Electric and magnetic field effects, including temperature, in a scalar self-interacting $\lambda\varphi^4$theory

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This talk is based on the following articles:

1) M. Loewe, D. Valenzuela, and R. Zamora; “Catalysis and inverse electric catalysis in a scalar theory”, Physical Review D 105, 036017 (2022)
2) M. Loewe and R. Zamora; “Renormalons in a scalar self-interacting theory: Thermal, thermomagnetic and thermoelectric corrections for all values of temperature”, Physical Review D 105, 076011 (2022)
3) M. Loewe, D. Valenzuela, R. Zamora, arXiv, 2207.12387 [hep-ph]

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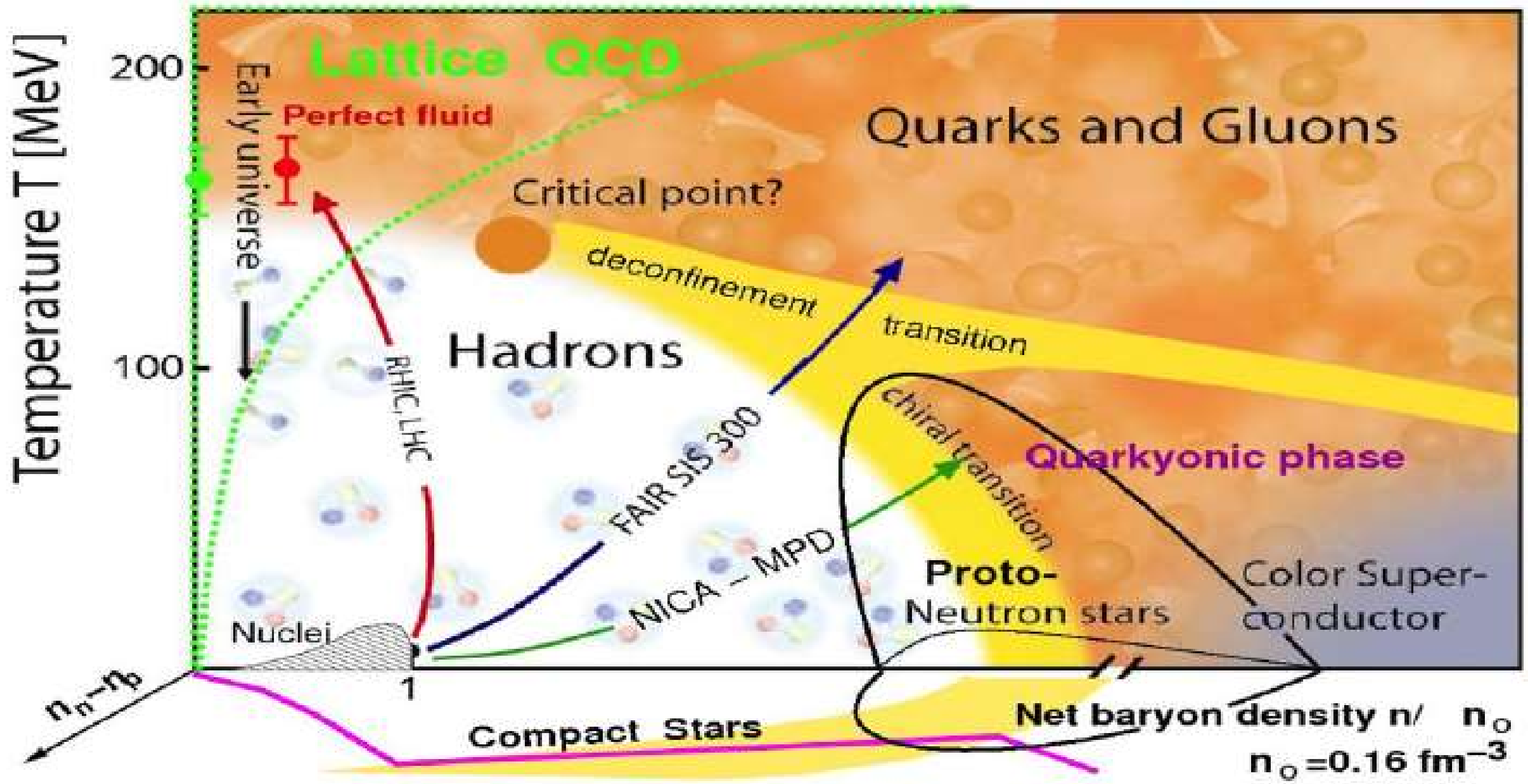
Relativistic heavy Ion collision experiments (RHIC, LHC, FAIR and NICA in the near future) open a window to explore initial stages of the universe.

A high temperature regime, huge magnetic fields, together with density effects, affect in a dramatic way the physics of strong interactions.

Several phase transitions occur: Deconfinement, Chiral Restoration, transition to a Quarkyonic phase

QCD phase diagram



Magnetic Fields im perpheral heavy ion collisions

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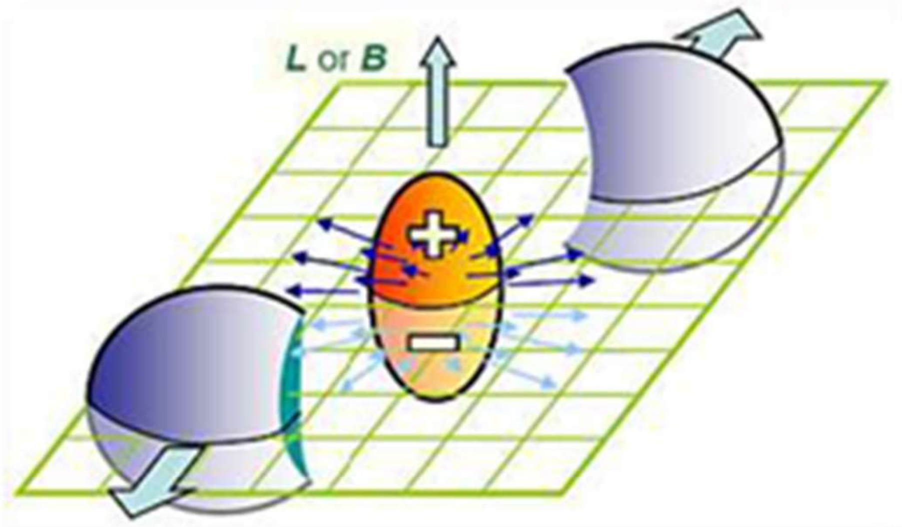
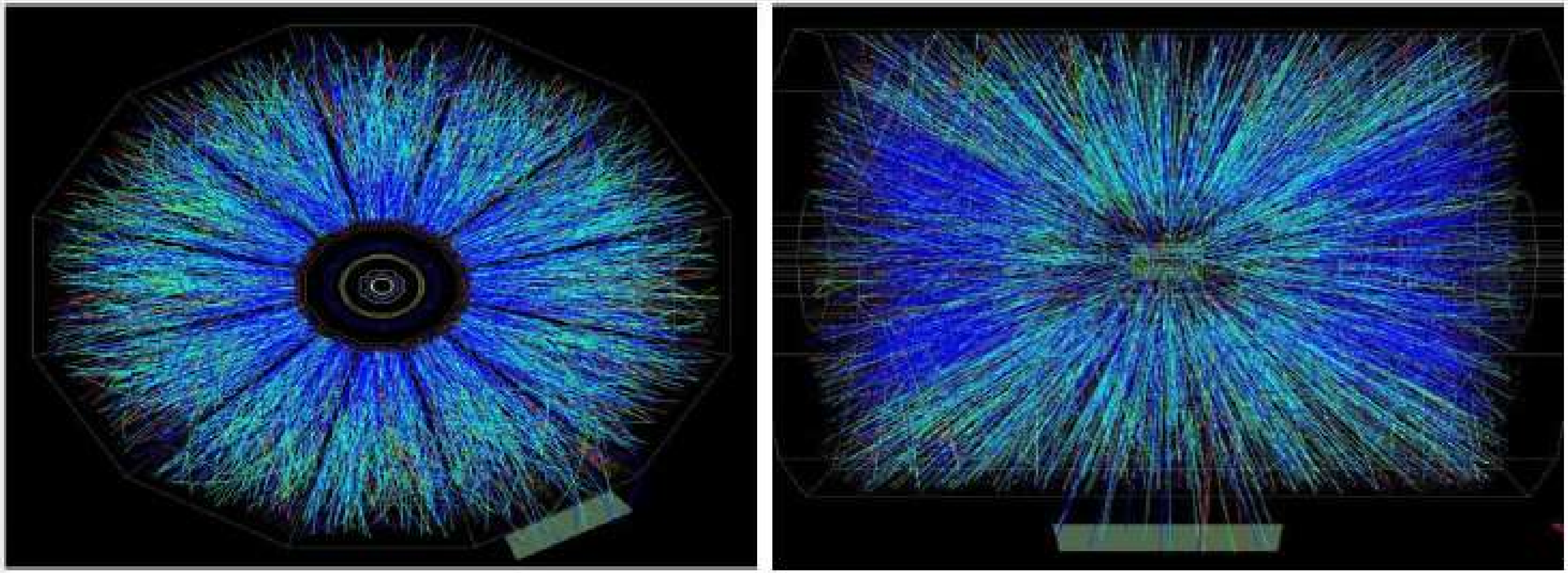
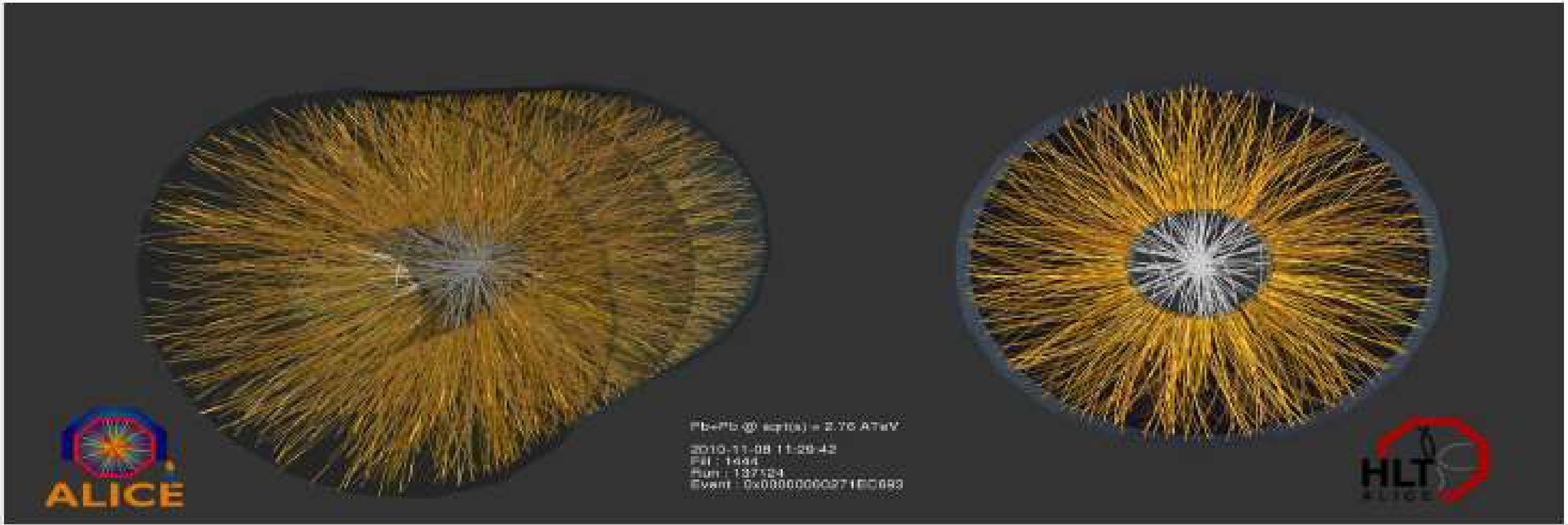
Peripheral Heavy ion collisions



Some experimental signals



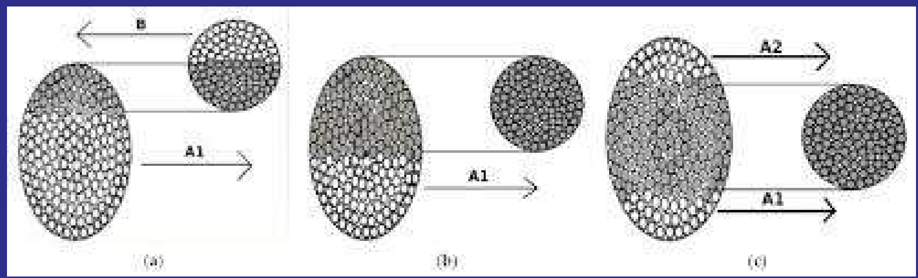
RHIC



LHC

Today I want to concéntrate on a different scenario where also huge electric fields appear: Asymmetric peripheral relativistic heavy ion collisions as, for example, Cu+Au, Cu+Pb collisions.

Due to the imbalance in the number of charges in each nuclei a strong electric field, $\perp$ to B, appears



Here we explore the phase diagram associated to symmetry breaking of a scalar theory in the presence of temperature and an external electric field. TOY MODEL!

For this we need the effective potential. So we start with the propagator for a charged scalar particle moving in the presence of a constant electric field

$$D(P,F) = \int_0^\infty ds e^{-m^2 s} \frac{e^{-sP_\mu \left(\frac{\tan Z}{Z}\right)^{\mu\nu} P_\nu}}{\sqrt{\det[\cos Z]}}$$

with

$$Z^{\mu\nu} = qF^{\mu\nu} s$$

A. Ahmad et al, Nucl. Phys.
B. B919, 9 (2017)

We take the electric field poynting into the z-axis

$$F^{\mu\nu} = iEf^{\mu\nu}$$

$$f^{\mu\nu} = \begin{bmatrix} 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

In this way we find

$$D(p) = \int_0^\infty ds \frac{e^{-s\left(\frac{\tanh(qiEs)}{qiEs} p_\parallel^2 + p_\perp^2 + m^2\right)}}{\cosh(qiEs)}$$

$p_\parallel$ and $p_\perp$ refer to $(p_4, 0, 0, p_3)$ and $(0, p_1, p_2, 0)$

In the weak electric field approximation

$$D(p) \approx \int_0^\infty dse^{-s(p^2+m^2)} \left[\frac{-(qEs)^2}{6} \left(-3 + 2sp_\parallel^2\right) + \frac{(qEs)^4}{360} \left(4sp_\parallel^2 [5sp_\parallel^2 - 27] + 75\right) - \frac{(qEs)^6}{45360} \times \left(2sp_\parallel^2 [14p_\parallel^2 s(10p_\parallel^2 s - 117) + 4311] - 3843\right) \ldots \right]$$

In the strong field region

$$D(p) \approx 2 \frac{e^{i \frac{p_\parallel^2}{qE}}}{p_\perp^2 + iqE + m^2}$$

Effective potential and symmetry restoration:

Our Model

$$\mathcal{L} = (D_\mu \phi)^\dagger D^\mu \phi + \mu^2 \phi^\dagger \phi - \frac{\lambda}{4}(\phi^\dagger \phi)^2$$

$$D_\mu = \partial_\mu + iqA_\mu$$

We take μ^2 > 0,
Λ > 0

We consider

$$A_\mu = -\delta_{\mu 0} x_3 E$$

Following the usual procedure:

$$\phi(x) = \frac{1}{\sqrt{2}}[\sigma(x) + i\chi(x)],$$

$$\phi^\dagger(x) = \frac{1}{\sqrt{2}}[\sigma(x) - i\chi(x)].$$

The σ field developes a vacuum expectation value

$$\sigma \to \sigma + v$$

After this shift we get

$$\mathcal{L} = -\frac{1}{2}[\sigma(\partial_\mu + iqA_\mu)^2\sigma] - \frac{1}{2}\left(\frac{3\lambda v^2}{4} - \mu^2\right)\sigma^2 - \frac{1}{2}[\chi(\partial_\mu + iqA_\mu)^2\chi] - \frac{1}{2}\left(\frac{\lambda v^2}{4} - \mu^2\right)\chi^2 + \frac{\mu^2}{2}v^2 - \frac{\lambda}{16}v^4 + \mathcal{L}_I,$$

with

$$\mathcal{L}_I = -\frac{\lambda}{16}(\sigma^4 + \chi^4 + 2\sigma^2\chi^2)$$

We will ignore the issue of the mass term generated for A^μ field, as well as issues regarding renormalization after symmetry breaking. From the previous Lagrangian we identify

$$m_\sigma^2 = \frac{3}{4}\lambda v^2 - \mu^2.$$
$$m_\chi^2 = \frac{1}{4}\lambda v^2 - \mu^2.$$

And the tree-level potential

$$V^{(\text{tree})} = -\frac{1}{2}\mu^2 v^2 + \frac{1}{16}\lambda^2 v^4.$$

Our goal is to calculate the one-loop effective potential, including temperature and electric field corrections

$$V^{(1-\mathrm{loop})} = \sum_{i=\sigma,\chi} \left(\frac{T}{2} \sum_n \int \frac{d^3p}{(2\pi)^3} \ln[D(\omega_n, p, m_i)]^{-1} \right),$$
$$= \sum_{i=\sigma,\chi} \left(\frac{T}{2} \sum_n \int dm_i^2 \int \frac{d^3p}{(2\pi)^3} D(\omega_n, p, m_i) \right)$$

Finite temperature effects will be handled in the imaginary time formalism

$$p_4 \to \omega_n = 2\pi n T, \quad n \in Z,$$

$$\int \frac{d^4p}{(2\pi)^4} f(p) \to T \sum_{n \in Z} \int \frac{d^3k}{(2\pi)^3} f(\omega_n, p)$$

It is possible to calculate this one-loop effective potential analitically, in the weak and strong field cases, for the whole range of temperature. The details are too technical to be presented here. We find

$$V_{\rm weak} = V^{(\rm tree)} + V^{(\rm 1\text{-}loop)}_{\rm weak} = -\frac{1}{2}\mu^2 v^2 + \frac{1}{16}\lambda^2 v^4$$
$$+ \sum_{i=\sigma,\chi}\left(-\frac{m_i^4}{64\pi^2}\left(\ln\left(\frac{\tilde{\mu}^2}{m_i^2}\right)+\frac{3}{2}\right)\right.$$
$$- \frac{m_i^2 T^2}{2\pi^2}\sum_{n=1}^{\infty}\frac{K_2(nm_i/T)}{n^2} - \frac{(qE)^2}{96\pi^2}\left[\ln\left(\frac{\tilde{\mu}^2}{m_i^2}\right)\right.$$
$$\left.\left.+1+\sum_{n=1}^{\infty}2K_0(nm_i/T)+\frac{2m_i}{T}\sum_{n=1}^{\infty}nK_1(nm_i/T)\right]\right)$$

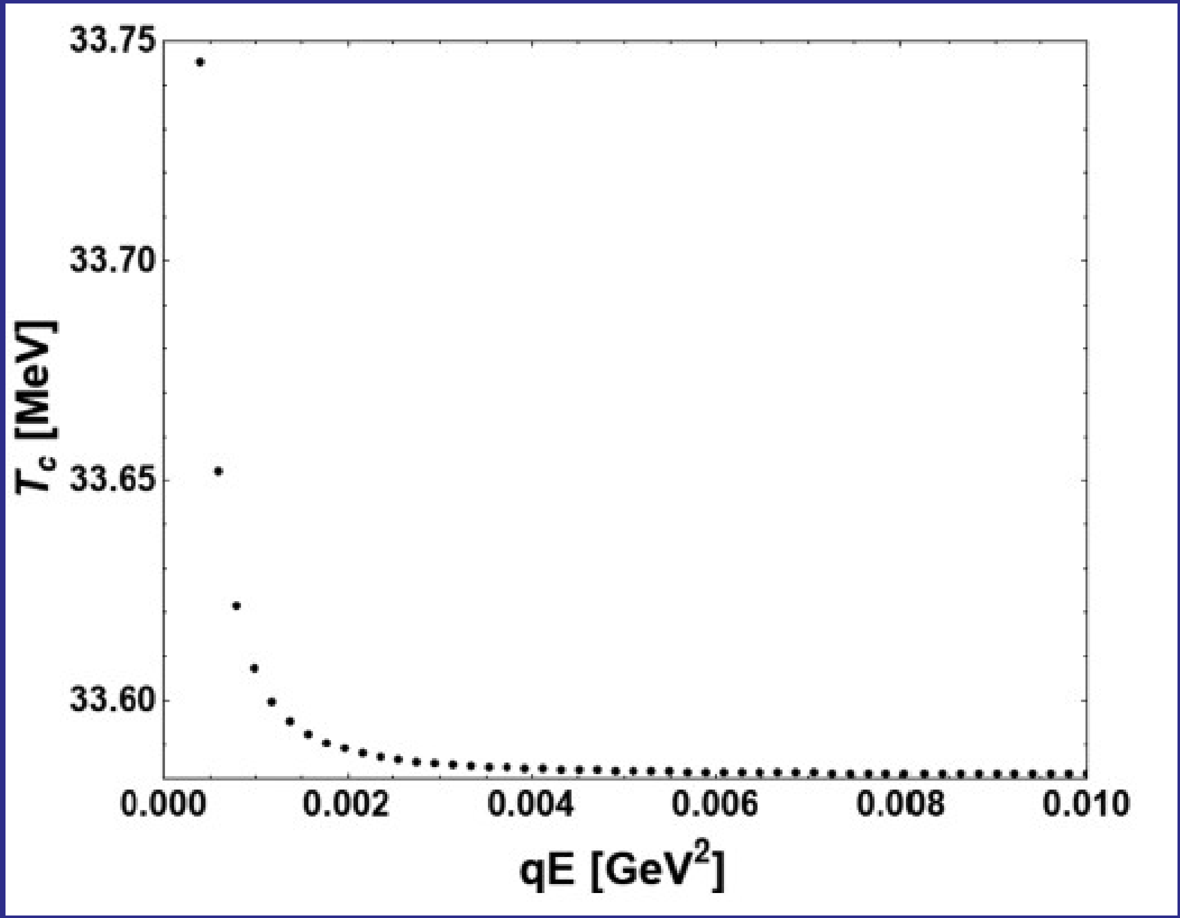
$\overline{\rm MS}$ scheme

$\tilde{\mu}$ is the ultraviolet renormalization scale

To find the critical temperature T_c such that for $T > T_c$ the symmetry is restored, we explore the evolution of the effective potential as function of the vacuum expectation value v for different values of T. When the first and second derivative vanish (in fact, all derivatives) we find T_c.

It is a second order phase transition

Electric inverse catalysis



Let us consider now the situation where we have both type of fields: Electric and Magnetic fields, perpendicular to each other. We assume (as usual) constant fields.

Once again, we start from

$$D(p) = \int_0^\infty dt\, e^{-m^2 t} \frac{e^{t p_\mu \left(\frac{\tan(Z)}{Z}\right)^{\mu\nu} p_\nu}}{\sqrt{\det(\cos(Z))}}$$

$$Z^{\mu\nu} \equiv qF^{\mu\nu} t.$$

$$F^{\mu\nu} = \begin{bmatrix} 0 & B_0 & 0 & iE_0 \\ -B_0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -iE_0 & 0 & 0 & 0 \end{bmatrix}$$

We find

$$D(p) = \int_0^\infty dt e^{-t\left(p_3^2 + \frac{1}{-E_0^2+B_0^2}(B_0 p_4 - iE_0 p_2)^2 + m^2\right)} \times \frac{e^{\frac{-\tanh\left(q\sqrt{-E_0^2+B_0^2}t\right)}{q\sqrt{-E_0^2+B_0^2}}\left(p_1^2 + \frac{1}{-E_0^2+B_0^2}(B_0 p_2 + iE_0 p_4)^2\right)}}{\cosh\left(q\sqrt{-E_0^2+B_0^2}t\right)}$$

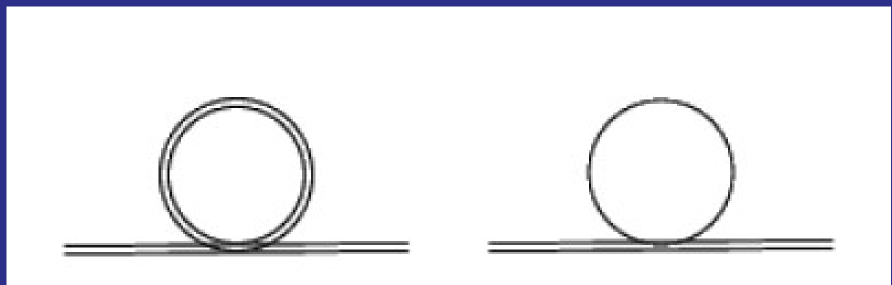
In the region of weak field intensities

$$D(p) \approx \frac{1}{p^2+m^2} + \frac{4ip_4p_2(qB)(qE)}{(p^2+m^2)^4} + \frac{(qE)^2(m^2+p^2-2(p_4^2+p_1^2))}{(p^2+m^2)^4} - \frac{(qB)^2(m^2+p^2-2p_\perp^2)}{(p^2+m^2)^4},$$

We will analize two different aspects: mass correction and effective potential (including the ring resumation)

For the mass of the σ meson we have

$$m_\sigma(T, B, E) = m_0 + \Pi_\sigma(T, B, E).$$

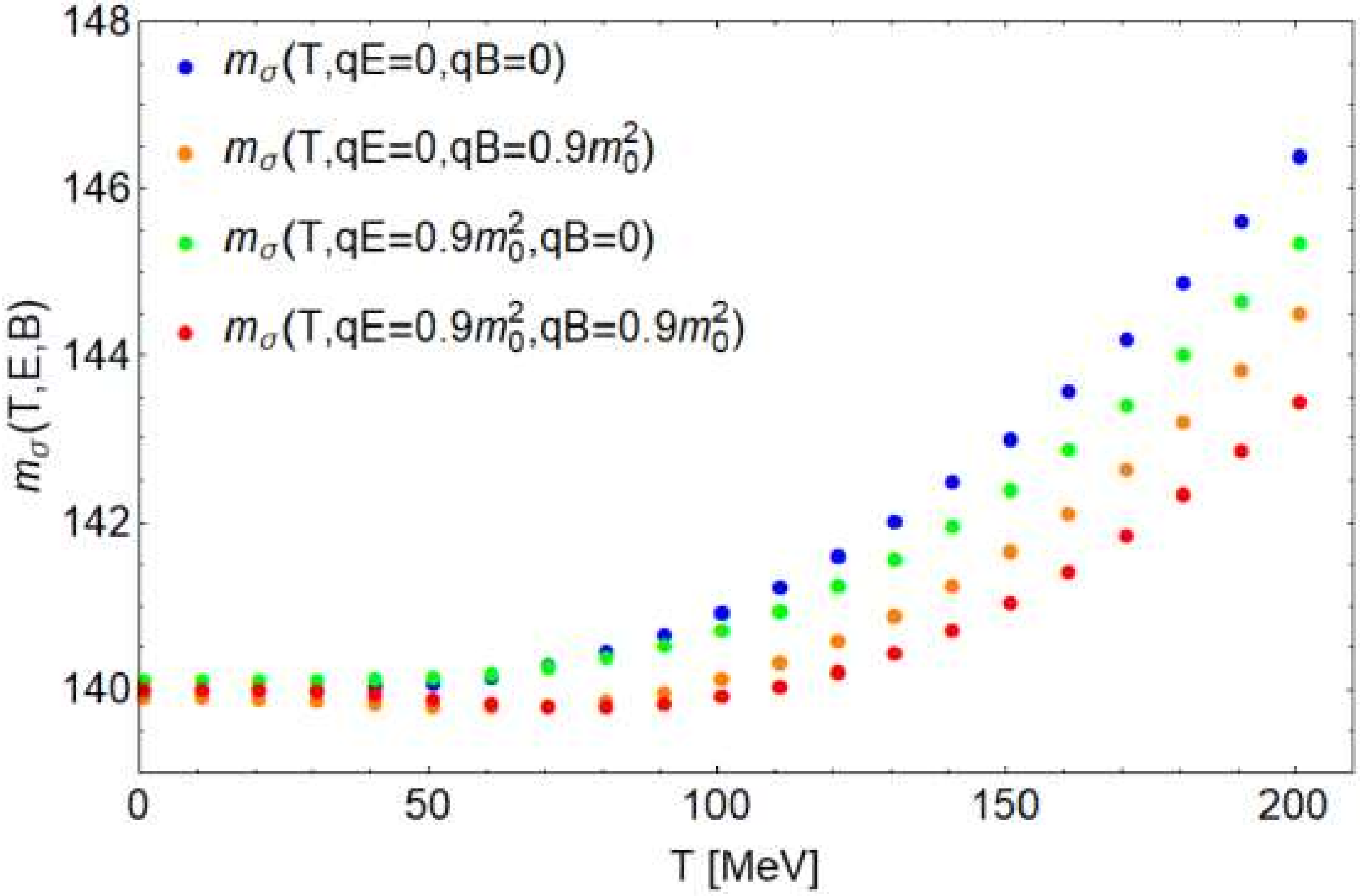


Taking our propagator, expanded in the small field strength region, in the previous diagrams we find

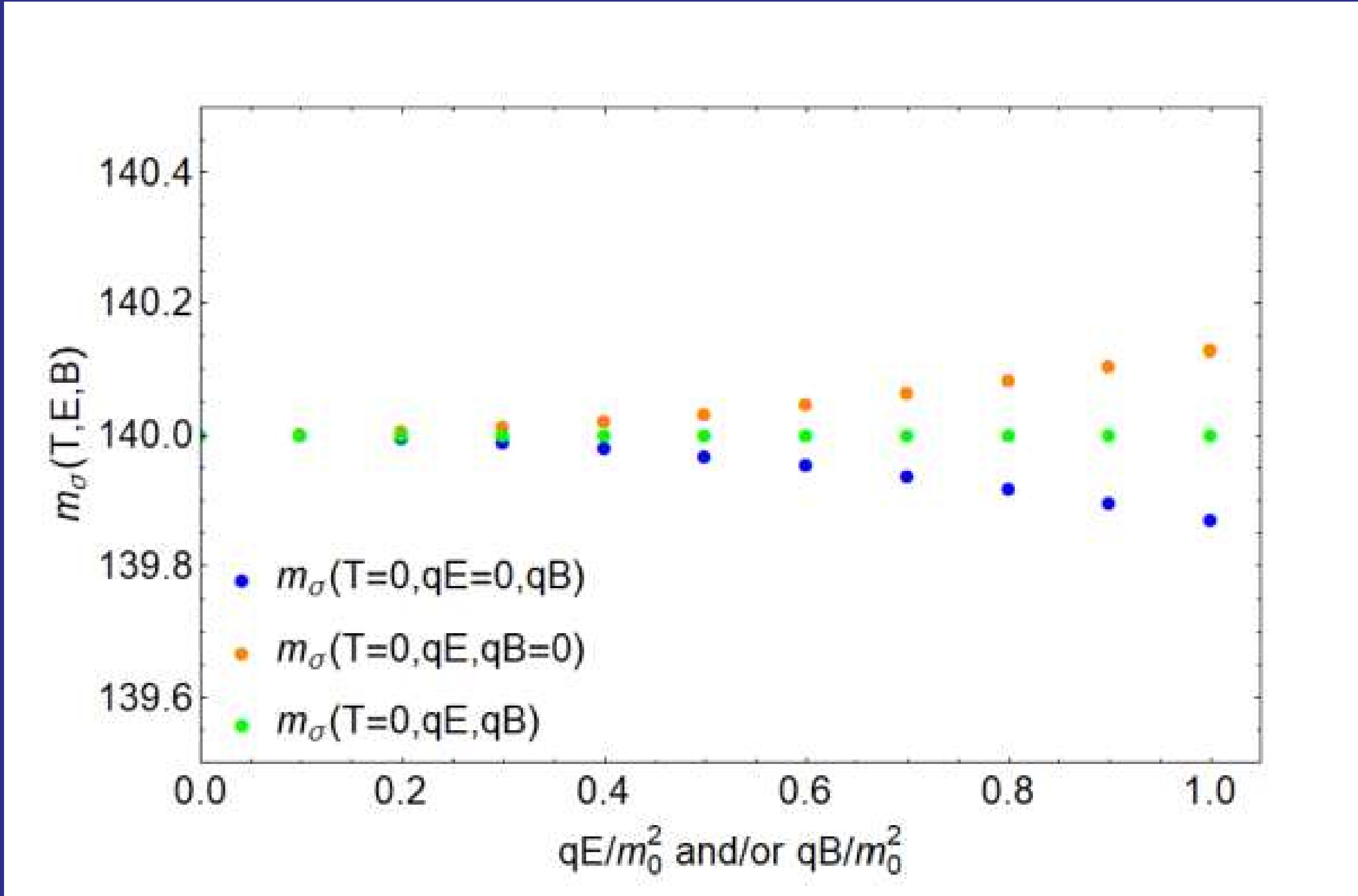
$$\Pi = -\frac{m^2}{16\pi^2}\left(\ln\left(\frac{\widetilde{\mu}^2}{m^2}\right)+1\right)+\frac{mT}{2\pi^2}\sum_{n=1}^{\infty}\frac{K_1(nm/T)}{n}$$

$$+\frac{[(qE)^2-(qB)^2]}{96\pi^2 m^2}+\frac{(qE)^2}{48\pi^2 T^2}\sum_{n=1}^{\infty} nK_2(nm/T)$$

$$-\quad\frac{5[(qE)^2+(qB)^2]}{48\pi^2 mT}\sum_{n=1}^{\infty} nK_1(nm/T)$$



m_σ(T,qE=0,qB=0)
m_σ(T,qE=0,qB=0.9m_0^2)
m_σ(T,qE=0.9m_0^2,qB=0)
m_σ(T,qE=0.9m_0^2,qB=0.9m_0^2)
m_σ(T,E,B)
T [MeV]
148
146
144
142
140
0
50
100
150
200



140.4
140.2
140.0
139.8
139.6
0.0
0.2
0.4
0.6
0.8
1.0
m_σ(T,E,B)
qE/m_0^2 and/or qB/m_0^2
m_σ(T=0,qE=0,qB)
m_σ(T=0,qE,qB=0)
m_σ(T=0,qE,qB)

Let us proceed with the effective potential

$$
\begin{aligned}
V^{(1\text{-loop})} &= \sum_{i=\sigma,\chi}\left(\frac{T}{2}\sum_n \int \frac{d^3p}{(2\pi)^3}\ln[D(\omega_n,p,m_i)]^{-1}\right)\\
&= \sum_{i=\sigma,\chi}\left(\frac{T}{2}\sum_n \int dm_i^2 \int \frac{d^3p}{(2\pi)^3} D(\omega_n,p,m_i)\right)\\
&\equiv \frac{T}{2}\sum_n \int dm^2 \int \frac{d^3p}{(2\pi)^3} D(\omega_n,p,m),
\end{aligned}
$$

Inserting our propagator we find

$$
\begin{aligned}
V^{(1\text{-loop})} = \sum_{i=\sigma,\chi} \Bigg(& -\frac{m_i^4}{64\pi^2}\left(\ln\left(\frac{\widetilde{\mu}^2}{m_i^2}\right)+\frac{3}{2}\right) \\
& -\frac{m_i^2 T^2}{2\pi^2}\sum_{n=1}^{\infty}\frac{K_2(nm_i/T)}{n^2} \\
& +\frac{(qB)^2-(qE)^2}{192\pi^2}\ln\left(\frac{\widetilde{\mu}^2}{m_i^2}\right) \\
& +\frac{5[(qE)^2+(qB)^2]}{48\pi^2}\sum_{n=1}^{\infty} nK_0(nm_i/T) \\
+ \; (qE)^2 \Bigg[& -\frac{1}{288\pi^2}-\frac{1}{24\pi^2}\sum_{n=1}^{\infty}\frac{K_0(nm_i/T)}{n} \\
& -\frac{m_i}{48\pi^2 T}\sum_{n=1}^{\infty}K_1(nm_i/T)\Bigg] + \frac{(qB)^2}{144\pi^2}\Bigg).
\end{aligned}
$$

Now we consider the ring contribution to the effective potential

$$V^{(\mathrm{ring})} = \frac{T}{2}\sum_n \int \frac{d^3p}{(2\pi)^3} \ln[1 + \Pi(\omega_n, p) D(\omega_n, p)].$$

Which can be written as

$$\begin{aligned}
V^{(\mathrm{ring})} &= \frac{T}{2}\sum_n \int \frac{d^3p}{(2\pi)^3} \ln\Big[(D(\omega_n, p)^{-1} + \Pi(\omega_n, p)) \\
&\times \; (D(\omega_n, p))\Big] \\
&= \frac{T}{2}\sum_n \int \frac{d^3p}{(2\pi)^3} \ln[D(\omega_n, p)] \\
&+ \; \frac{T}{2}\sum_n \int \frac{d^3p}{(2\pi)^3} \ln[D(\omega_n, p)^{-1} + \Pi(\omega_n, p)],
\end{aligned}$$

Therefore we have

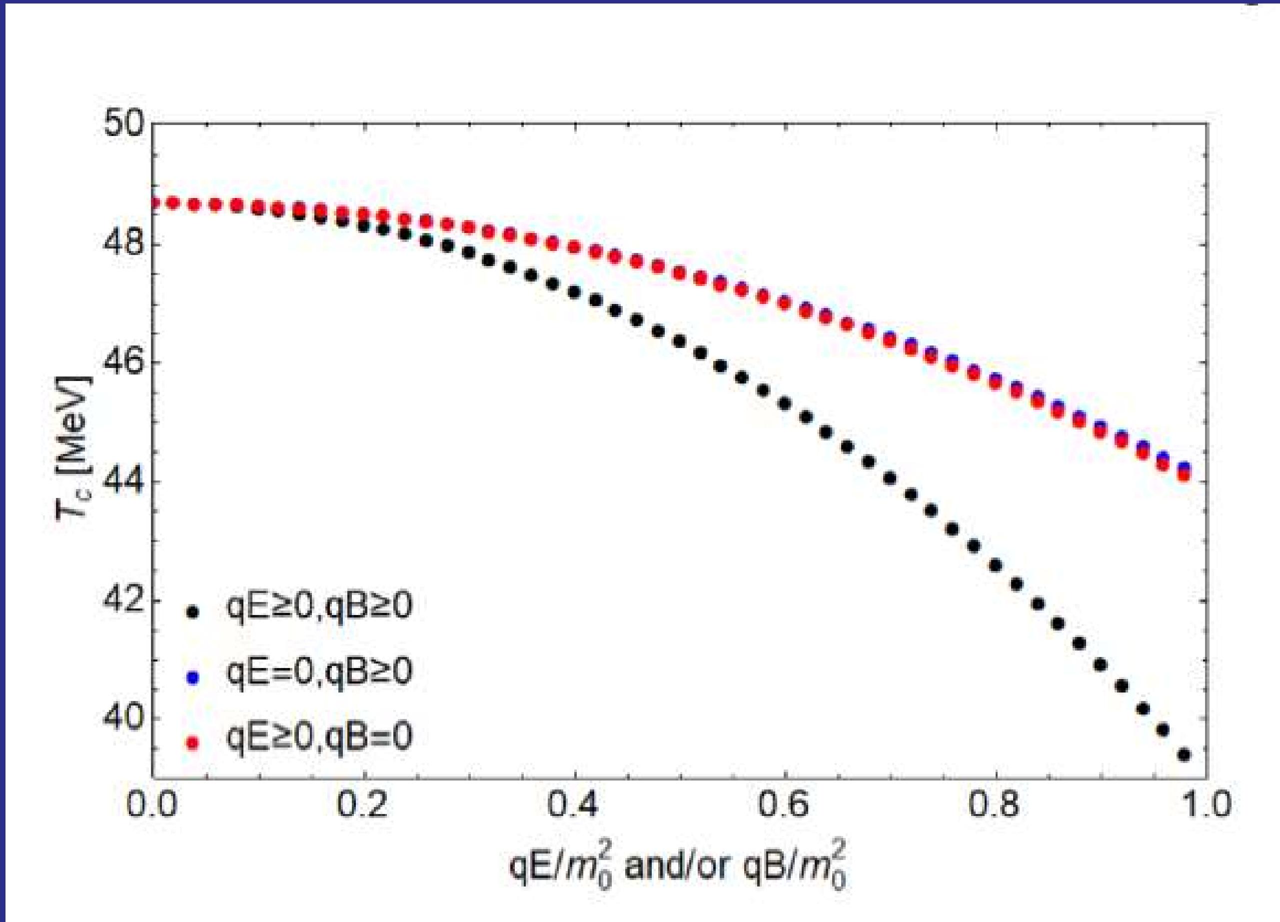
$$V^{(\text{1-loop})} + V^{(\text{ring})} = \frac{T}{2}\sum_n \int \frac{d^3p}{(2\pi)^3} \ln\left[D(\omega_n, p)^{-1} + \Pi(\omega_n, p)\right]$$

And we do not need to calculate anything else. We have a long expression for the effective potetial which allows us
To find the evolution of the critical temperature with the intensities of the fields.

$$
\begin{aligned}
&V^{(\text{tree})} + V^{(\text{1-loop})} + V^{(\text{ring})} = \\
&-\frac{1}{2}\mu^2 v^2 + \frac{1}{16}\lambda^2 v^4 \\
&\sum_{i=\sigma,\chi} \Bigg(-\frac{(m_i^2+\Pi_i)^2}{64\pi^2}\left(\ln\left(\frac{\widetilde{\mu}^2}{m_i^2+\Pi_i}\right)+\frac{3}{2}\right) \\
&-\frac{(m_i^2+\Pi_i)T^2}{2\pi^2}\sum_{n=1}^{\infty}\frac{K_2(n\sqrt{m_i^2+\Pi_i}/T)}{n^2} \\
&+\frac{(qB)^2-(qE)^2}{192\pi^2}\ln\left(\frac{\widetilde{\mu}^2}{m_i^2+\Pi_i}\right) \\
&+\frac{5[(qE)^2+(qB)^2]}{48\pi^2}\sum_{n=1}^{\infty} nK_0(n\sqrt{m_i^2+\Pi_i/T}) \\
&(qE)^2\Bigg[-\frac{1}{288\pi^2}-\frac{1}{24\pi^2}\sum_{n=1}^{\infty}\frac{K_0(n\sqrt{m_i^2+\Pi_i}/T)}{n} \\
&-\frac{m_i}{48\pi^2 T}\sum_{n=1}^{\infty}K_1(n\sqrt{m_i^2+\Pi_i}/T)\Bigg]+\frac{(qB)^2}{144\pi^2}\Bigg)
\end{aligned}
$$

We used μ-tilde = 150 MeV

Evolution of the critical temperature



Conclusions and prospects:

In the weak electric field región, only electric field, we find inverse electric catalysis (IEC). The rate of diminishing of T_c becomes slower when the electric field starts to grow. Our results are valid for the whole Temperature range.
This effect is analogous to what happens in the pure magnetic case (A. Ayala, A. Mizher, M. Loewe and R. Zamora: *Phys.Rev.D* 90 (2014) 3, 036001)

In the strong electric field region, however, the situation is not clear, since the effective potential becomes imaginary and we were not able to find Electric catalysis as was reported in the frame of the NJL model, fermión propagator, (W. R. Tavares, R. L. S. Farías, and S. S. Avancini: Phys. Rev. D 101 (2020) 016017)

When both type of fields are present (Electric and Magnetic) we found IMEC (Inverse magnetic-electric catalysis) in the weak field región.
It is crucial for the analiticity to include ring resummation in the effective potential, avoiding the appearance of imaginary mass terms.

Mass at T=0 grows with the electric field and diminishes with the magnetic field intensity. For high values of temperature, mass grows as function of the intensity of both fields.

THANK YOU