Workshop on Electromagnetic Effects in Strongly Interacting Matter
October 25-28, 2022 ICTP-SAIFR, São Paulo

Effects of Strong Electric and Magnetic Fields in Superdense Matter

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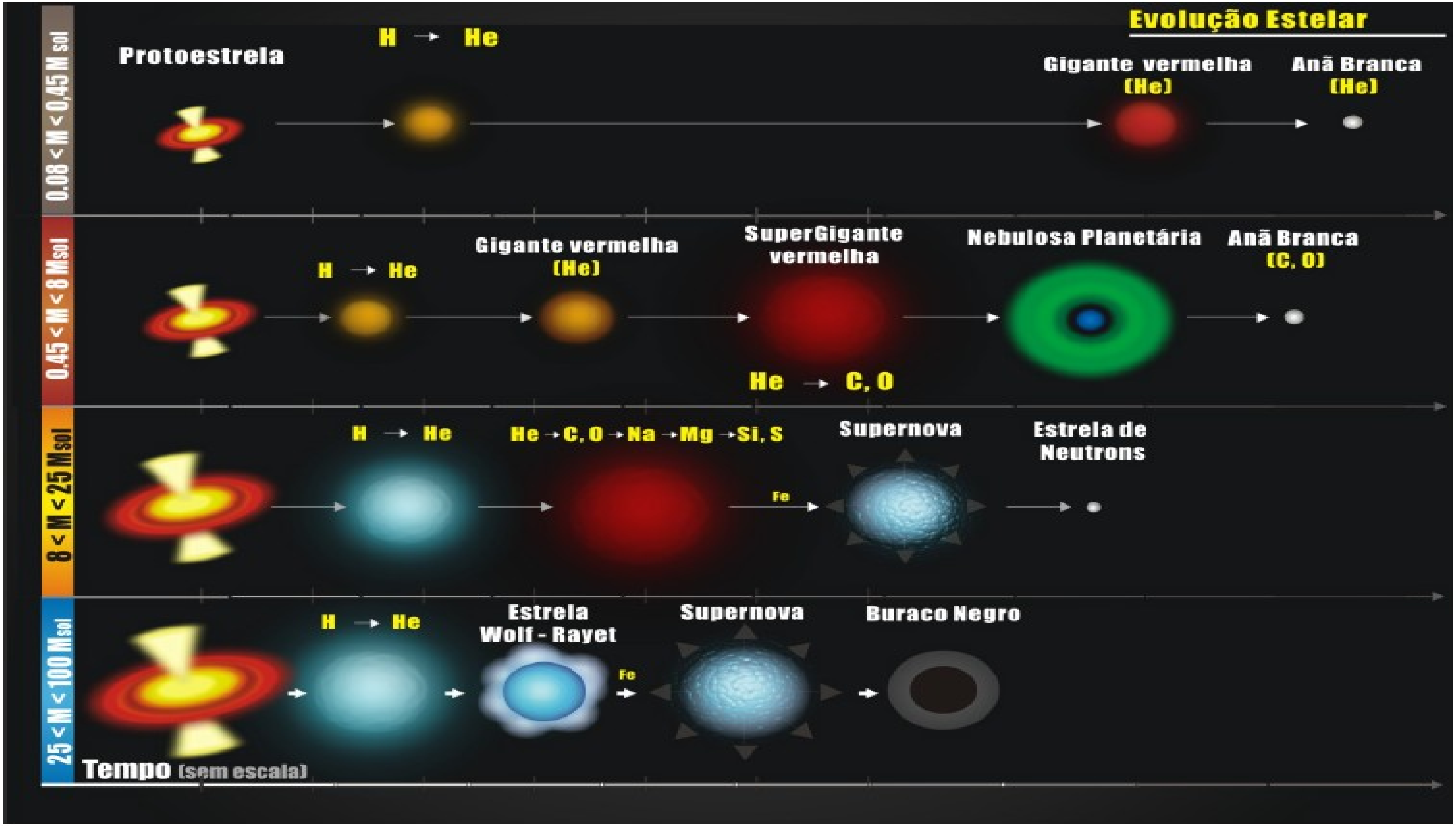
Thematic Projetc FAPESP "Superdense Matter in the Universe"
ITA, INPE, IAG-USP, UNIFESP, UFABC, UNESP, IFSP (2014 -2020)

Outline

I - Introduction to Compact Stars

II - Strong Magnetic Fields in White Dwarf and Neutron Stars

III - Strong Electric Fields in Quark Stars



Evolução Estelar
0.08 < M < 0,45 M sol
Protoestrela
H → He
Gigante vermelha
(He)
Anã Branca
(He)
0,45 < M < 8 Msol
H → He
Gigante vermelha
(He)
SuperGigante
vermelha
He → C, O
Nebulosa Planetária
Anã Branca
(C, O)
8 < M < 25 Msol
H → He
He → C, O → Na → Mg → Si, S
Fe
Supernova
Estrela de
Neutrons
25 < M < 100 Msol
H → He
Estrela
Wolf - Rayet
Fe
Supernova
Buraco Negro
Tempo (sem escala)

Nuclear Astrophysics – Compact Stars

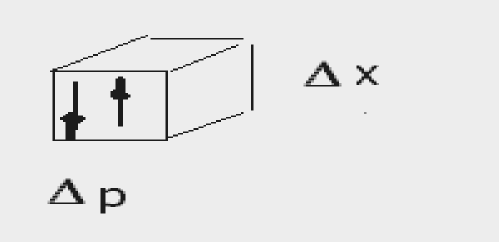
Name	M/M_{Sun}	R (Km)	ρ (g/cm^3)	Pressure (dynas/cm^2)
Neutrons Stars	1 – 2	11 – 13	5×10^{14}	10^{32} - 10^{35}
White Dwarfs	1	5400	3×10^{6}	10^{25} - 10^{28}
Sun	1	7×10^{5}	1.4	10^{17}

Quantum Stars

Ideal gas (Boltzmann) $PV = NRT$

When $T \rightarrow 0$ $P \rightarrow 0$ (thermal excitation)

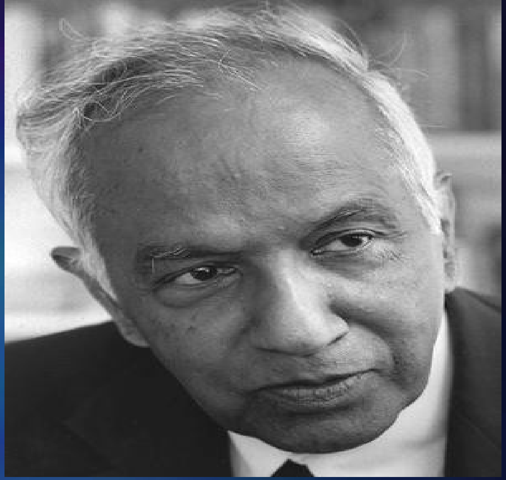
But if we increase the density, each fermion particle is forced to occupy only one energy state due to Pauli Principle.
This degeneracy regime provides pressure even at T=0!



Thus, the pressure of the free electron Fermi gas is

$$P \rightarrow \frac{\hbar^2}{m}\left(\frac{N}{V}\right)^{\frac{5}{3}} (\,non \quad relativistic\,)$$

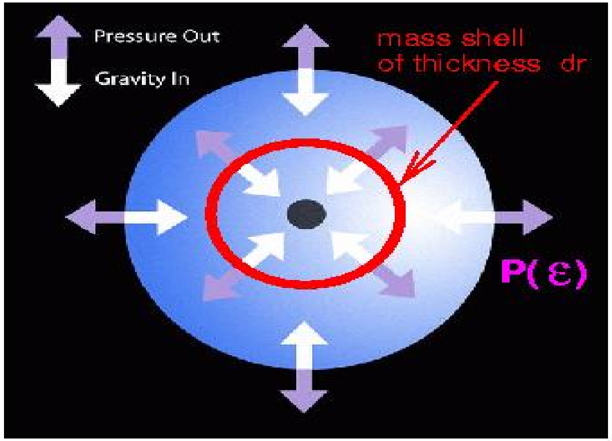
$$P \rightarrow \frac{\hbar^2}{m}\left(\frac{N}{V}\right)^{\frac{4}{3}} (\,ultra\ relativistic\,)$$



S. Chandrasekhar
(1934)

There will be stellar structure supported by this degeneracy pressure since gravitation compress the material enough (final stages)

Hydrostatic Equilibrium

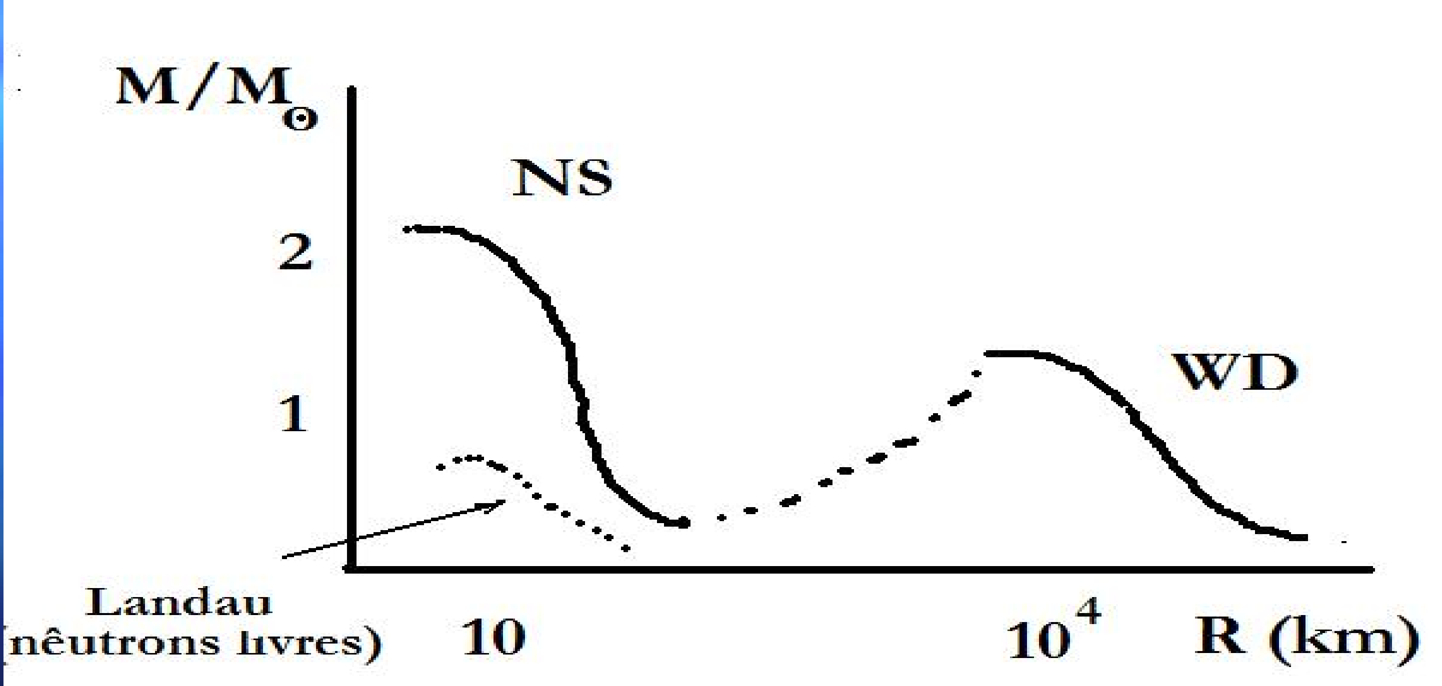


$$[P(r+dr) - P(r)]\, 4\pi r^2 = -\frac{G\, m(r)\, dm}{r^2}$$

with $dm = 4\pi r^2 \varepsilon\, dr$

$$\frac{dP}{dr} = -G\,\frac{m\,\varepsilon}{r^2}\,\frac{\left(1 + 4\pi r^3 \frac{P}{m}\right)\left(1 + \frac{P}{\varepsilon}\right)}{1 - 2mG/r}$$

Tolman, Oppenheimer, Volkoff, 1939


M/M_⊙
NS
2
1
WD
Landau
(nêutrons livres)
10
10^4
R (km)

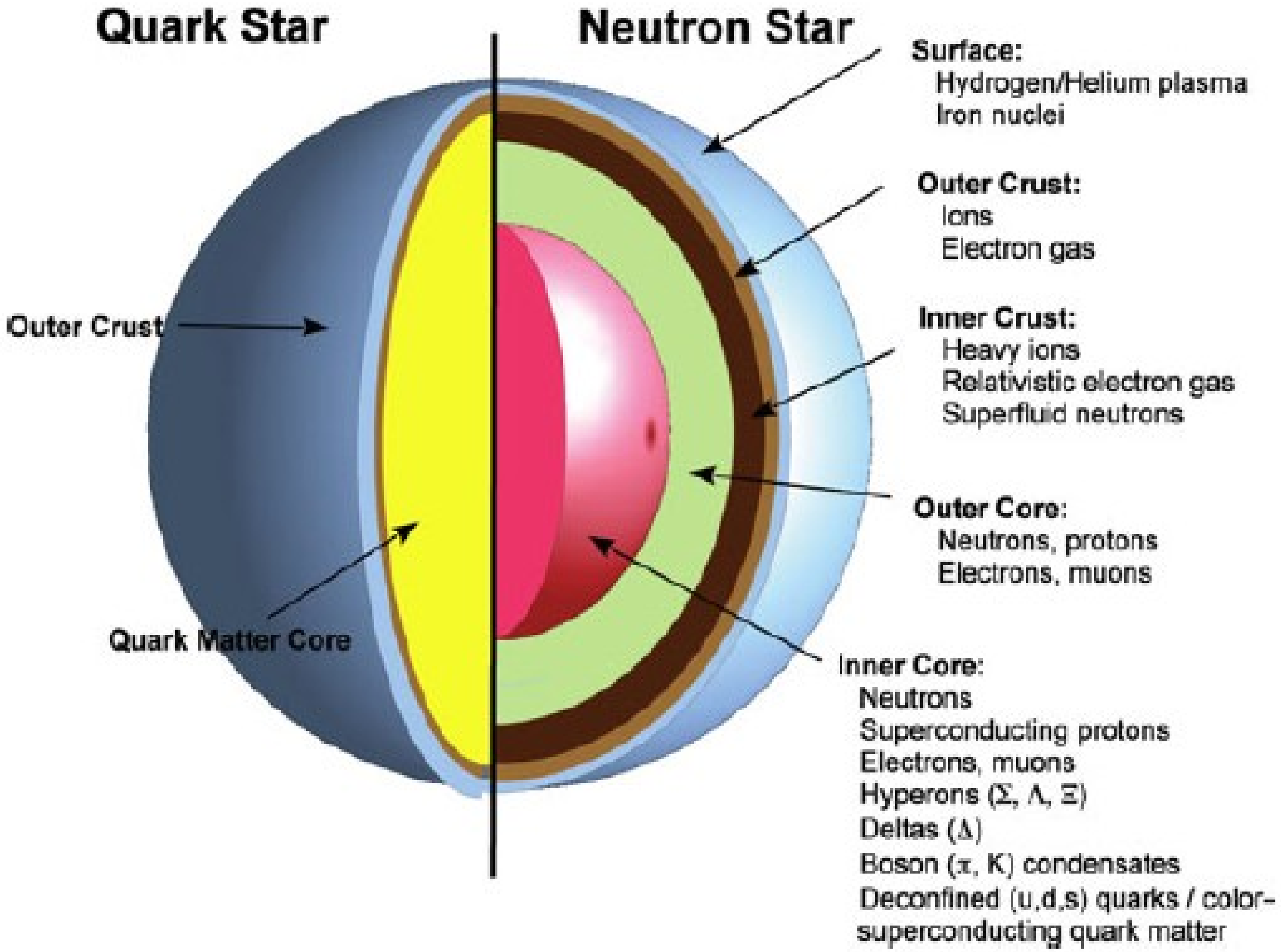


Figure 1. Schematic structures of quark stars and neutron stars.

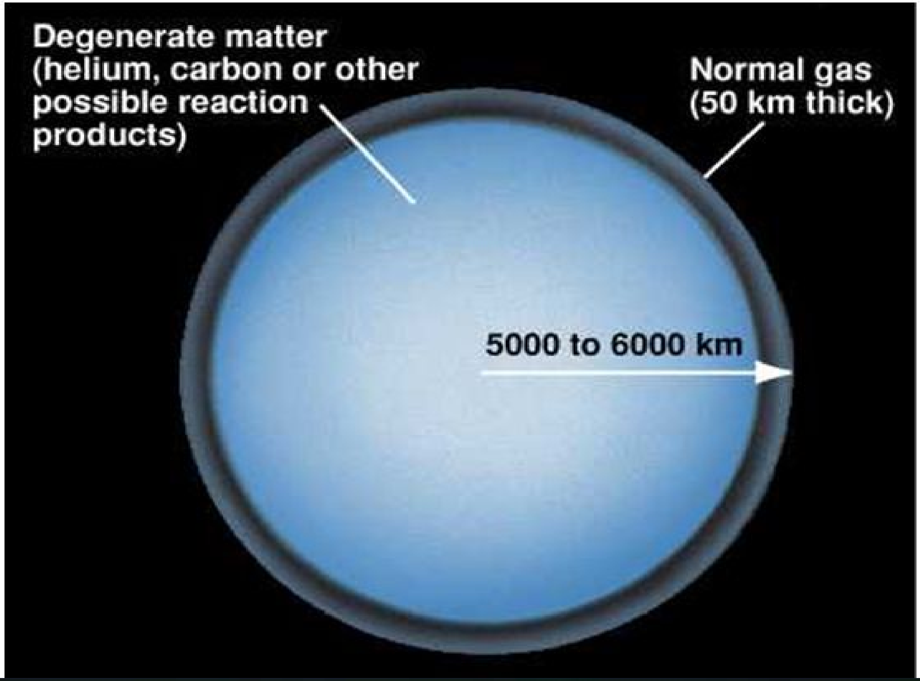
Table 1. Theoretical properties of quark stars and neutron stars compared.

Quark Stars	**Neutron Stars**
Made entirely of deconfined up, down, strange quarks, and electrons	Nucleons, hyperons, boson condensates, deconfined quarks, electrons, and muons
Quarks ought to be color superconducting	Superfluid neutrons Superconducting protons
Energy per baryon $\lesssim 930$ MeV	Energy per baryon > 930 MeV
Self-bound ($M \propto R^3$)	Bound by gravity
Maximum mass $\sim 2\,M_\odot$	Same
No minimum mass	$\sim 0.1\,M_\odot$
Radii $R \lesssim 10 - 12$ km	$R \gtrsim 10 - 12$ km
Baryon number $B \lesssim 10^{57}$	$10^{56} \lesssim B \lesssim 10^{57}$
Electric surface fields $\sim 10^{18}$ to $\sim 10^{19}$ V/cm	Absent
Can be bare (pure quark stars) or enveloped in thin nuclear crusts (mass $10^{-5}\,M_\odot$)	Not possible Always possess nuclear crusts
Density of crust is less than neutron drip i.e., posses only outer crusts	Density of crust above neutron drip i.e., posses inner and outer crusts
Form two-parameter stellar sequences	Form one-parameter stellar sequences

White Dwarfs

Composition

- Heavy nuclei are pulled below the surface, while hydrogen rises to the top, layered above the helium



Chandrasekhar limit

The relativistic expression for pressure is:

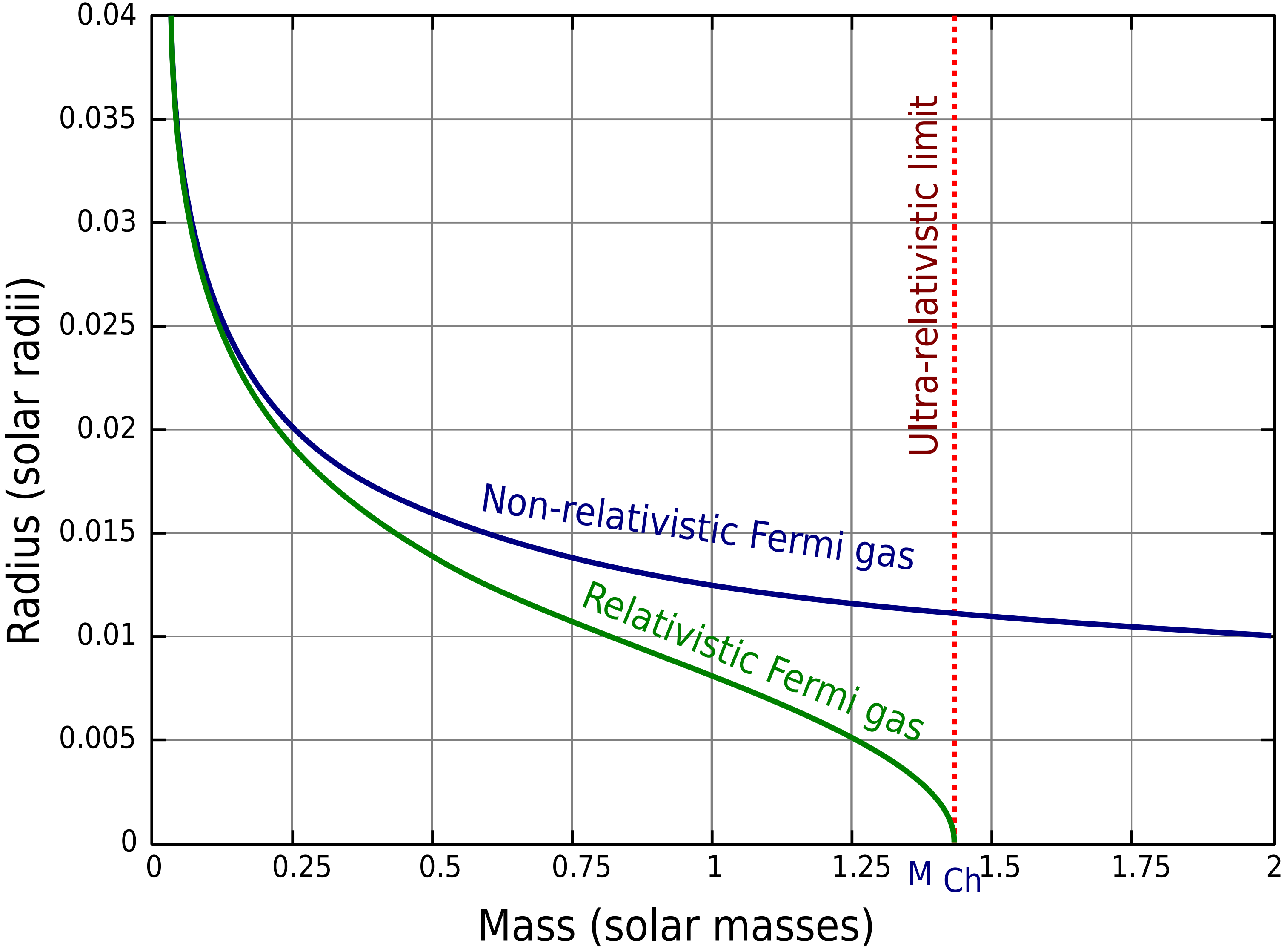
$$P = \frac{(3\pi^2)^{1/3}}{4}\hbar c\left[\left(\frac{Z}{A}\right)\frac{\rho}{m_H}\right]^{4/3}$$

This leads to the *Chandrasekhar mass limit*:

$$M_{Ch} \approx \frac{3\sqrt{2\pi}}{8}\left(\frac{\hbar c}{G}\right)^{3/2}\left[\left(\frac{Z}{A}\right)\frac{1}{m_H}\right]^2$$

(contains elements of quantum mechanics, relativity, and gravity!)

A more careful calculation shows: $M_{Ch} \approx 1.44 M_{Sun}$


Radius (solar radii)
0.04
0.035
0.03
0.025
0.02
0.015
0.01
0.005
0
0
0.25
0.5
0.75
1
1.25
M Ch
1.5
1.75
2
Mass (solar masses)
Ultra-relativistic limit
Non-relativistic Fermi gas
Relativistic Fermi gas

Scale of magnetic and electric fields to change matter equation of state and stellar mass and radius

Pressure of Magnetic ($B^2/8\pi$) and Electric fields ($E^2/8\pi$)

Quark and Neutron Star – Pressure 10^{32} - 10^{35} dynas/cm^2

B > 10^{16} G - strong effect B ~ 10^{18} G

E > 10^{16} statV/cm ~ 10^{18} V/cm – strong effect E ~ 10^{20} V/cm

White Dwarf – Pressure 10^{25} - 10^{28} dynas/cm^2

B > 10^{13} G - strong effect B ~ 10^{15} G

E > 10^{15} V/cm - strong effect E ~ 10^{17} V/cm

Critical magnetic field

$$B_{\rm crit} = m_e^2 c^3/(e\hbar) \simeq 4.4 \times 10^{13}\ {\rm G}$$

Critical Eletric field - $E_{crit} = 1.3 \times 10^{16}$ V/cm

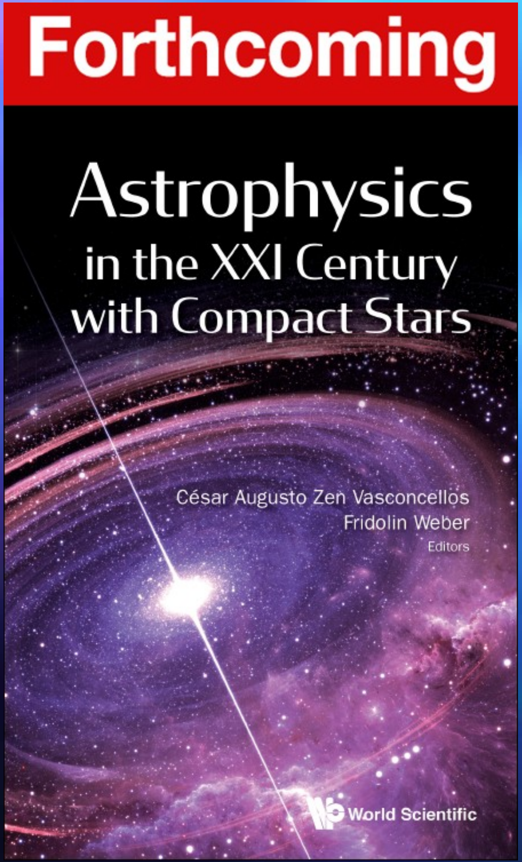
Maximum surface magnetic fields

High B pulsars B~10^{13} G ;Magnetars B~ 10^{14} - 10^{15} G ???

White Dwarfs B~10^{8} to 10^{9} G (very magnetic)

II - Strong Magnetic Fields in Neutron Stars and White Dwarfs

Magnetic White Dwarfs



Contents

Electron Gas under Strong B Fields

$$\mathcal{E}_e = \sum_{n=0}^{n_{\max}} \frac{eB}{(2\pi)^2\hbar c}\left[\mu_e\sqrt{\mu_e^2 - s_e(n)^2} + s_e(n)^2 \ln\frac{\mu_e + \sqrt{\mu_e^2 - s_e(n)^2}}{s_e(n)}\right] + \frac{B^2}{8\pi}.$$

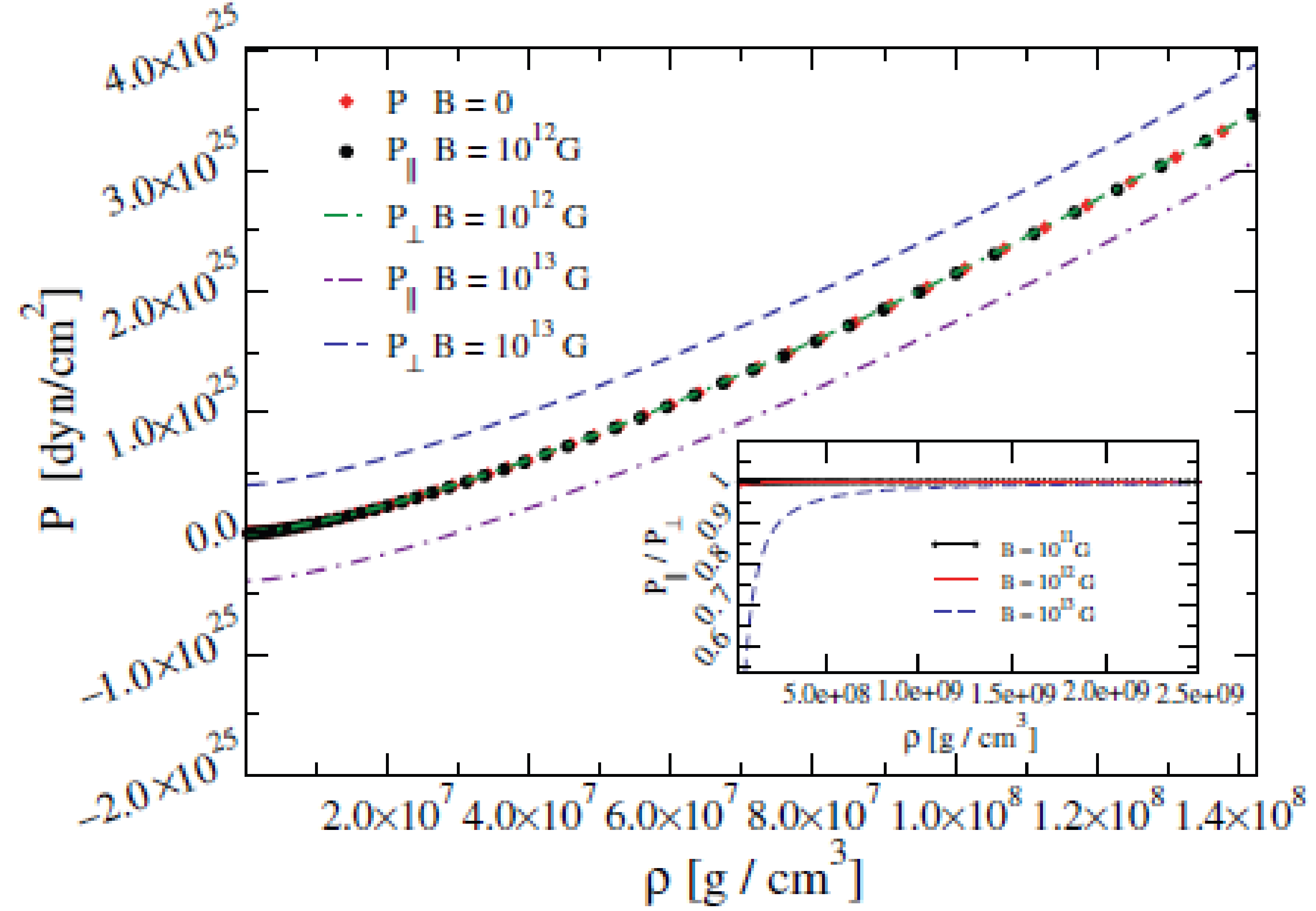
$$s_e(n) = \sqrt{m_e^2 c^4\left(1 + 2\frac{B}{B_c}n\right)},$$

$$P_\parallel = \sum_{n=0}^{n_{\max}} g_n \frac{eB}{(2\pi)^2\hbar c}\left[\mu_e\sqrt{\mu_e^2 - s_e(n)^2} - s_e(n)^2 \ln\frac{\mu_e + \sqrt{\mu_e^2 - s_e(n)^2}}{s_e(n)}\right] - \frac{B^2}{8\pi}.$$

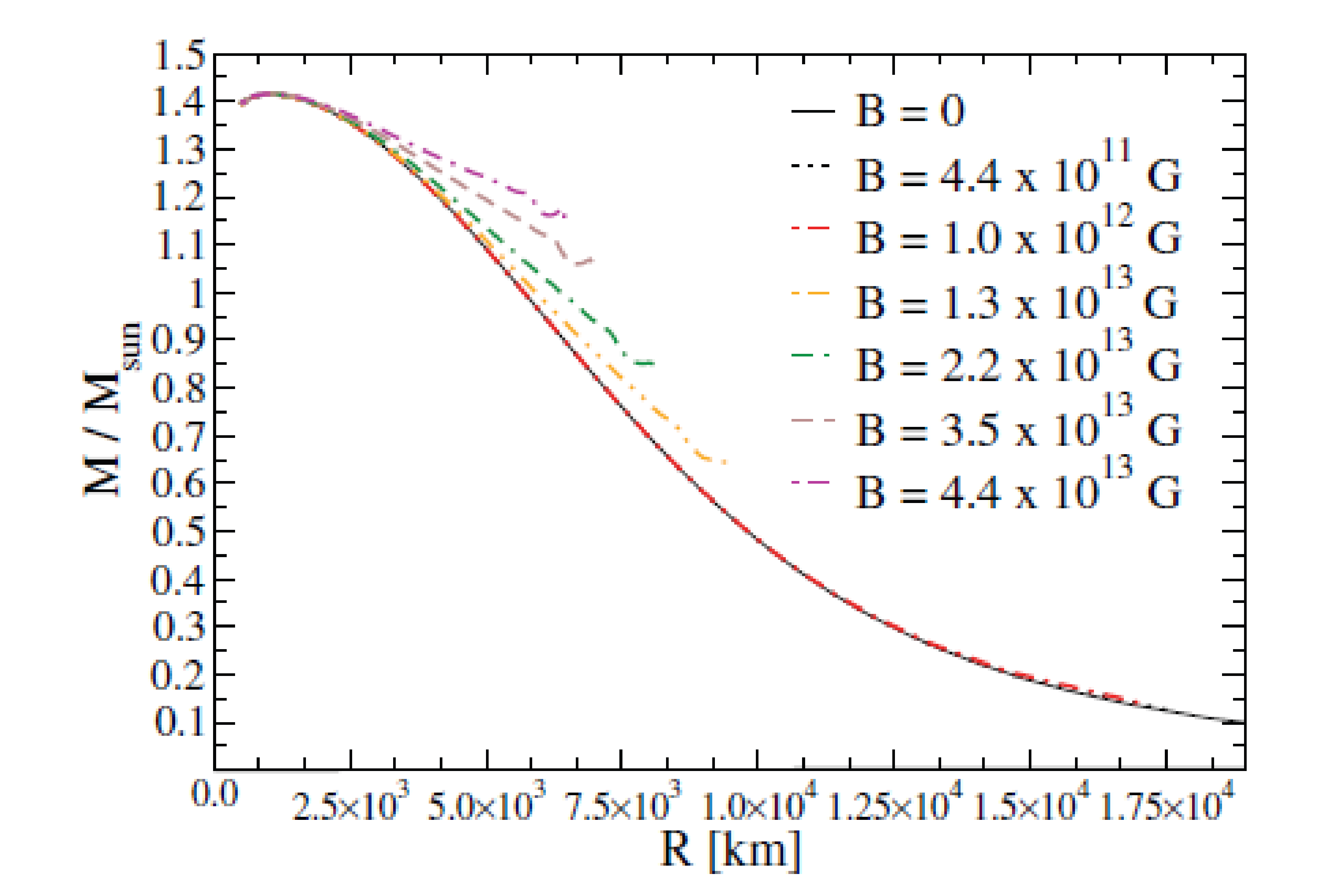
The perpendicular pressure is given by

$$P_\perp = -\Omega_e - B\mathcal{M}_e + \frac{B^2}{8\pi},$$

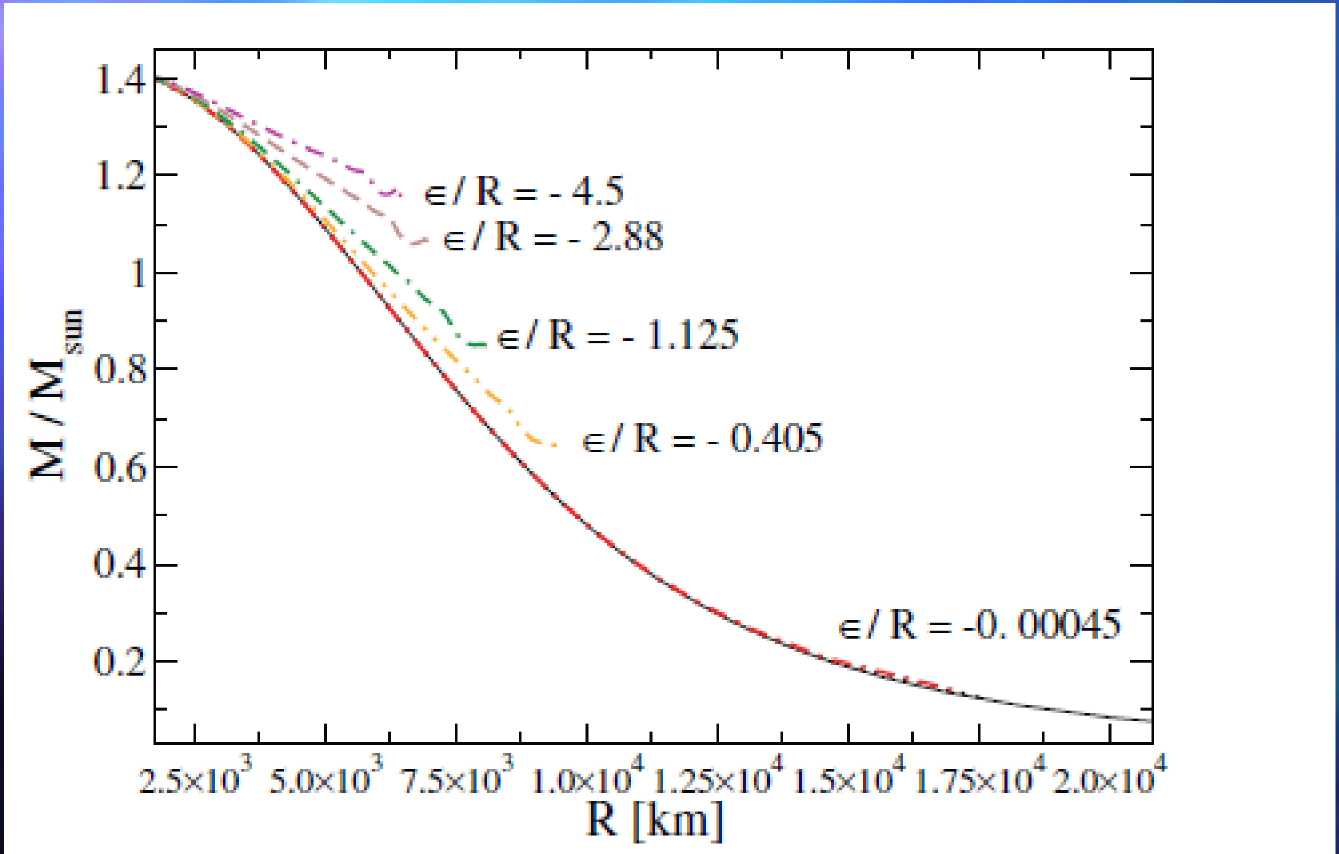
$$P_\perp = \sum_{n=0}^{n_{\max}} g_n \frac{eB}{(2\pi)^2\hbar c}\left[\mu_e\sqrt{\mu_e^2 - s_e(n)^2} - s_e(n)^2 \times \ln\frac{\mu_e + \sqrt{\mu_e^2 - s_e(n)^2}}{s_e(n)}\right] - \frac{e m_e}{4\pi^2}\left(\sum_{n=0}^{n_{\max}} g_n\left[\mu_e\sqrt{\mu_e^2 - s_e(n)^2} - \left[s_e(n)^2 + 2s_e(n)\mathcal{C}_e\right]\ln\frac{\mu_e + \sqrt{\mu_e^2 - s_e(n)^2}}{s_e(n)}\right]\right) + \frac{B^2}{8\pi},$$



P B = 0
P∥ B = 10^12 G
P⊥ B = 10^12 G
P∥ B = 10^13 G
P⊥ B = 10^13 G
P [dyn/cm^2]
ρ [g / cm^3]
4.0×10^25
3.0×10^25
2.0×10^25
1.0×10^25
0.0
-1.0×10^25
-2.0×10^25
2.0×10^7 4.0×10^7 6.0×10^7 8.0×10^7 1.0×10^8 1.2×10^8 1.4×10^8
P∥ / P⊥
B = 10^11 G
B = 10^12 G
B = 10^13 G
5.0e+08 1.0e+09 1.5e+09 2.0e+09 2.5e+09
ρ [g / cm^3]


1.5
1.4
1.3
1.2
1.1
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
M / M_sun
0.0
2.5×10^3
5.0×10^3
7.5×10^3
1.0×10^4
1.25×10^4
1.5×10^4
1.75×10^4
R [km]
B = 0
B = 4.4 x 10^11 G
B = 1.0 x 10^12 G
B = 1.3 x 10^13 G
B = 2.2 x 10^13 G
B = 3.5 x 10^13 G
B = 4.4 x 10^13 G

Deformation of the Star under Strong Magnetic Fields



$$\frac{\epsilon}{R} = \frac{R_{eq} - R_{p}}{R},$$

Pycononuclear, inverse beta decay reactions.

Maximum Mass

$$\rho_B(A,Z,B) = \frac{A}{Z}\frac{m}{\lambda_e}\frac{B^{3/2}}{\sqrt{2}\pi^2}$$

Local Stability

$$B > B^{\beta}(A,Z) \equiv \frac{1}{2}\left(\frac{\mu_e^{\beta}(A,Z)}{mc^2}\right)^2$$

$$\mu_e \geq \mu_e^{\beta}(A,Z) \equiv \Delta(A,Z-1) - \Delta(A,Z) + mc^2$$

$${}^{A}_{Z}X + e^- \rightarrow {}^{A}_{Z} Y + \nu_e$$

$$\rho_\beta > \rho_\beta^{min} = \frac{B^{\beta}(A,Z)^{3/2}}{\sqrt{2}\pi^2\lambda^3}\frac{A}{Z}m$$

$${}^{A}_{Z}X + {}^{A}_{Z}X \rightarrow {}^{2A}_{2Z} Y$$

$$\rho_{pyc} > \rho_{pyc}^{min} = \frac{B^{\beta}(2A,2Z)^{3/2}}{\sqrt{2}\pi^2\lambda^3}\frac{A}{Z}m$$

Nuclear Reactions Constraints

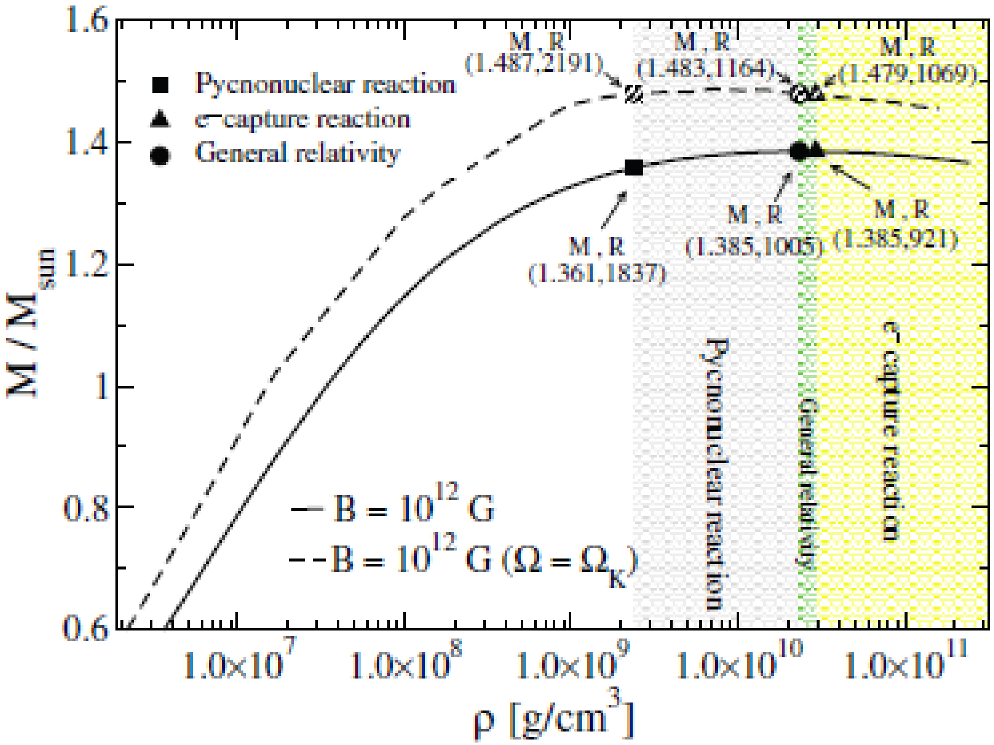


Fig. 4. Gravitational mass as a function of central mass density for a ^{12}C WD with a magnetic field of $B = 1.0 \times 10^{12}$ G. The colored regions represent areas where WD matter is subject to pycnonuclear reactions, an axisymmetric general relativistic instability, and electron capture reactions.

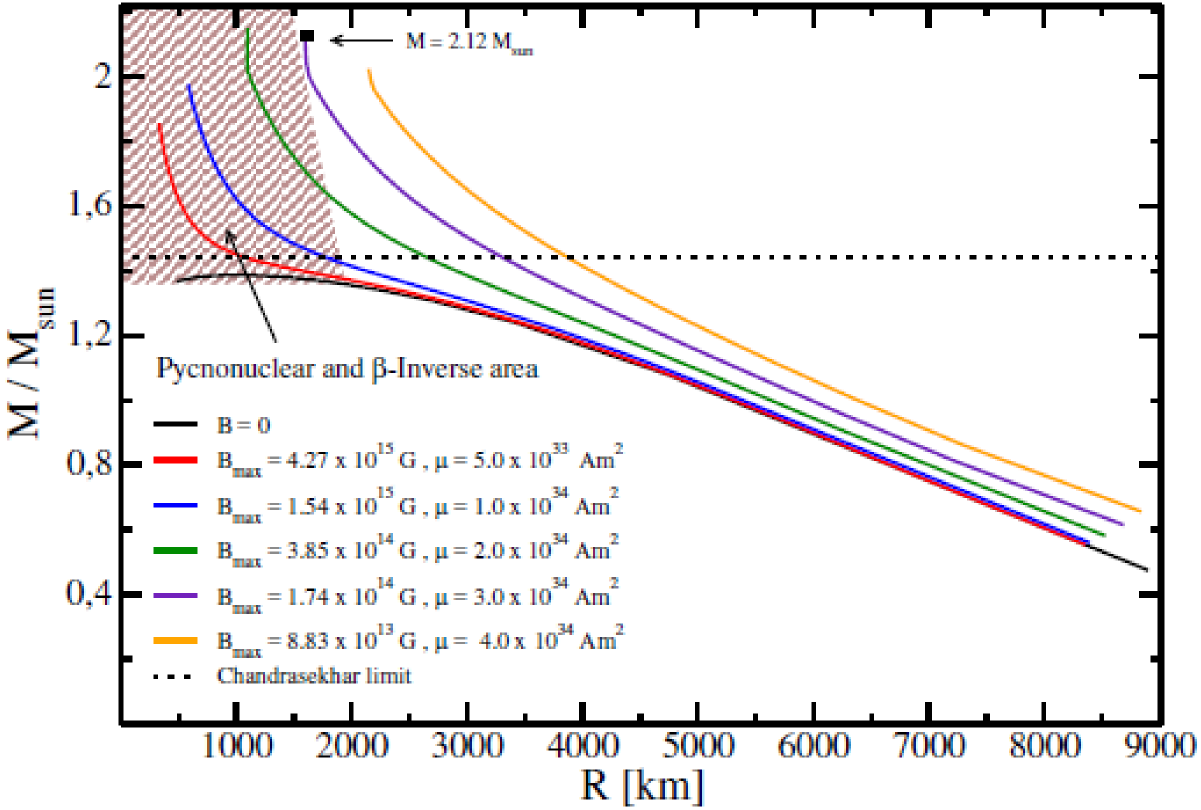


Figure : Mass-radius relationship for magnetized white dwarfs assuming different magnetic dipole moment, μ. The black lines represents the mass-radius relationship of non-magnetized white dwarfs. The horizontal line represents the Chandrasekhar mass limit for spherical stars. Also shown are the values of the central magnetic field $B_{\rm max}$ (with the corresponding magnetic dipole moment μ) reached at the center of maximum mass stars (end point of the curve with fixed μ). White dwarfs located in the yellow region are subject to pyco-nuclear or inverse β-decay reactions.

Heavy Magnetic Neutron Stars

Ishfaq A. Rather, Usuf Rahaman, V. Dexheimer,

A. A. Usmani, and S. K. Patra, ApJ 917, 1 (2021)

Table 3
Magnetic Field at Low Densities B_s (Corresponding to Stellar Surfaces) and at High Values of Densities B_c Calculated for the DD-MEX EoS With and Without Hyperons

	Nucleonic Star		Hyperonic Star	
μ (Am^2)	B_s (G)	B_c (G)	B_s (G)	B_c (G)
5×10^{30}	1.01×10^{15}	2.59×10^{16}	6.65×10^{15}	1.96×10^{16}
5×10^{31}	8.98×10^{16}	2.28×10^{17}	5.83×10^{16}	1.89×10^{17}
10^{32}	1.79×10^{17}	4.55×10^{17}	1.12×10^{17}	3.77×10^{17}

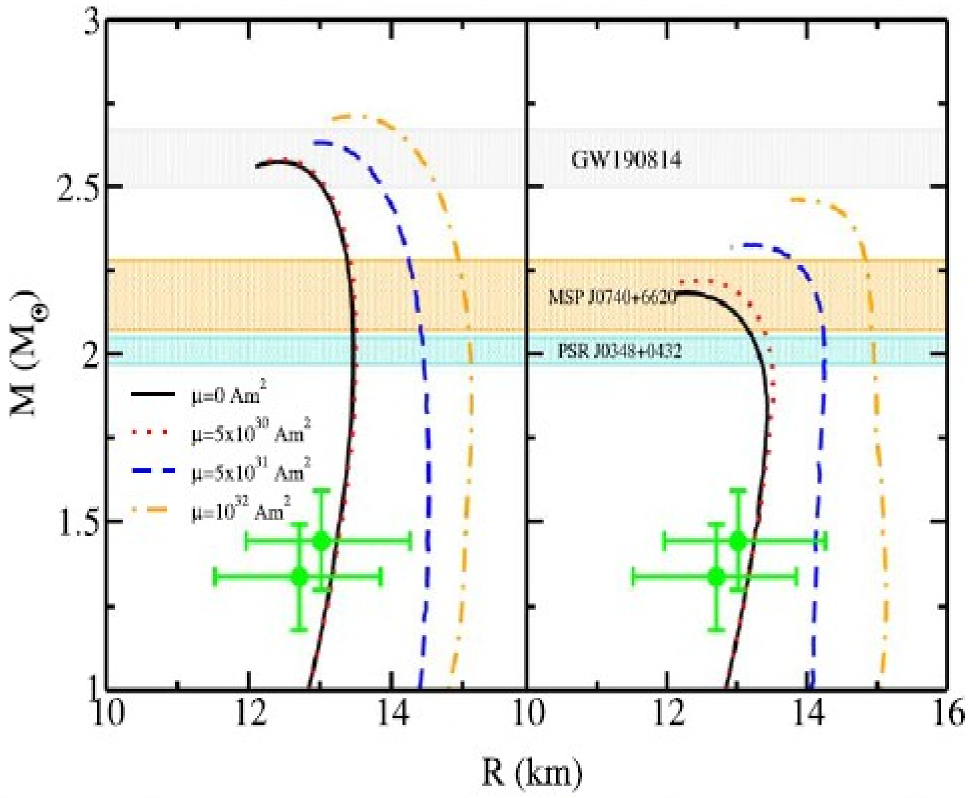


Figure 7. Relation between mass and circumferential radius for an NS without magnetic field and with magnetic field effects considering different magnetic dipole moments without hyperons (left panel) and with hyperons (right panel) using the DD-MEX parameter set. The colored areas show the recent constraints inferred from GW 190814, MSP J0740+6620, and PSR J0348+0432 (Antoniadis et al. 2013; Cromartie & Fonseca 2019; Abbott et al. 2020a). The constraints on the mass–radius limits inferred from NICER (Miller et al. 2019; Riley et al. 2019) are also shown.

III - Strong Electric Fields in Quark Stars

Electrically charged strange quark stars Rodrigo Picanço Negreiros, Fridolin Weber, Manuel Malheiro, and Vladimir Usov Phys. Rev. D 80, 083006 (2009)

$$\rho_{ch}(r) = \kappa \exp\left(-\left((r - r_g)/b\right)^2\right),$$

$$4\pi \int_{-\infty}^{+\infty} \rho_{ch}(r) r^2 dr = \sigma ,$$

$$8\pi\kappa = \sigma \left(\sqrt{\pi}b^3/4 + r_g b^2 + \sqrt{\pi} r_g^2 b/2\right)^{-1}$$

$$\frac{dQ}{dr} = \frac{r^2\sigma \exp\left(-((r - r_g)/b)^2\right) \exp(\Lambda/2)}{2(\sqrt{\pi}b^3/4 + r_g b^2 + \sqrt{\pi} r_g^2 b/2)}$$

Field Equation of Relativistic Charged Stars

The energy-momentum tensor for a spherically symetric charged Neutron Star is:

$$T^{\mu}_{\upsilon} = \left(P + \rho c^2\right) u^{\mu} u_{\nu} + P \;\; \delta^{\mu}_{\upsilon} + \frac{1}{4\pi}\left(F^{\mu\;\alpha} F_{\alpha\;\upsilon} - \frac{1}{4}\delta^{\mu}_{\upsilon} \;\; F_{\alpha\;\beta} F^{\alpha\;\beta}\right)$$

The Field equations are the following:

$$e^{-\lambda}\left(\frac{1}{r}\frac{d\nu}{dr} + \frac{1}{r^2}\right) - \frac{1}{r^2} = \frac{8\pi G}{c^4}\left(P + \frac{E^2}{8\pi}\right)$$

$$e^{-\lambda}\left(\frac{1}{r}\frac{d\lambda}{dr} - \frac{1}{r^2}\right) + \frac{1}{r^2} = \frac{8\pi G}{c^4}\left(\rho c^2 - \frac{E^2}{8\pi}\right)$$

The Bekenstein Equation (1971)

The electric charge profile we introduce is radially symmetric, and for this reason the star remains spherically symmetric and follows Schawarzschild-like coordinates, given by

$$ds^2 = -e^{\nu}dt^2 + e^{\lambda}dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2, \qquad (5)$$

being ν, and λ only dependent on the radius. The inclusion of the electromagnetic tensor in the stress-momentum tensor leads to the Maxwell-Einstein equations that describe the stellar exterior structure [41]

$$\frac{dq}{dr} = 4\pi\rho_e r^2 e^{\lambda/2}, \qquad (6)$$

$$\frac{dm}{dr} = 4\pi\varepsilon r^2 + \frac{q}{r}\frac{dq}{dr}, \qquad (7)$$

$$\frac{dP}{dr} = -(P+\varepsilon)\left[4\pi r P + \frac{m}{r^2} - \frac{q^2}{r^3}\right]e^{\lambda} + \frac{q}{4\pi r^4}\frac{dq}{dr}, \qquad (8)$$

$$\frac{d\nu}{dr} = -\frac{2}{(P+\varepsilon)}\left[\frac{dP}{dr} - \frac{q}{4\pi r^4}\frac{dq}{dr}\right], \qquad (9)$$

being q the local charge, and m the mass. The potential metric e^{λ} is described as

$$e^{\lambda} = \left[1 - \frac{2m}{r} + \frac{q^2}{r^2}\right]^{-1}. \qquad (10)$$

Pressure and Electric Field in the Layer

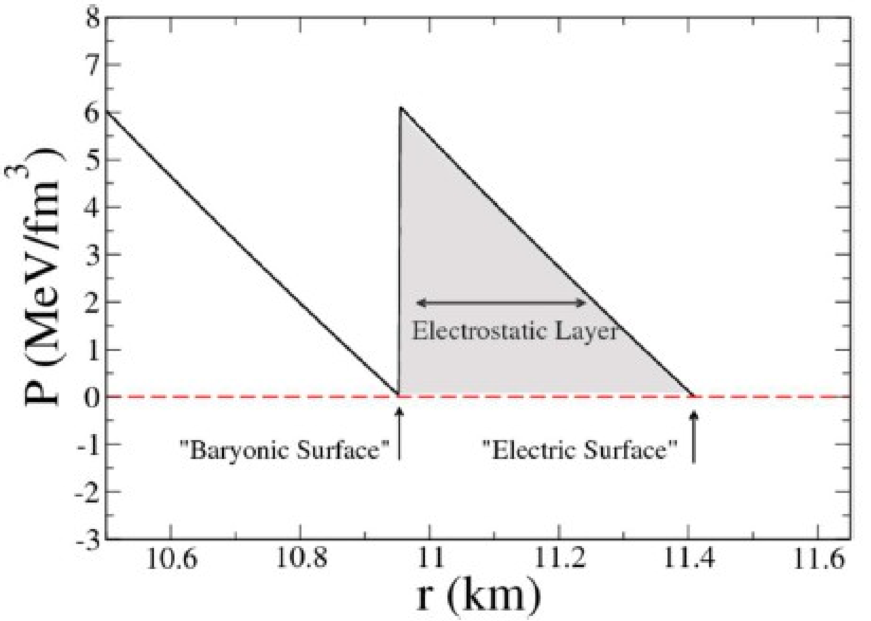


FIG. 3: (Color Online) Pressure profile for maximum-mass star for the σ = 1000 sequence. We show the definitions of "Baryonic Pressure" (where the baryon pressure goes to zero), "Electric Pressure" where the total pressure of the star vanishes, and the electrostatic layer between them.

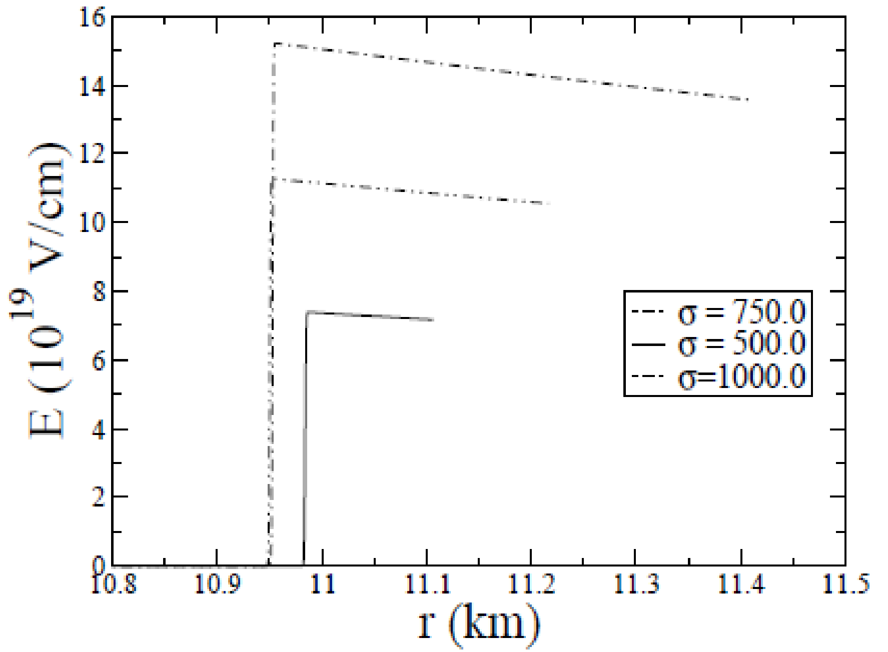


FIG. 4: Electric field profile inside maximum-mass stars (plotted in the relevant region).

Mass, Charge and Surface Electric Field

TABLE I: Properties of electrically charged maximum-mass strange quark stars. The quantities R and M denote their radii and gravitational masses, respectively. The stars carry given electric charges, Q, which give rise to electric stellar surface fields E.

σ	R (km)	M ($M_\odot$)	Q ($\times 10^{17} C$)	E (10^{19} V/cm)
0	10.99	2.02	0	0
500	11.1	2.07	989	7.1
750	11.2	2.15	1486	10.5
1000	11.4	2.25	1982	13.5

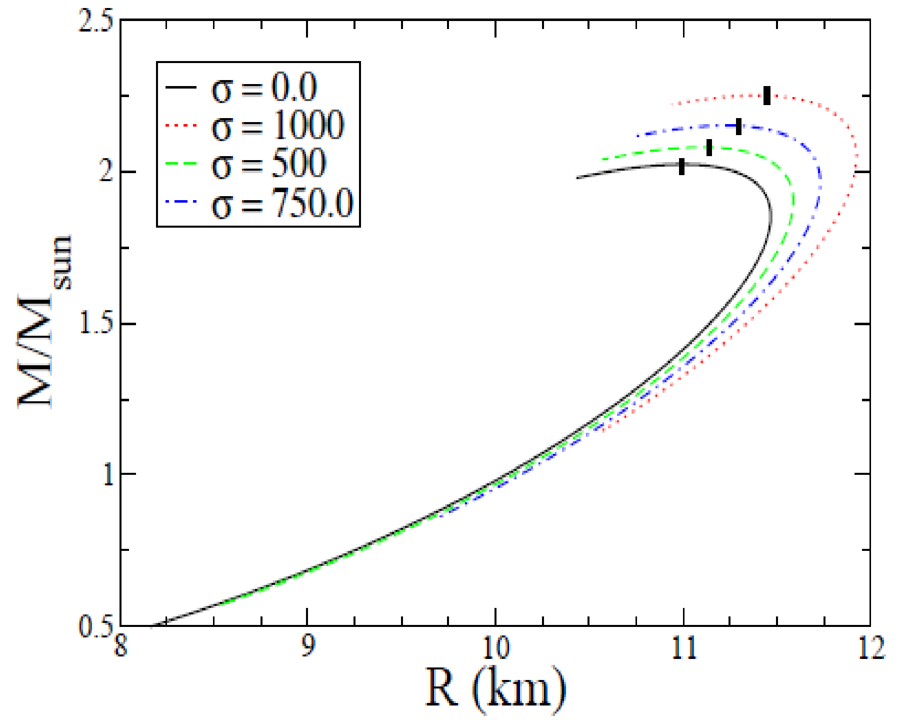


FIG. 1: (Color Online) Mass-radius relationship of electrically charged strange stars. Tick marks denote the maximum-mass star of each sequence, whose properties are given in Table I.

Properties of bare strange stars associated with surface electric fields

Rodrigo Picanço Negreiros, Igor N. Mishustin, Stefan Schramm, and Fridolin Weber Phys. Rev. D 82, 103010 (2010)

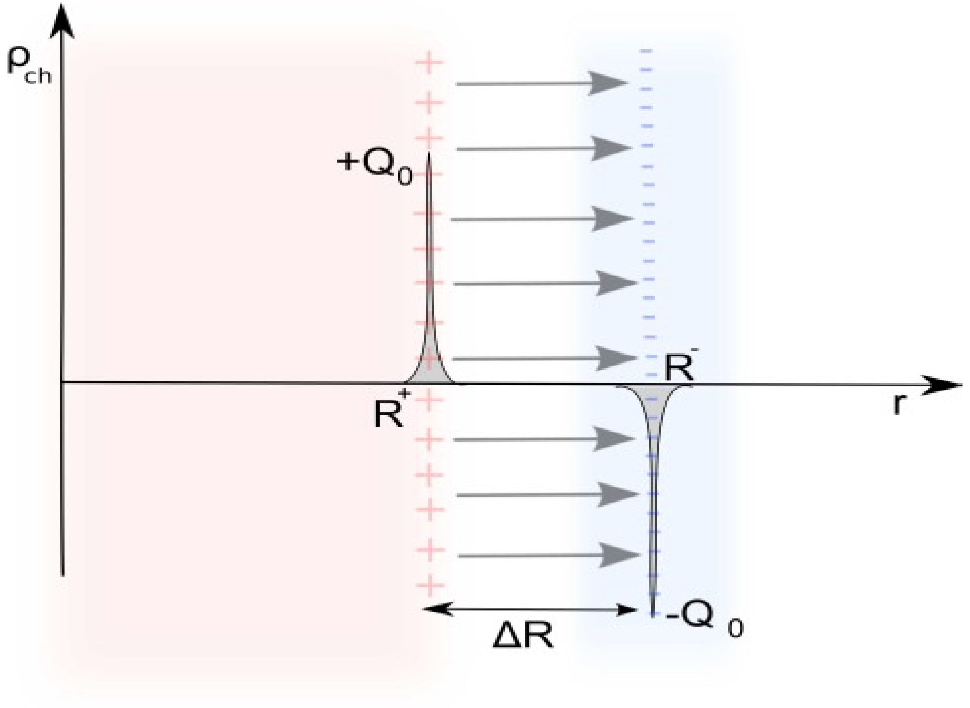


FIG. 1. (color online) Schematic representation of the electric charge distribution on the surface of a bare strange star. The core surface (R^+) becomes positively charged as the electrons (R^-) extend beyond the star's surface, giving rise to an electric dipole layer of width $\sim 10^3$ fm [1–3, 9, 18].

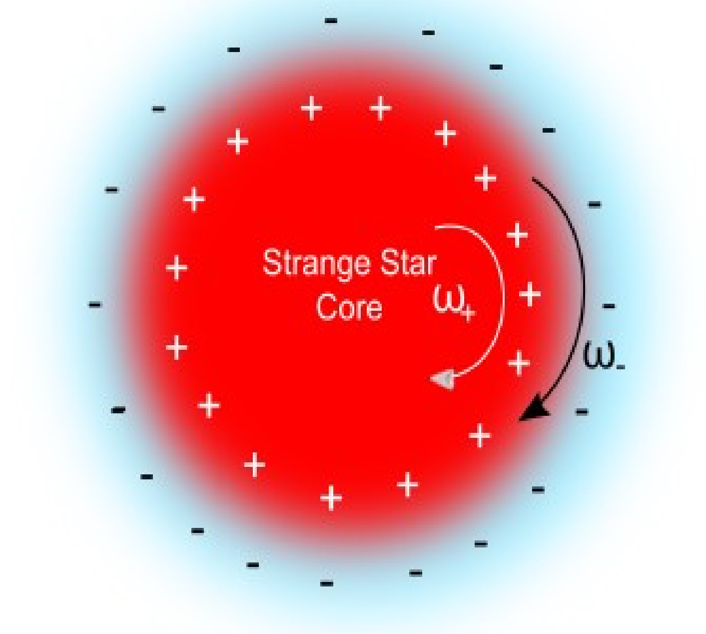


FIG. 3. (color online) Schematic illustration of the formation of electric currents at the surface of a rotating strange star. ω_+ and ω_- are the frequencies at which the core and electron layer are rotating, respectively.

Electrostatic contribution to the star mass

$$Q(r) = \begin{cases} +Q_0 \text{ for } R^+ \le r \le R^- \, , \\ 0 \quad \text{elsewhere} \, , \end{cases}$$

$$M_{\mathcal{E}} = \frac{Q_0^2}{2}\int_{R_+}^{R^-}\frac{dr}{r^2} = \frac{Q_0^2}{2}\frac{\Delta R}{R^+ \, R^-} \, ,$$

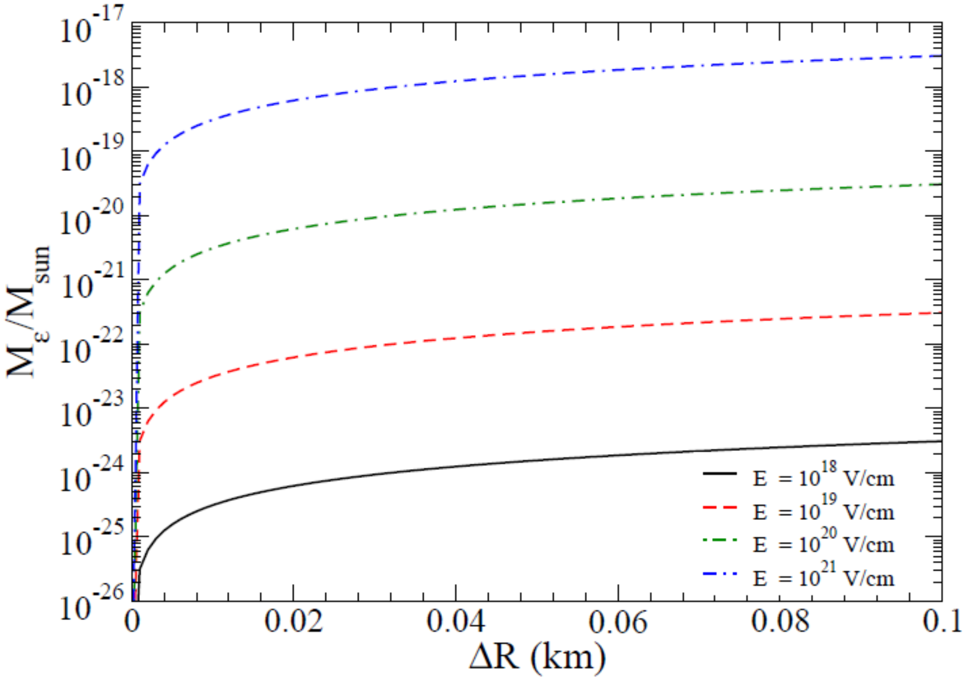


FIG. 2. (color online) Increase of gravitational mass due to electric dipole energy, as a function of dipole width. The calculations are performed for a strange star with radius of 10 km and for the surface electric fields indicated.

CHEERS - ITA

COSMOLOGY & HIGH ENERGY EMISSION FROM RELATIVISTIC STARS

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Prof. Jaziel
Goulart
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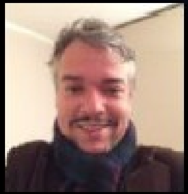

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Lapola
PhD student -

Valéria Vale
Master student -
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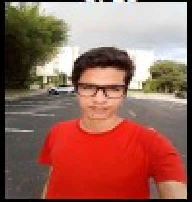
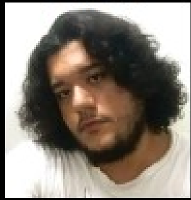


Marcos Cunha
Master student -
ITA



Pedro Ricardo
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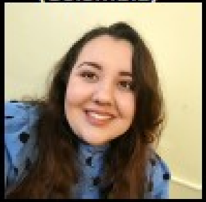
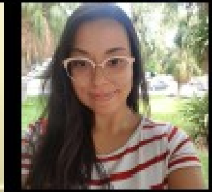
ITA



Sarah Borges
PhD student - UWM
(USA)



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