Statistical interaction for quasi-particles

Some topological content

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ICTP-SAIFR:

Electromagnetic effects in Strongly interacting matter

Table of Contents

Table of Contents

General remarks

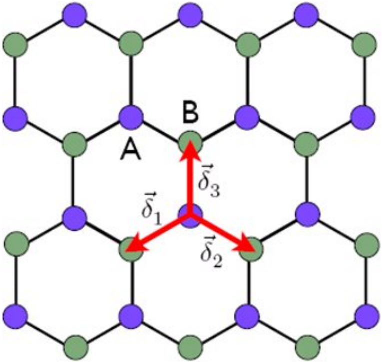
- Some materials such as Graphene, Edge states in HgTe/CdTe quantum wells, Silicene and Transition metal dichalcogenides (TMD), can be described by means of a two band model.
- Their low energy regime is described by Massless/Massive Lorentz broken modified Dirac equation.

General remarks

- There is also a recent realization of Haldane model in *Fe*-based honeycomb ferromagnetic insulators.
- Each Brilouin zone presents some points of minimal energy gap in the lattice.
- Expanding the solution near these points leads to a Dirac-like model with Lorentz breaking.
- The solutions around these points define the so-called Valleys. QUASI-PARTICLES.
- CAN BE USED IN TECHNOLOGY. The so-called valleytronics!
- May present robust topological states. Useful to store information in quantum computing.

General remarks

- The graphene is a two-dimensional material composed by carbon atoms.
- Valence electrons in a sp^2 hibridization and the remaining one is in a p_z orbital. We consider just the interaction between the NON-equivalent A and B sites. E. C. MARINO
- The sp^2 orbitals: Planar and form angles of 120^0 between them.



General remarks

- The crystal basis is defined as $\delta_3 = \frac{a}{\sqrt{3}} e_y$, $\delta_1 = -\frac{a}{\sqrt{3}}(\frac{\sqrt{3}}{2} e_x + \frac{1}{2} e_y)$ and $\delta_2 = \frac{a}{\sqrt{3}}(\frac{\sqrt{3}}{2} e_x - \frac{1}{2} e_y)$,
- With $\frac{a}{\sqrt{3}} = 0,142nm$, which connect one basis element to its first neighbors.
- The Hamiltonian presents a tight binding structure in which the p_z electrons are governed by the Hamiltonian (WALLACE 1942)

$$H = -t \sum_{R;i=1,2,3;\ \sigma=\pm} \left(C_B^{\dagger}(R + \delta_i, \sigma) C_A(R, \sigma) + H.C. \right) \qquad (1)$$

- t Has ORDER $3e.v$ GOERBIG

Tight binding Hamiltonian

- The Hamiltonian can be rewritten, in momentum space, as

$$\begin{aligned} H &= \sum_{k,\sigma} \big(-tC_A(k,\sigma) C_B^\dagger(k,\sigma) \sum_i e^{ik.\delta_i} + H.C. \big) \\ &= \sum_{k,\sigma} \psi^\dagger(k,\sigma) \begin{pmatrix} 0 & \phi \\ \phi^* & 0 \end{pmatrix} \psi(k,\sigma) \end{aligned} \tag{2}$$

- With $\phi(k) = -t \sum_l e^{ik.\delta_l}$ and $\psi^\dagger(k,\sigma) \equiv \big(C_A^\dagger(k,\sigma), C_B^\dagger(k,\sigma)\big)$.
- The energy eigenvalues define the two band structure

$$E(k) = \pm|\phi(k)| = \pm \sqrt{\sum_{l,j} e^{ik.(\delta_l - \delta_j)}} \tag{3}$$

General remarks

- Brillouin zone corners: $K_{\pm} = \pm\frac{4\pi e_x}{3a}$. Low energy: $\phi(K_+) = \phi(K_-) = 1 + e^{i\frac{2\pi}{3}} + e^{-i\frac{2\pi}{3}} = 0$.
- Near the Dirac points $K = K_{\pm} + p$ which, after a canonical transformation

$$H_{K_+} = \psi^{\dagger}_{K_+}(k,\sigma) h_{K_+} \psi_{K_+}(k,\sigma) \quad (4)$$

$$H_{K_-} = \psi^{\dagger}_{K_-}(k,\sigma) h_{K_-} \psi_{K_-}(k,\sigma) \quad (5)$$

with Dirac-like Hamiltonian

$$h_{K_+} = v\left(p_x\sigma_x + p_y\sigma_y\right) + \mathcal{O}(p^2) \quad (6)$$

- Effective velocity $v = \frac{\sqrt{3}ta}{2} \sim \frac{c}{300}$ for graphene
- Material dependent (t, a, v) Lorentz Breaking.
- Quasi-particles on the valleys $K_{\pm}$.

Dirac Cones in Brillouin zone

- Accurate description until the cut-off

$$\Lambda < \frac{hv}{a} \tag{7}$$

- **v is the fermi-velocity and a is associated to the lattice size**

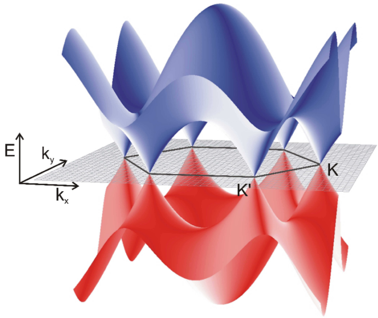


Figure: Low energy points for graphene-like model

General remarks

- If the symmetry between the A and B of a graphene-like lattice sites is broken, a staggered local energy arises leading to an energy gap.
- Arises due to: Spin-orbit interaction OR different atoms in A and B lattices.
- This is the case for(TMD) materials. It presents a strong electron-hole interaction.
- Monoatom: For Silicene $|m_{\pm}| \sim 1,55 meV$ and for Germanene $|m_{\pm}| \sim 23,9 meV$.
- It implies in an additional mass term $m_{K_{\pm}} = \pm m$ for the quasi-particles. It does not violate discrete symmetries.

General remarks

- The recent experimentally realized Haldane model consists of this kind of lattice with an additional zero averaged inter-plaquete magnetic flux.
- HALDANE: $|m_{K_+}| \neq |m_{K_-}|$ is allowed (may violate discrete symmetries.).
- Associated to non-vanishing MAGNETIC flux scenario.

Current-current interactions in discrete symmetry breaking scenario ($\vec{B}_{ext} \neq 0$)

- $\bar{\psi}_A \gamma_\mu \psi_A = J_A$
- For now on:$\hbar = c = 1$. Then, $v < 1$ and dimensionless.
- Statistical-1:

$$\mathcal{L} \to \mathcal{L} - i \frac{e^2}{2K} \frac{\left(J_1^\mu + J_2^\mu\right) \epsilon_{\mu\alpha\nu} \partial^\alpha \left(J_1^\nu + J_2^\nu\right)}{\Box} \tag{8}$$

- J_A^μ, with $A = 1, 2$ denote the valley current operators.
- Statistical-2:

$$\mathcal{L} \to \mathcal{L} - i \sum_A \frac{e^2}{2K} (-1)^A \left(J_A^\mu\right) \frac{\epsilon_{\mu\alpha\nu}}{\Box} \partial^\alpha \left(J_A^\nu\right) \tag{9}$$

.

- Statistical-3:

$$\mathcal{L} \to \mathcal{L} - i \frac{e^2}{2K} \left(J_1^\mu\right) \frac{\epsilon_{\mu\alpha\nu}}{\Box} \partial^\alpha \left(J_2^\nu\right) \tag{10}$$

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Table of Contents

Statistical interaction for quasi-particles

- Our model includes all the mentioned models for quasi-particles excitations. ZHANG
- We investigate a non-trivial statistical interaction between them. It can be written in terms of a gauge field.
- TYPE-1 STATISTICAL INTERACTION!
- Bidimensional representation: $(\gamma_0 = \sigma_3, \gamma_1 = i\sigma_1, \gamma_2 = i\sigma_2)$.

$$\mathcal{L} = \frac{\mathcal{K}}{2}\epsilon^{\mu\nu\beta}\mathcal{C}_\mu\partial_\nu\mathcal{C}_\beta + B\partial_\mu\mathcal{C}^\mu + \sum_{A=1}^{2}\left[i\bar{\psi}_A\tilde{\gamma}^\mu\partial_\mu\psi_A - m_A\bar{\psi}_A\psi_A\right]$$

$+\mathrm{eC}_\mu \sum_{A=1}^{2} \bar{\psi}_A\tilde{\gamma}^\mu\psi_A$

- Violates discrete symmetry. Can describe the effect of a magnetic impurity. With the vertex and propagator $(\tilde{k}_\mu = (k_0, v\vec{k}))$

$$\mathcal{C}_\mu\psi\tilde{\gamma}^\mu\psi = \mathcal{C}^\mu\left(\bar{\psi}\gamma_0\psi\delta^0_\mu + v\bar{\psi}\gamma_j\psi\delta^j_\mu\right) \quad ; \quad S^A(k) = \frac{i\left(\tilde{k} + m_A\right)}{\tilde{k}^2 - m_A^2} \quad (11)$$

- Material dependent Lorentz breaking due to v.

Statistical interaction for quasi-particles

- We consider the vacuum corrections and include the effect of the external magnetic field **a posteriori**, in the renormalized structure.
- By means of Peierls substitution
- $\hbar = c = 1$
- Chern Simons action is not gauge invariant.
- However, for $\mathcal{K} = e^2\nu/2\pi$, with $\nu = 1, 2, 3....$
- The quantum functional generator $\mathcal{Z}$ is indeed invariant
- **perturbative for big enough** ν
- It is also a motivation to apply the Nakanishi B -field non perturbative Heisenberg approach in this scenario

Statistical interaction for quasi-particles

- The bosonic self energies(relativistic $v = 1$):

$$\Pi^F_{\mu\nu}(k^2) = \sum_A \left(g_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right) \frac{e^2 k^2}{16\pi} \Big[\frac{1}{\sqrt{k^2}} (1 + \frac{4m_A^2}{k^2}) \ \log \frac{1 - \sqrt{k^2/4m_A^2}}{1 + \sqrt{k^2/4m_A^2}} \tag{12}$$

$$+ \frac{4m_A}{k^2} \Big] - i m_A \epsilon_{\mu\nu\alpha} k^\alpha \Big(\frac{e^2}{4\pi\sqrt{k^2}} \log \frac{1 - \sqrt{k^2/4m_A^2}}{1 + \sqrt{k^2/4m_A^2}} \Big)$$

The $\Pi^F_{[xy]}(k \to 0)$ **is related to a shift of the transverse generalized conductivity of the form** (ONE LOOP exactness even for $v \neq 1$ D.DUDAL)

$$\sigma_{xy} = \Big(\mathcal{K} + \frac{e^2}{4\pi} (sign(m_1) + sign(m_2)) \Big) \tag{13}$$

if one adds the two valleys with masses m_1 and m_2.

Lorentz violation

- For the case $v \neq 1$, one can use the same projectors as in thermal field theoryD.DUDAL and KAPUSTA

$$P_T^{ij} = \delta^{ij} - \frac{k^i k^j}{\vec{k}^2} \quad ; \quad P_L^{\mu\nu} = -\eta^{\mu\nu} + \frac{k^\mu k^\nu}{k^2} - P_T^{\mu\nu} \tag{14}$$

- It is possible to show that the polarization tensor can be written in terms of them as

$$\Pi_{\mu\nu}(\ v^2\vec{k}^2\ ) = F\ P_{L\ \mu\nu} + G\ P_{T\ \mu\nu} + mE_{\mu\nu}(\vec{k}^2)\sqrt{\vec{k}^2}\Pi^2(v^2\vec{k}^2) \tag{15}$$

Radiative corrections for $v \neq 1$

- We consider the static case in order to derive the potentials created due to a point charge.
- Then, we have

$$G = -\Pi_1(v\vec{k}, 0) = \sum_i \frac{e^2 v^2 \vec{k}^2}{16\pi|m_i|} \quad ; \quad F = -\frac{\Pi_1}{v^2}(v\vec{k}, 0) = \sum_i \frac{e^2 \vec{k}^2}{16\pi|m_i|} \tag{16}$$

- The conclusion regarding $\Pi_{[\mu\nu]}$ at low energies is kept.
- The Hall-like transport properties for LOW ENERGY **are independent of the microscopic details of the sample as the value of** v.
- Even for **Non-ISOTROPIC** case:

$$k_\mu = (k_0, v_1 k_1, v_2 k_2) \tag{17}$$

- THE CONCLUSION REGARDING THE TRANSVERSE CONDUCTIVITY IS KEPT!

The potential and the magnetic field due to test charge q

- Considering $v \neq 1$ velocity of these quasi particles, the broken Lorentz potentials due to a static charge are

$$< \mathcal{C}_0(r) >= \frac{q}{\tilde{n}v^4} K_0(m'.r)$$
$$= \frac{q}{\tilde{n}v^4} \frac{e^{-m'r}\pi^{1/2}}{\sqrt{2m'r}} \left[1 - \frac{1}{8m'r}\left(1 - \frac{9}{16m'r}(1 - \frac{25}{24m'r})\right)\right]$$

- **Anyons**: Path dependent Aharonov-Bohm effect:

$$\mathcal{B}_C(r) = \frac{q}{\tilde{\mathcal{K}}}\left[2\pi\delta^2(\vec{r}) - 2\pi\delta^2(\vec{r}) + {m'}^2 K_0(m'.r)\right] \qquad (18)$$

- Radiative Cancellation of delta-like behaviour

- with

$$\tilde{n} = -\frac{e^2}{16\pi}\left(\frac{1}{|m_1|} + \frac{1}{|m_2|}\right), c_i = \frac{e^2}{4\pi|m_i|}, m' = \left(\mathcal{K} + m_1 c_1' + m_2 c_2'\right)/\tilde{n}v^2 \tag{19}$$

and

$$\tilde{\mathcal{K}} \equiv \mathcal{K} + \frac{e^2}{4\pi}(sign(m_1) + sign(m_2)) \tag{20}$$

Statistical interaction for quasi-particles $v = 1$(YET.)!!

- The renormalized fermionic response at low energies One Valley:

$$\Sigma_F(p \to 0) = -\frac{e^2}{4\pi\mathcal{K}}\left(2m - \frac{p^\mu p_\mu}{|m|} + sign(m)\not{p}\right) \quad (21)$$

- **The latter leads to a fermionic valley with Chern number** (SHEN)

$$n_{chern} \propto F(\mathcal{S}^{-1}, \mathcal{S}, \partial_{k_i}\mathcal{S}, \partial_{k_i}\mathcal{S}^{-1})|_{E=0} \quad (22)$$

- Then, same qualitative topological properties as the SHEN model with just $\vec{p}^2$.

$$n_{chern} = \frac{1}{2}\left[sign\left[m_R\left(1 - \frac{e^2}{4\pi\mathcal{K}}sign(m_R)\right)\right] + sign\left(\frac{e^2}{4\pi\mathcal{K}|m_R|}\right)\right] \quad (23)$$

The model present localized boundary states if $n_{chern} \neq 0$.

Localized boundary states

- SHEN this general solution can be adapted to our case

$$AE\psi(p) = \left(\tilde{M}\sigma_z - C\left(p_x^2 - p_y^2\right)^2\sigma_z + A\left(\sigma_x p_x - i\sigma_y p_y\right)\right)\psi(p) \quad (24)$$

- **Extended states localized near the boundary**

$$\psi(p) = \begin{bmatrix} c(p_x) \\ d(p_x) \end{bmatrix} \left(e^{-y\Lambda_1} - e^{-y\Lambda_2}\right) \quad (25)$$

with $\vec{\sigma} = (\sigma_x, \sigma_y, \sigma_z)$ denoting the Pauli matrices. We are assuming the definition

$$\tilde{M} \equiv m_R\left(1 - \frac{e^2}{4\pi\mathcal{K}} sign(m_R)\right) + \frac{e^2 E^2}{4\pi\mathcal{K}|m_R|} \quad (26)$$

- Condition for existence of the solution

$$n_{chern} \neq 0 \rightarrow \Lambda_1\Lambda_2 > 0 \quad (27)$$

Localized boundary states

- For Discrete symmetry on matter sample:($m_1 = -m_2$, one valley becomes TOPOLOGICAL and the other just TRIVIAL.)
  TYPE 1 INTERACTION.
- For Discrete symmetry on matter sample:TYPE 2 INTERACTION: needs 2 auxiliary boson fields **WITH OPPOSITE normalization**.
- BOTH VALLEYS BECOME TOPOLOGICAL with the opposite drift velocity.
- TYPE-3 INTERACTION: generates NO Fermion mass correction. TRIVIAL TOPOLOGY. (CAN BE IMPLEMENTED VIA **BF** action:)

$$\mathcal{L} \sim \mathcal{K} B^{\mu} \epsilon_{\mu\alpha\nu} \partial^{\alpha} A^{\nu} + A_{\mu} J_1^{\mu} + B_{\mu} J_2^{\mu} \tag{28}$$

$T > 0$:Adding ghosts

- For $T > 0$, even decoupled Abelian ghosts are important
- In Landau gauge: $\Box\eta_{\mu\nu} = \epsilon_{\mu\beta\gamma}\partial^{\beta}\epsilon_{\gamma\alpha\nu}\partial^{\alpha}$. (On the space of transverse vector fields.)
- It means that free CS (No d.o.f,no Free Hamitonian.)photons have non-trivial partition function as well as true photons? But there are no observable asymptotic CS excitations with positive norm.
- **Remove it by means of decoupled bosonic and fermionic ghost sector**

$$\mathcal{L}_{ghost} = \bar{c}\Box c + \left(\phi\sqrt{\Box}\phi\right) \tag{29}$$

Then, for interacting theory, this gauge field contributes just for the radiative corrections of the partition function.

Overview and future objectives

- Recover the correct units
- Interesting to also achieve an $v \neq 1$ version for fermionic response.
- We performed something qualitatively "analogous" to LAMB shift .
- Vacuum effects for effective model for Hall effect on a given class of samples. for low energy.
- Perform Peierls SUBSTITUTION $p_\mu \to p_\mu - eA_\mu(x)$ to evaluate the effect of a non-zero external magnetic field in the RENORMALIZED vacuum corrected structure.
- Then, use standard approach to obtain the generalized Landau levels for this structure,