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Issues related to regularizing thermo and magnetic contributions within nonrenormalizable theories

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Workshop on Electromagnetic Effects in Strongly Interacting Matter

FAPESP ICTP unesp IFT - UNESP CLAF I·AN QCD Network of Networks Inter-American

CNPq Conselho Nacional de Desenvolvimento Científico e Tecnológico

FAPERGS

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In Collaboration with:

- R. M. Nunes - UFSM - Brazil
- S. Avancini, M.B. Pinto, W.R. Tavares - UFSC - Brazil
- T. Restrepo (UFRJ) - Brazil
- G. Krein - IFT - UNESP - Brazil
- V.T Salvador - Unicamp - Brazil

This talk was based on:

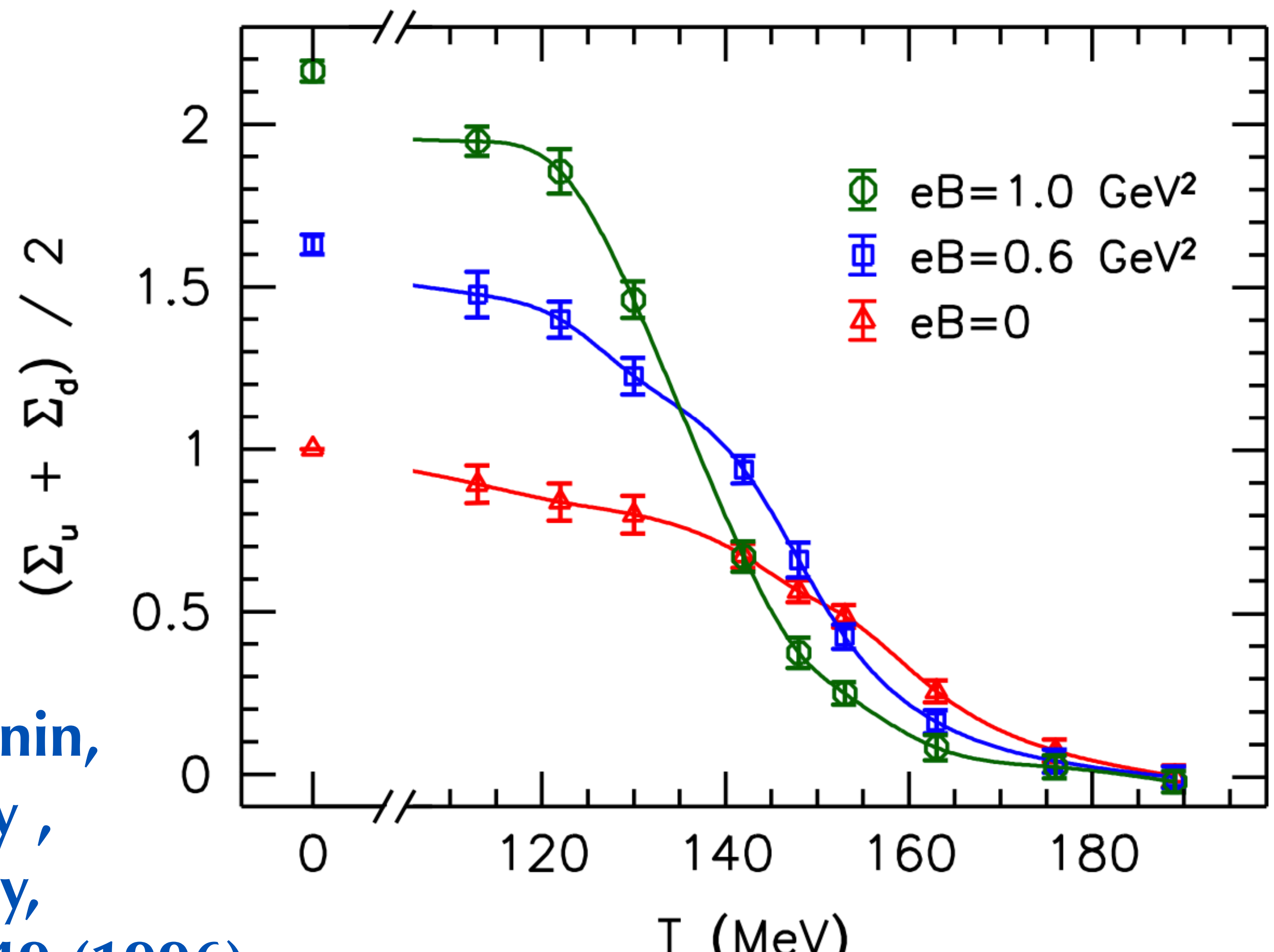
- **Eur. Phys.J.A (2021) 57:278**
- **Eur. Phys. J. C (2022) 82:674**
- **PRD 103, 056009 (2021)**

Outline

- Motivation
- The importance of implementing a proper regularization procedure in order to treat thermo and magnetic contributions within non renormalizable theories
- Magnetic Field Independent Regularization (MFIR)
- Vacuum Magnetic Regularization (VMR)
- Thermo-magnetic effects on the magnetization: NJL X lattice
- Quark AMM effects: chiral symmetry restoration
- Conclusions and perspectives

B Effects on QCD phase transitions?

$$\Lambda^2_{QCD} \sim (200\,\mathrm{MeV})^2 \sim 2 \times 10^{18}\,\mathrm{G}$$

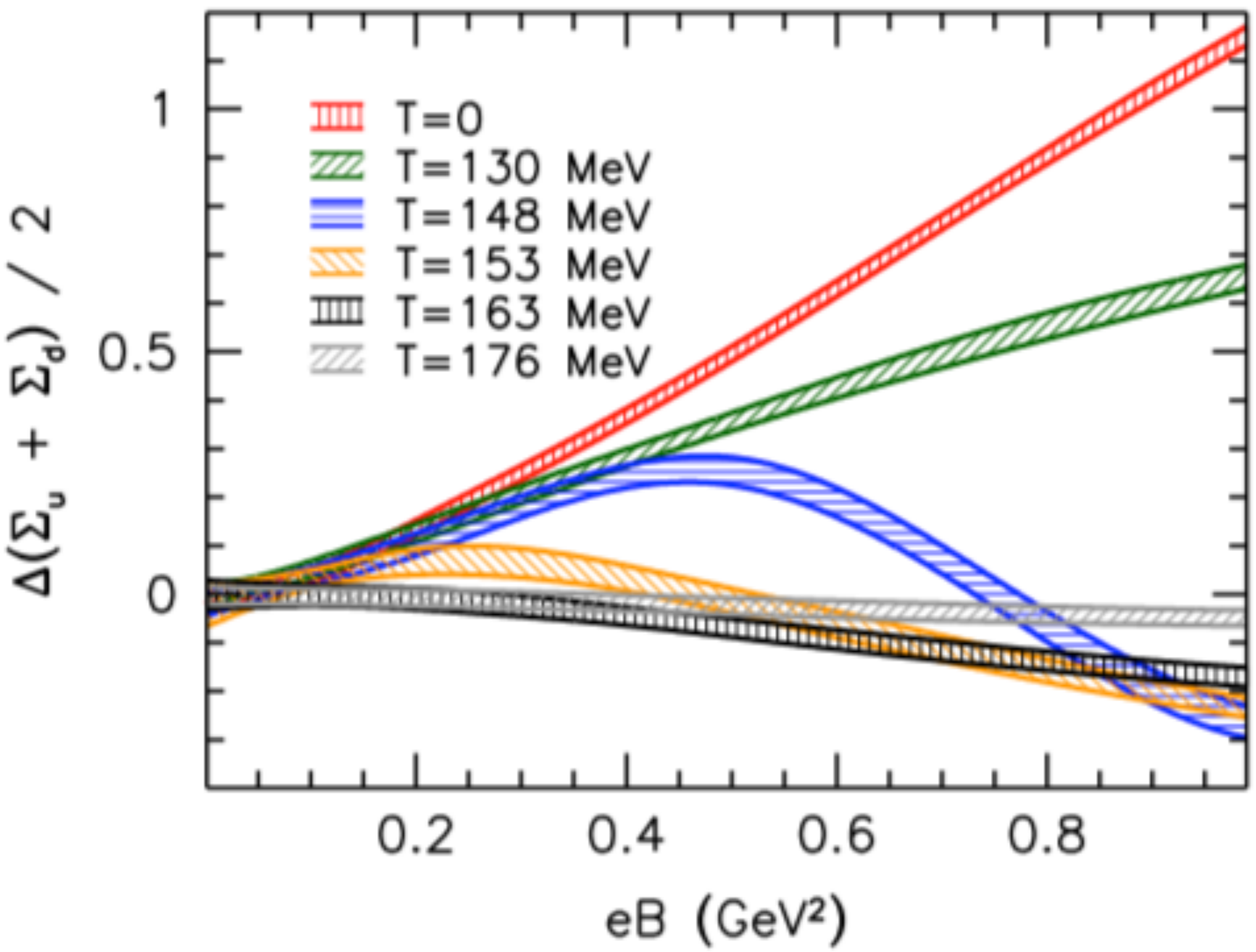


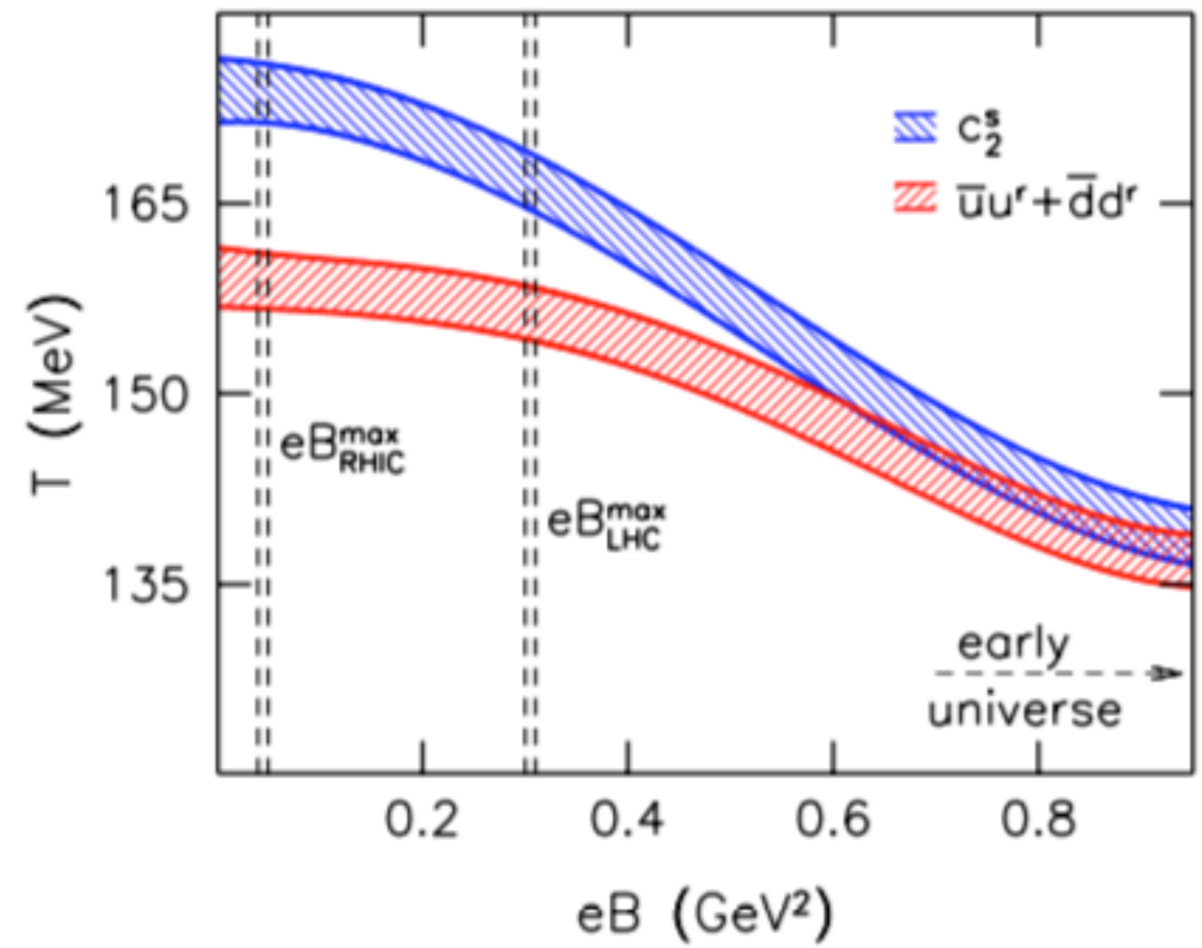
IMC: Bali, Bruckmann, Endrodi, Fodor, Katz et al.
JHEP 02 (2012) 044
Phys.Rev.D 86 (2012) 071502

MC: V.P. Gusynin, V.A. Miransky , I.A. Shovkovy,
Nucl. Phys. B 462 249 (1996)

B Effects on QCD phase transitions?

MC and IMC



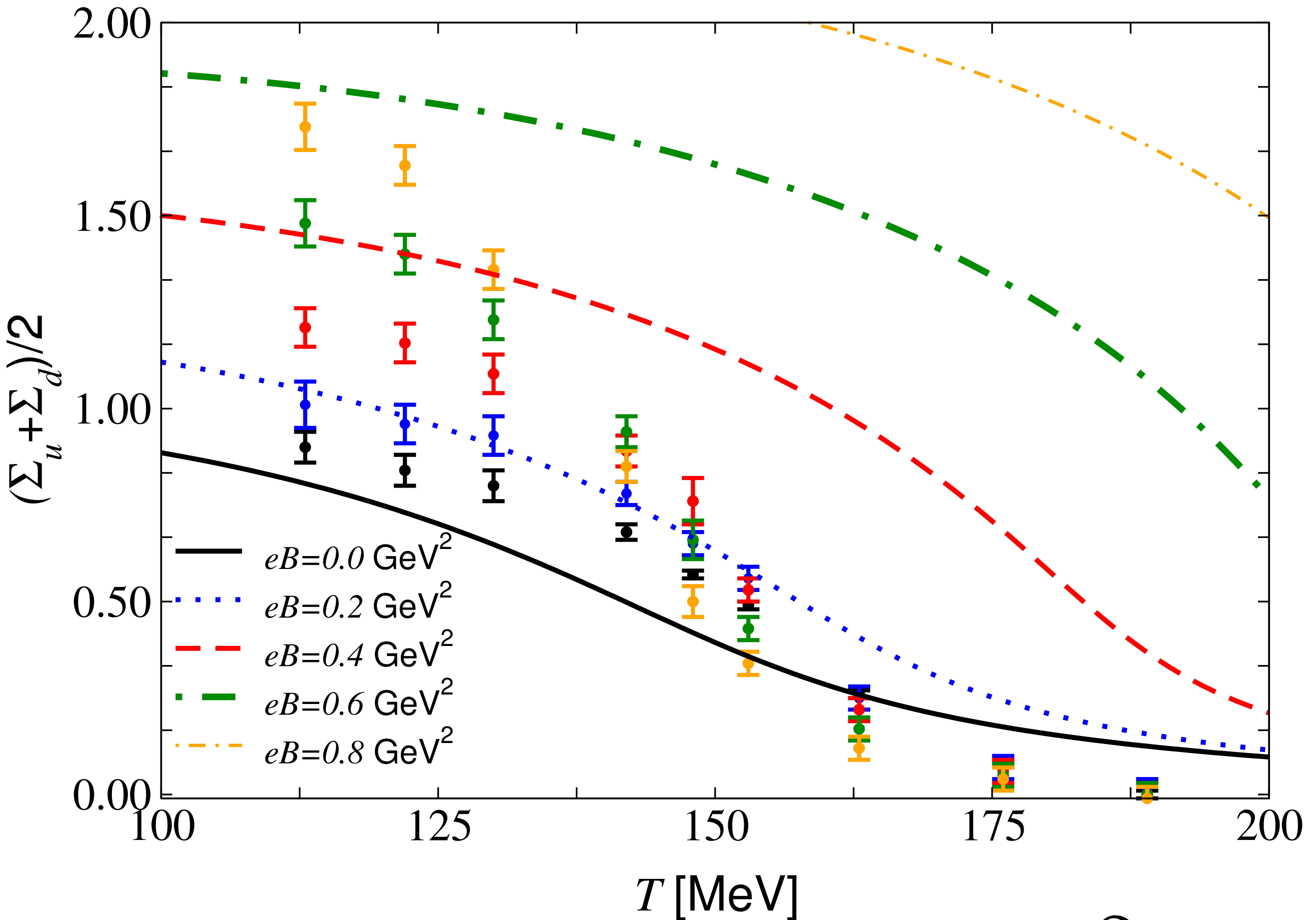


Failure of ALL effective models in providing inverse magnetic catalysis!

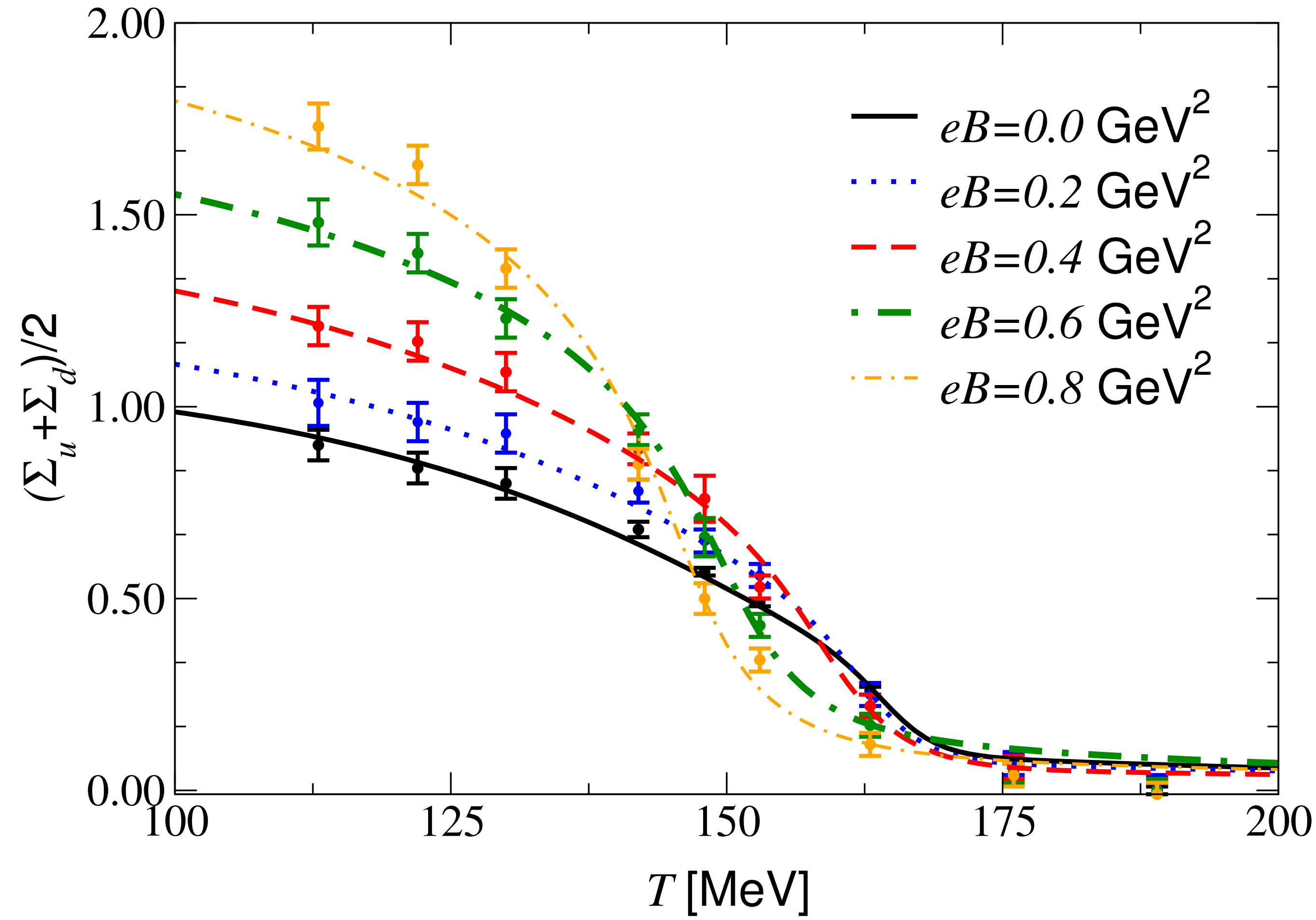
G. S. Bali et al., JHEP **1202**, 044 (2012)

SU(2) NJL + Thermo-Magnetic effects G(B,T)

G(0,0)



G(B,T)



$$\Sigma_f(B,T) = \frac{2m_f}{m_\pi^2 f_\pi^2}\left[\langle\bar{\psi}_f\psi_f\rangle - \langle\bar{\psi}_f\psi_f\rangle_0\right] + 1$$

RLSF, K.P. Gomes, M.B. Pinto, G. Krein, Phys. Rev. C 90, 025203 (2014).

RLSF, V.S. Timoteo, S.S. Avancini, M.B. Pinto and G. Krein Eur. Phys. J. A (2017) 53: 101

RLSF, W. Tavares, S.S. Avancini, V.S. Timoteo, G. Krein and M.B. Pinto, Eur. Phys. J. A (2021) 57: 278

B Effects on QCD phase transitions?

Inverse magnetic catalysis: how much do we know about?

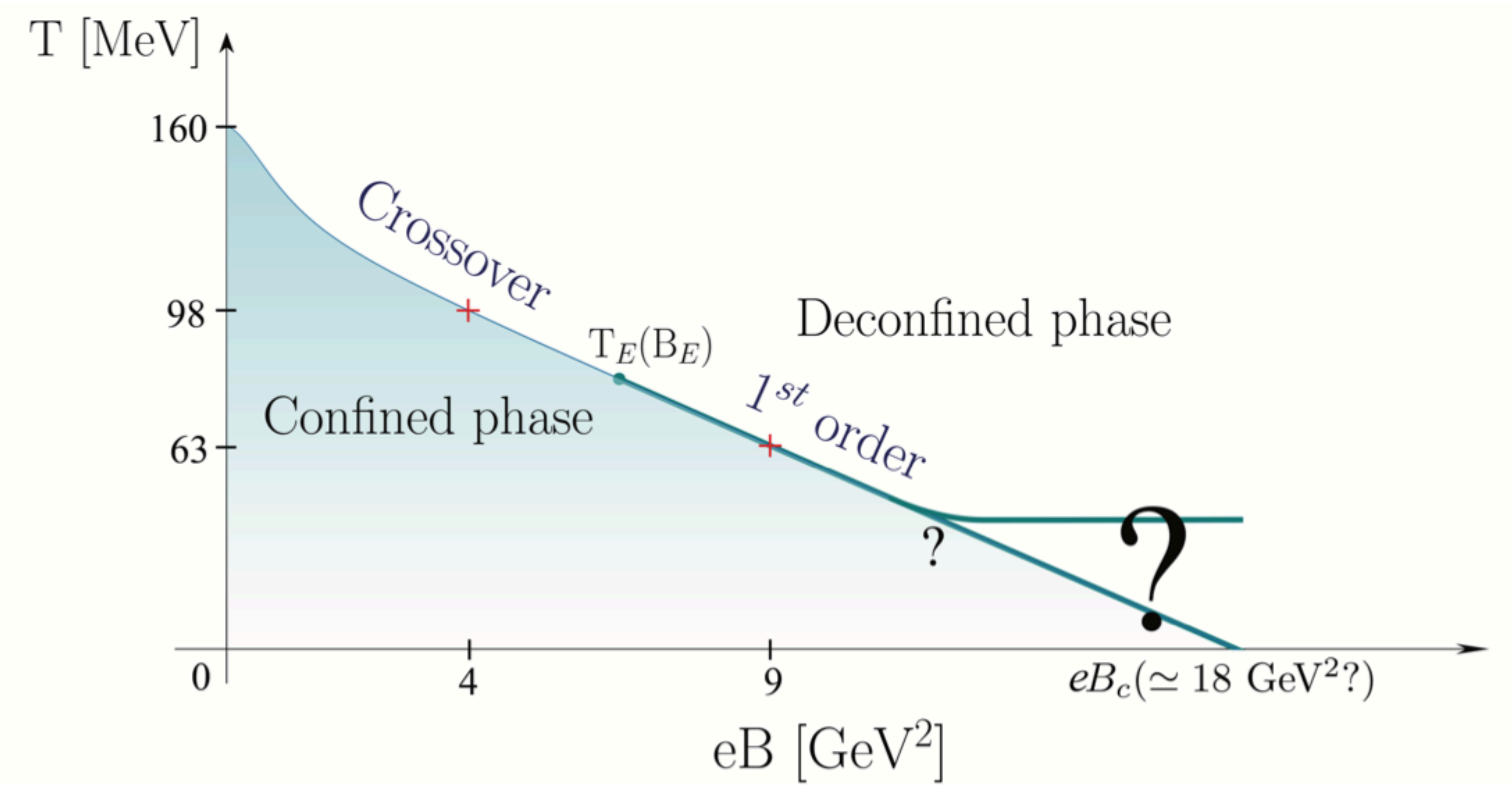
A. Bandyopadhyay, R.L.S. Farias, *Eur. Phys. J. ST* 230 (2021) 3, 719-728,
B. e-Print: 2003.11054 [hep-ph]

Other possible explanations for IMC:

- Competition of B effects on sea and valence quarks, F. Bruckmann, G. Endrodi, T. G. Kovacs, *JHEP* 04 (2013) 112

- Inclusion of plasma screening effects that capture the physics of collective, long-wave modes, and thus describe a prime property of plasmas near transition lines, namely, long distance correlations. A.Ayala, L.A. Hernadez, M.Lowe, C, Villavicencio, EPJA, 57, 234 (2021)

B Effects on QCD phase transitions?



M. D'Elia , L. Maio, F. Sanfilippo, A. Stanzione, *Phys. Rev. D* **105** , 034511 (2022).

SU(2) Nambu—Jona-Lasinio model (NJL)

$$\mathcal{L}_{NJL} = \bar{\psi}\left(\not{D} - m\right)\psi + G\left[(\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma_5\vec{\tau}\psi)^2\right] - \frac{1}{4}F^{\mu\nu}F_{\mu\nu}$$

$$D^\mu = (i\partial^\mu - QA^\mu)$$

$$F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$$

$Q=\mathrm{diag}(q_u = 2e/3,\ q_d = -e/3)$

good **chiral** physics, pions,…
BUT no confinement

✓ **strong magnetic field background that is constant and homogeneous!**

G, Λ and m_c → m_π, f_π and $\langle\bar{\psi}\psi\rangle$

natural units: $1\mathrm{GeV}^2 \approx 5.34 \times 10^{19}\,\mathrm{G}$ and $e = \sqrt{\frac{4\pi}{137}}$

Y. Nambu and G. Jona-Lasinio, Phys. Rev. **122**, 345 (1961)

NJL at finite B

At B=0
$$\mathcal{F} = \frac{(M-m_c)^2}{4G} - N_c \sum_{f,s} \int \frac{d^4p}{(2\pi)^4} \ln\left[p^2 + M^2\right]$$

By using the replacement $\vec{p}^{\,2} \to p_3^2 + 2k|q_f|B$

$$\int \frac{d^4p}{(2\pi)^4} \to \int_{-\infty}^{\infty} \frac{dp_3}{2\pi} \int_{-\infty}^{\infty} \frac{dp_4}{2\pi} \sum_{k=0}^{\infty} \alpha_k$$

$$\alpha_k = 2 - \delta_{k0}$$

$$\begin{aligned} \mathcal{F} &= \frac{(M-m_c)^2}{4G} \\ &- N_c \sum_f \frac{|q_f|B}{2\pi} \sum_{k=0}^{\infty} \alpha_k \int_{-\infty}^{\infty} \frac{dp_3}{2\pi} \int_{-\infty}^{\infty} \frac{dp_4}{2\pi} \ln\left[p_4^2 + p_3^2 + 2k|q_f|B + M^2\right] \end{aligned}$$

And the gap equation: $\partial\mathcal{F}/\partial M = 0 \longrightarrow \infty$

We need a regularization procedure!

Which procedure/method is more appropriate?

Is there any criteria?

Noncovariant Regularizations

Form factors: $$\sum_{k=0}^{\infty}\int_{-\infty}^{\infty}\frac{dp_3}{2\pi}\rightarrow\sum_{k=0}^{\infty}\int_{-\infty}^{\infty}\frac{dp_3}{2\pi}\,U_\Lambda(p_3^2+2k|q_f|B)$$

- ✓ Lorenztian: $$U_\Lambda^{(Lor\,N)}(x)=\left[1+\left(\frac{x}{\Lambda^2}\right)^N\right]^{-1}$$

- ✓ Wood-Saxon: $$U_\Lambda^{(WS\alpha)}(x)=\left[1+\exp\left(\frac{x/\Lambda^2-1}{\alpha}\right)\right]^{-1}$$

- ✓ Fermi-Dirac: $$U_\Lambda^{\mathrm{FD}}(x)=\frac{1}{2}\left[1-\tanh\left(\frac{\frac{x}{\Lambda}-1}{\alpha}\right)\right]$$

- ✓ 3D sharp cutoff

RLSF, S.S. Avancini, N. Scoccola, W.R. Tavares, PRD **99**, 116002 (2019).

MFIR - Magnetic Field Independent Regularization

- ✓ D. Ebert, K. G. Klimenko, M. A. Vdovichenko, and A. S. Vshivtsev, Phys. Rev. D 61, 025005 (1999);
- ✓ M.A. Vdovichenko, A.S. Vshivtsev and K.G. Klimenko, Yad. Fiz. 63, 542 (2000) [Phys. At. Nucl. 63, 470 (2000)].
- ✓ D. Ebert and K.G. Klimenko, Nucl. Phys. **A728**, 203 (2003).
- ✓ D. P. Menezes, M. B. Pinto, S. S. Avancini, A. P. Martínez, and C. Providência, Phys. Rev. C **79**, 035807 (2009).
- ✓ P. G. Allen, A. G. Grunfeld, and N. N. Scoccola, Phys. Rev. D **92**, 074041 (2015).
- ✓ D.C.Duarte, P.G.Allen, R.L.S.Farias, P.H.A.Manso, R.O.Ramos and N. N. Scoccola, Phys. Rev. D **93**, 025017 (2016).
- ✓ S. S. Avancini, W. R. Tavares and M. B. Pinto, Phys. Rev. D **93**, 014010 (2016).
- ✓...

MFIR - Magnetic Field Independent Regularization

$$\mathcal{F}_{vac} = \frac{N_c N_f}{8\pi^2}\left\{M^4 \ln\left[\frac{(\Lambda+\epsilon_\Lambda)}{M}\right] - \epsilon_\Lambda \Lambda\left[\Lambda^2+\epsilon_\Lambda^2\right]\right\}$$

Vacuum NJL

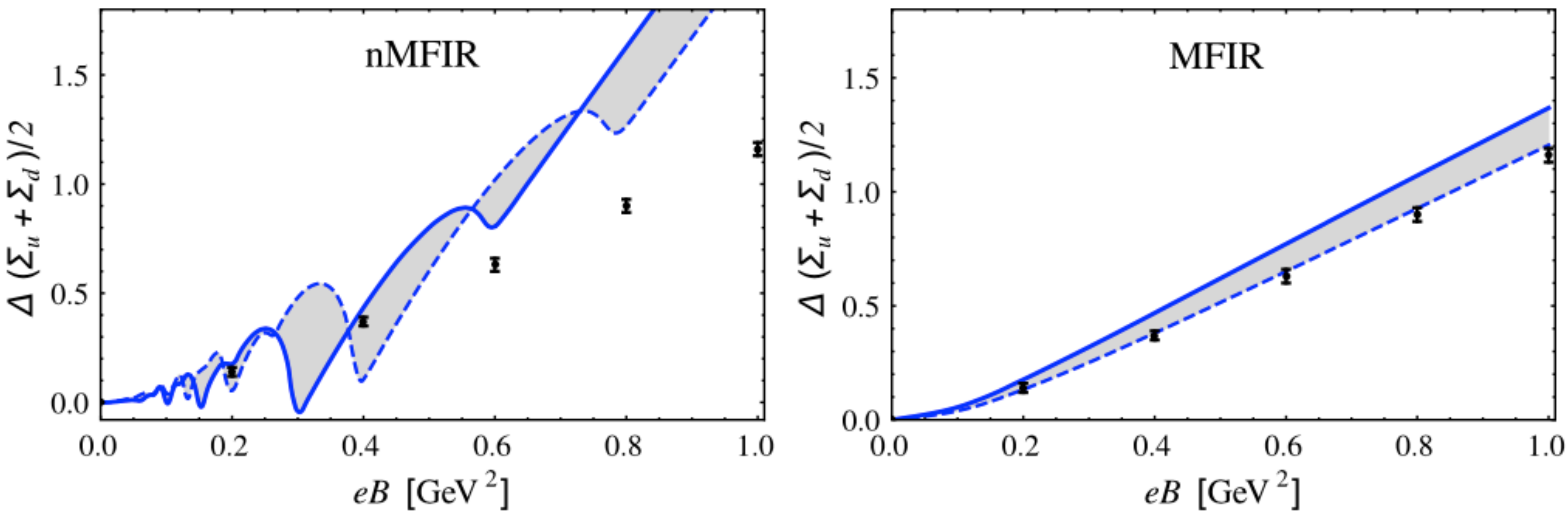
$$\mathcal{F}_{mag} = -\sum_{f=u}^{d}\frac{N_c(|q_f|B)^2}{2\pi^2}\left\{\zeta'[-1,x_f] - \frac{1}{2}[x_f^2 - x_f]\ln x_f + \frac{x_f^2}{4}\right\}$$

Finite

Fermi-Dirac Form Factor

$$U_\Lambda^{\mathrm{FD}}(x) = \frac{1}{2}\left[1 - \tanh\left(\frac{\frac{x}{\Lambda} - 1}{\alpha}\right)\right]$$

$$245\,\mathrm{MeV} < -\bar{\Phi}_0^{1/3} < 260\,\mathrm{MeV}$$



Lattice data: G. S. Bali, F. Bruckmann, G. Endrodi, Z. Fodor, S. D. Katz and A. Schafer, Phys. Rev. D **86**, 071502(R) (2012)

RLSF, S.S. Avancini, N. Scoccola, W.R. Tavares, PRD **99**, 116002 (2019).

Covariant Regularizations:

- ✓ **4D sharp cutoff**
- ✓ **Proper time**
- ✓ **Pauli-Villars**

RLSF, S.S. Avancini, N. Scoccola, W.R. Tavares, PRD **99**, 116002 (2019).

Goldstone Theorem + MFIR

At B=0: [&]

$$m_\pi^2 = -\frac{m}{M}\frac{1}{4iGN_cN_fI(m_\pi^2)}$$

At finite B: [&&]

$$m_\pi^2(B) = -\frac{m_0}{M(B)}\frac{(2\pi)^3}{\sum_{n=0} g_n \sum_{q=u,d} i2G\beta_q N_c I_n(m_\pi^2)}$$

[&] S. P. Klevansky, Rev. Mod. Phys. **64**, 649 (1992)

[&&] W.R. Tavares, S.S. Avancini and M.B. Pinto, *Phys. Rev. D* **93** (2016) 1, 014010

VMR + PNJL SU(2)

$$\Omega_{\rm MFIR}(M,\Phi,T,B) = \mathcal{U}(\Phi,T) + \frac{(M-m_c)^2}{4G} + \frac{N_c N_f}{8\pi^2}\int_{\frac{1}{\Lambda^2}}^{\infty}\frac{ds}{s^3}e^{-M^2 s} - N_c\sum_{f=u,d}\frac{(|q_f|B)^2}{2\pi^2}\left[\zeta'(-1,x_f) - \frac{1}{2}[x_f^2 - x_f]\ln x_f + \frac{x_f^2}{4}\right]$$
$$+ \frac{1}{8\pi^2}\sum_{f=u,d}(|q_f|B)^2\int_0^{\infty}\frac{ds}{s^2}e^{-\frac{M^2 s}{|q_f|B}}\coth(s)\left\{2\sum_{n=1}^{\infty}e^{-\frac{|q_f|Bn^2}{4sT^2}}(-1)^n\left[2\cos\left(n\cos^{-1}\frac{3\Phi-1}{2}\right)+1\right]\right\},$$

$$\Omega_{\rm VMR}(M,\Phi,T,B) = \Omega_{\rm MFIR}(M,\Phi,T,B) + \frac{N_c}{24\pi^2}\sum_{q_f=u,d}(|q_f|B)^2\left[\ln\left(\frac{\Lambda^2}{2|q_f|B}\right) + 1 - \gamma_E\right]$$

No M dependence!

RLSF, S.S. Avancini, M.B.Pinto, T.E. restrepo and W.R. Tavares, PRD **103**, 056009 (2021)

VMR and MFIR frameworks

vacuum magnetic regularization

$$\frac{N_c}{8\pi^2}\sum_{q_f=u,d}(|q_f|B)^2\int_{\frac{|q_f|B}{\Lambda^2}}^{\infty}\frac{ds}{s^2}e^{-\frac{M^2 s}{|q_f|B}}\coth(s)$$

$$=\frac{N_cN_f}{8\pi^2}\int_{\frac{1}{\Lambda^2}}^{\infty}\frac{ds}{s^3}e^{-M^2 s}$$

$$+\frac{N_c}{24\pi^2}\sum_{q_f=u,d}(|q_f|B)^2\left[\ln\left(\frac{\Lambda^2}{2|q_f|B}\right)+1-\gamma_E\right]$$

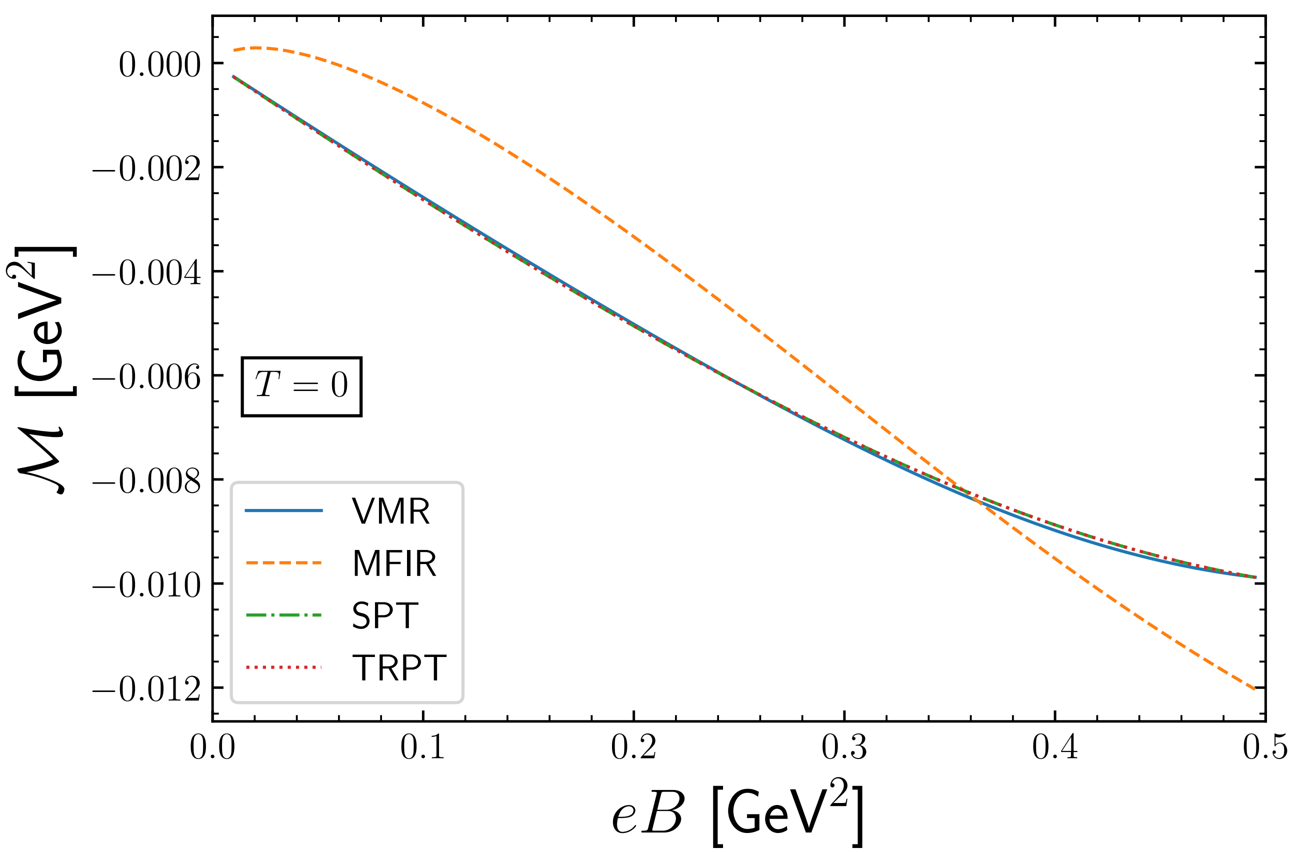
usually neglected

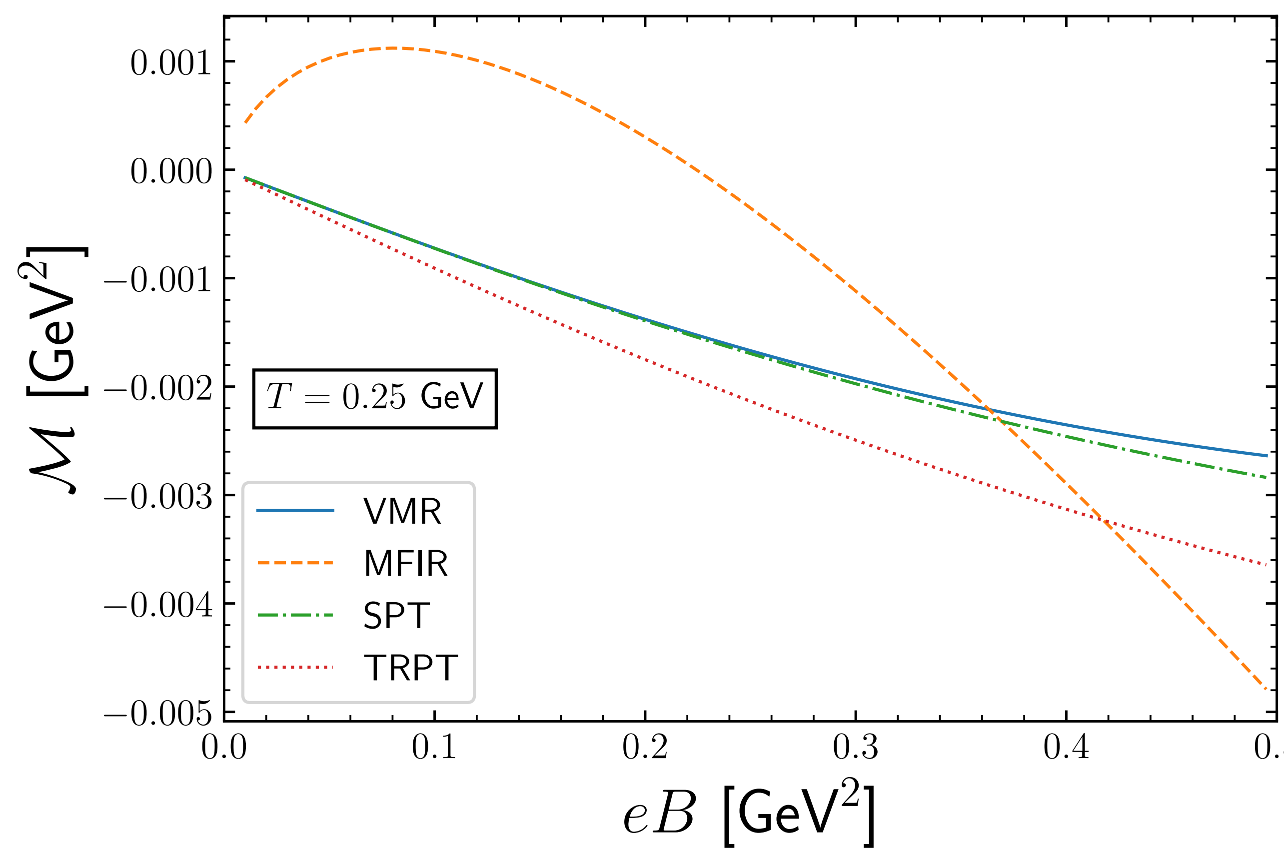
$$-N_c\sum_{f=u,d}\frac{(|q_f|B)^2}{2\pi^2}\left[\zeta'(-1,x_f)-\left[x_f^2-x_f\right]\frac{\ln x_f}{2}+\frac{x_f^2}{4}\right]$$

where $x_f=\frac{M^2}{2|q_f|B}$, and ζ represents the Hurwitz-Riemann zeta function

RLSF, S.S. Avancini, M.B.Pinto, T.E. restrepo and W.R. Tavares, PRD **103**, 056009 (2021)

Magnetization





$$\mathcal{M} = \frac{\partial P}{\partial(eB)}$$

Diamagnetic behavior?

RLSF, S.S. Avancini, M.B.Pinto, T.E. restrepo and W.R. Tavares, PRD **103**, 056009 (2021)

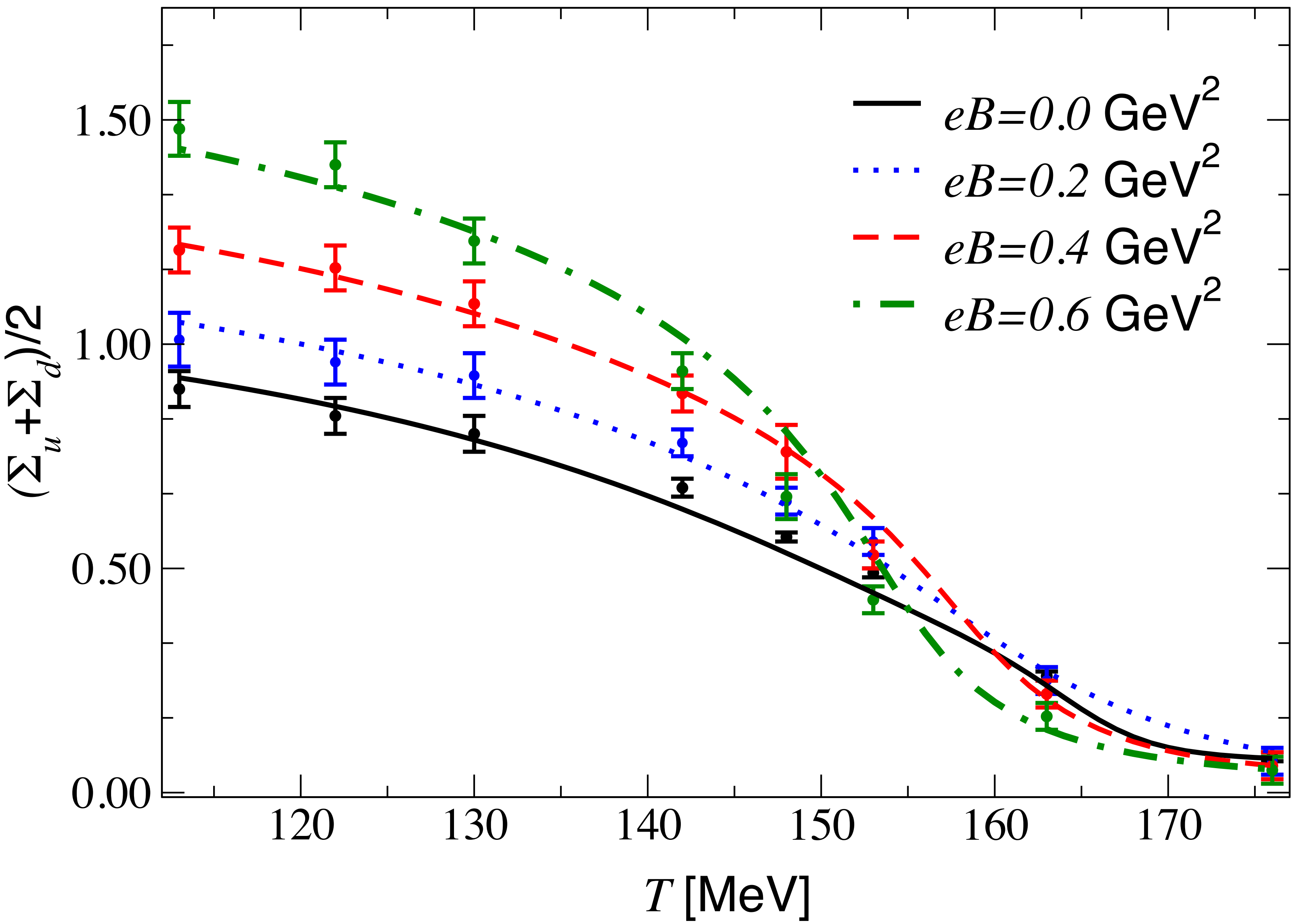
Renormalized magnetization

At this point, a digression concerning the magnetic character of the QCD vacuum is in order since **LQCD evaluations, at T = 0, have shown that the vacuum is paramagnetic in contradiction to our present findings.**

The VMR can indeed be reconciled with the LQCD results provided that one uses the same definition for the renormalized magnetization

$$\mathcal{M}^r \cdot eB = \mathcal{M} \cdot eB - (eB)^2 \lim_{eB \to 0} \frac{\mathcal{M} \cdot eB}{(eB)^2} \bigg|_{T=0}$$

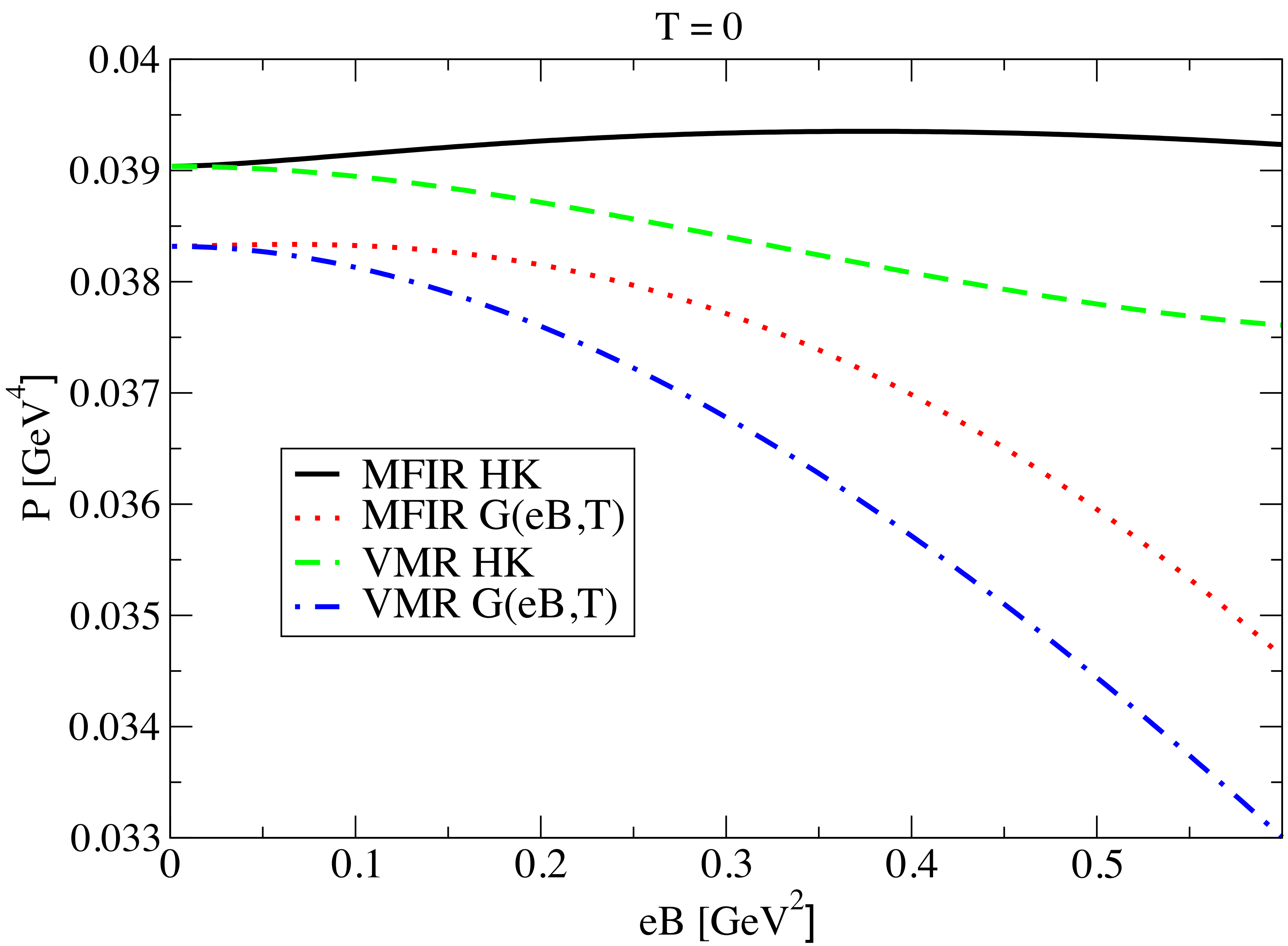
G(eB,T) in SU(3) NJL model



Lattice data: G. S. Bali, F. Bruckmann, G. Endrodi, Z. Fodor, S. D. Katz and A. Schafer, Phys. Rev. D **86**, 071502(R) (2012)

RLSF, W.R.Tavares, S.S. Avancini, M.B.Pinto, V.S.Timoteo and G. Krein, Eur. Phys.J.A (2021) 57:278

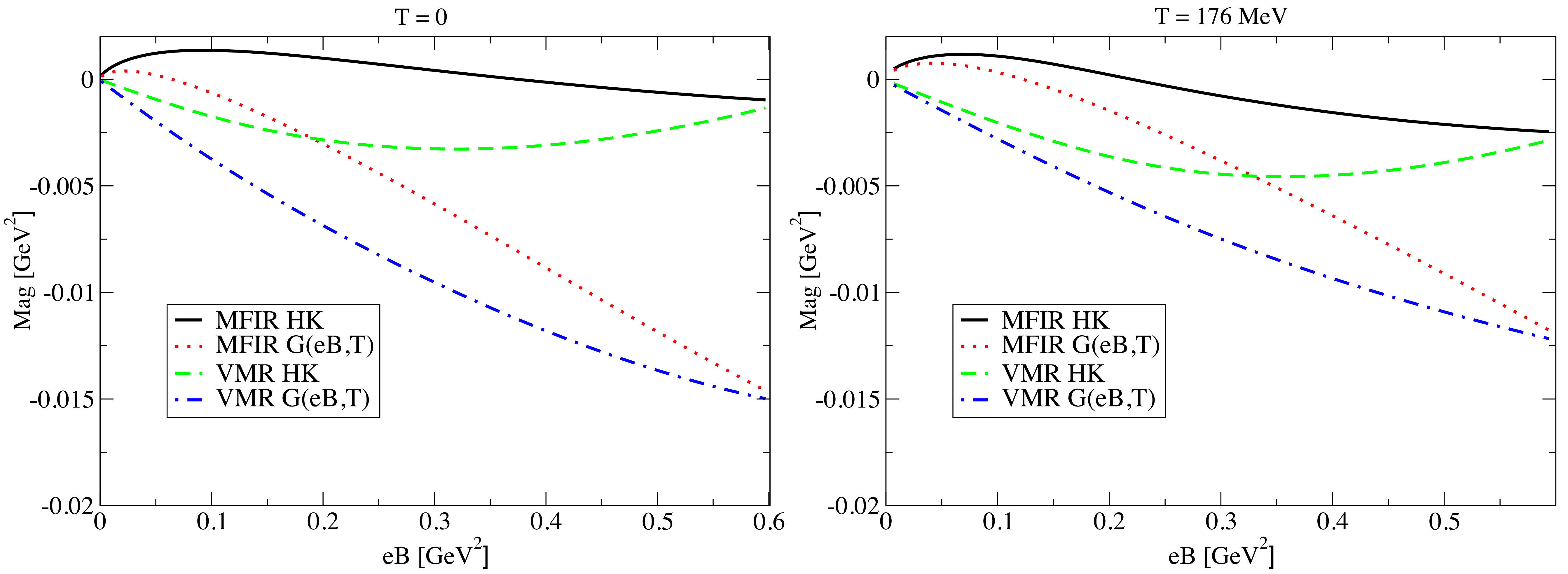
Pressure: MFIR X VMR in SU(3) NJL



HK parametrization, T. Hatsuda and T. Kunihiro, Phys. Rep. **247**, 221 (1994)

RLSF, W.R.Tavares, S.S. Avancini, M.B.Pinto, V.S.Timoteo and G. Krein, Eur. Phys.J.A (2021) 57:278

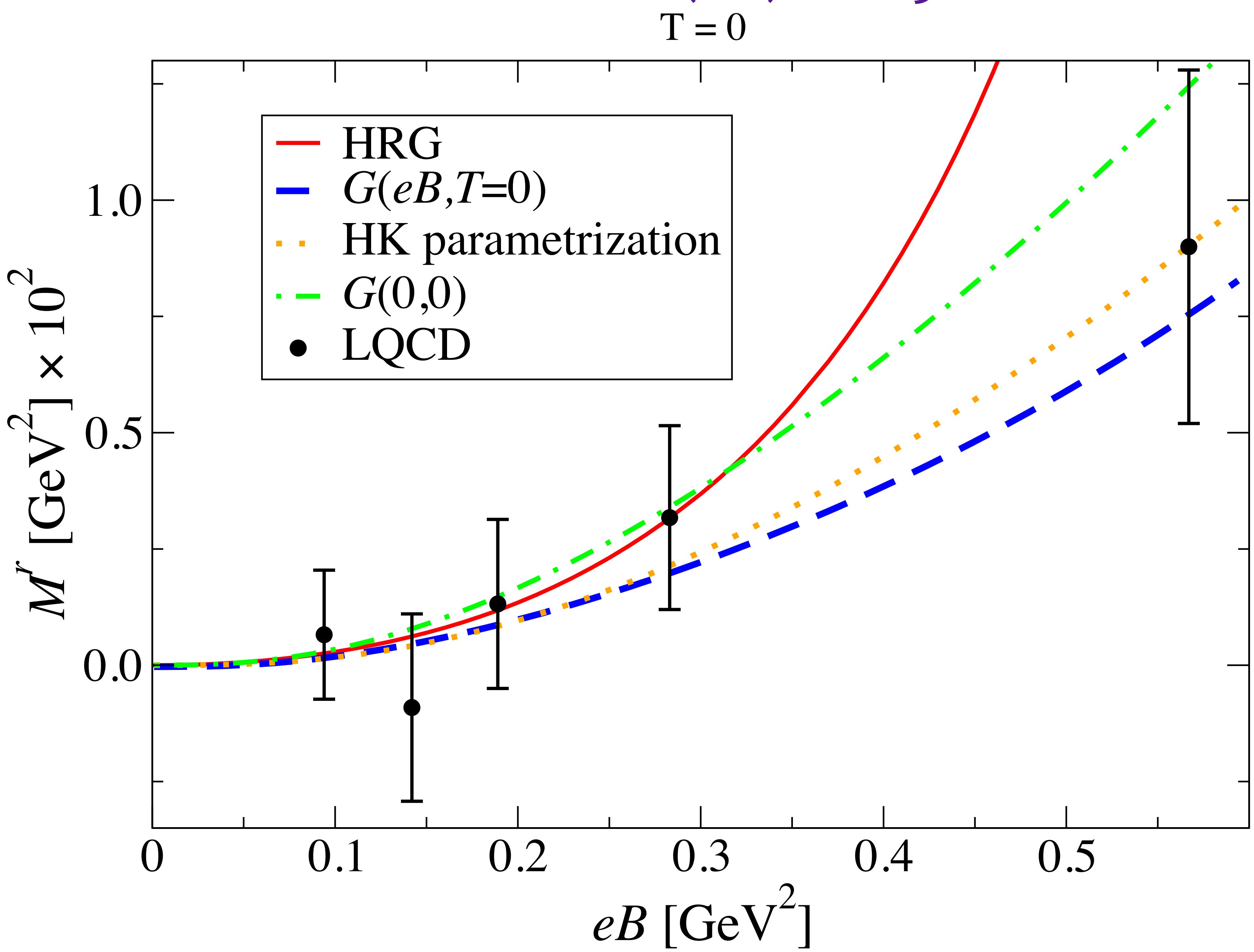
Magnetization: MFIR X VMR in SU(3)



HK parametrization, T. Hatsuda and T. Kunihiro, Phys. Rep. **247**, 221 (1994)

RLSF, W.R.Tavares, S.S. Avancini, M.B.Pinto, V.S.Timoteo and G. Krein, Eur. Phys.J.A (2021) 57:278

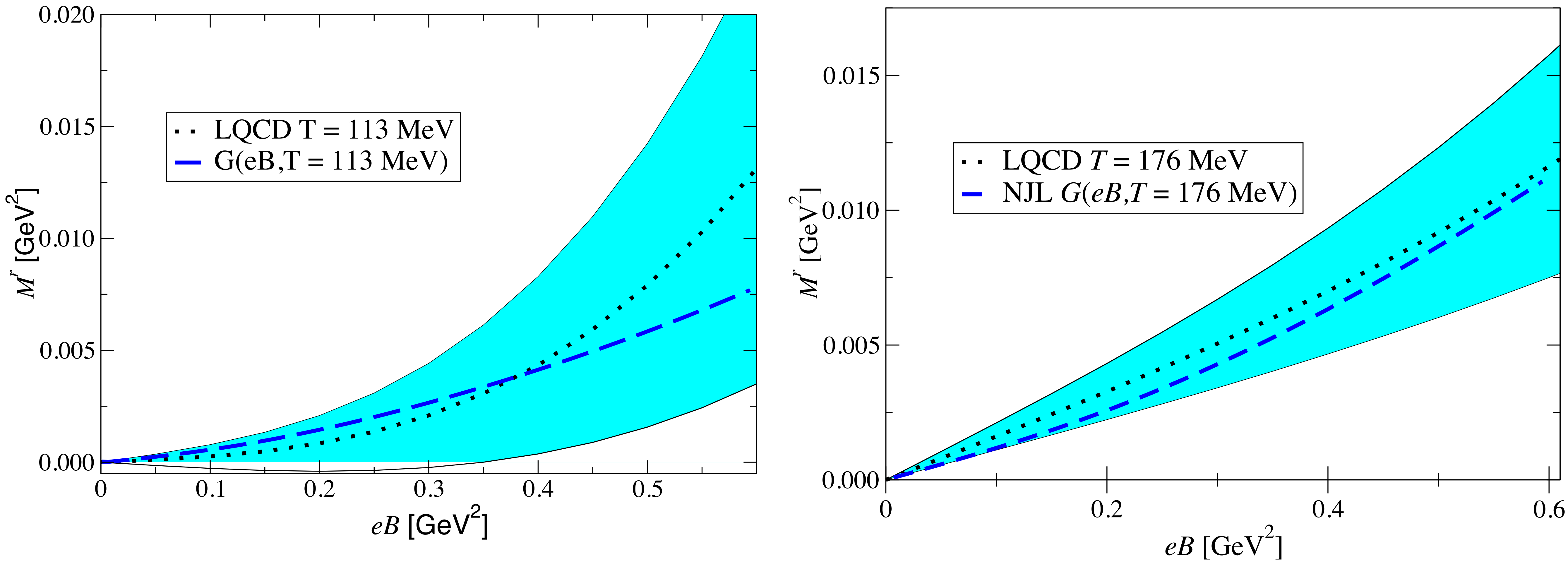
Renormalized magnetization: VMR in SU(3) NJL



Lattice data, G. Bali, F. Bruckmann, G. Endrodi, F. Gruber, and A. Schaefer, JHEP 04, 130 (2013).

RLSF, W.R.Tavares, S.S. Avancini, M.B.Pinto, V.S.Timoteo and G. Krein, Eur. Phys.J.A (2021) 57:278

Renormalized magnetization: VMR in SU(3) NJL



Lattice data, G. S. Bali, F. Bruckmann, G. Endrodi, and A. Schafer, Phys. Rev. Lett. **112**, 042301 (2014).

RLSF, W.R.Tavares, S.S. Avancini, M.B.Pinto, V.S.Timoteo and G. Krein, Eur. Phys.J.A (2021) 57:278

SU(2) NJL + AMM

$$\mathcal{L} = \bar{\psi}\left(i\not{D} - \hat{m} + \frac{1}{2}\hat{a}\sigma^{\mu\nu}F^{\mu\nu}\right)\psi$$
$$+G\left[(\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma_5\vec{\tau}\psi)^2\right], \qquad (1)$$

where A^μ, $F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$ are respectively the electromagnetic gauge and tensor fields, G represents the coupling constant, $\vec{\tau}$ are isospin Pauli matrices, Q is the diagonal quark charge [1] matrix, Q=diag(q_u= 2/3, q_d=-1/3), $D^\mu = (\partial^\mu + ieQA^\mu)$ is the covariant derivative, ψ is the quark fermion field, and $\hat{m}$ represents the bare quark mass matrix,

$$\psi = \begin{pmatrix} \psi_u \\ \psi_d \end{pmatrix}, \quad \hat{m} = \begin{pmatrix} m_u & 0 \\ 0 & m_d \end{pmatrix}. \qquad (2)$$

We consider here m_u=$m_d = m$ and choose the Landau gauge, $A^\mu = \delta_{\mu 2}x_1 B$, which satisfies $\nabla \cdot \vec{A} = 0$ and $\nabla \times \vec{A} = \vec{B} = B\hat{e}_3$, i. e., resulting in a constant magnetic field in the z-direction.

Phys. Rev. D 99 (2019) 11, 116025; Phys. Rev D 90, 105030 (2014)

SU(2) NJL + AMM

In the mean field approximation, the lagrangian $\mathcal{L}$ is denoted by

$$\mathcal{L} = \bar{\psi}\left(i\not{D} - M + \frac{1}{2}\hat{a}\sigma_{\mu\nu}F^{\mu\nu}\right)\psi - \frac{(M-m)^2}{4G}, \quad (3)$$

where the constituent quark mass is defined by

$$M = m - 2G\left\langle\bar{\psi}\psi\right\rangle. \quad (4)$$

where $\langle\bar{\psi}\psi\rangle$ is the chiral quark condensate. The AMM factor $\hat{a}$ is given by $\hat{a}$=diag(a_u, a_d) with $a_f = q_f\alpha_f\mu_B$, where $f = u, d$ represents the quark flavor. In the one-loop level approximation, the previous quantities are given by

$$\alpha_f = \frac{\alpha_e q_f^2}{2\pi}, \quad \alpha_e = \frac{1}{137}, \quad \mu_B = \frac{e}{2M}.$$

$$\kappa_f = \frac{\alpha_f}{2M}$$

Phys. Rev. D 99 (2019) 11, 116025; Phys. Rev D 90, 105030 (2014)

SU(2) NJL + AMM

$$\Omega = \frac{(M-m)^2}{4G} - N_c \sum_f \frac{|e_f B|}{\beta} \sum_{n=0}^{\infty} \sum_{s\in\{\pm 1\}} \int_{-\infty}^{\infty} \frac{dp_z}{4\pi^2} \{\beta E_{nfs} - \ln(1-n^+) - \ln(1-n^-)\}$$

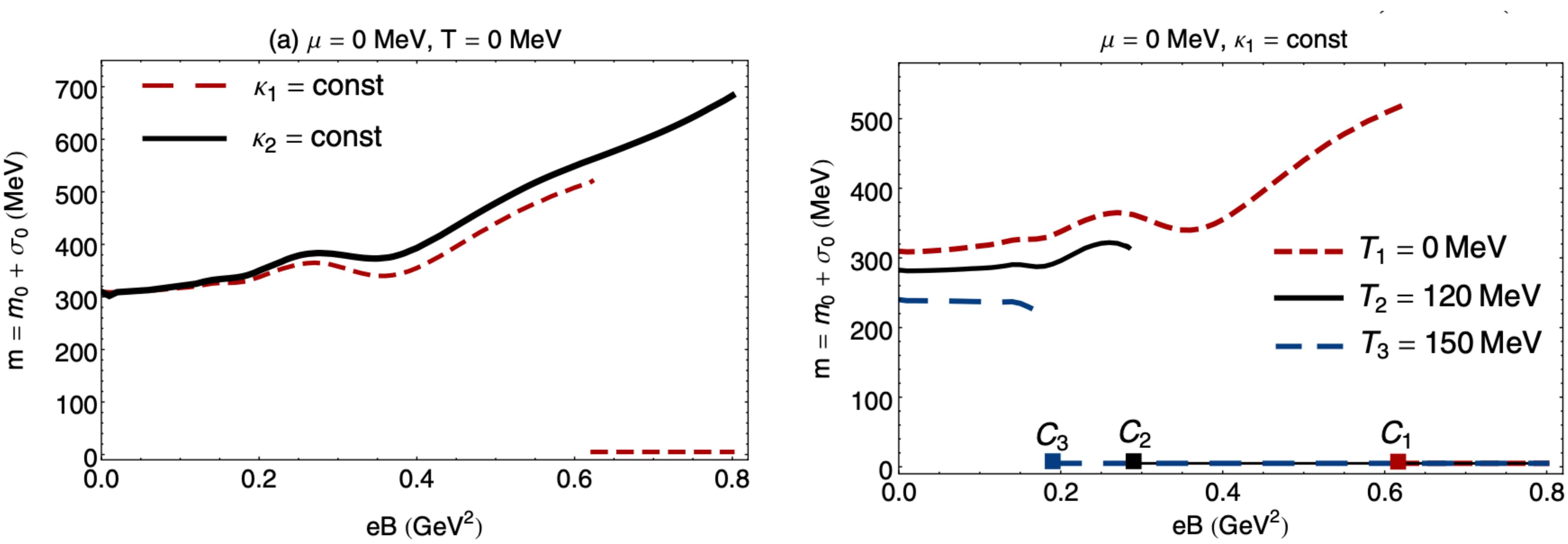
$$E_{nfs} = \left[p_z^2 + \left\{\left(\sqrt{|e_f B|(2n+1-s\xi_f) + M^2} - s\kappa_f e_f B\right)^2\right\}\right]^{\frac{1}{2}}$$

We need a regularization procedure!

Which procedure/method is more appropriate?

Phys. Rev. D 99 (2019) 11, 116025; Phys. Rev D 90, 105030 (2014)

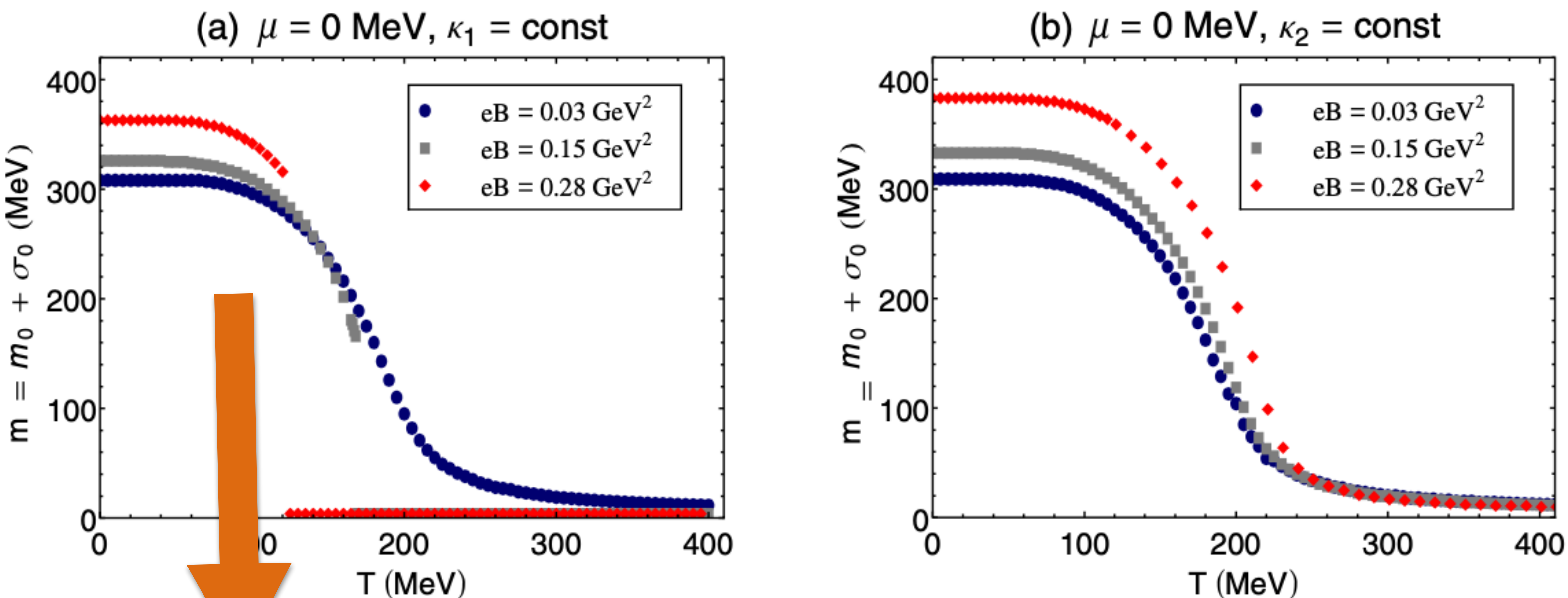
Non Physical oscillations and CP



$$f_{\Lambda,B}^{(p,s)} = \frac{1}{1+\exp\left(\frac{\sqrt{p_3^2+|q_f eBp|[2p+1-s\xi_f]}-\Lambda}{A}\right)}$$

Phys. Rev D 90, 105030 (2014)

AMM + chiral symmetry restoration

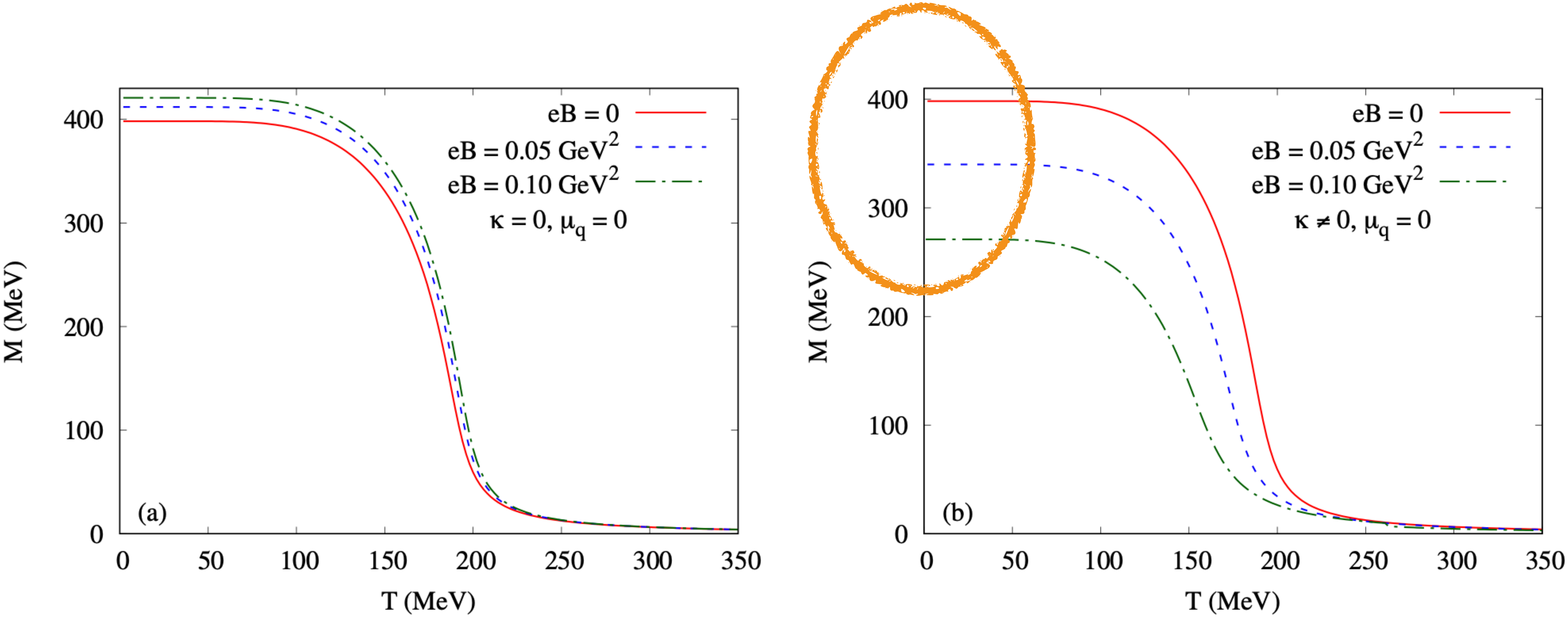


IMC, first order phase transition

$$f_{\Lambda,B}^{(p,s)} = \frac{1}{1 + \exp\left(\frac{\sqrt{p_3^2+|q_f eBp|[2p+1-s\xi_f]}-\Lambda}{A}\right)}$$

Phys. Rev D 90, 105030 (2014)

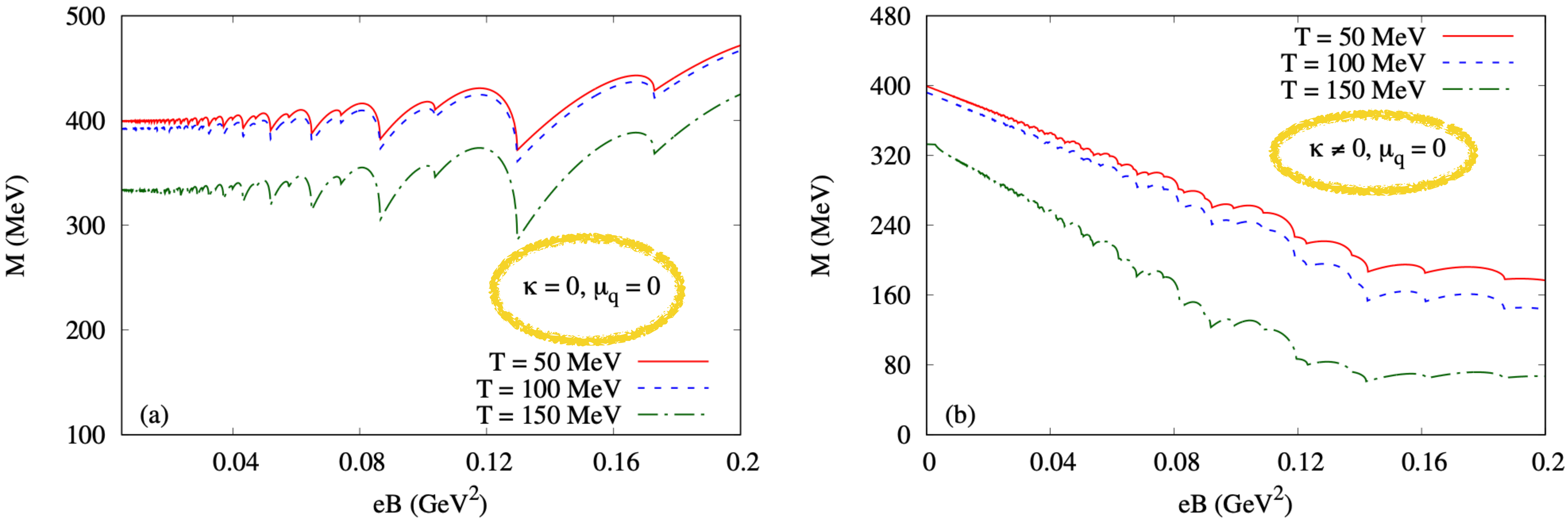
IMC at T=0 ?



$$\Lambda_z = \sqrt{\Lambda^2 - \vec{p}_\perp^{\,2}} \qquad \vec{p}_\perp^{\,2} = \left(\sqrt{\left|e_f B\right| (2n + 1 - s\xi_f) + M^2} - s\kappa_f e_f B\right)^2 - M^2$$

***Phys. Rev. D* 99 (2019) 11, 116025**

Non Physical oscillations

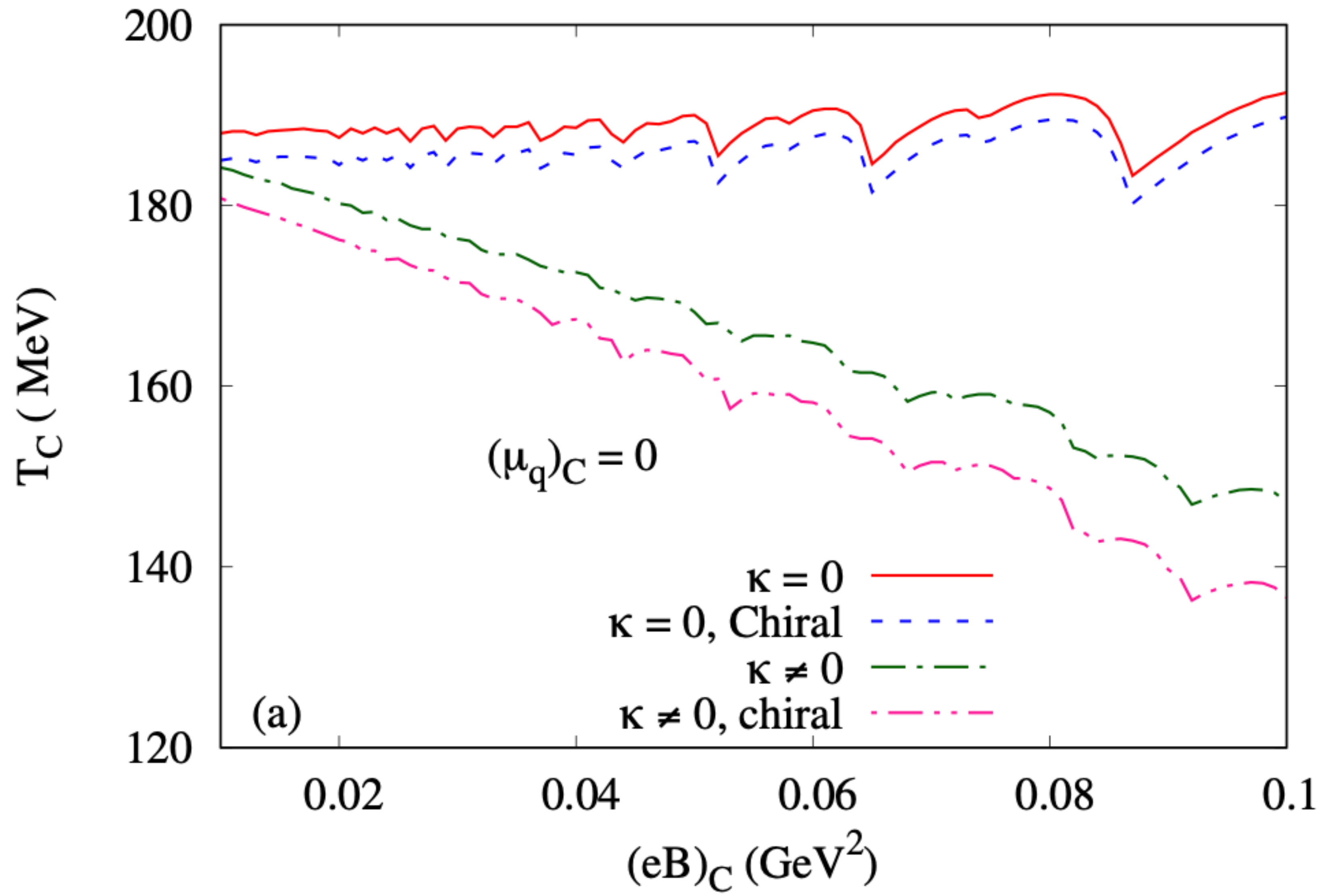


$$\Lambda_z = \sqrt{\Lambda^2 - \vec{p}_\perp^{\,2}}$$

$$\vec{p}_\perp^{\,2} = \left(\sqrt{\left|e_f B\right|(2n + 1 - s\xi_f) + M^2} - s\kappa_f e_f B\right)^2 - M^2$$

***Phys. Rev. D* 99 (2019) 11, 116025**

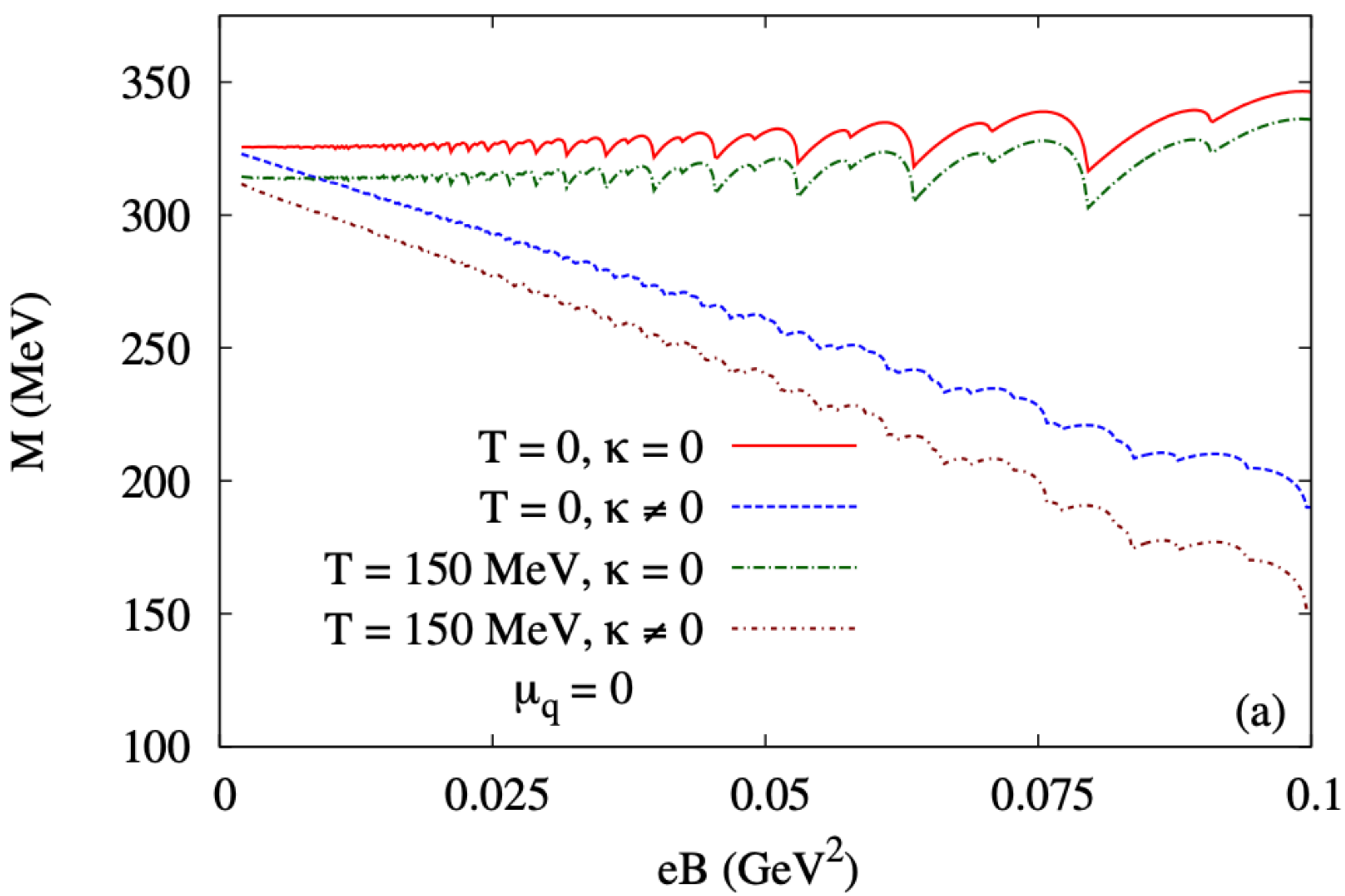
Non Physical oscillations



$$\Lambda_z = \sqrt{\Lambda^2 - \vec{p}_\perp^2} \qquad \vec{p}_\perp^2 = \left(\sqrt{|e_f B|(2n + 1 - s\xi_f) + M^2} - s\kappa_f e_f B\right)^2 - M^2$$

Phys. Rev. D **99 (2019) 11, 116025**

Non Physical oscillations



Same problem in SU(2) PNJL

***Eur.Phys.J.A* 56 (2020) 8, 213**

We and other authors in the literature have shown in several works that these **nonphysical oscillations disappear** when the divergent terms are disentangled from the pure magnetic contributions by using the MFIR/VMR schemes:

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[3] S.S. Avancini, W.R. Tavares, M.B. Pinto, Phys. Rev. D 93(1), 014010 (2016).

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[18] M. Coppola, P. Allen, A. Grunfeld, N. Scoccola, Phys. Rev. D 96(5), 056013 (2017).
[19] D.C. Duarte, **R.L.S. Farias**, P.H. Manso, R.O. Ramos, J. Phys. Conf. Ser. 706(5), 052010 (2016).

…

We adapt the non- regularized QED effective lagrangian in a constant external magnetic field with the AMM of the electron, in the one-loop approximation, to the SU(2) NJL model.

$$\mathcal{L}(B) = \sum_{f=u,d} \frac{N_c}{8\pi^2} \int_0^\infty \frac{ds}{s^3} e^{-i\mathcal{K}_{0f}^2 s} \frac{q_f eBs}{\sin(q_f eBs)} \times \cos(2M\eta_f Bs),$$

where we have adopted the following definitions

$$\mathcal{K}_{0f}^2 = M^2 + a_f^2 B^2,$$
$$\eta_f = -q_f(\alpha_f + 1)\mu_B.$$

$k_u^{[1]} = 0.29016\ \mathrm{GeV}^{-1}$, $k_d^{[1]} = 0.35986\ \mathrm{GeV}^{-1}$,
$\alpha_u^{[1]} = 0.242$, $\alpha_d^{[1]} = 0.304$,

and the second set $\kappa^{[2]}$ is given by

$k_u^{[2]} = 0.00995\ \mathrm{GeV}^{-1}$, $k_d^{[2]} = 0.07975\ \mathrm{GeV}^{-1}$,
$\alpha_u^{[2]} = 0.006$, $\alpha_d^{[2]} = 0.056$,

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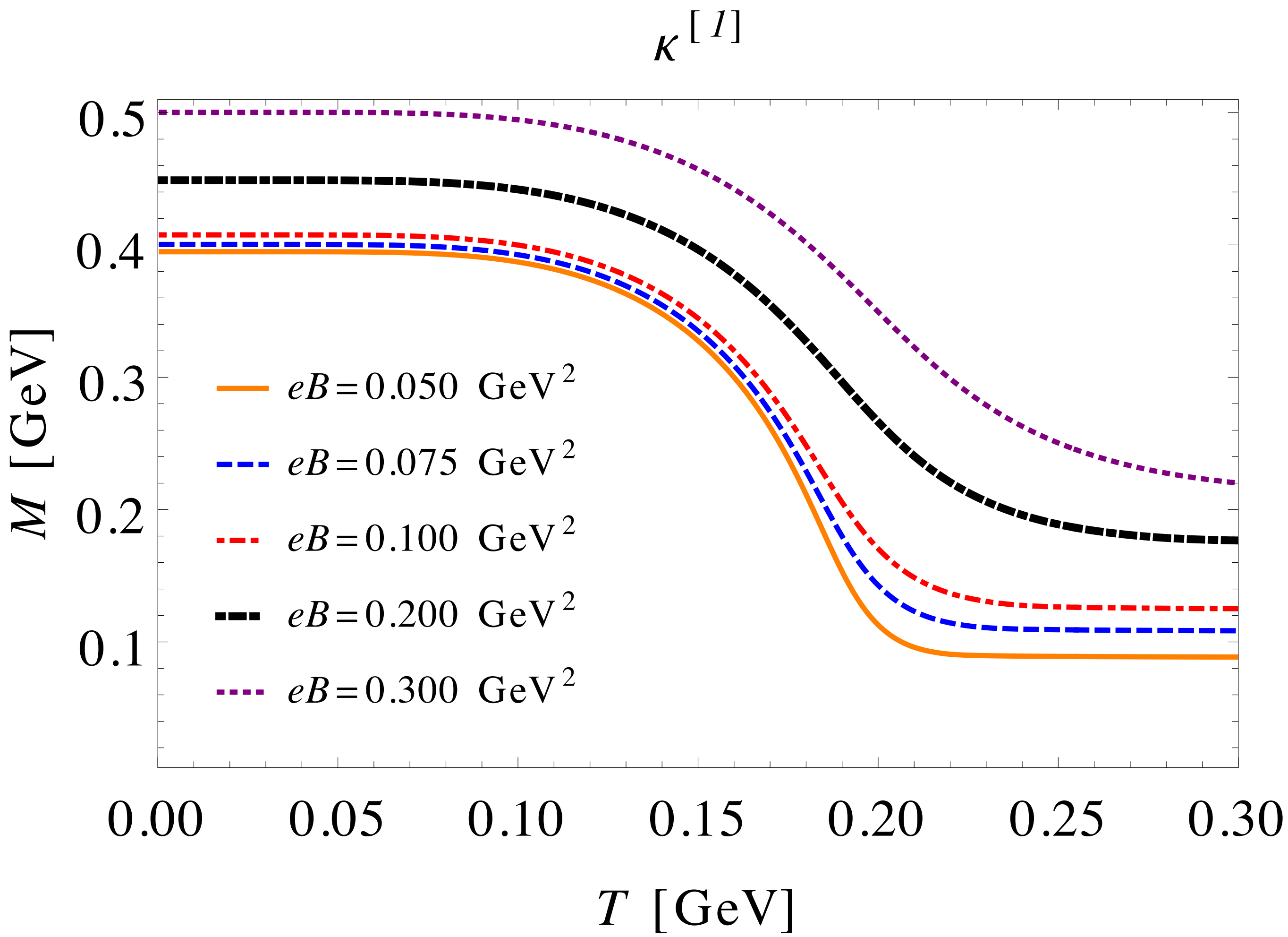
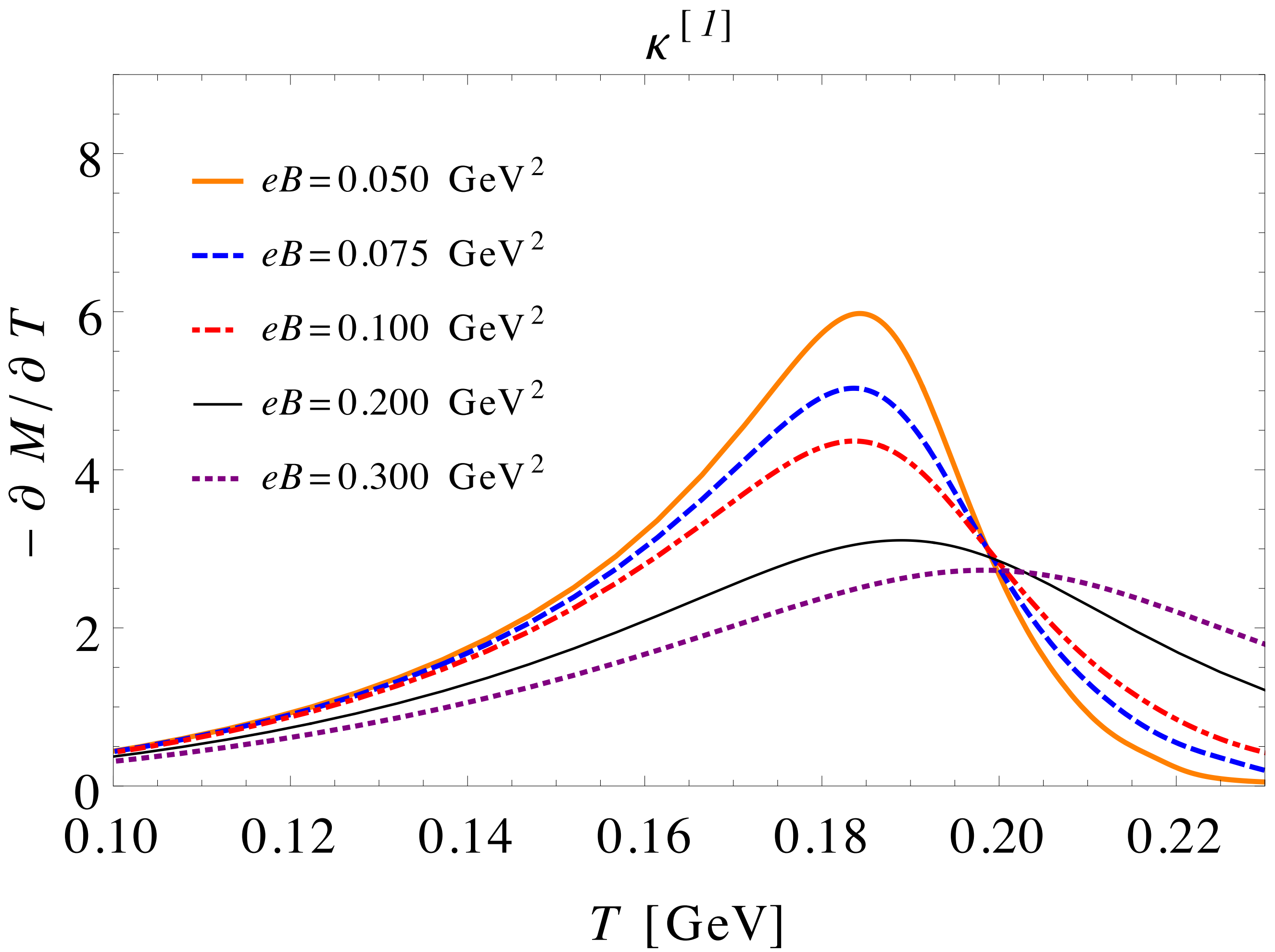
Eur. Phys. J. C (2022) 82:674

AMM => IMC or MC ?





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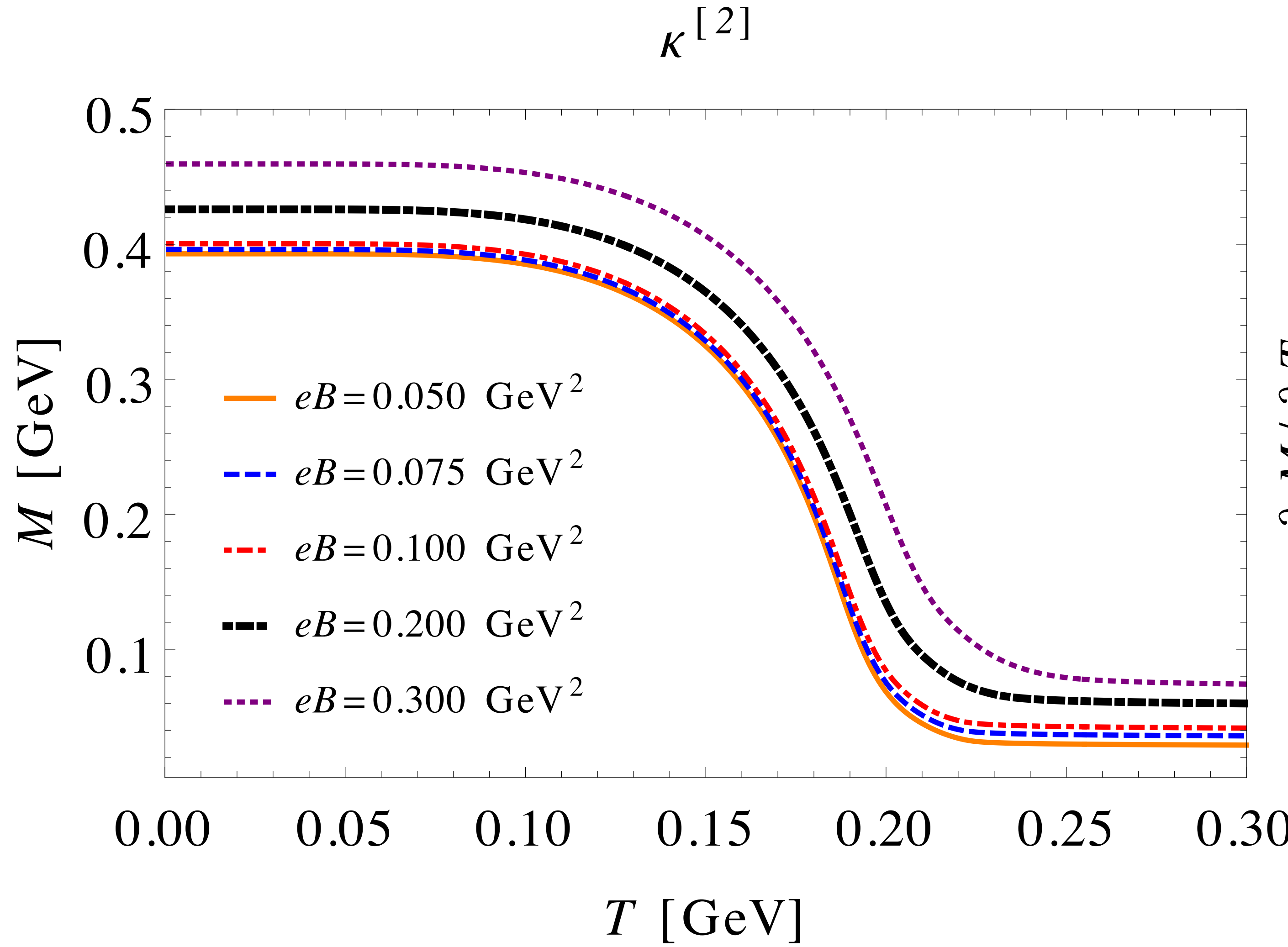

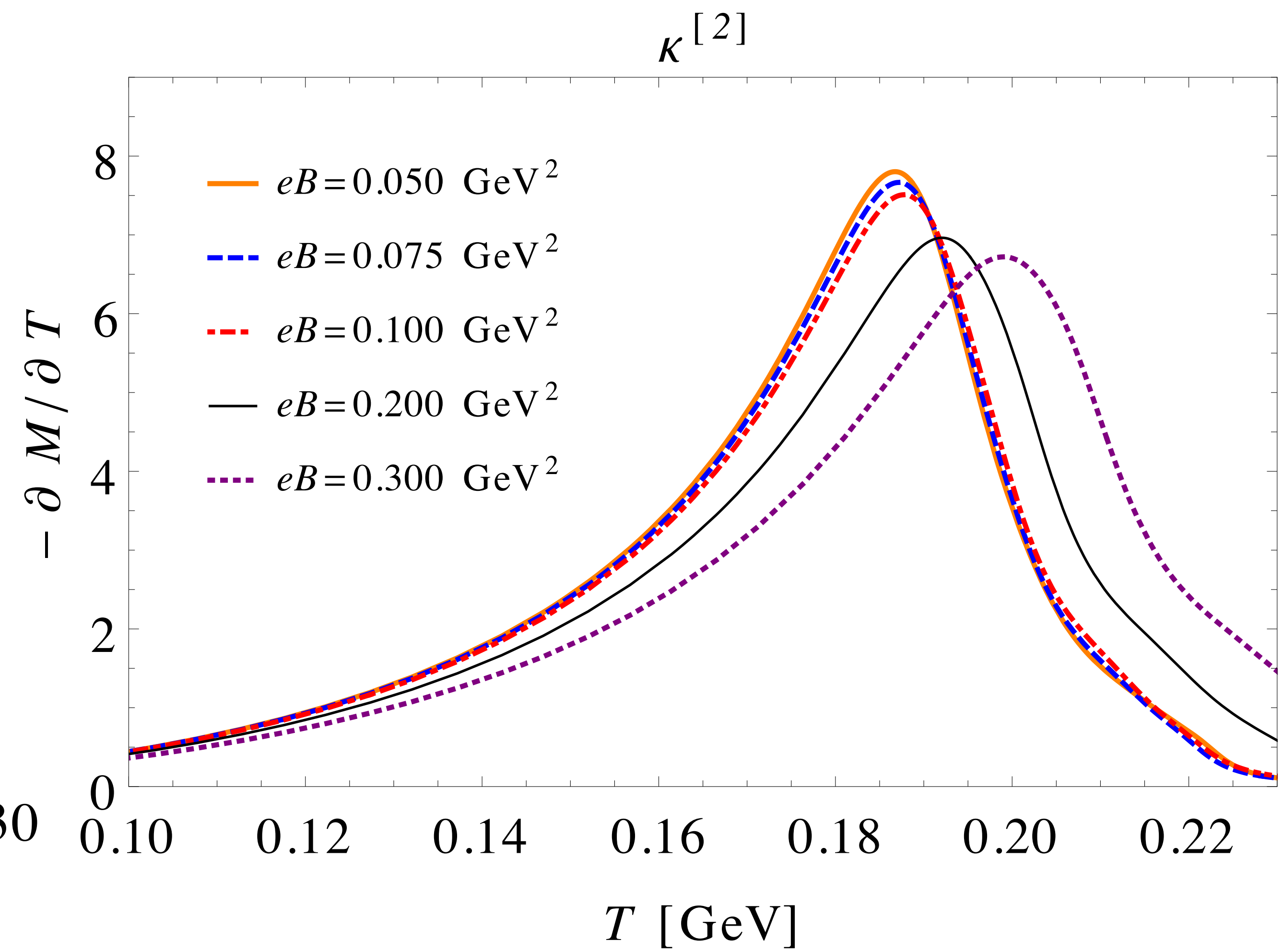

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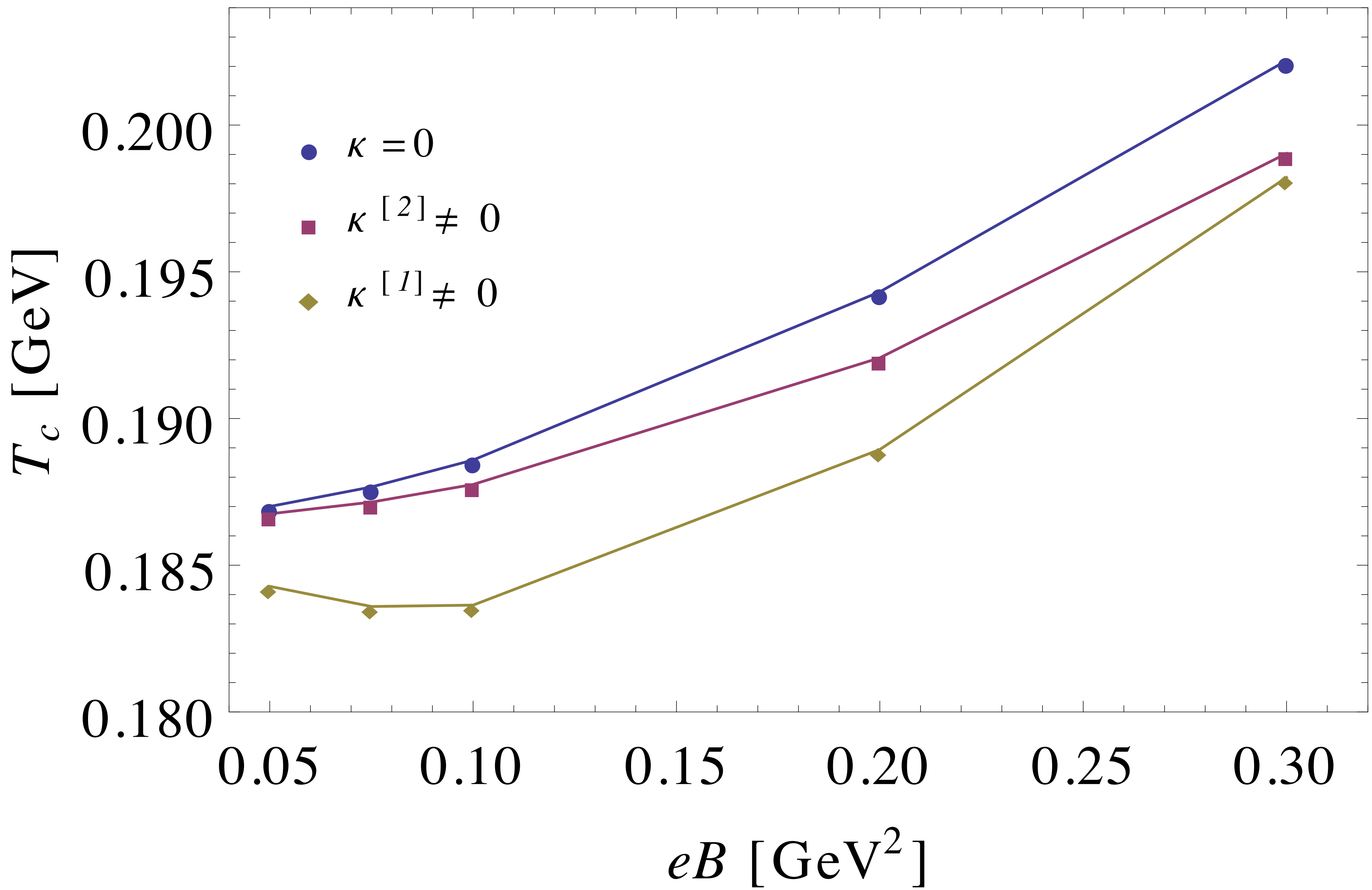
AMM => IMC or MC ?



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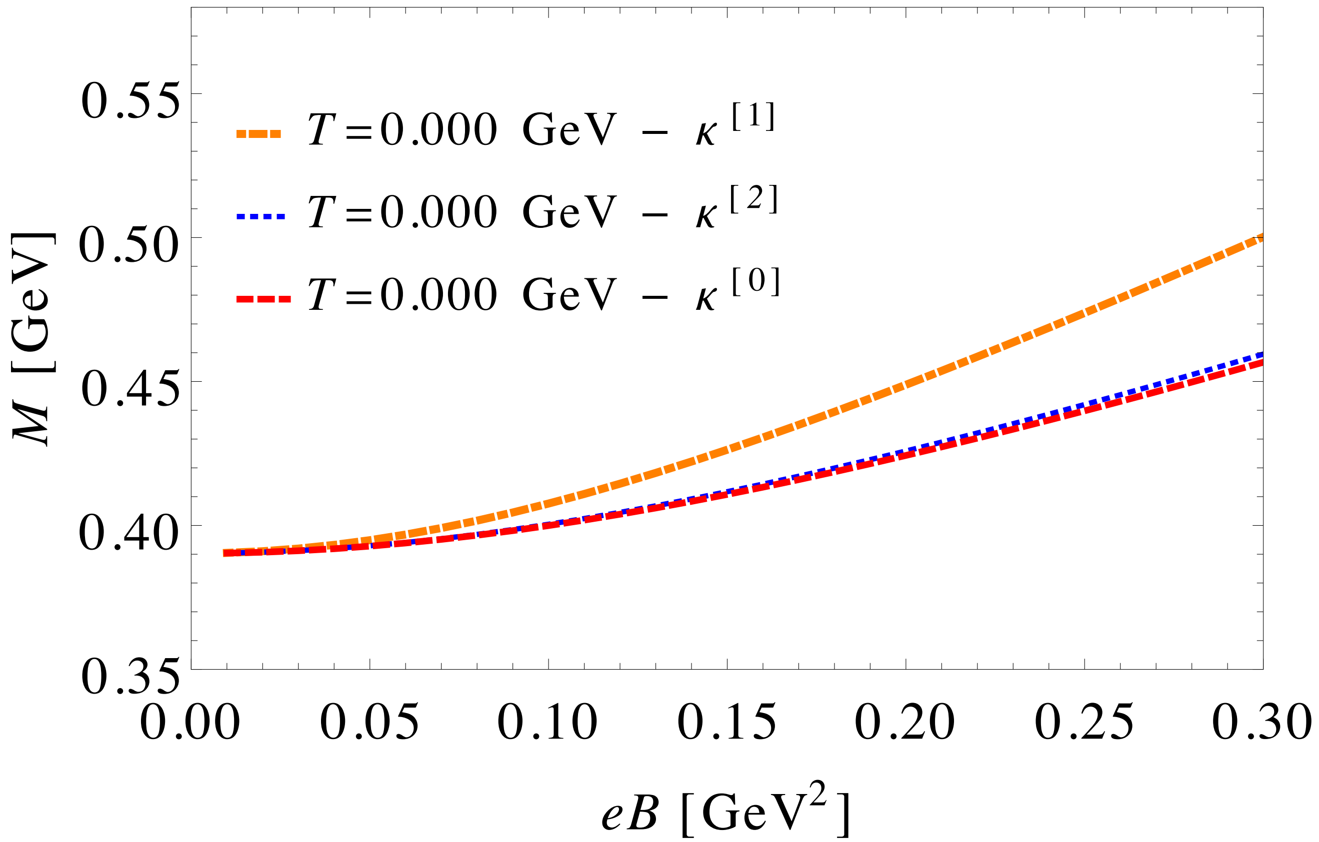
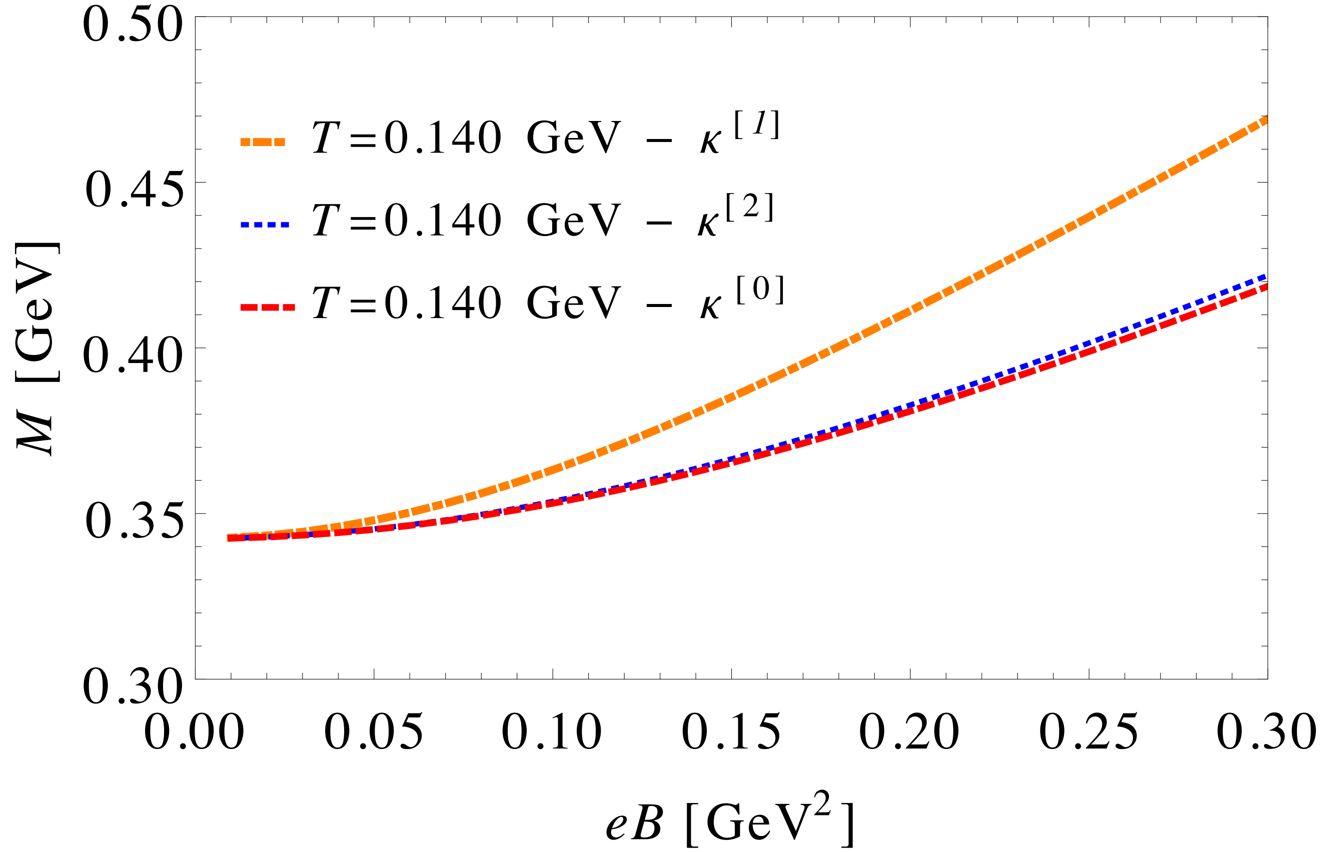
We avoid non Physical oscillations





NO first order phase transition!

Ricardo L.S. Farias, William R. Tavares, Rodrigo M. Nunes, Sidney S. Avancini,
Eur. Phys. J. C (2022) 82:674

NJL + AMM -> First order phase transition?

- **AMM proportional to quark condensate!**

  e-Print: 2205.08169 [hep-ph]

- **Effective photon-quark-antiquark vertex function: very small AMM**

  Phys.Rev.D **103** (2021) 116008

- ** **We found the explanation for this first order phase transition!**

  ** **William R. Tavares, Sidney S. Avancini, Rafael P. Cardoso and RLSF, in preparation...**

Violation of Goldstone Theorem

NJL + VMR + B + T + AMM:

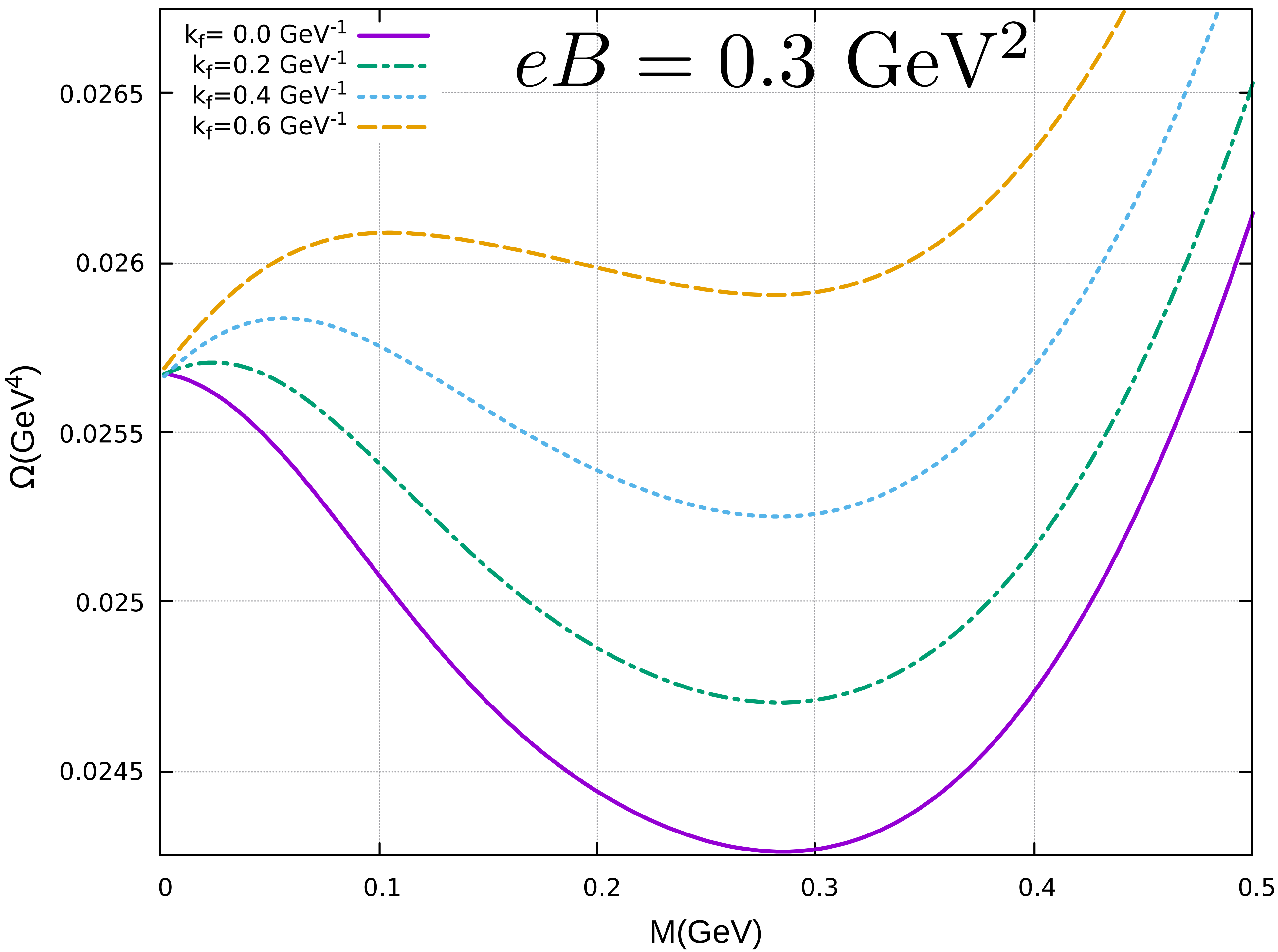
$$m_\pi^2(B) = -\frac{m_0}{M(B)} \frac{(2\pi)^3}{\sum_{n=0} g_n \sum_{q=u,d} i2G\beta_q N_c I_n(m_\pi^2)}$$ **+ adicional terms**

These violating terms can be removed with an appropriated expansion:

$$\frac{|q_f e B_f|}{M^2} \ll 1$$

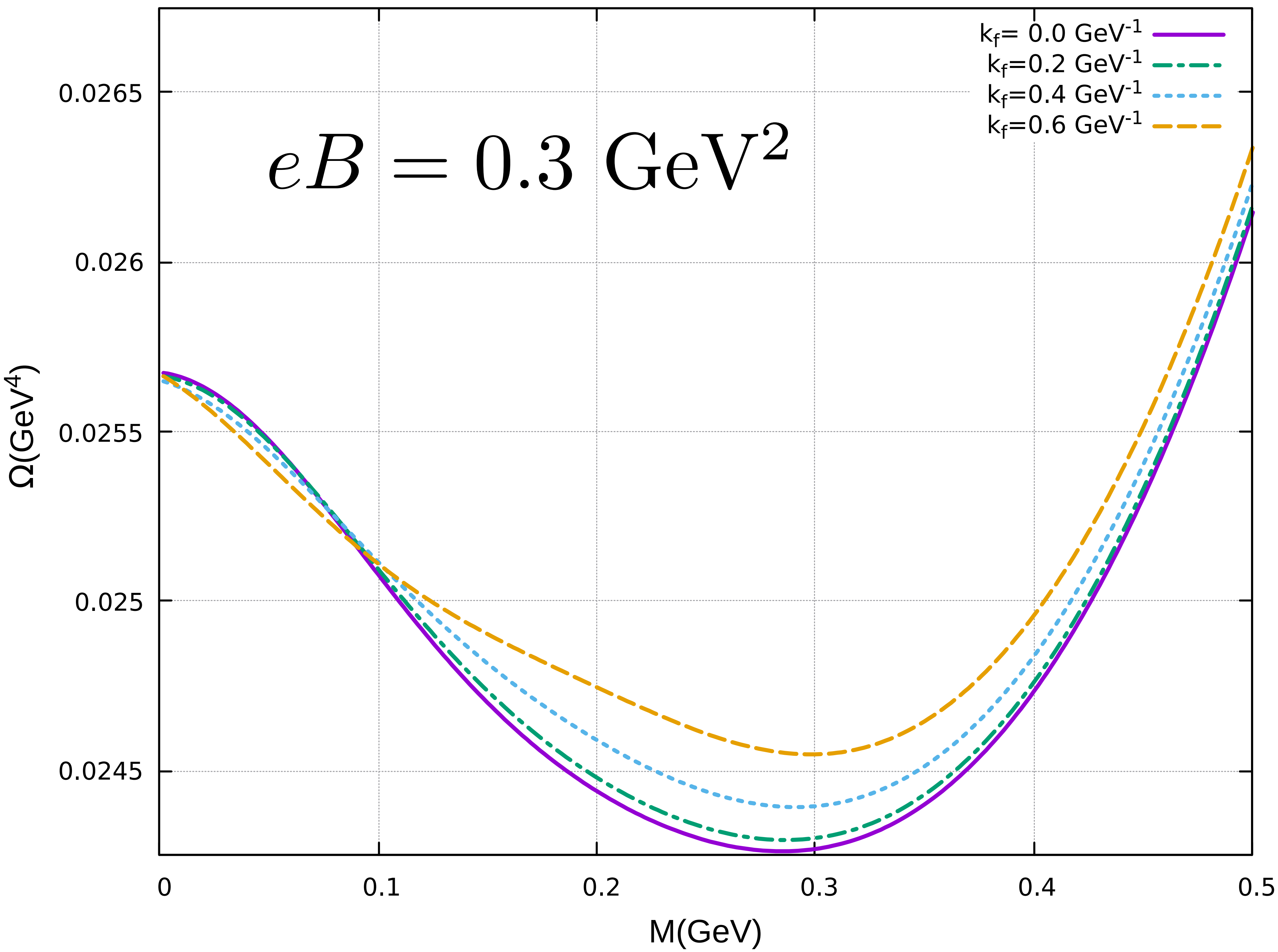
William R. Tavares, Sidney S. Avancini, Rafael P. Cardoso and RLSFarias, in preparation…

Violation of Goldstone Theorem



William R. Tavares, Sidney S. Avancini, Rafael P. Cardoso and RLSFarias, in preparation…

Goldstone Theorem satisfied!



William R. Tavares, Sidney S. Avancini, Rafael P. Cardoso and RLSFarias, in preparation…

Conclusions

✓ The thermo-magnetic dependence of G(B,T) is obtained by fitting lattice QCD predictions for the chiral transition order parameter

✓ **MFIR/VMR scheme avoid some unphysical results**, and this choice of regularization provide to us some different results from most of the regularizations prescriptions of the current literature.

✓ SU(3) NJL model + G(eB,T) is in agreement with lattice simulations: indicating a paramagnetic behavior for the QCD vacuum

Conclusions

- ✓ For a sizable value of κ, we observe a smoothly decrease of $T_{\rm pc}$ for the magnetic field region: $eB <$ 0.1 GeV^2.
- ✓ Our results also shows that the $T_{\rm pc}$ is always bigger in the case of $\kappa = 0$ than the finite κ case.
- ✓ Therefore, one main effect of the AMM is to decrease the values of $T_{\rm pc}$ as we increase the value of κ

Conclusions

- ✓ The magnetic catalysis also holds for low temperatures with no oscillations, which is expected in the MFIR or VMR inspired regularization procedures

- ✓ This work can clarify that these non-physical oscillations are an artifact of some regularization prescriptions that entangled the magnetic medium with the vacuum. Furthermore, these oscillations cannot be confused with de Haas-van Alphen oscillations.

- ✓ Our results when we consider nonvanishing quark AMM show that chiral symmetry restoration happens always as a smooth crossover and never turns into a first order phase transition.

Perspectives

✓ AMM effects in the thermodynamical properties of magnetized, ***dense*** and hot quark matter with VMR scheme

✓ PNJL + AMM + VMR

✓ Thermo-magnetic effects in AMM: $\kappa_f = \kappa_f(eB, T)$

Thank you for your attention!