Electric and magnetic field effects, including temperature, in a scalar self-interacting $\lambda\phi^4$ theory

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Relativistic heavy ion collision experiments (RHIC, LHC, FAIR and NICA in the near future) open a window to explore initial stages of the universe.

A high temperature regime, huge magnetic fields, together with density effects, affect in a dramatic way the physics of strong interactions.

Several phase transitions occur: Deconfinement, Chiral Restoration, transition to a Quarkyonic phase
QCD phase diagram
Magnetic Fields im peripheral heavy ion collisions
Some experimental signals
Today I want to concentrate on a different scenario where also huge electric fields appear: Asymmetric peripheral relativistic heavy ion collisions as, for example, Cu+Au, Cu+Pb collisions.

Due to the imbalance in the number of charges in each nuclei a strong electric field, $\perp$ to B, appears.
Here we explore the phase diagram associated to symmetry breaking of a scalar theory in the presence of temperature and an external electric field. TOY MODEL!

For this we need the effective potential. So we start with the propagator for a charged scalar particle moving in the presence of a constant electric field

\[
D(P, F) = \int_0^\infty ds e^{-m^2s} \frac{e^{-sP_\mu (\tan Z)^\mu \nu P_\nu}}{\sqrt{\det[\cos Z]}}
\]

with

\[
Z^{\mu \nu} = q F^{\mu \nu}
\]

We take the electric field pointing into the z-axis

\[ F_{\mu\nu} = i E f_{\mu\nu} \]

\[ f_{\mu\nu} = \begin{bmatrix} 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \]

In this way we find

\[ D(p) = \int_0^\infty ds e^{-s \left( \frac{\tanh(qiEs)}{qiEs} p_\parallel^2 + p_\perp^2 + m^2 \right) / \cosh(qiEs)} \]

\( p_\parallel \) and \( p_\perp \) refer to \((p_4, 0, 0, p_3)\) and \((0, p_1, p_2, 0)\)
In the weak electric field approximation

\[
D(p) \approx \int_0^\infty dse^{-s(p^2+m^2)} \left[ \frac{-(qEs)^2}{6} \left( -3 + 2sp_\parallel^2 \right) \\
+ \frac{(qEs)^4}{360} \left( 4sp_\parallel^2[5sp_\parallel^2 - 27] + 75 \right) - \frac{(qEs)^6}{45360} \\
\times \left( 2sp_\parallel^2[14p_\parallel^2 s(10p_\parallel^2 s - 117) + 4311] - 3843 \right) \right] 
\]

In the strong field region

\[
D(p) \approx 2\frac{e^{\frac{p_\parallel^2}{qE}}}{p_\perp^2 + iqE + m^2}
\]
Effective potential and symmetry restoration: 
Our Model

\[ \mathcal{L} = (D_\mu \phi)^\dagger D^\mu \phi + \mu^2 \phi^\dagger \phi - \frac{\lambda}{4} (\phi^\dagger \phi)^2 \]

\[ D_\mu = \partial_\mu + iqA_\mu \]

We take \( \mu^2 > 0, \ Lambda > 0 \)

We consider

\[ A_\mu = -\delta_{\mu 0} x_3 E \]

Following the usual procedure:

\[ \phi(x) = \frac{1}{\sqrt{2}} [\sigma(x) + i\chi(x)] , \]

\[ \phi^\dagger(x) = \frac{1}{\sqrt{2}} [\sigma(x) - i\chi(x)] . \]
The σ field develops a vacuum expectation value

\[
\sigma \rightarrow \sigma + \nu
\]

After this shift we get

\[
\mathcal{L} = -\frac{1}{2} \left[ \sigma (\partial_\mu + iqA_\mu)^2 \sigma \right] - \frac{1}{2} \left( \frac{3\lambda v^2}{4} - \mu^2 \right) \sigma^2 \\
- \frac{1}{2} \left[ \chi (\partial_\mu + iqA_\mu)^2 \chi \right] - \frac{1}{2} \left( \frac{\lambda v^2}{4} - \mu^2 \right) \chi^2 + \frac{\mu^2}{2} v^2 \\
- \frac{\lambda}{16} v^4 + \mathcal{L}_I,
\]

with

\[
\mathcal{L}_I = -\frac{\lambda}{16} (\sigma^4 + \chi^4 + 2\sigma^2 \chi^2)
\]
We will ignore the issue of the mass term generated for $A^\mu$ field, as well as issues regarding renormalization after symmetry breaking. From the previous Lagrangian we identify

$$m_\sigma^2 = \frac{3}{4} \lambda v^2 - \mu^2,$$
$$m_\chi^2 = \frac{1}{4} \lambda v^2 - \mu^2.$$

And the tree-level potential

$$V^{(\text{tree})} = -\frac{1}{2} \mu^2 v^2 + \frac{1}{16} \lambda^2 v^4.$$

Our goal is to calculate the one-loop effective potential, including temperature and electric field corrections.
Finite temperature effects will be handled in the imaginary time formalism

\[ V^{(1\text{-loop})} = \sum_{i=\sigma,\chi} \left( \frac{T}{2} \sum_n \int \frac{d^3 p}{(2\pi)^3} \ln[D(\omega_n, p, m_i)]^{-1} \right), \]

\[ = \sum_{i=\sigma,\chi} \left( \frac{T}{2} \sum_n \int dm_i^2 \int \frac{d^3 p}{(2\pi)^3} D(\omega_n, p, m_i) \right) \]

\[ p_4 \rightarrow \omega_n = 2\pi n T, \quad n \in \mathbb{Z}, \]

\[ \int \frac{d^4 p}{(2\pi)^4} f(p) \rightarrow T \sum_{n \in \mathbb{Z}} \int \frac{d^3 k}{(2\pi)^3} f(\omega_n, p) \]
It is possible to calculate this one-loop effective potential analytically, in the weak and strong field cases, for the whole range of temperature. The details are too technical to be presented here. We find

\[
V_{\text{weak}} = V^{(\text{tree})} + V^{(1\text{-loop})}_{\text{weak}} = -\frac{1}{2} \mu^2 v^2 + \frac{1}{16} \lambda^2 v^4 \\
+ \sum_{i=\sigma,\chi} \left( -\frac{m_i^4}{64\pi^2} \ln \left( \frac{\tilde{\mu}^2}{m_i^2} \right) + \frac{3}{2} \right) \\
- \frac{m_i^2 T^2}{2\pi^2} \sum_{n=1}^{\infty} \frac{K_2(nm_i/T)}{n^2} - \frac{(qE)^2}{96\pi^2} \left[ \ln \left( \frac{\tilde{\mu}^2}{m_i^2} \right) \\
+ 1 + \sum_{n=1}^{\infty} 2K_0(nm_i/T) + \frac{2m_i}{T} \sum_{n=1}^{\infty} nK_1(nm_i/T) \right] \right)
\]

\(\tilde{\mu}\) is the ultraviolet renormalization scale
To find the critical temperature $T_c$ such that for $T > T_c$ the symmetry is restored, we explore the evolution of the effective potential as function of the vacuum expectation value $v$ for different values of $T$. When the first and second derivative vanish (in fact, all derivatives) we find $T_c$. It is a second order phase transition.
Let us consider now the situation where we have both type of fields: Electric and Magnetic fields, perpendicular to each other. We assume (as usual) constant fields.

Once again, we start from

$$D(p) = \int_0^\infty dt \ e^{-m^2 t} \ e^{tp_\mu \left( \frac{\tan(Z)}{Z} \right)_{\mu\nu} p_\nu} \sqrt{\det(\cos(Z))}$$

$$Z^{\mu\nu} \equiv q F^{\mu\nu} t$$

$$F^{\mu\nu} = \begin{bmatrix} 0 & B_0 & 0 & iE_0 \\ -B_0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -iE_0 & 0 & 0 & 0 \end{bmatrix}$$
We find

\[ D(p) = \int_0^\infty dt e^{-t} \left( p_3^2 + \frac{1}{-E_0^2 + B_0^2} (B_0 p_4 - iE_0 p_2)^2 + m^2 \right) \]

\[ \times e^{-\tanh\left( q\sqrt{-E_0^2 + B_0^2} t \right)} \left( p_1^2 + \frac{1}{-E_0^2 + B_0^2} (B_0 p_2 + iE_0 p_4)^2 \right) \]

\[ \times \frac{\sqrt{-E_0^2 + B_0^2} t)}{\cosh(q\sqrt{-E_0^2 + B_0^2} t)} \]

In the region of weak field intensities

\[ D(p) \approx \frac{1}{p^2 + m^2} + \frac{4i p_4 p_2 (qB)(qE)}{(p^2 + m^2)^4} \]

\[ + \frac{(qE)^2 (m^2 + p^2 - 2(p_4^2 + p_1^2))}{(p^2 + m^2)^4} \]

\[ - \frac{(qB)^2 (m^2 + p^2 - 2p_1^2)}{(p^2 + m^2)^4} \]
We will analyze two different aspects: mass correction and effective potential (including the ring resumation)

For the mass of the σ meson we have

\[ m_\sigma(T, B, E) = m_0 + \Pi_\sigma(T, B, E). \]
Taking our propagator, expanded in the small field strength region, in the previous diagrams we find

\[
\Pi = - \frac{m^2}{16\pi^2} \left( \ln \left( \frac{\tilde{\mu}^2}{m^2} \right) + 1 \right) + \frac{mT}{2\pi^2} \sum_{n=1}^{\infty} \frac{K_1(nm/T)}{n} \\
+ \frac{[ (qE)^2 - (qB)^2 ]}{96\pi^2 m^2} + \frac{(qE)^2}{48\pi^2 T^2} \sum_{n=1}^{\infty} n K_2(nm/T) \\
- \frac{5[ (qE)^2 + (qB)^2 ]}{48\pi^2 mT} \sum_{n=1}^{\infty} n K_1(nm/T)
\]
Let us proceed with the effective potential

\[ V^{(1\text{-loop})} = \sum_{i=\sigma,\chi} \left( \frac{T}{2} \sum_n \int \frac{d^3p}{(2\pi)^3} \ln[D(\omega_n, p, m_i)]^{-1} \right) \]
\[ = \sum_{i=\sigma,\chi} \left( \frac{T}{2} \sum_n \int dm_i^2 \int \frac{d^3p}{(2\pi)^3} D(\omega_n, p, m_i) \right) \]
\[ \equiv \frac{T}{2} \sum_n \int dm^2 \int \frac{d^3p}{(2\pi)^3} D(\omega_n, p, m). \]
Inserting our propagator we find

\[
V^{(1\text{-loop})} = \sum_{i=\sigma,\chi} \left( -\frac{m_i^4}{64\pi^2} \left( \ln \left( \frac{\tilde{\mu}^2}{m_i^2} \right) + \frac{3}{2} \right) - \frac{m_i^2 T^2}{2\pi^2} \sum_{n=1}^{\infty} \frac{K_2(nm_i/T)}{n^2} \right.

+ \left( \frac{(qB)^2 - (qE)^2}{192\pi^2} \right) \ln \left( \frac{\tilde{\mu}^2}{m_i^2} \right)

\left. + \frac{5[(qE)^2 + (qB)^2]}{48\pi^2} \sum_{n=1}^{\infty} nK_0(nm_i/T) \right)

+ \left( qE \right)^2 \left[ -\frac{1}{288\pi^2} - \frac{1}{24\pi^2} \sum_{n=1}^{\infty} \frac{K_0(nm_i/T)}{n} \right] \left. - \frac{m_i}{48\pi^2 T} \sum_{n=1}^{\infty} K_1(nm_i/T) \right] + \left( \frac{(qB)^2}{144\pi^2} \right).
\]
Now we consider the ring contribution to the effective potential

\[ V^{(\text{ring})} = \frac{T}{2} \sum_n \int \frac{d^3 p}{(2\pi)^3} \ln \left[ 1 + \Pi(\omega_n, p) D(\omega_n, p) \right]. \]

Which can be written as

\[
V^{(\text{ring})} = \frac{T}{2} \sum_n \int \frac{d^3 p}{(2\pi)^3} \ln \left[ (D(\omega_n, p)^{-1} + \Pi(\omega_n, p)) \right] \\
\times (D(\omega_n, p)) \\
= \frac{T}{2} \sum_n \int \frac{d^3 p}{(2\pi)^3} \ln [D(\omega_n, p)] \\
+ \frac{T}{2} \sum_n \int \frac{d^3 p}{(2\pi)^3} \ln [D(\omega_n, p)^{-1} + \Pi(\omega_n, p)],
\]
Therefore we have

\[ V^{(1\text{-loop})} + V^{(\text{ring})} = \frac{T}{2} \sum_n \int \frac{d^3p}{(2\pi)^3} \ln[D(\omega_n, p)^{-1} + \Pi(\omega_n, p)] \]

And we do not need to calculate anything else. We have a long expression for the effective potential which allows us to find the evolution of the critical temperature with the intensities of the fields.
We used $\mu\text{-tilde} = 150$ MeV.
Evolution of the critical temperature
Conclusions and prospects:

In the weak electric field region, only electric field, we find inverse electric catalysis (IEC). The rate of diminishing of $T_c$ becomes slower when the electric field starts to grow. Our results are valid for the whole temperature range. This effect is analogous to what happens in the pure magnetic case (A. Ayala, A. Mizher, M. Loewe and R. Zamora: *Phys. Rev. D* 90 (2014) 3, 036001).

In the strong electric field region, however, the situation is not clear, since the effective potential becomes imaginary and we were not able to find Electric catalysis as was reported in the frame of the NJL model, fermión propagator, (W. R. Tavares, R. L. S. Farías, and S. S. Avancini: *Phys. Rev. D* 101 (2020) 016017).
When both type of fields are present (Electric and Magnetic) we found IMEC (Inverse magnetic-electric catalysis) in the weak field region. It is crucial for the analiticy to include ring resummation in the effective potential, avoiding the appearance of imaginary mass terms.

Mass at $T=0$ grows with the electric field and diminishes with the magnetic field intensity. For high values of temperature, mass grows as function of the intensity of both fields.

THANK YOU