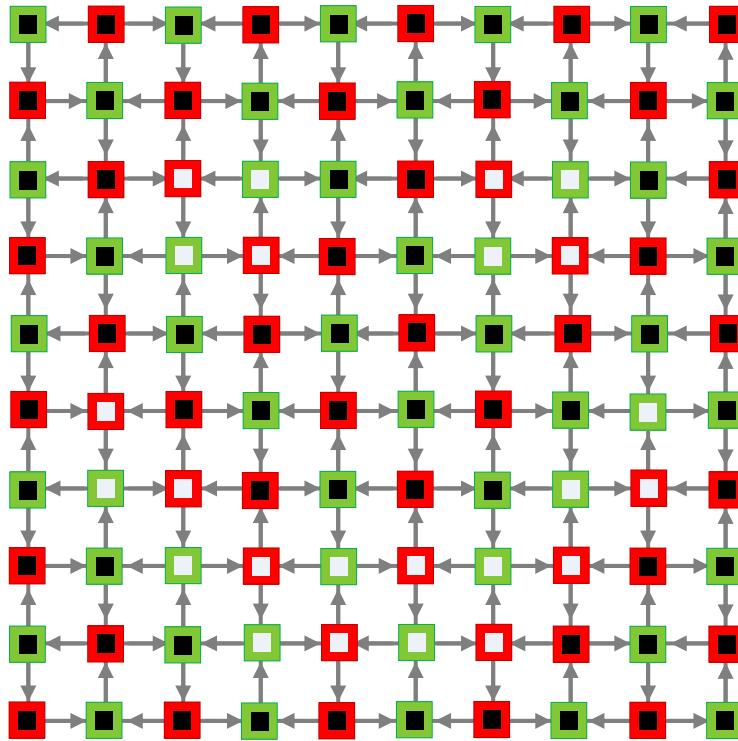
Back: Holographic



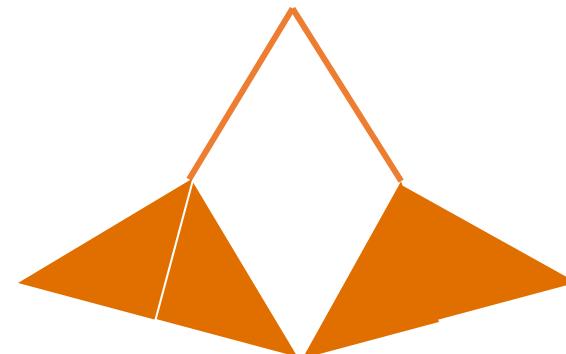
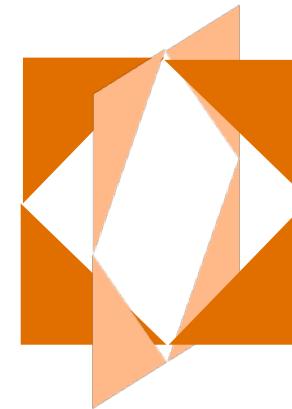
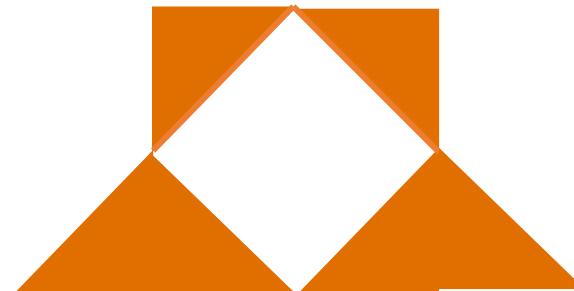
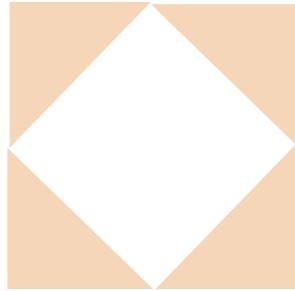
Side: Motif Stacking



Exercise 1: Design your initial (15 min)



Exploit (Anti)-Ferromagnetic order

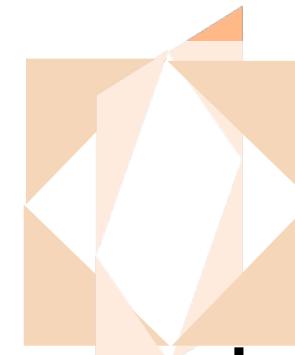
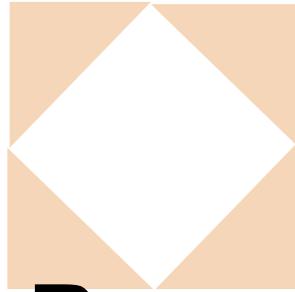


Meeussen et al. Nat. Phys. 2020

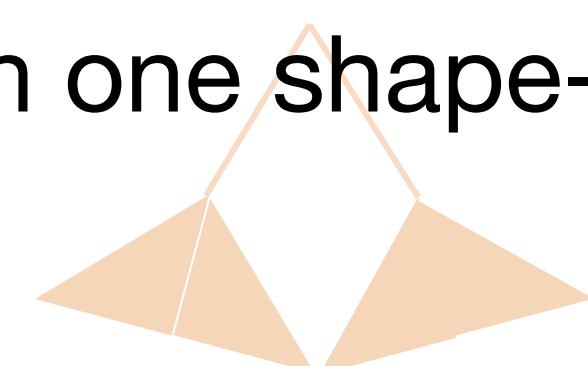
Coulais et al. Nature 2021
Van Mastrigt et al., PRL 2022

Bossart, Dykstra et al. PNAS 2021
Van Mastrigt et al., PRL 2022

Exploit (Anti)-Ferromagnetic order



Beyond (Anti)-Ferromagnetic order?



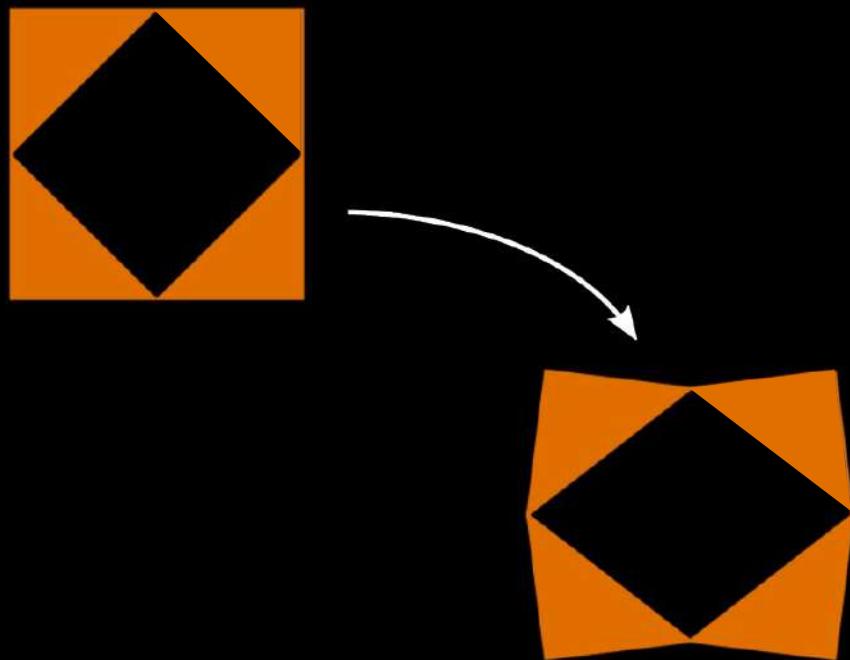
More than one shape-change?

Meeussen et al. Nat. Phys. 2020

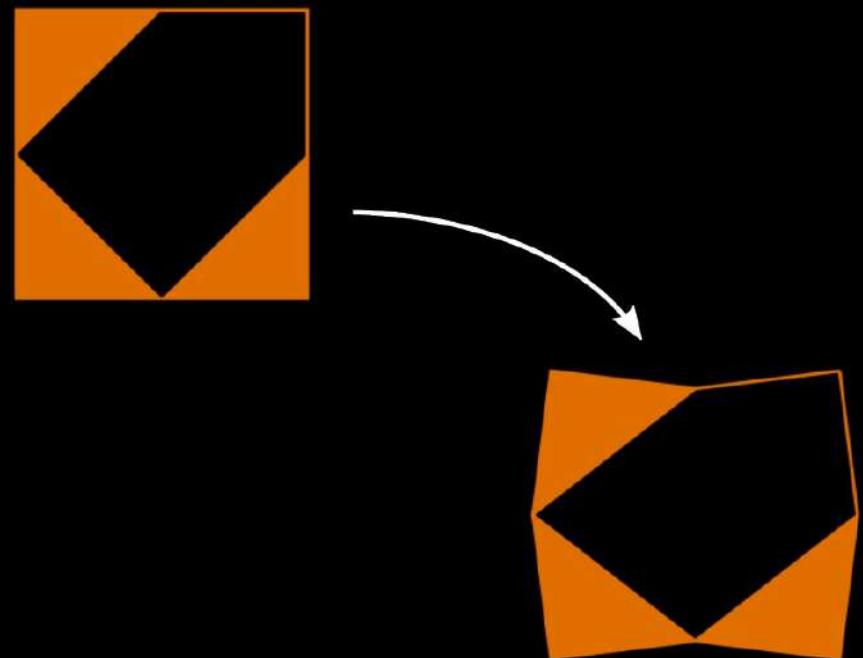
Coulais et al. Nature 2021
Van Mastrigt et al., PRL 2022

Bossart, Dykstra et al. PNAS 2021
Van Mastrigt et al., PRL 2022

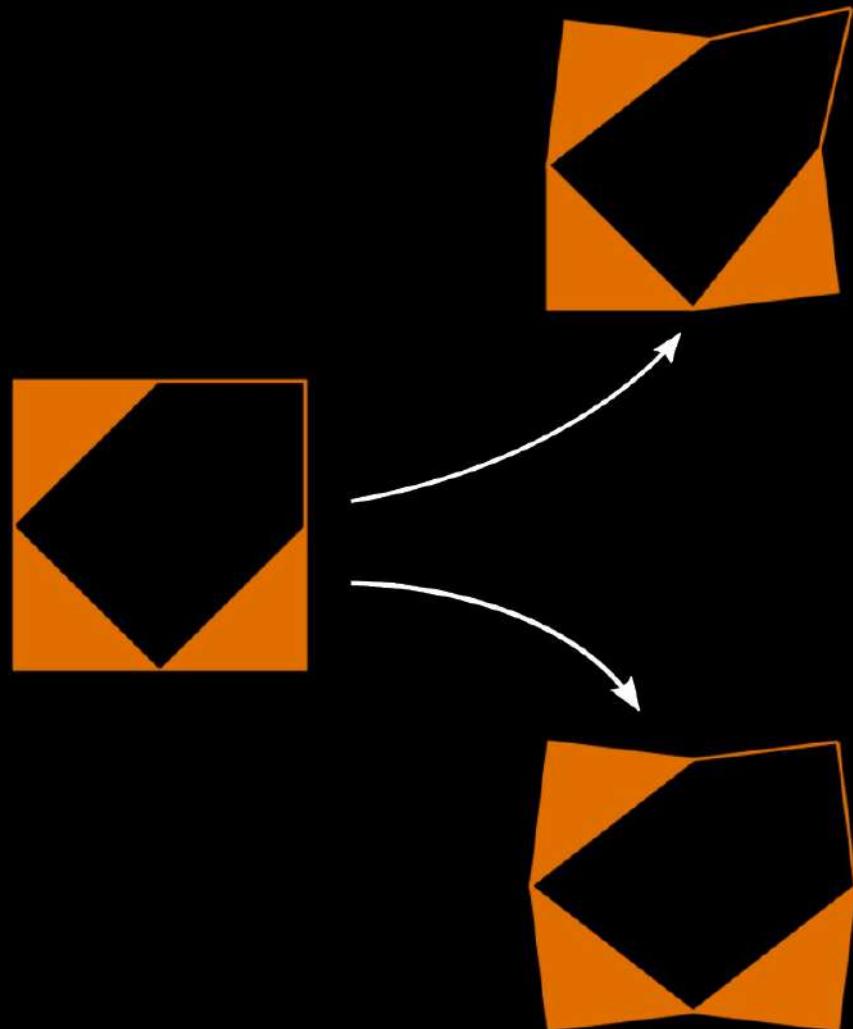
Multimode Combinatorial Metamaterials



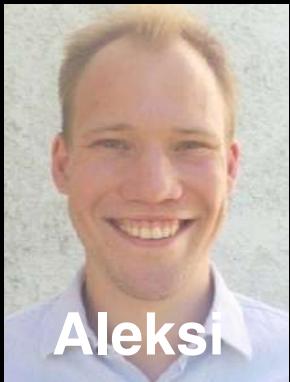
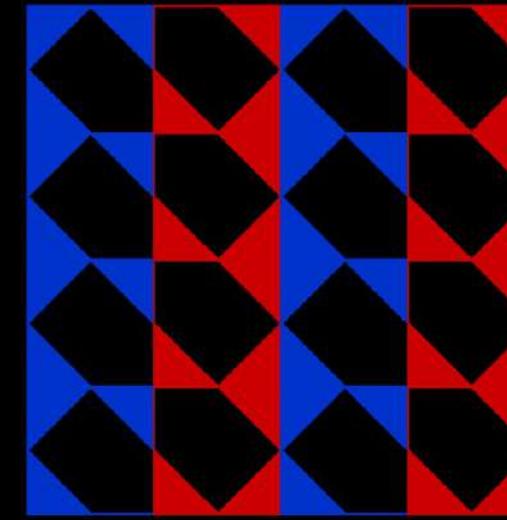
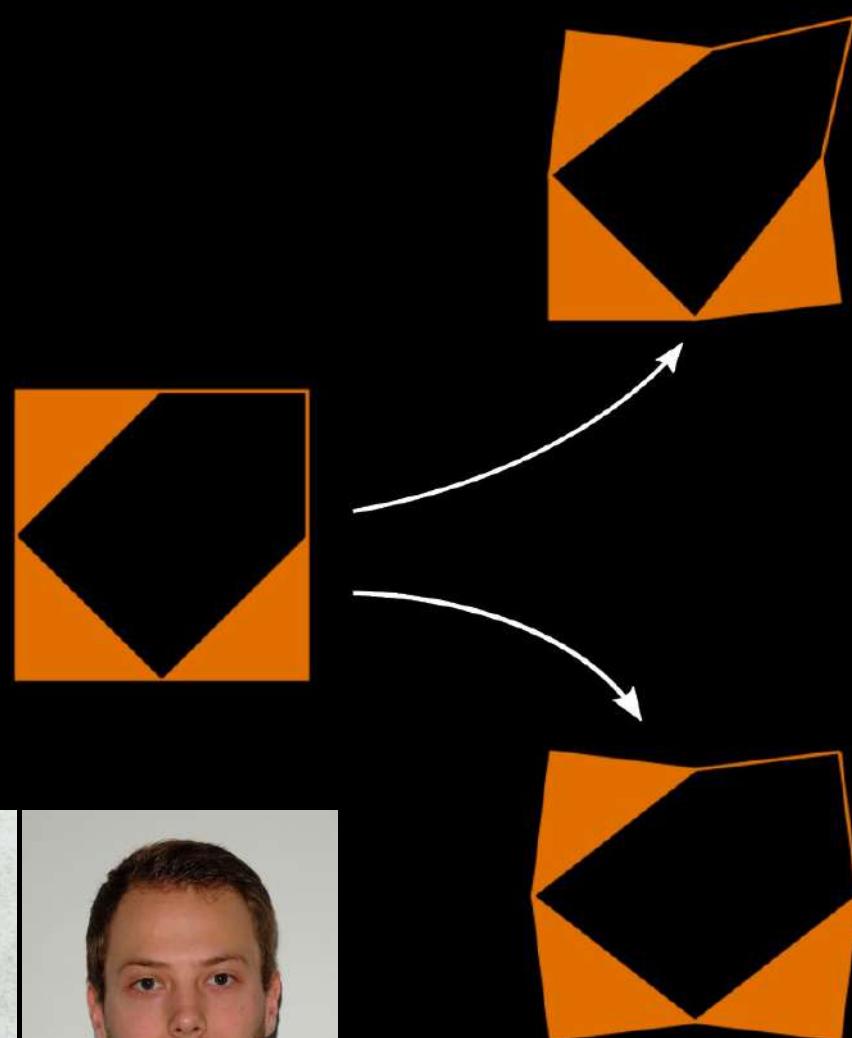
Multimode Combinatorial Metamaterials



Multimode Combinatorial Metamaterials



Multimode Combinatorial Metamaterials



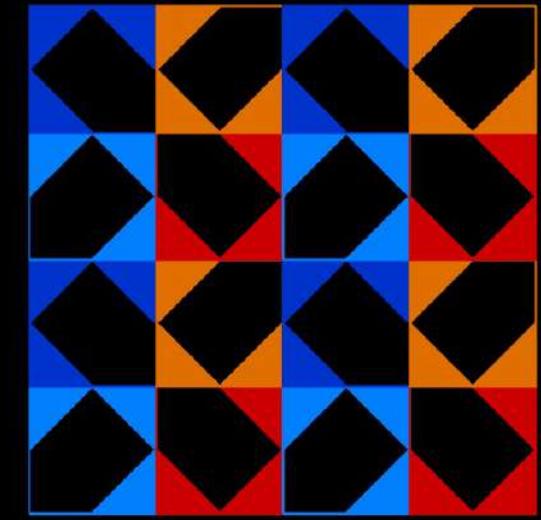
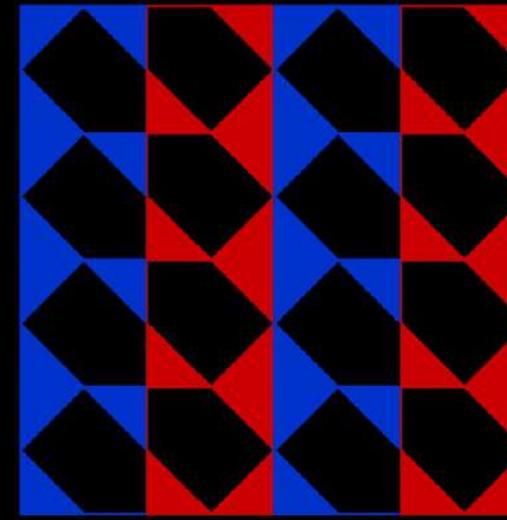
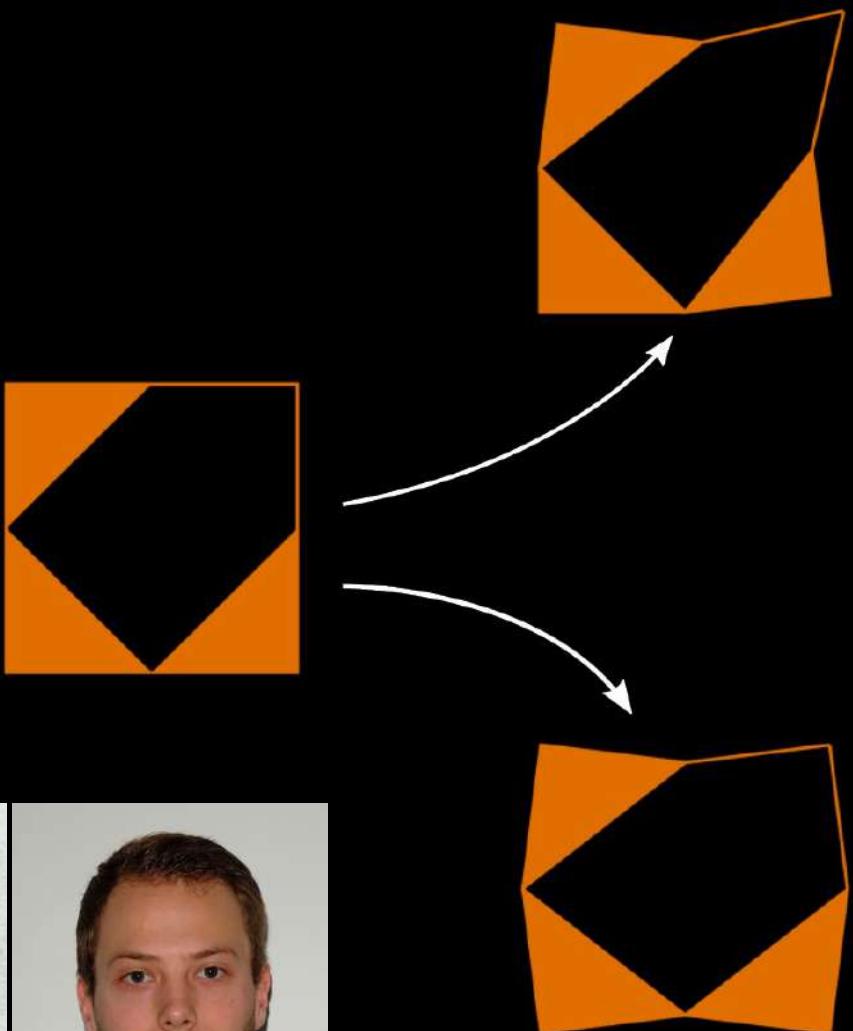
Aleksi



David

Bossart, Dykstra et al. PNAS 2021

Multimode Combinatorial Metamaterials



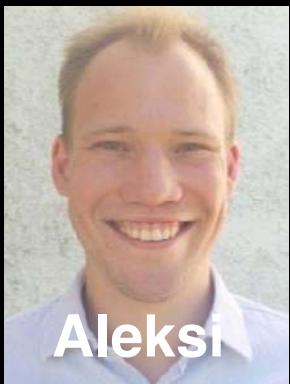
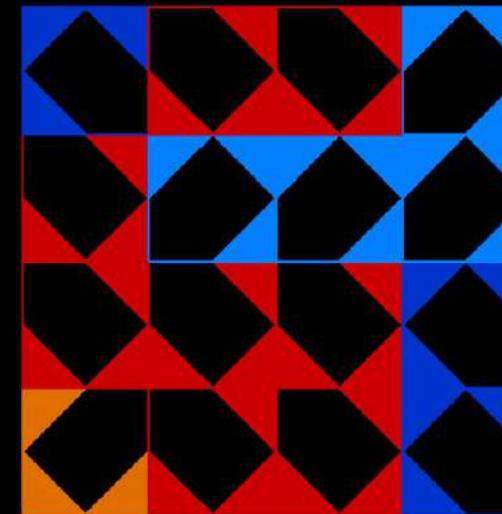
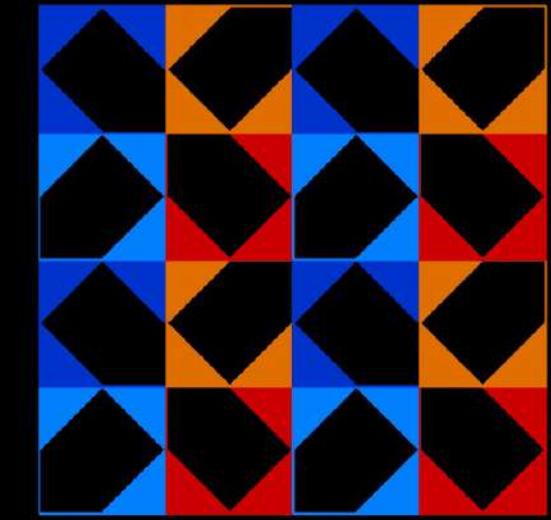
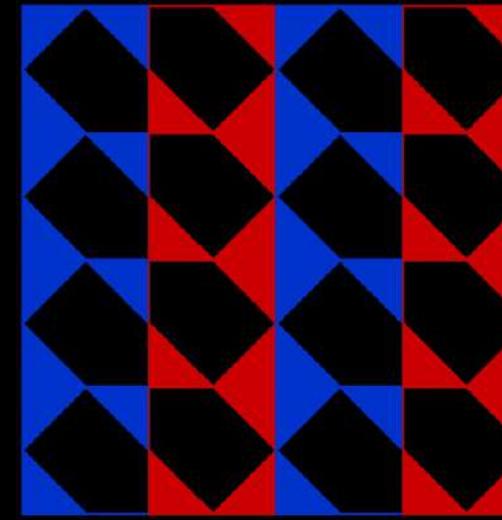
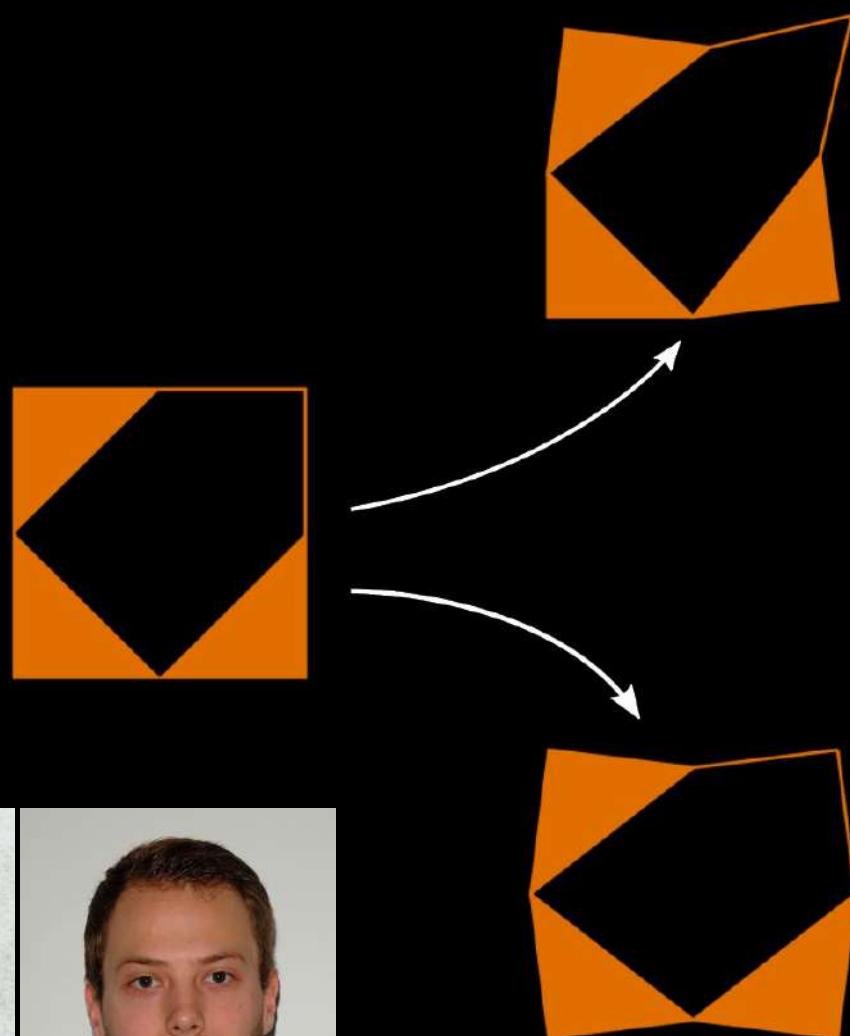
Aleksandr



Dykstra

Bossart, Dykstra et al. PNAS 2021

Multimode Combinatorial Metamaterials



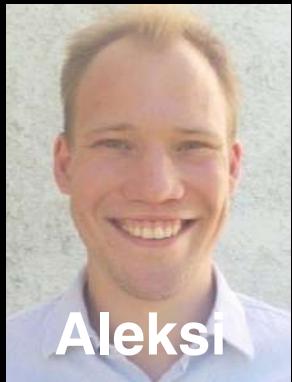
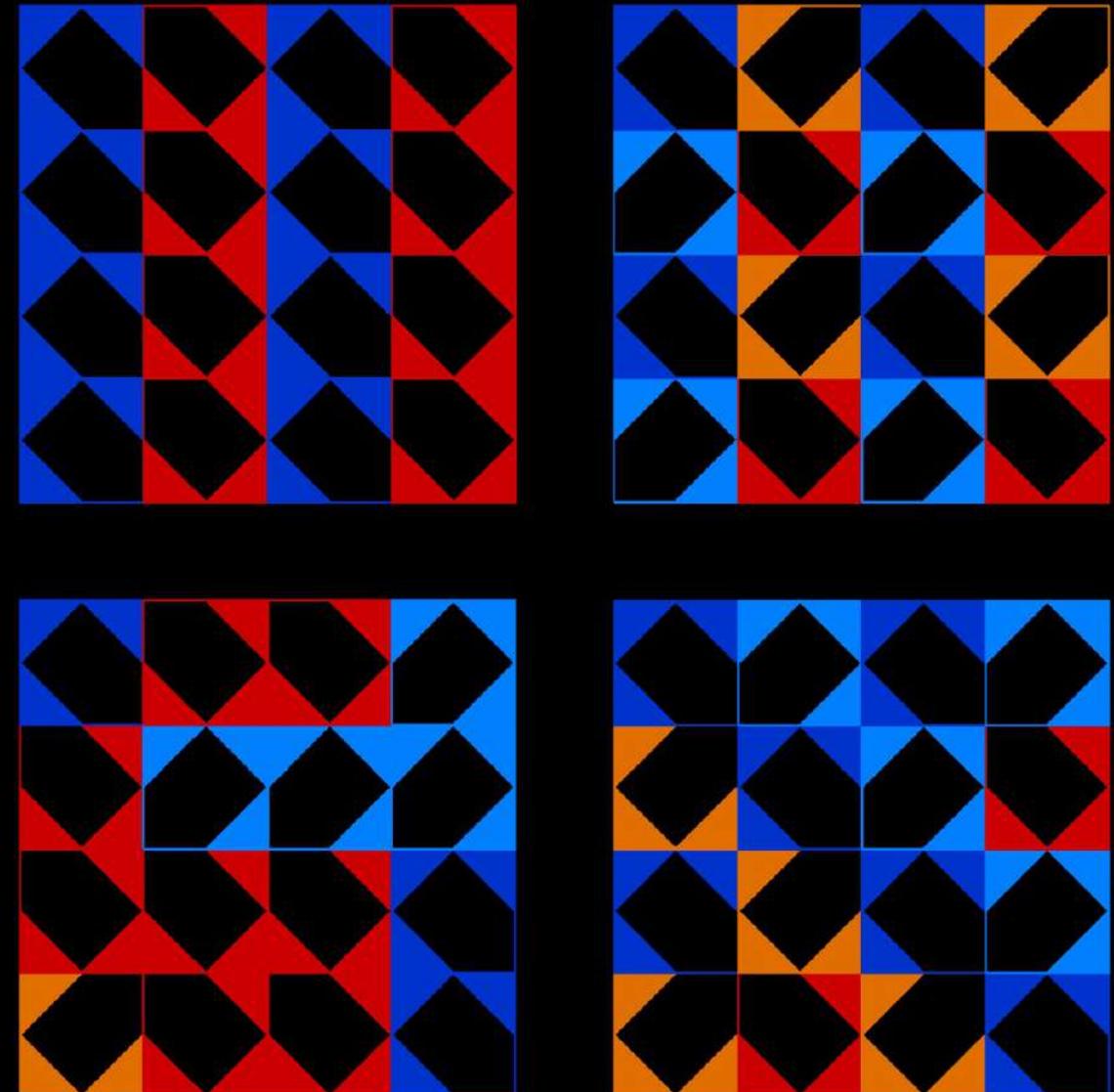
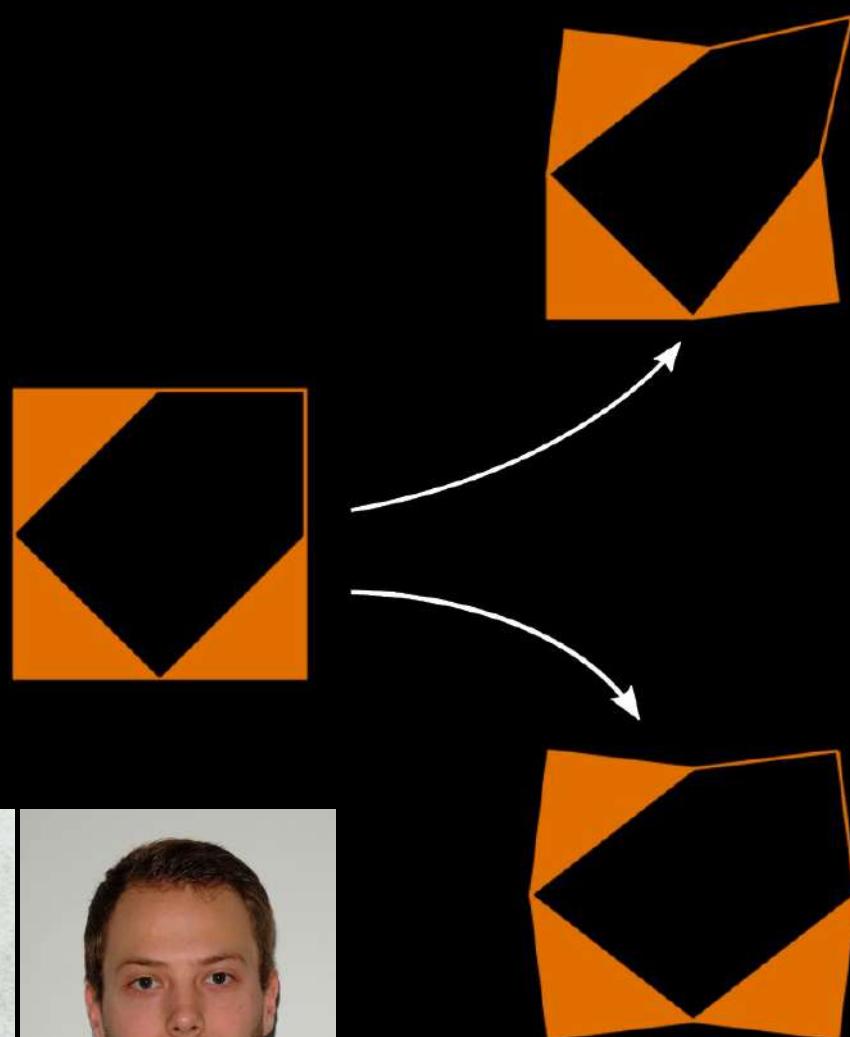
Aleksandr



Dykstra

Bossart, Dykstra et al. PNAS 2021

Multimode Combinatorial Metamaterials



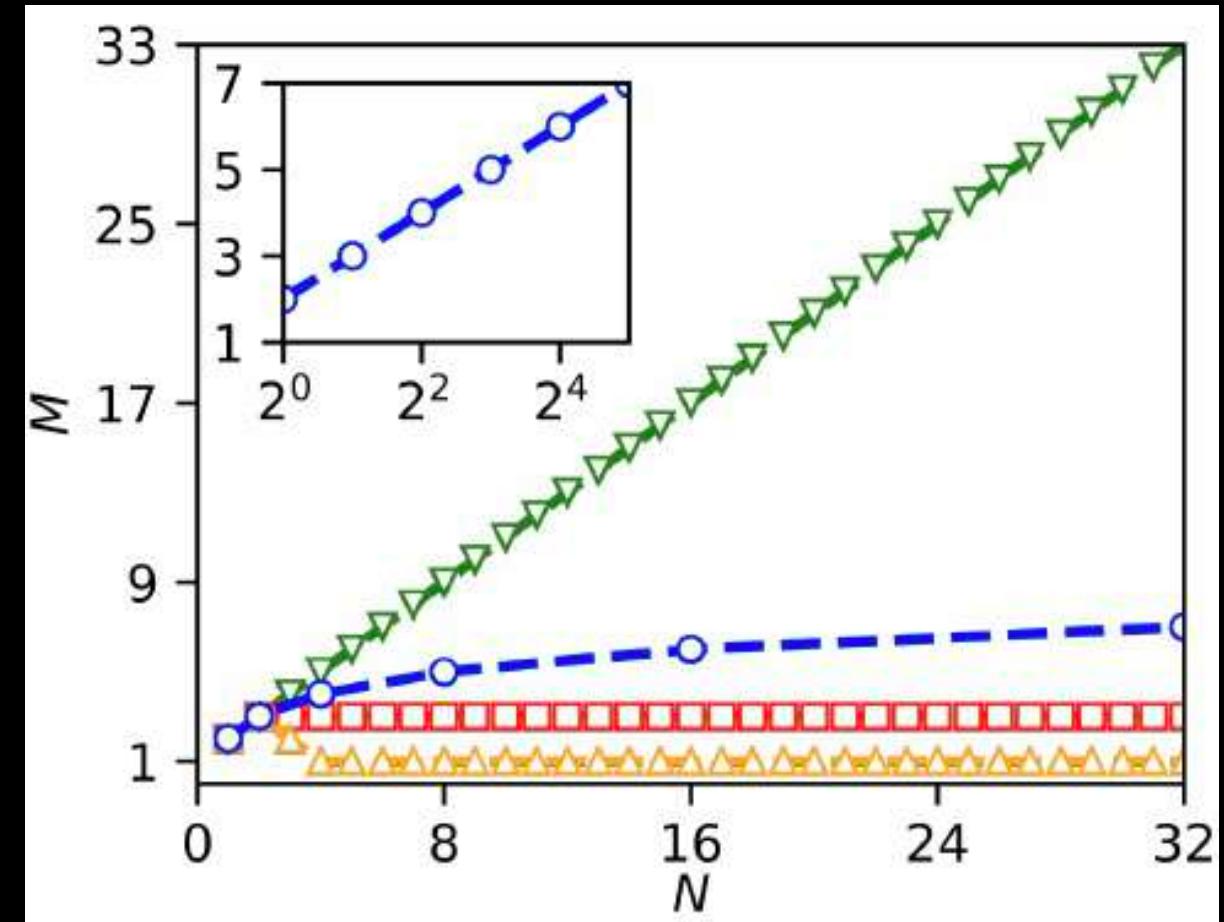
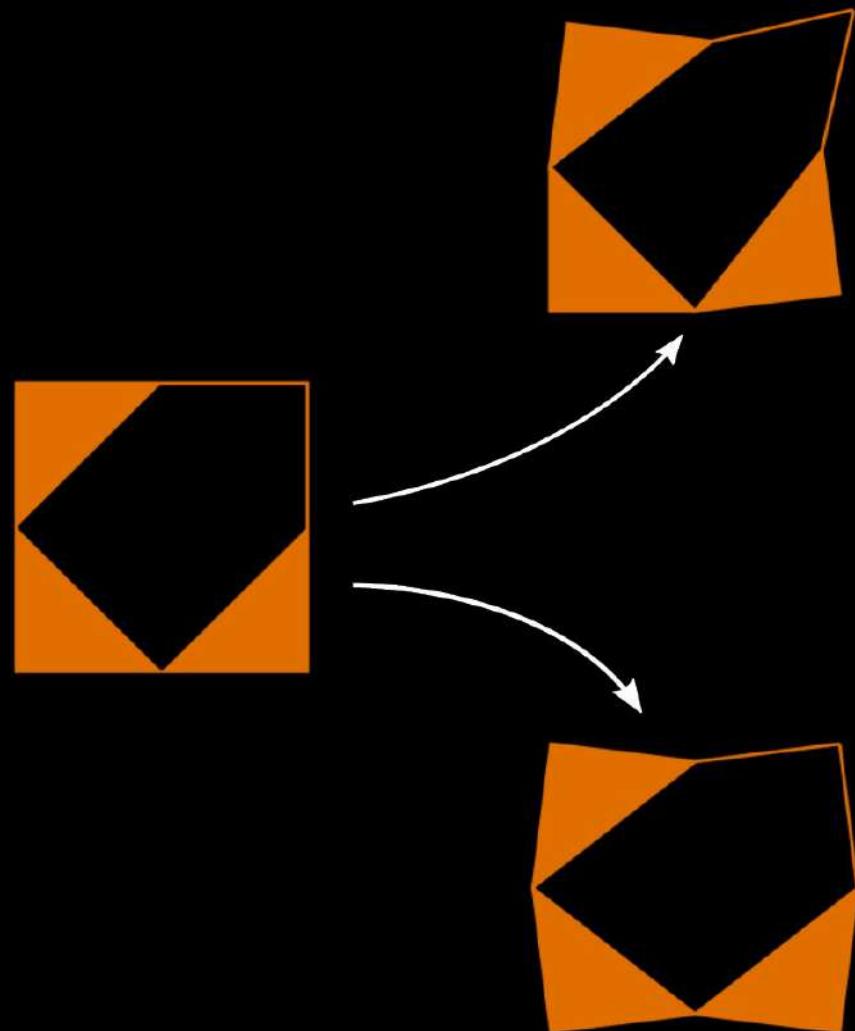
Aleksi



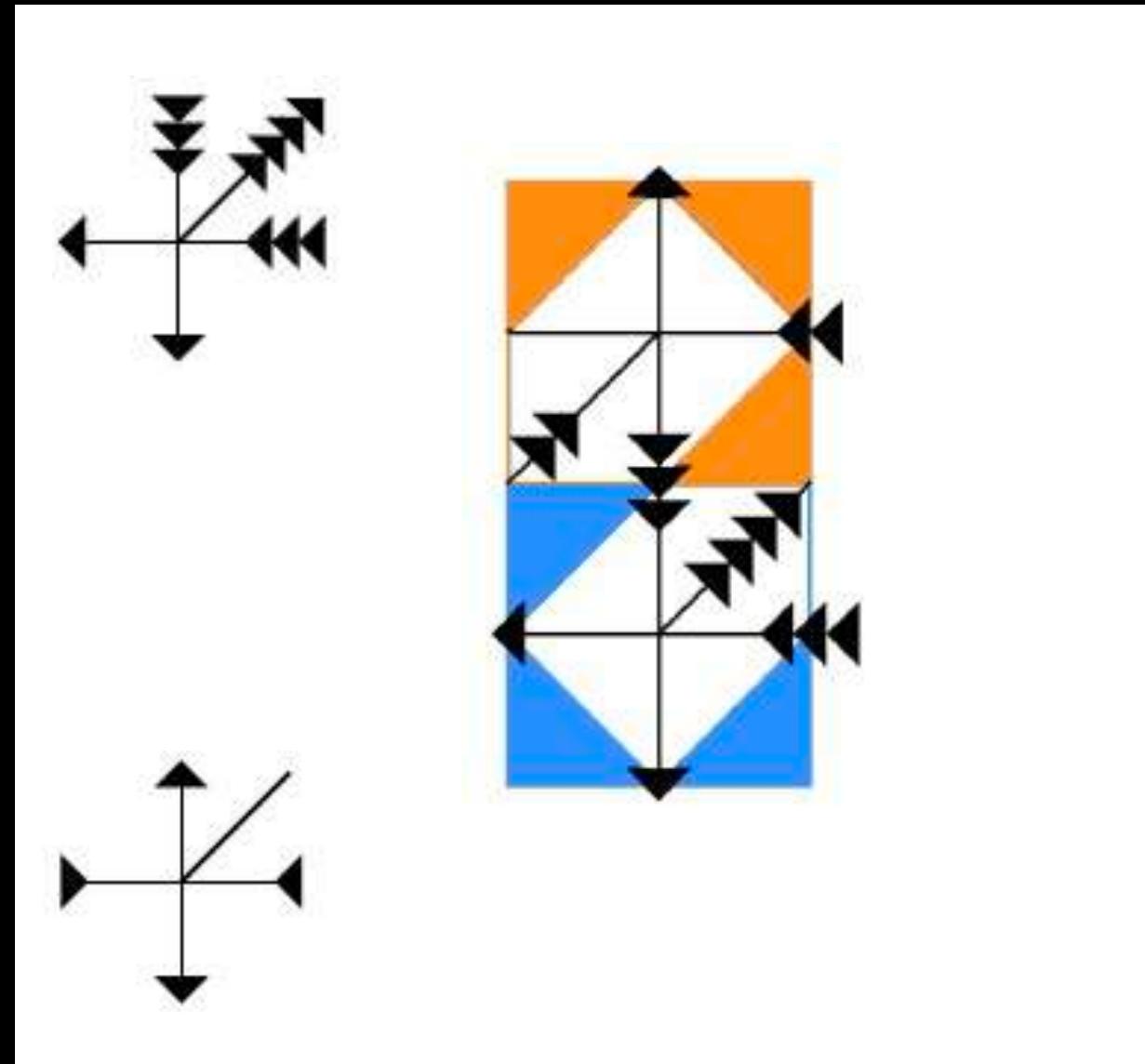
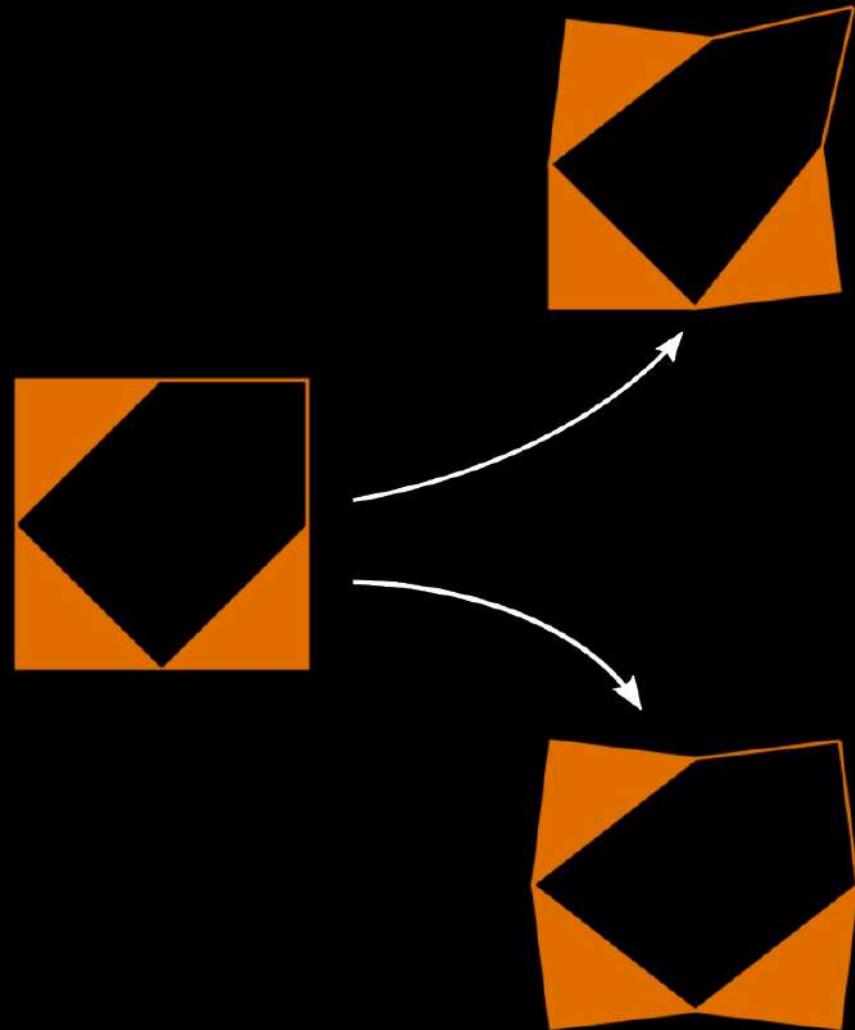
David

Bossart, Dykstra et al. PNAS 2021

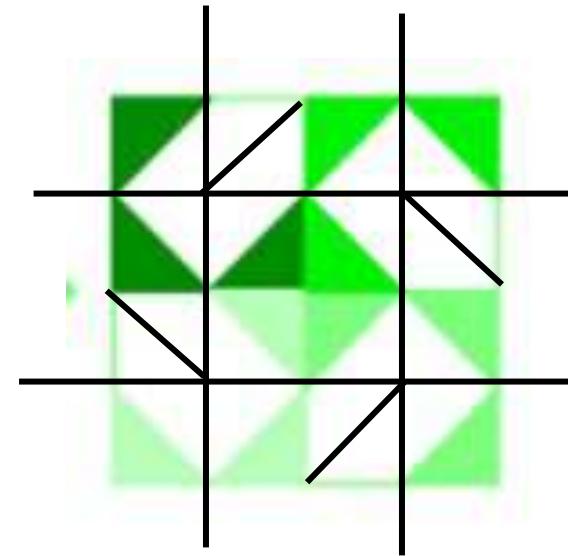
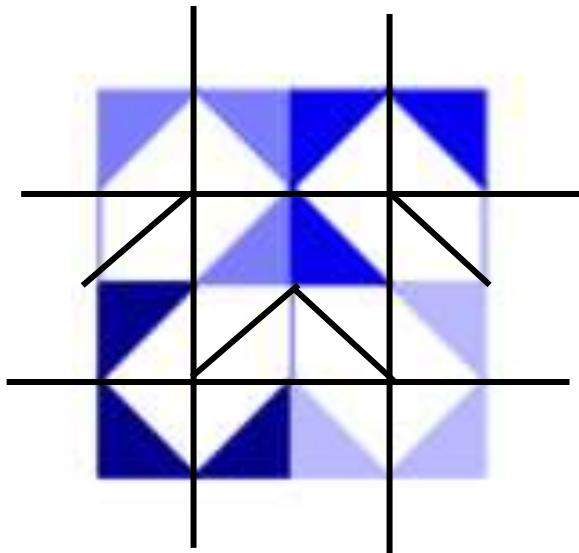
Multimode Combinatorial Metamaterials



Multimode Combinatorial Metamaterials

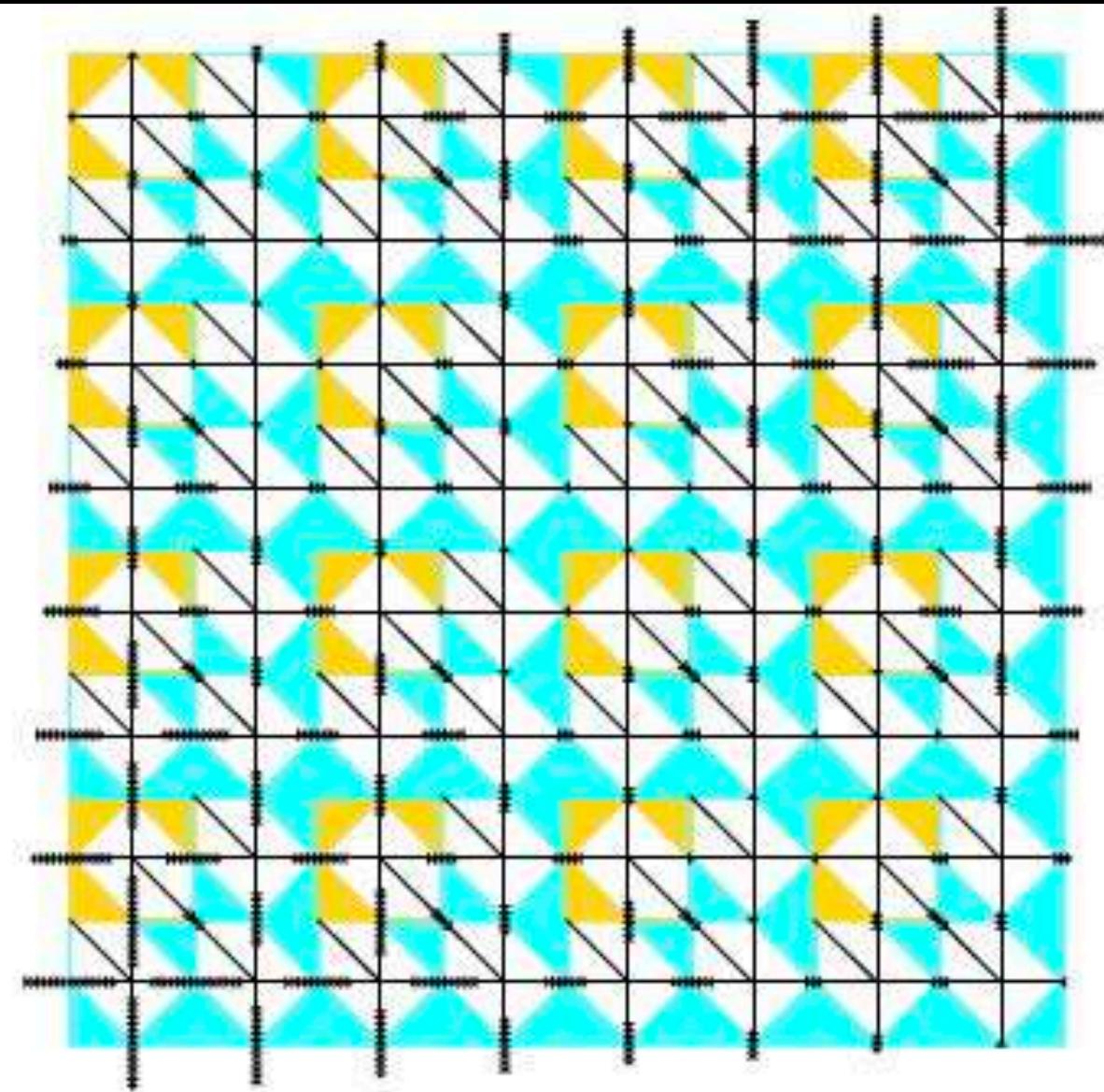
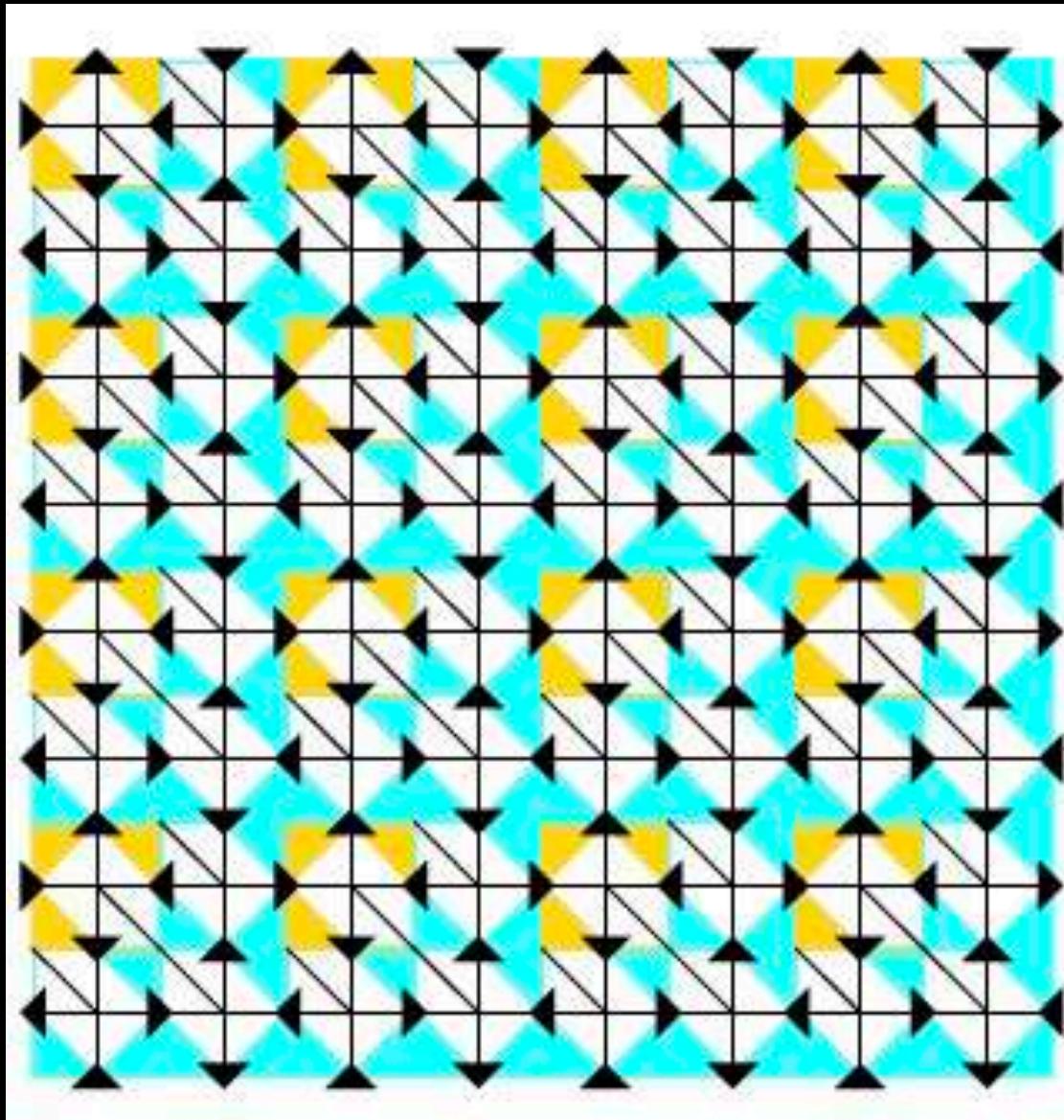


Exercise 3 (15 min)

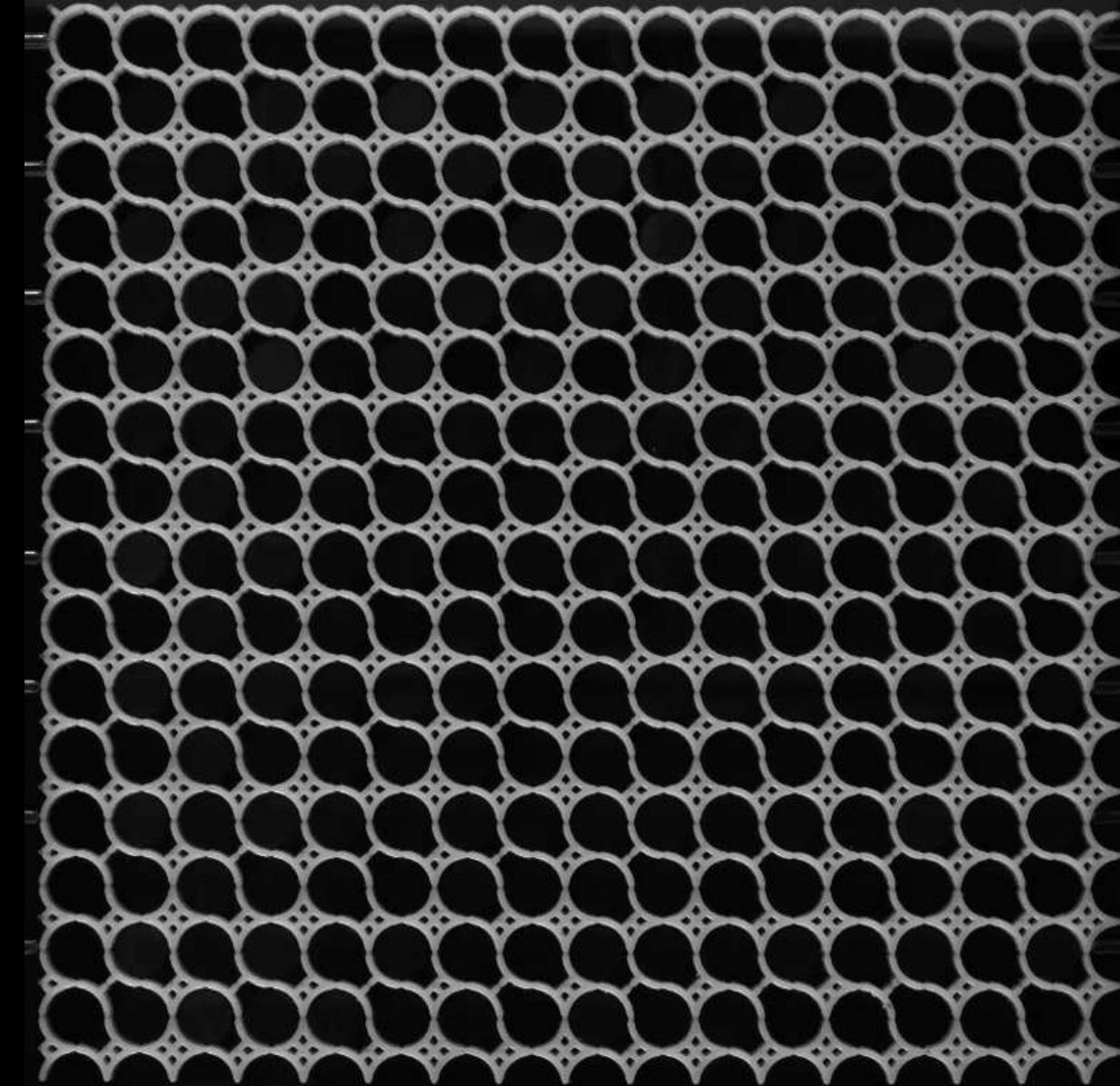
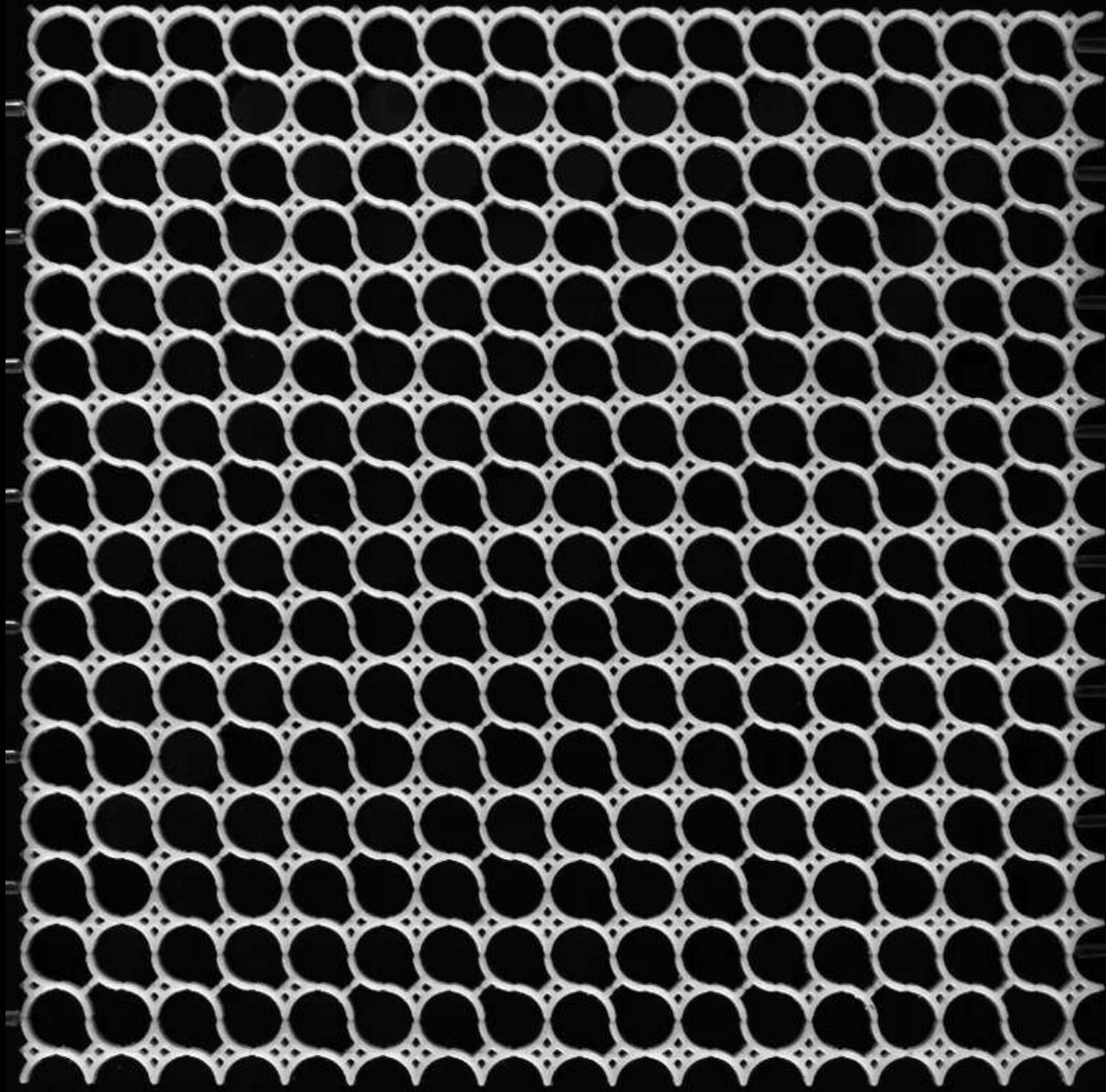


Use the graph based representation to determine the shape of the zeros modes

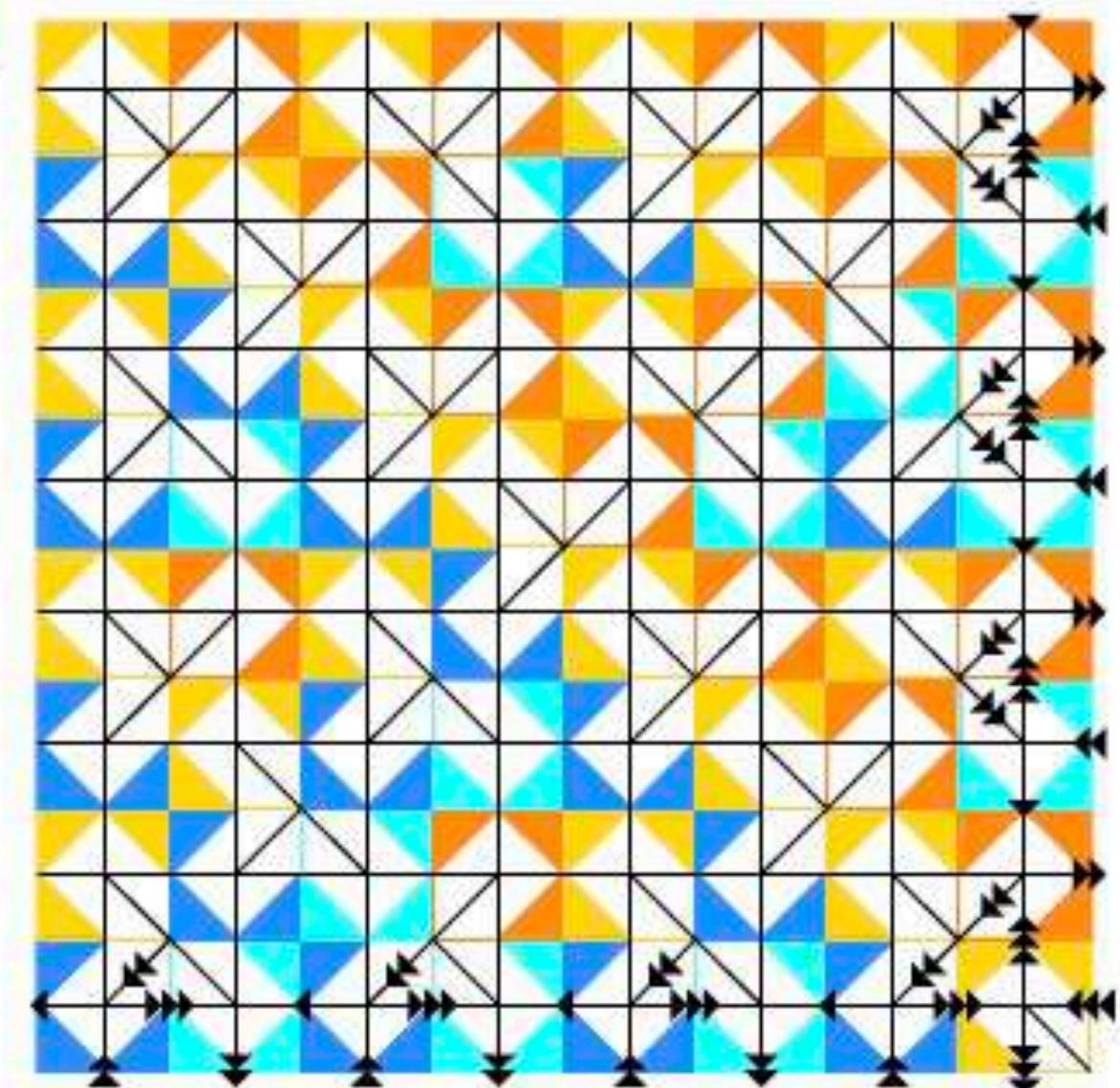
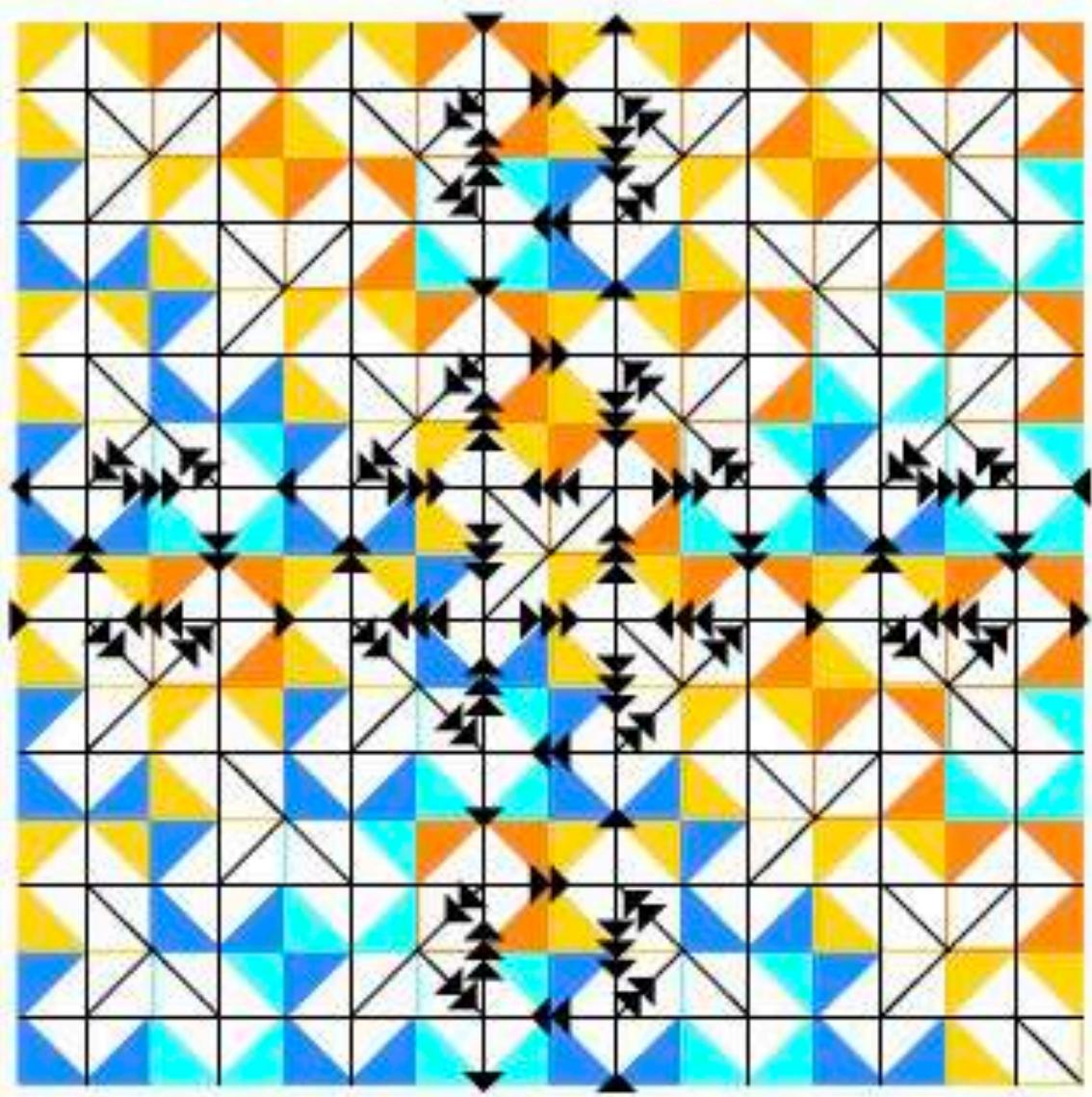
Multimode Combinatorial Metamaterials



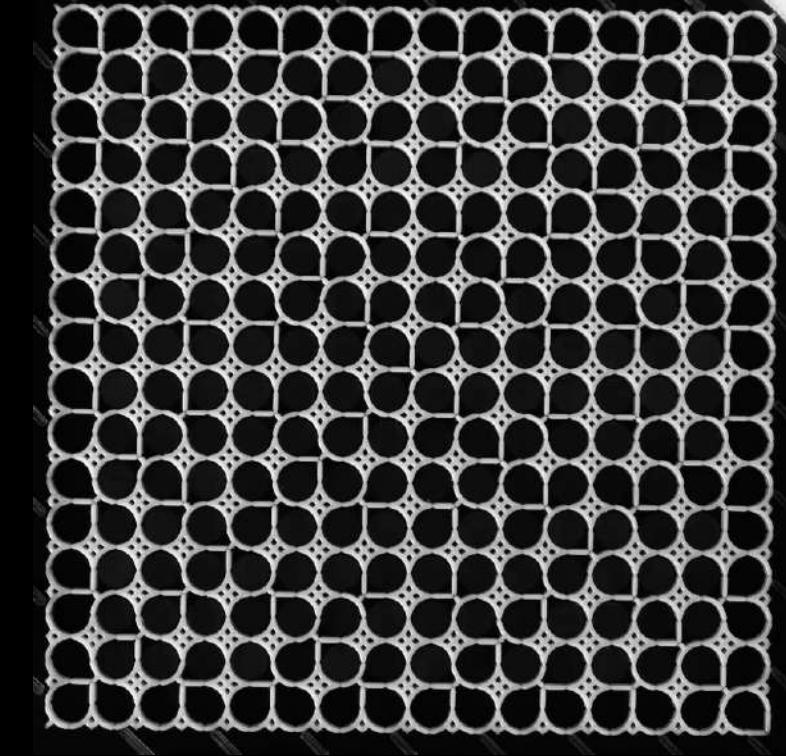
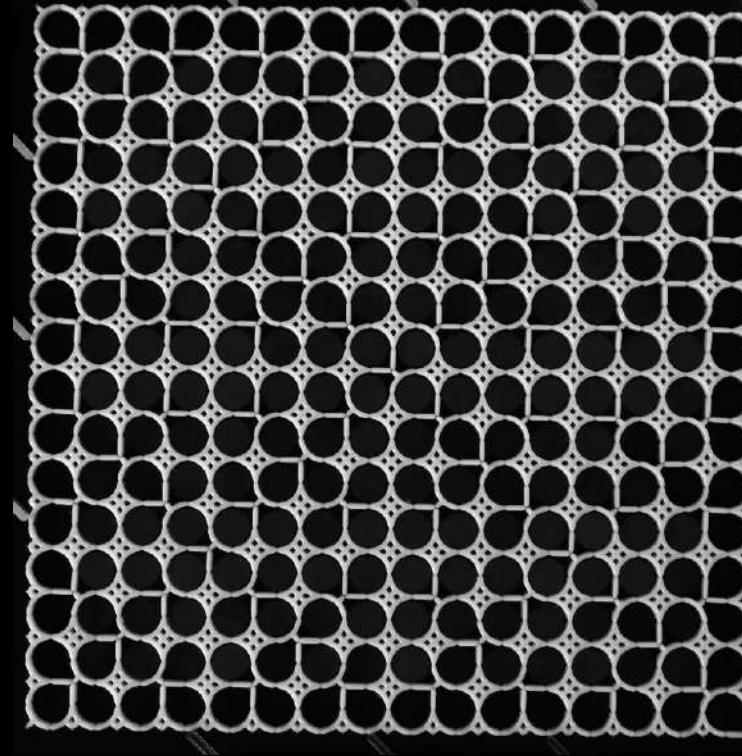
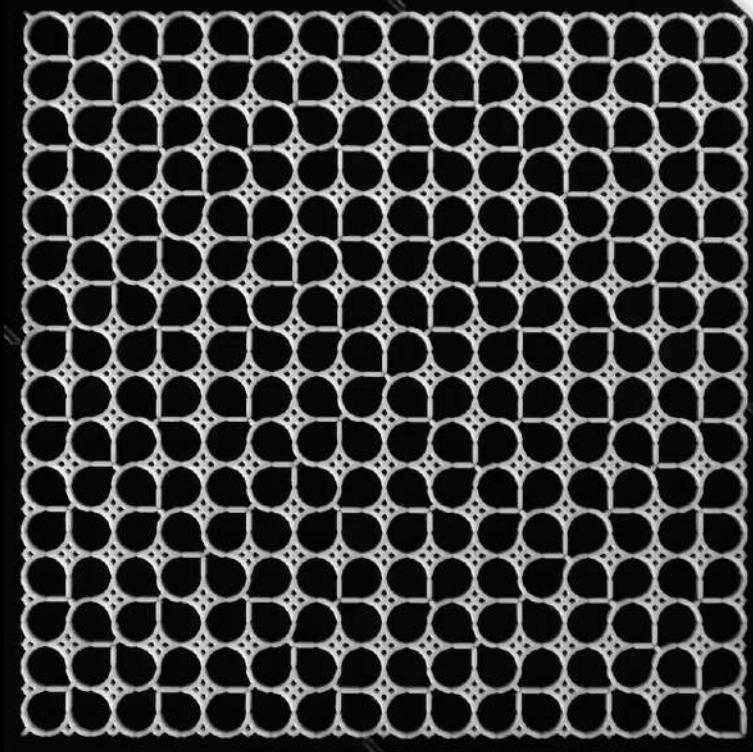
Multimode Combinatorial Metamaterials



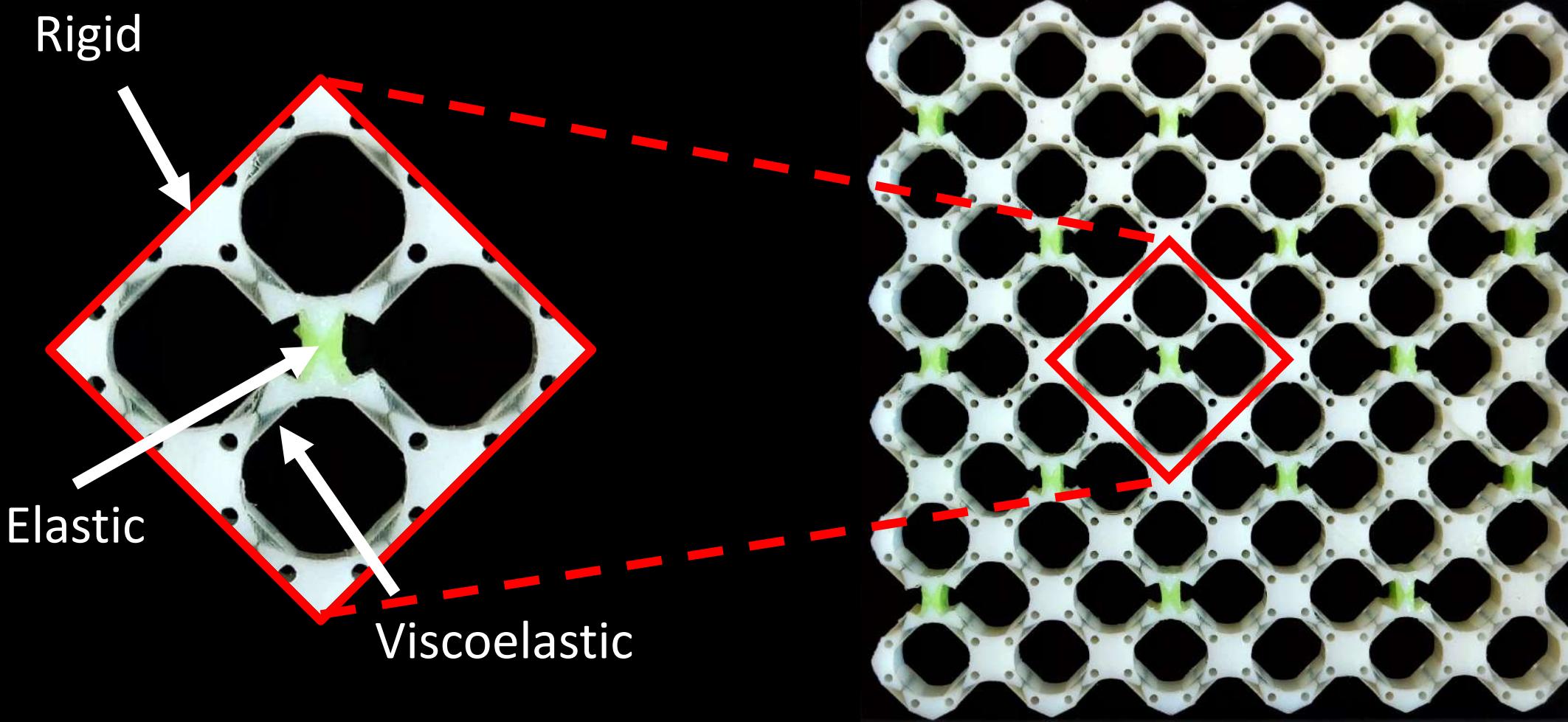
Multimode Combinatorial Metamaterials



Multimode Combinatorial Metamaterials

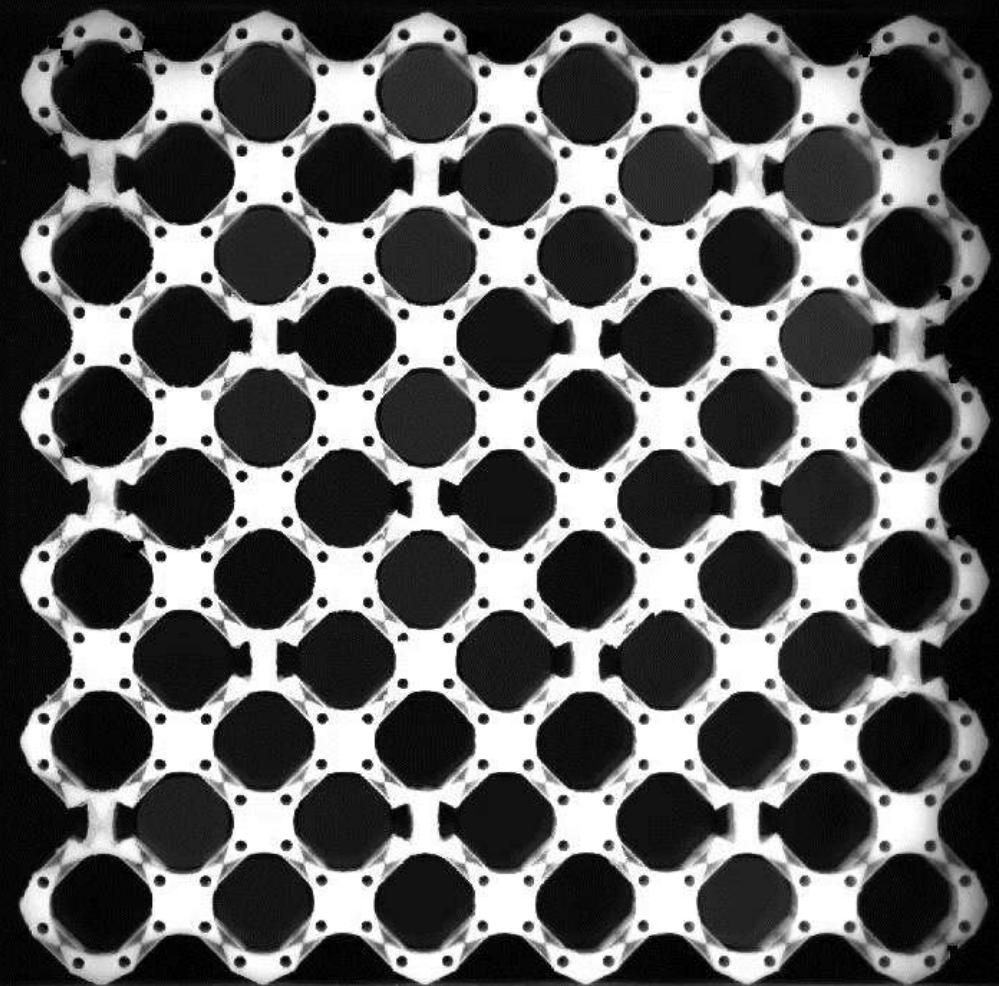


Multifunctional Metamaterials ?

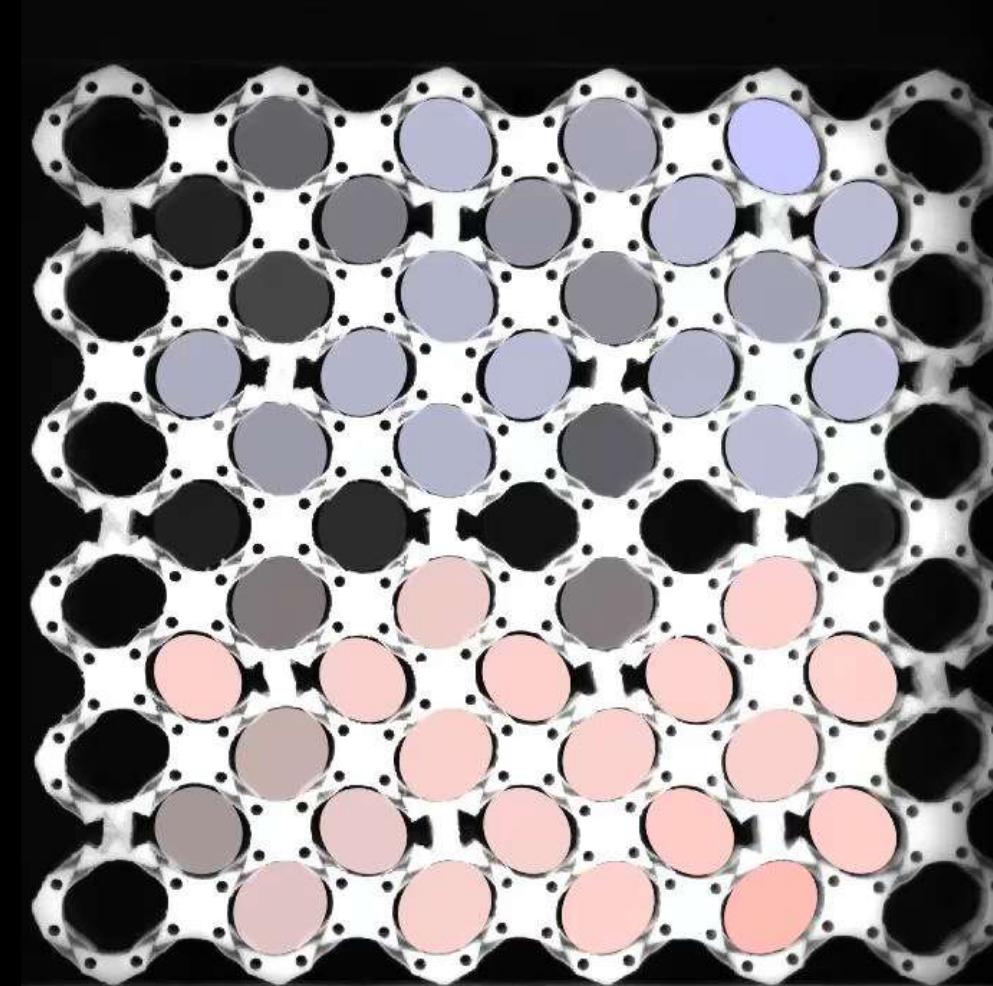


Multifunctional Metamaterials !

Slow



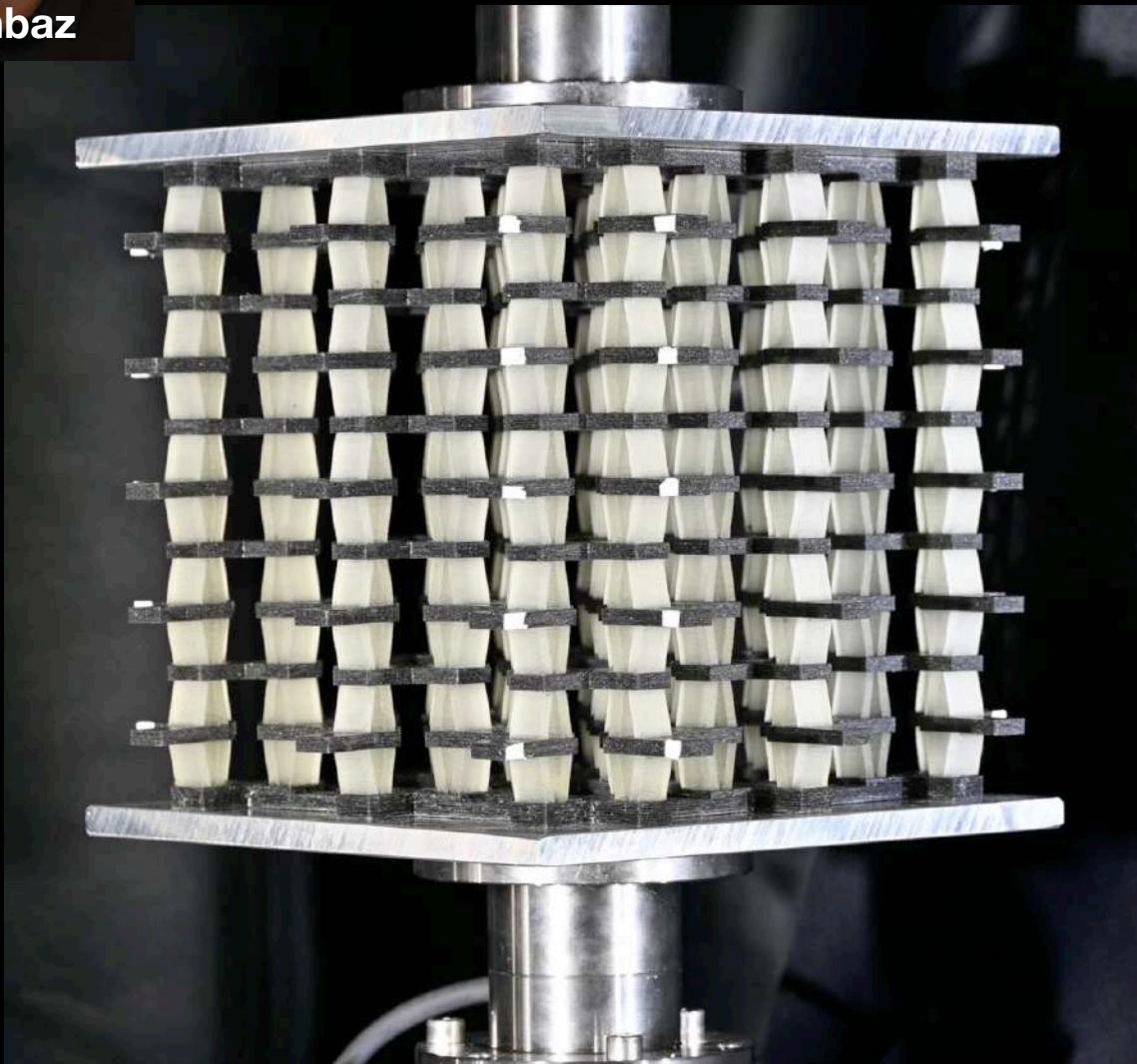
Fast



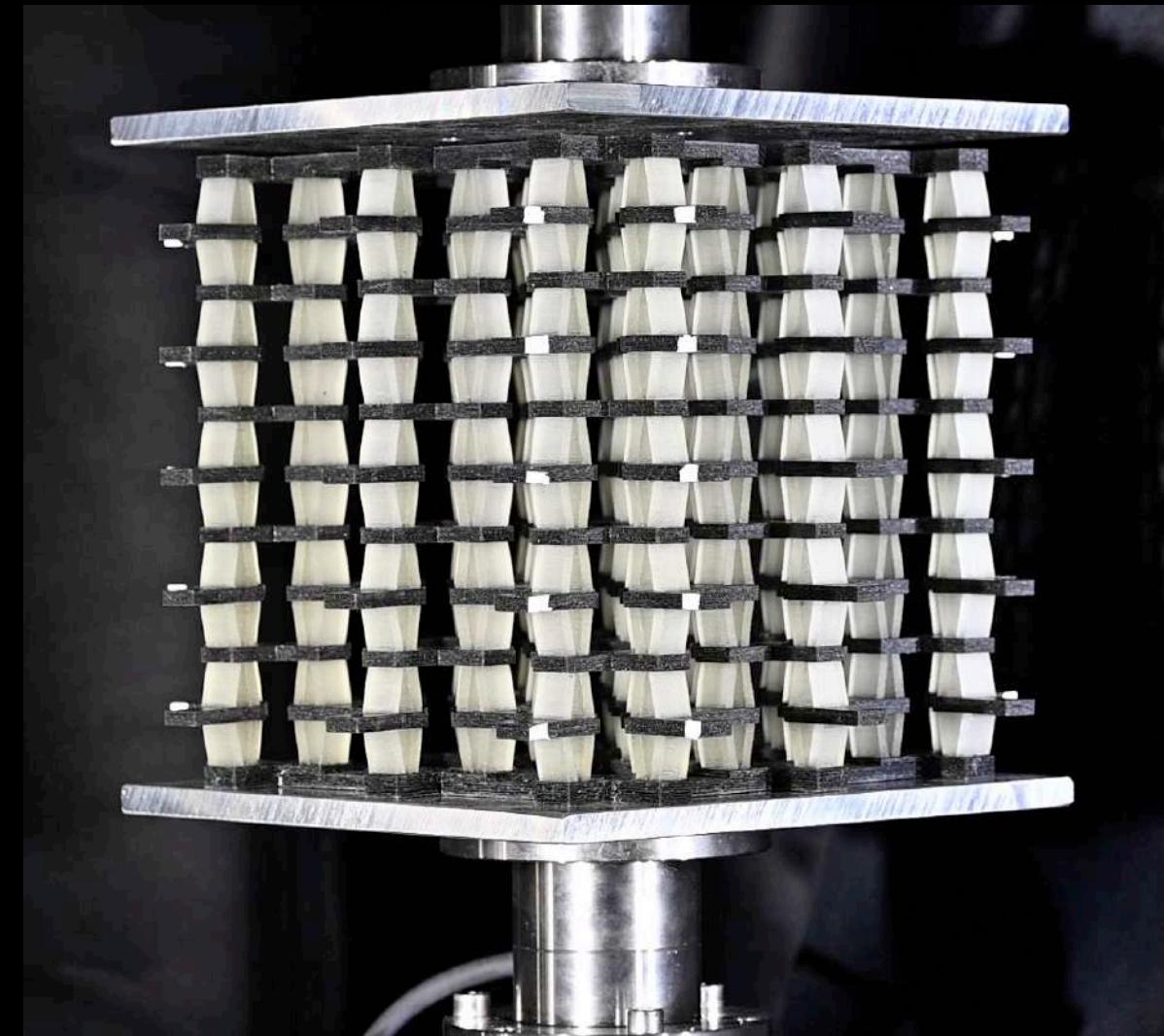


Multifunctional 3D Metamaterials

Slow



Fast



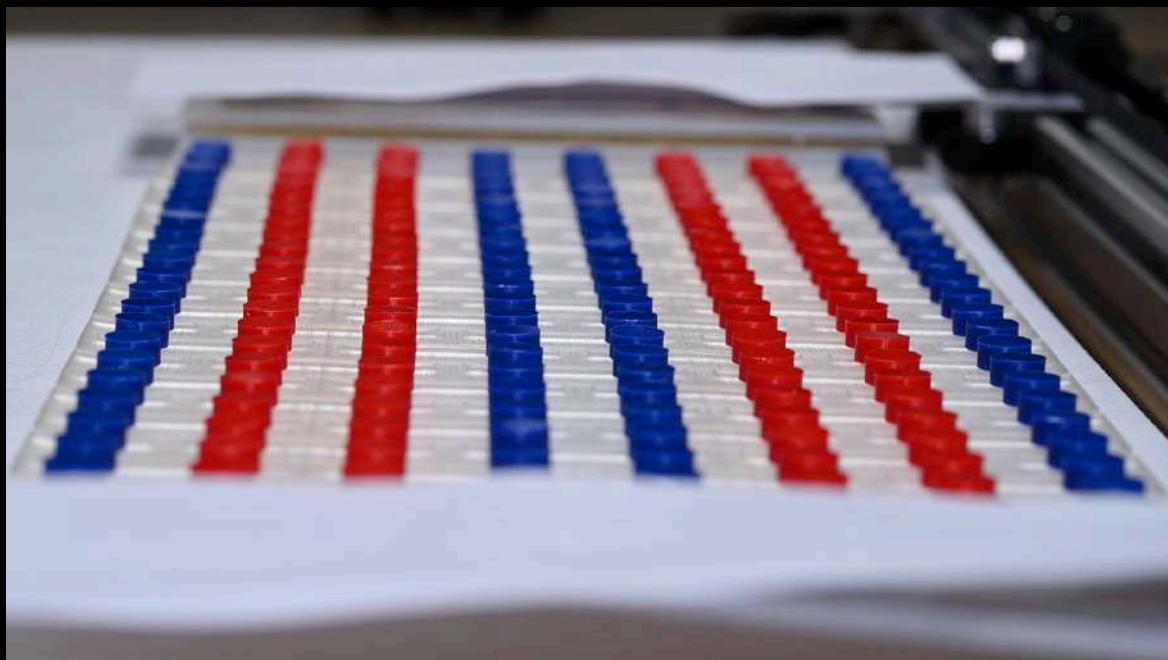
Shahram
Janbaz



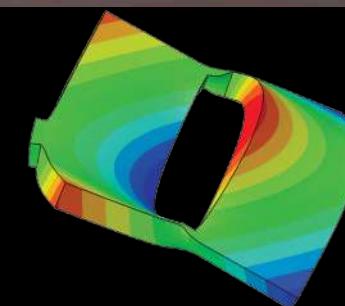
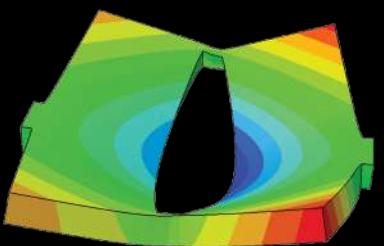
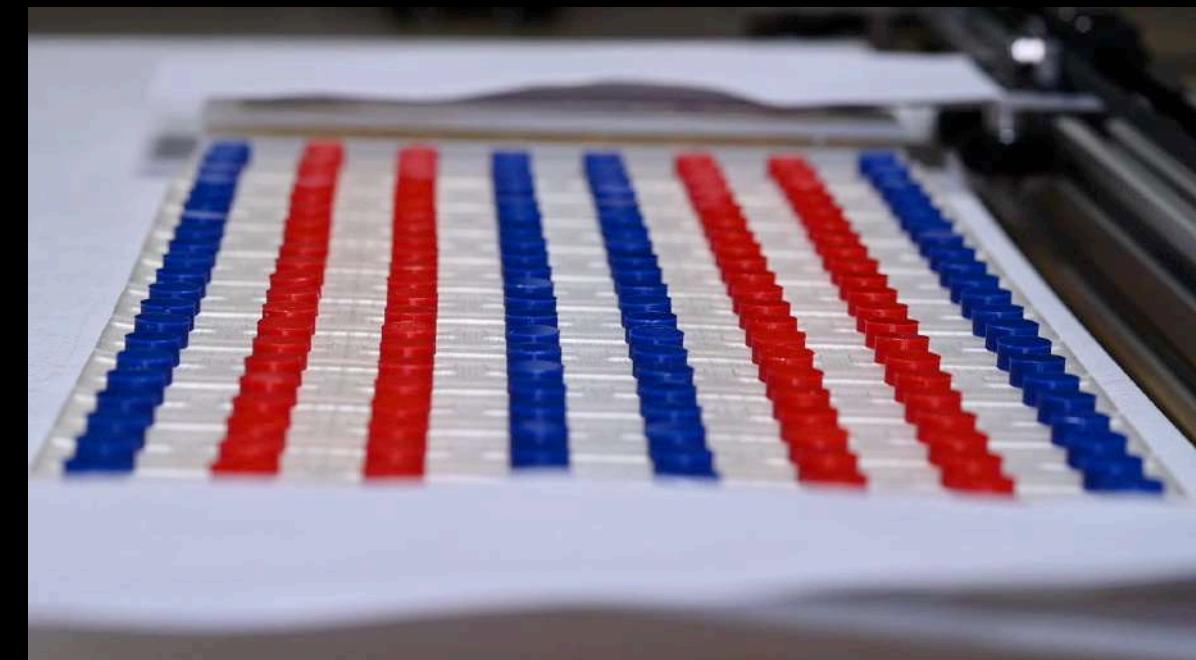
Multitextured Kirigami

Shahram
Janbaz

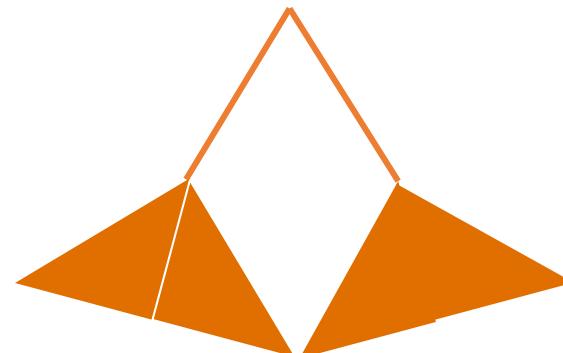
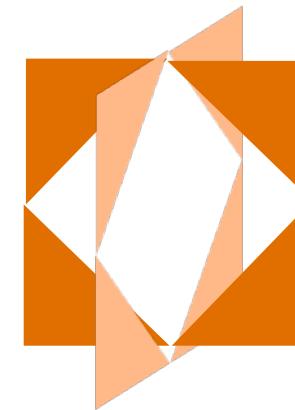
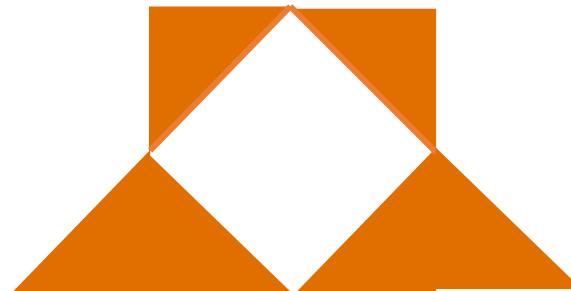
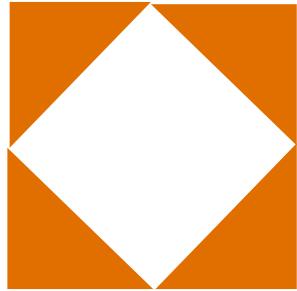
Slow



Fast



Combinatorial metamaterials

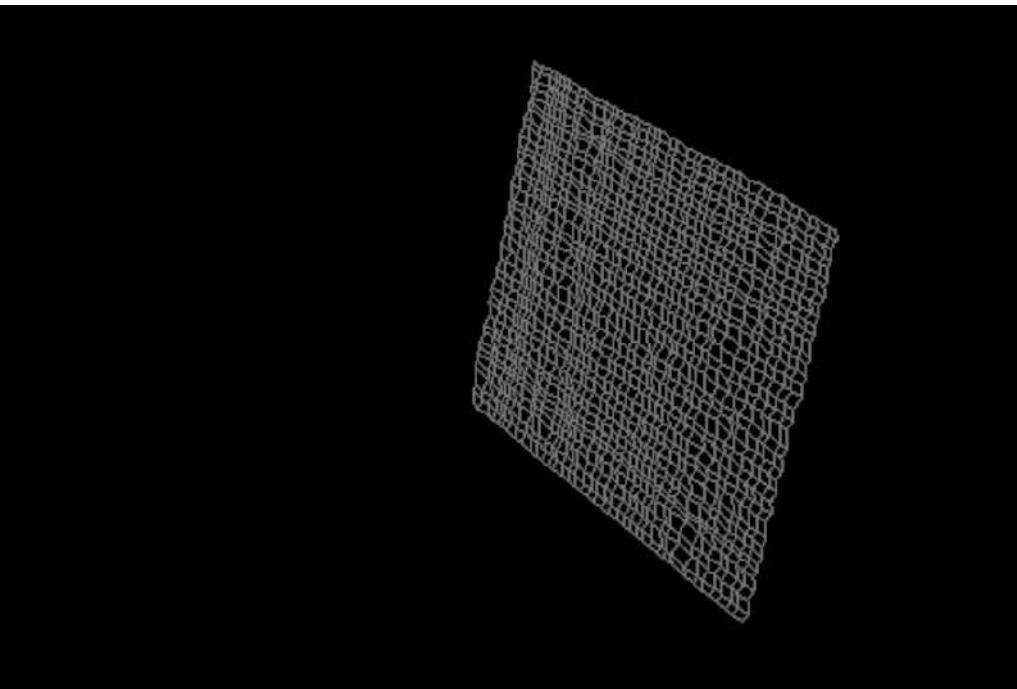
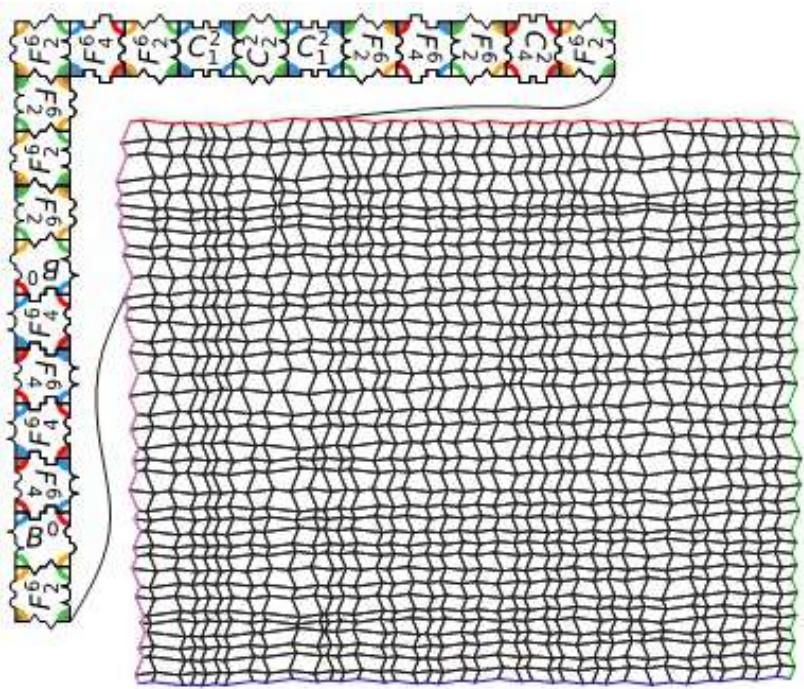


Meeussen et al. Nat. Phys. 2020

Coulais et al. Nature 2021
Van Mastrigt et al., PRL 2022

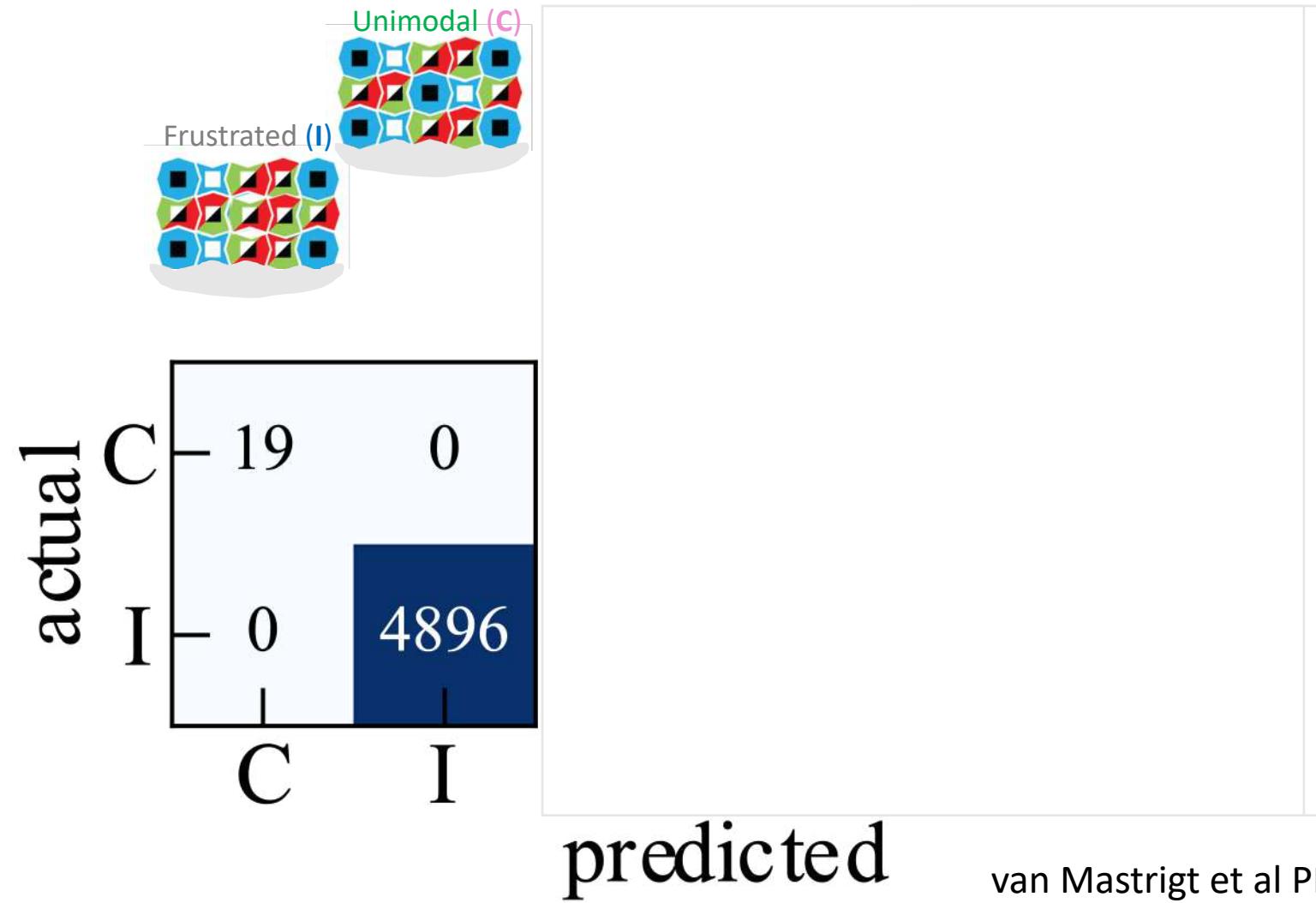
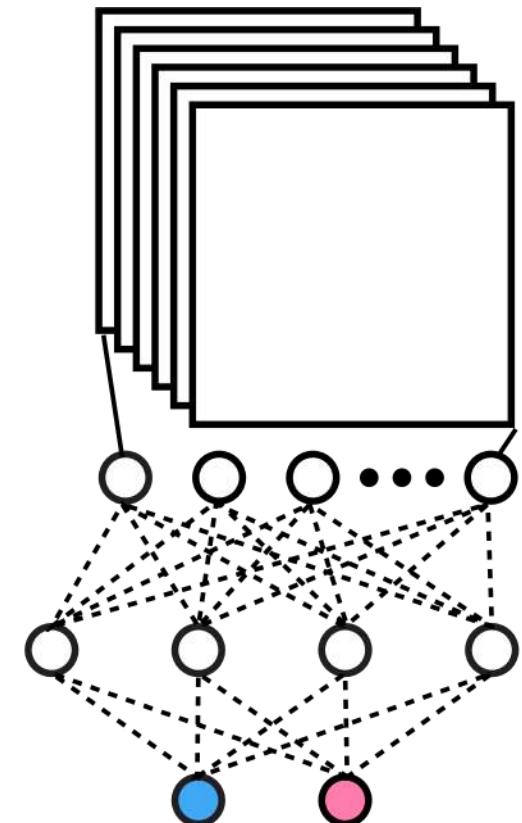
Bossart, Dykstra et al. PNAS 2021
Van Mastrigt et al., PRL 2022

Combinatorial Design of Pluripotent Origami

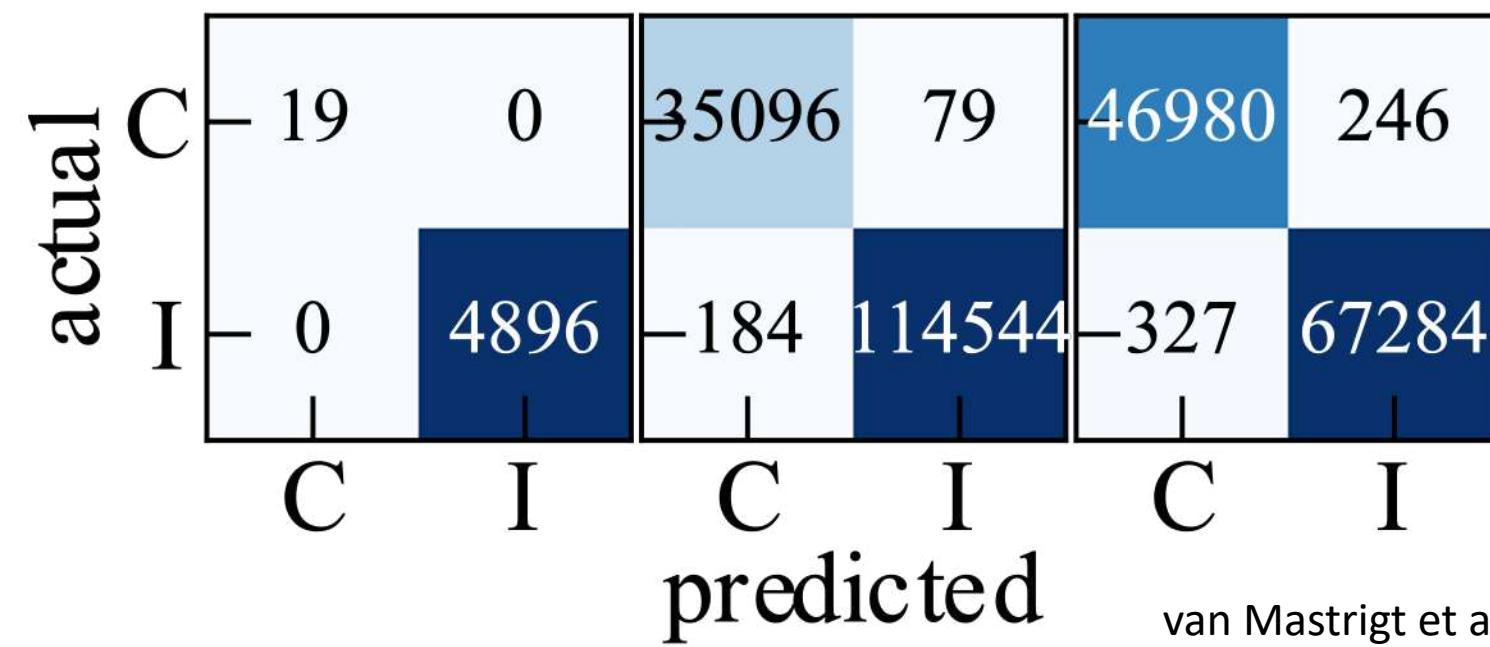
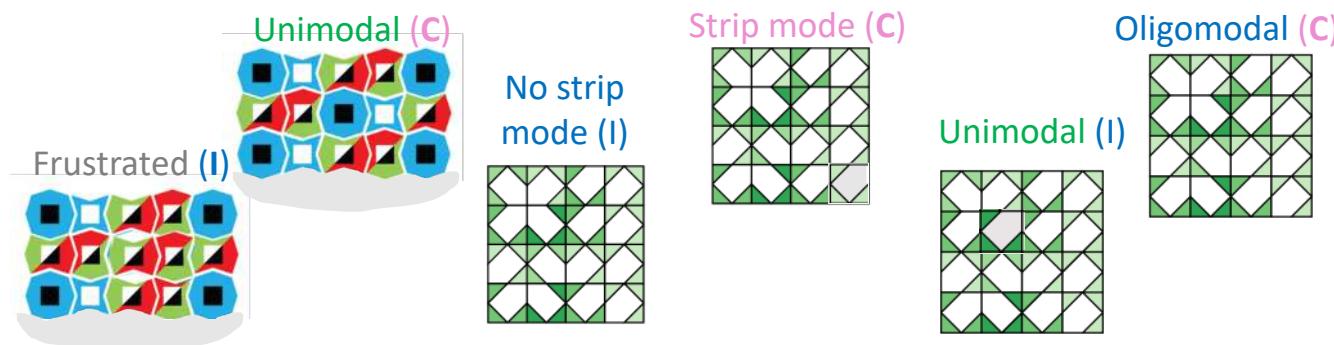
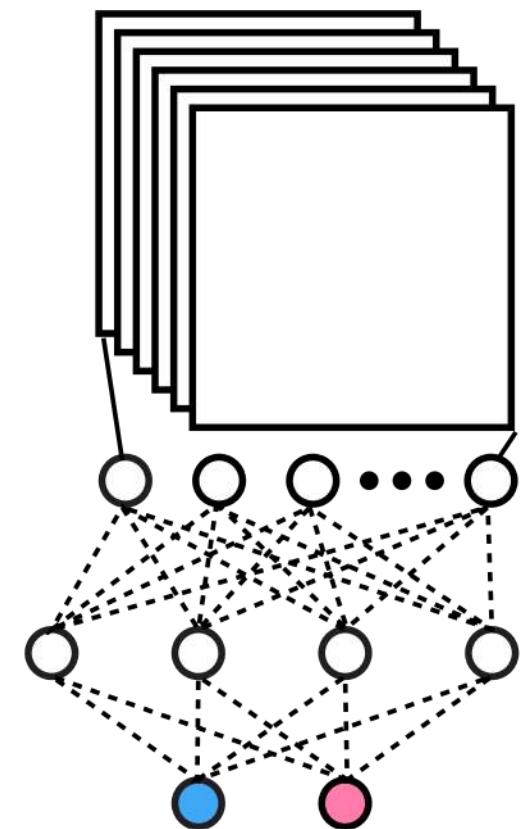




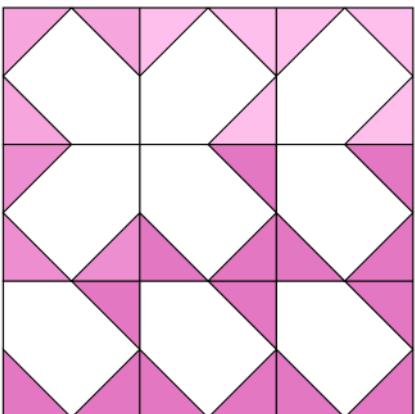
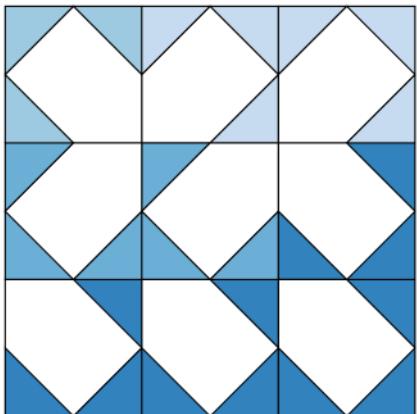
Needles in hay-stacks -> Machine Learning



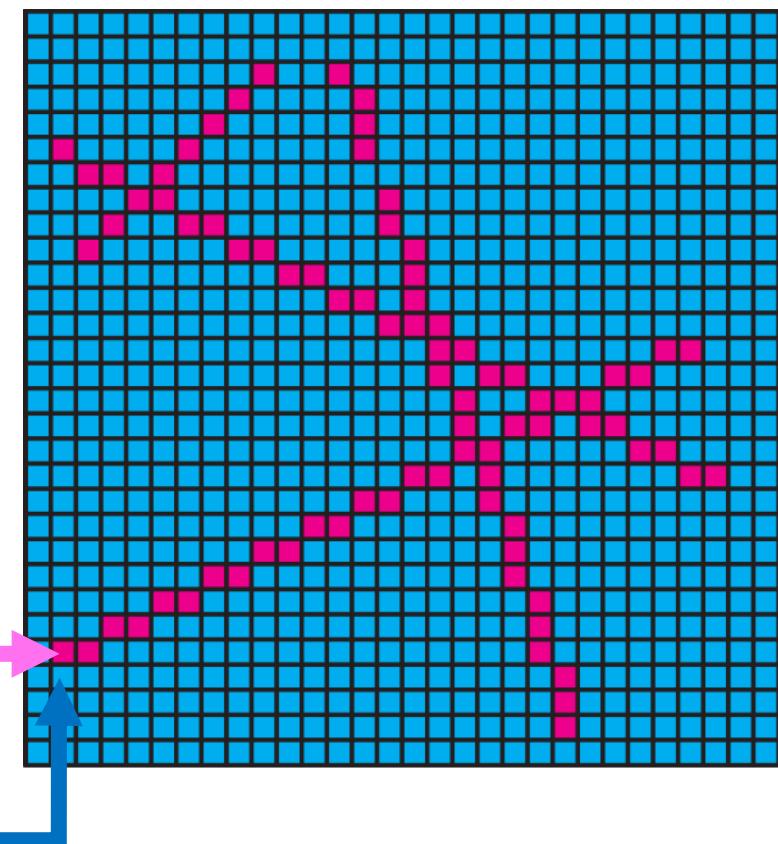
Needles in hay-stacks -> Machine Learning



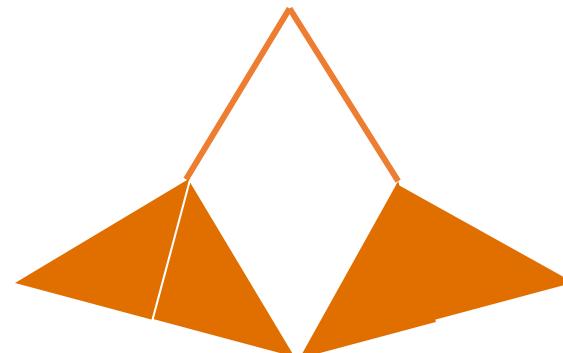
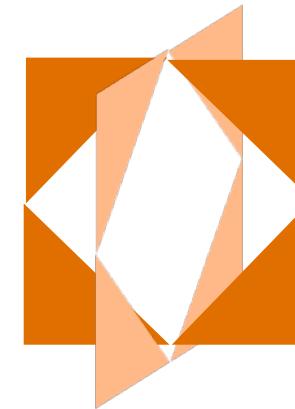
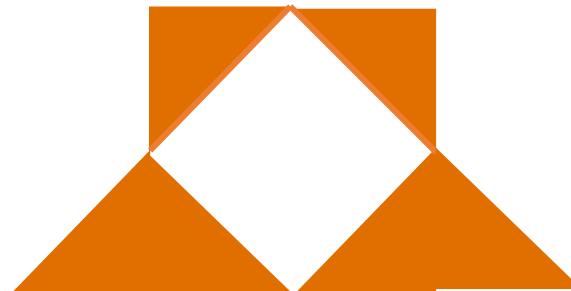
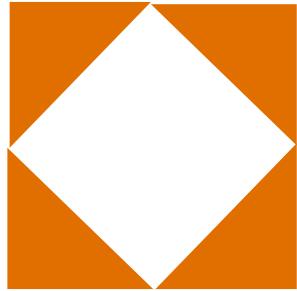
Needles in hay-stacks -> Machine Learning



Monte Carlo
sampling



Combinatorial metamaterials



Meeussen et al. Nat. Phys. 2020

Coulais et al. Nature 2021
Van Mastrigt et al., PRL 2022

Bossart, Dykstra et al. PNAS 2021
Van Mastrigt et al., PRL 2022