QCD pressure at finite temperature and high magnetic fields

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Workshop on Electromagnetic Effects in Strongly Interacting Matter

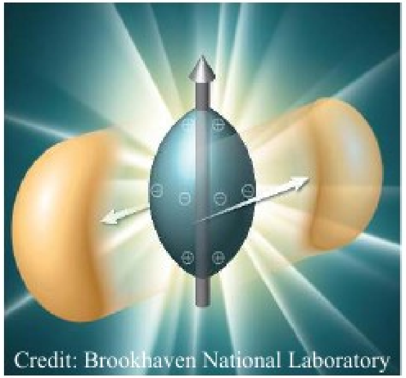
2022

Outline

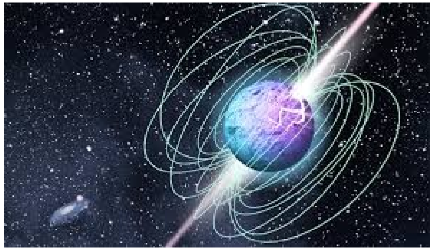
- Motivation
- Quark pressure at high B
- QCD coupling constant
- Results
- Conclusions

Strongly interacting matter at high magnetic fields

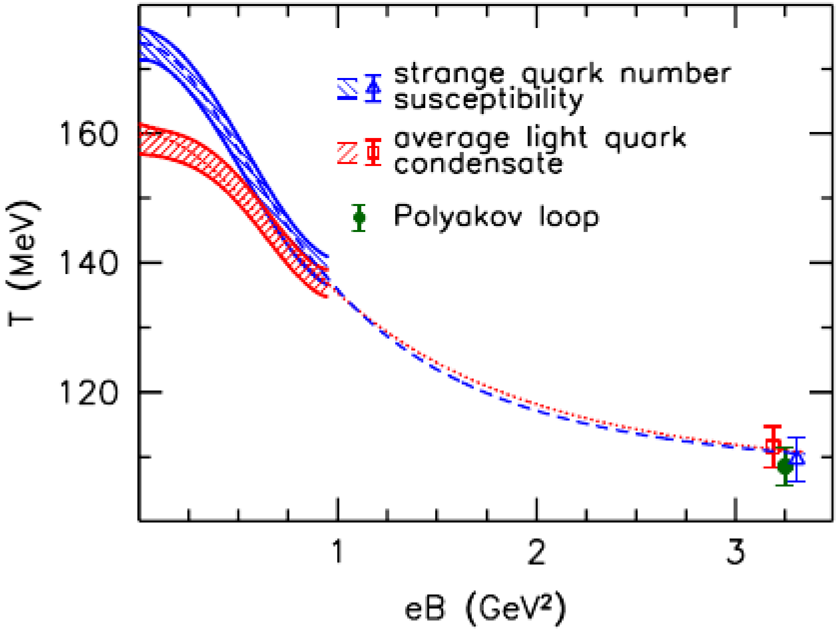
- Non frontal heavy ion collisions $(|eB|)^{1/2} \sim 100$ MeV



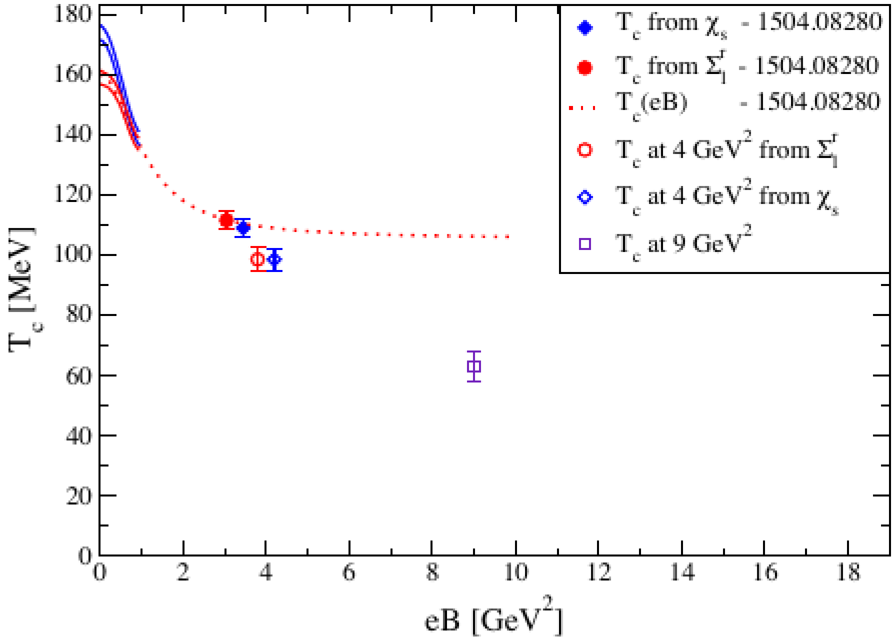
- Magnetars $(|eB|)^{1/2} \sim 1\text{-}10$ MeV



Phase diagram

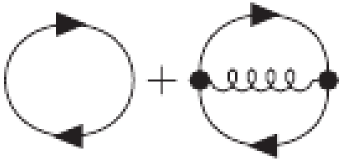


G. Endrodi, JHEP07(2015)173

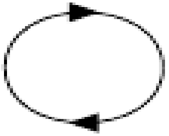


M. D‘ Elia *et al*, PhysRevD.105.034511(2022)

Quark pressure at finite B

$P^{PT} =$  $+ \mathcal{O}(g^2)$

Pure magnetic contribution

$\frac{P_{free}}{N_c} =$ 

$$= \sum_f \frac{(q_f B)^2}{2\pi^2} \left[\zeta'(-1, x_f) - \zeta'(-1, 0) + \frac{1}{2}\left(x_f - x_f^2\right) \ln x_f + \frac{x_f^2}{4} \right]$$

$$+ T \sum_{n,f} \frac{q_f B}{\pi} \left(1 - \delta_{n,0}/2\right) \int \frac{dp_z}{2\pi} \left\{ \ln\left(1 + e^{-\beta[E(n,p_z) - \mu_f]}\right) + \ln\left(1 + e^{-\beta[E(n,p_z) + \mu_f]}\right) \right\}$$

$$x_f \equiv m_f^2/(2 q_f B)$$

$$E^2(n, p_z) = p_z^2 + m_f^2 + 2 q_f B n.$$

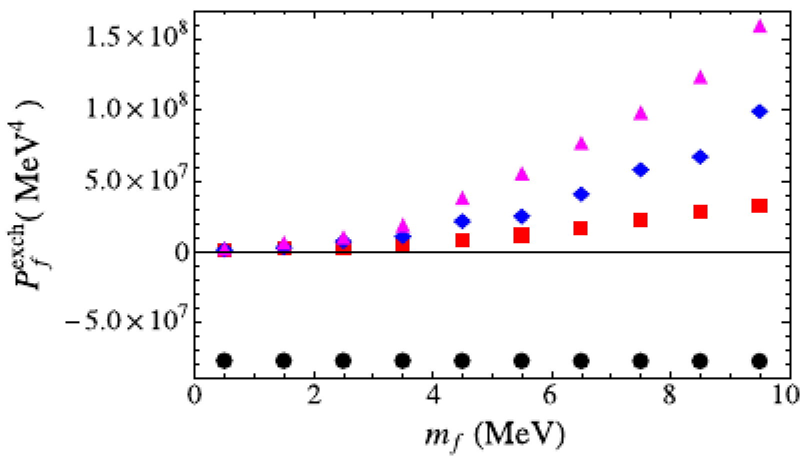
- Other renormalization schemes, G. Endrodi, JHEP04(2013)023
- A recent discussion in the context of efective models, S. Avancini *et al*, PhysRevD.103.056009(2021)

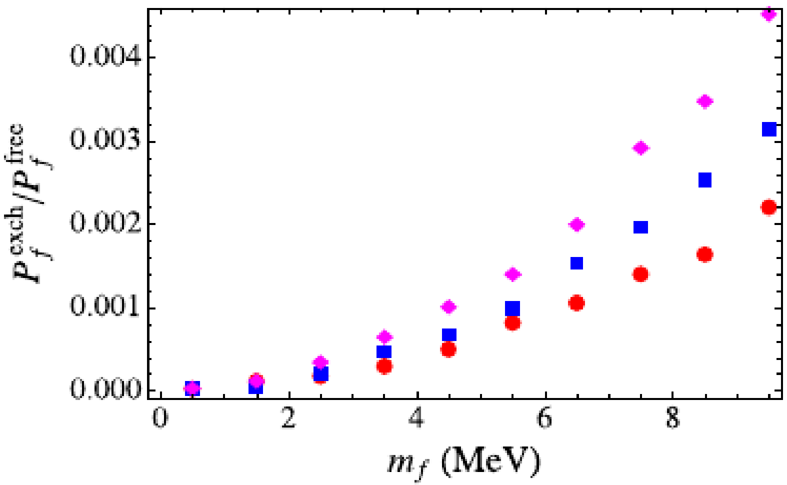
- At high B, lowest Landau level (LLL) aproximation is valid , $n=0$, $m^2<T^2<|eB|$

$$\frac{P^{LLL}_{free}}{N_c} = -\sum_f \frac{(q_f B)^2}{2\pi^2}\left[x_f \ln\sqrt{x_f}\right] + T\sum_f \frac{q_f B}{2\pi}\int\frac{dp_z}{2\pi}\left\{\ln\left(1+e^{-\beta[E(0,p_z)-\mu_f]}\right)\right.$$
$$\left. + \ln\left(1+e^{-\beta[E(0,p_z)+\mu_f]}\right)\right\}$$

$$\frac{P^{LLL}_{exch}}{N_c} = \frac{1}{2}g^2\left(\frac{N_c^2-1}{2N_c}\right)m_f^2\left(\frac{q_f B}{2\pi}\right)\int\frac{dm_k}{2\pi}m_k e^{-\frac{m_k^2}{2q_f B}}$$
$$\times\int\frac{dp_z dq_z dk_z}{(2\pi)^3}(2\pi)\delta(k_z-p_z+q_z)\frac{1}{\omega E_p E_q}\left\{\frac{\omega\Sigma_+}{E_-^2-\omega^2}+\frac{\omega\Sigma_-}{E_+^2-\omega^2}\right.$$
$$+2\left[\frac{E_+}{E_+^2-\omega^2}-\frac{E_-}{E_+^2-\omega^2}\right]n_B(\omega)N_F(1)-\left[\frac{2\left(E_q+\omega\right)}{\left(E_--\omega\right)\left(E++\omega\right)}\right]N_f(1)$$
$$\left.-2\frac{E_+}{E_+^2-\omega^2}n_B(\omega)-\frac{1}{E_+-\omega}\right\}$$

J.-P. Blaizot, E. S. Fraga, L. F. Palhares, Phys. Lett B 722 (2013)





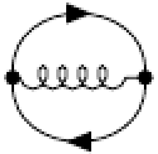
$T = 100\, MeV\,,\, B/m_\pi^2 = 50\,,\, 100\,,\, 200$

J.-P. Blaizot, E. S. Fraga, L. F. Palhares, Phys. Lett B 722 (2013)

What happens if one performs the momentum integrals before summing the Matsubara frequencies?

Matsubara's frequency sum

$$\frac{P^{LLL}_{exch}}{N_c} =$$



$$: \frac{1}{2} g^2 \left(\frac{N_c^2 - 1}{2N_c}\right) T^2 m_f^2 \left(\frac{q_f B}{2\pi}\right) \sum_{l,n_2} \int \frac{dm_k}{2\pi} m_k e^{-\frac{m_k^2}{2q_f B}} \times \frac{\mathcal{E}_l - \mathcal{E}_{n_2}}{\mathcal{E}_l \mathcal{E}_{n_1} \mathcal{E}_{n_2} \left|\mathcal{E}_l - \mathcal{E}_{n_2}\right| \left(\left|\mathcal{E}_l - \mathcal{E}_{n_2}\right| + \mathcal{E}_{n_1}\right)}$$

$$\mu = 0$$

$$\mathcal{E}_l = \sqrt{\omega_l^2 + m_k^2},\ \mathcal{E}_{n_1} = \sqrt{(\omega_{n_2} + \omega_l)^2 + m_f^2} \qquad \mathcal{E}_{n_2} = \sqrt{\omega_{n_2}^2 + m_f^2}.$$

QCD coupling

$$\alpha_s(\Lambda) = \frac{\alpha_s(\Lambda_0^2)}{1 + b_1 \ln(\Lambda^2/\Lambda_0^2)}$$

How does α_s change with B?

- Renormalization group analisis

$$\alpha_s(|eB|) = \frac{\alpha_s(\Lambda^2)}{1 + b_1 \ln(\frac{\Lambda^2}{\Lambda^2 + |eB|})} \qquad \boxed{T=0}$$

A. Ayala *et al*, PhysRevD.98.031501(2018)

- Color Coulomb potencial

$$\frac{\alpha_s^0}{4\pi}\Pi_q^B(\mathbf{k}) = k_3^2 \frac{\alpha_s^0(\mu_0)}{3\pi} \sum_{i=1}^{N_f} \frac{|q_i B|}{\tau} \exp\left(\frac{-k_\perp^2}{2|q_i B|}\right).$$

E. J. Ferrer *et al*, PhysRevD.91.054006(2015)

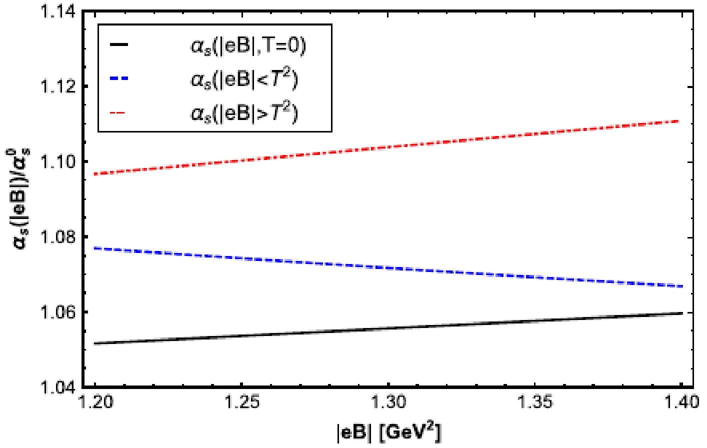
- Ansat? $\Lambda = \sqrt{(2\pi T)^2 + eB}.$

$$|eB| < T^2$$

$$\alpha_s(|eB|) = \frac{\alpha_s(Q^2 + \widetilde{|eB|})}{1 + b_1 \alpha_s(Q^2 + \widetilde{|eB|}) \ln\left(\frac{Q^2 + |eB|}{Q^2 + |eB| + T^2}\right)}$$

$$|eB| > T^2$$

$$\alpha_s(|eB|) = \frac{\alpha_s(Q^2 + \tilde{T}^2)}{1 + b_1 \alpha_s(Q^2 + \tilde{T}^2) \ln\left(\frac{Q^2 + T^2}{Q^2 + T^2 + |eB|}\right)}$$

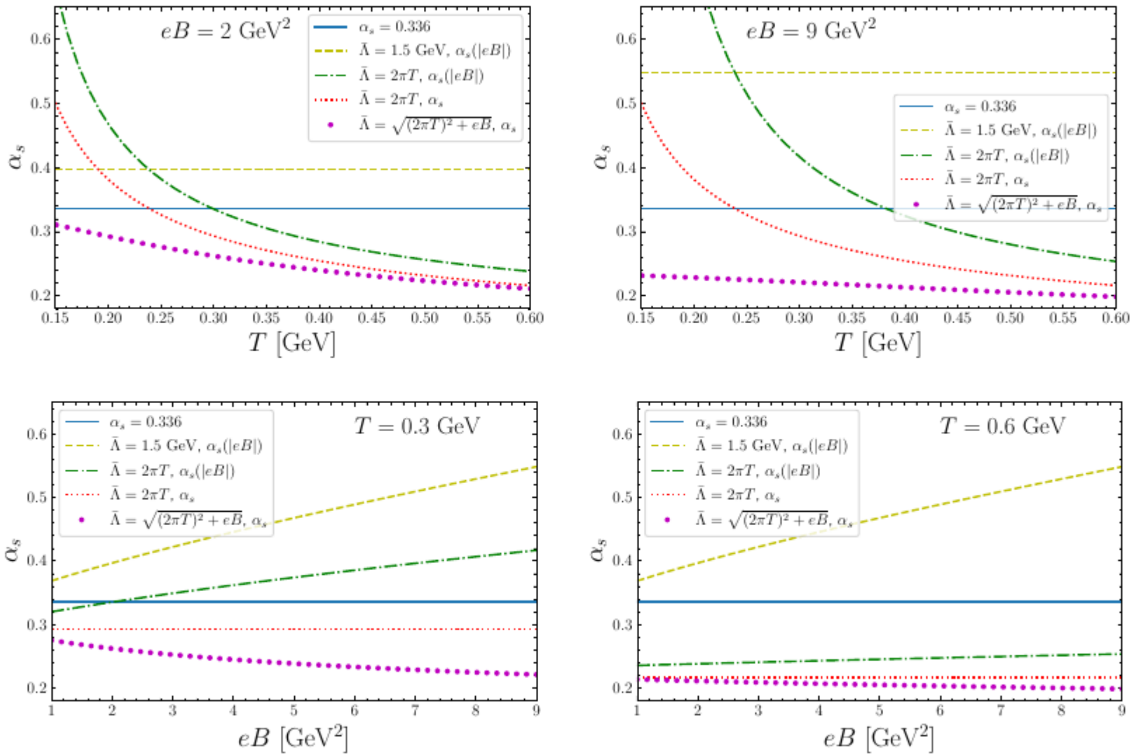


A. Ayala *et al*, PhysRevD.98.031501(2018)

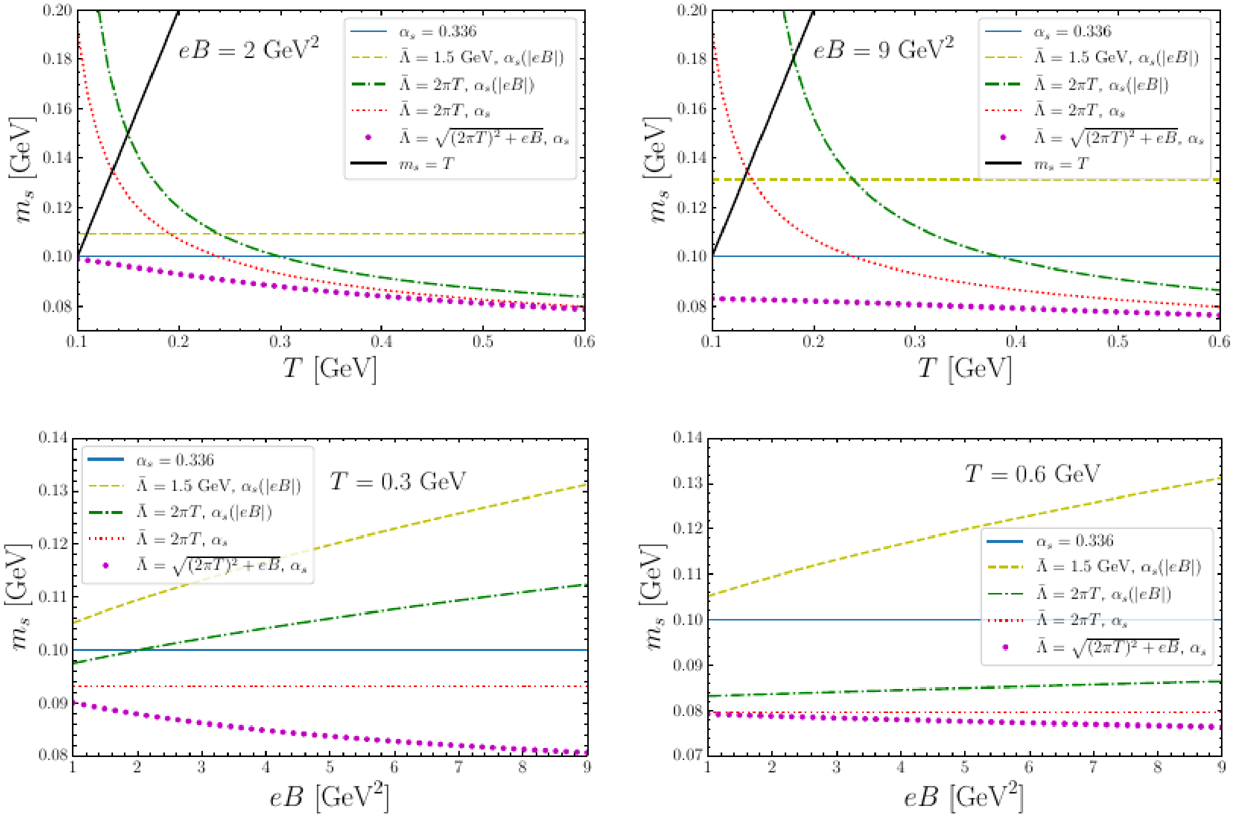
$$\alpha_s(|eB|) = \frac{\alpha_s(\Lambda^2)}{1 + b_1 \ln(\frac{\Lambda^2}{\Lambda^2 + |eB|})}$$ with $\Lambda = 2\pi T$.

B. Karmakar *et al*, PhysRevD.99.094002(2019)

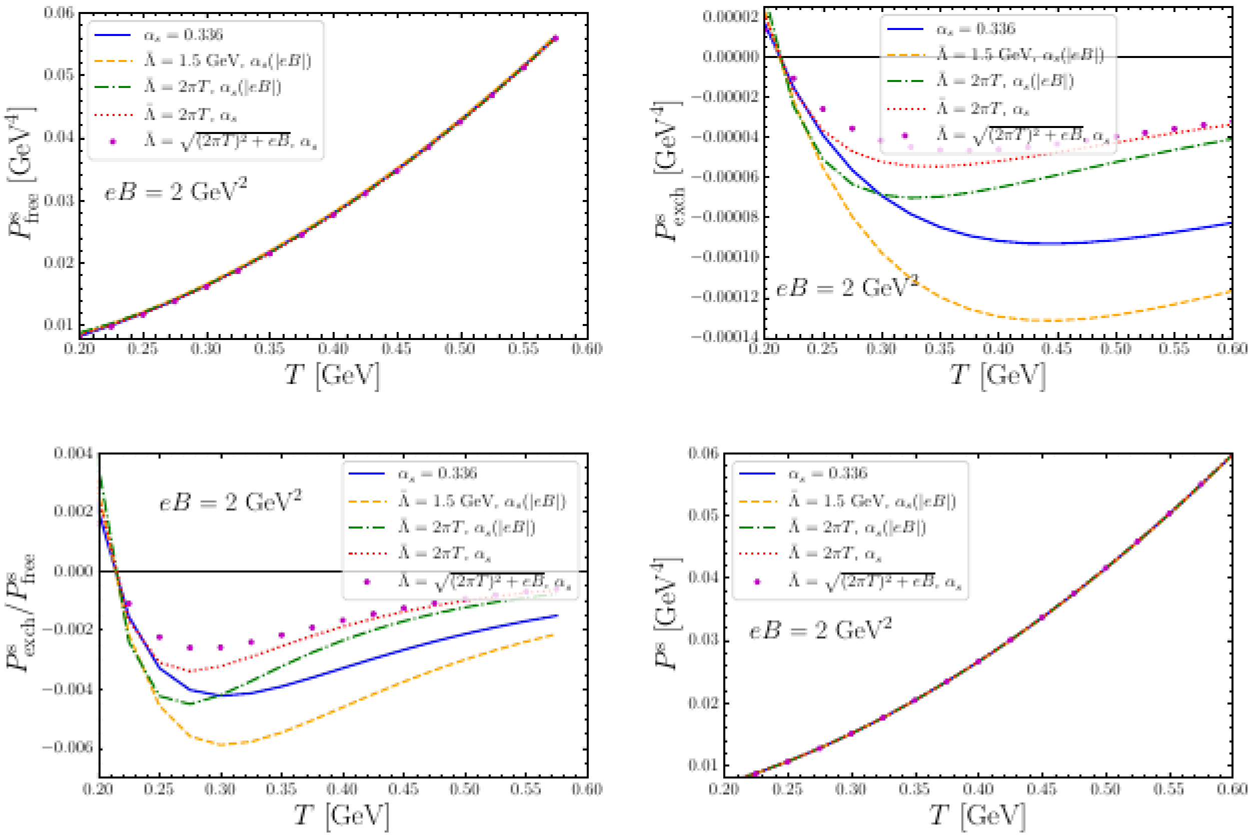
Results



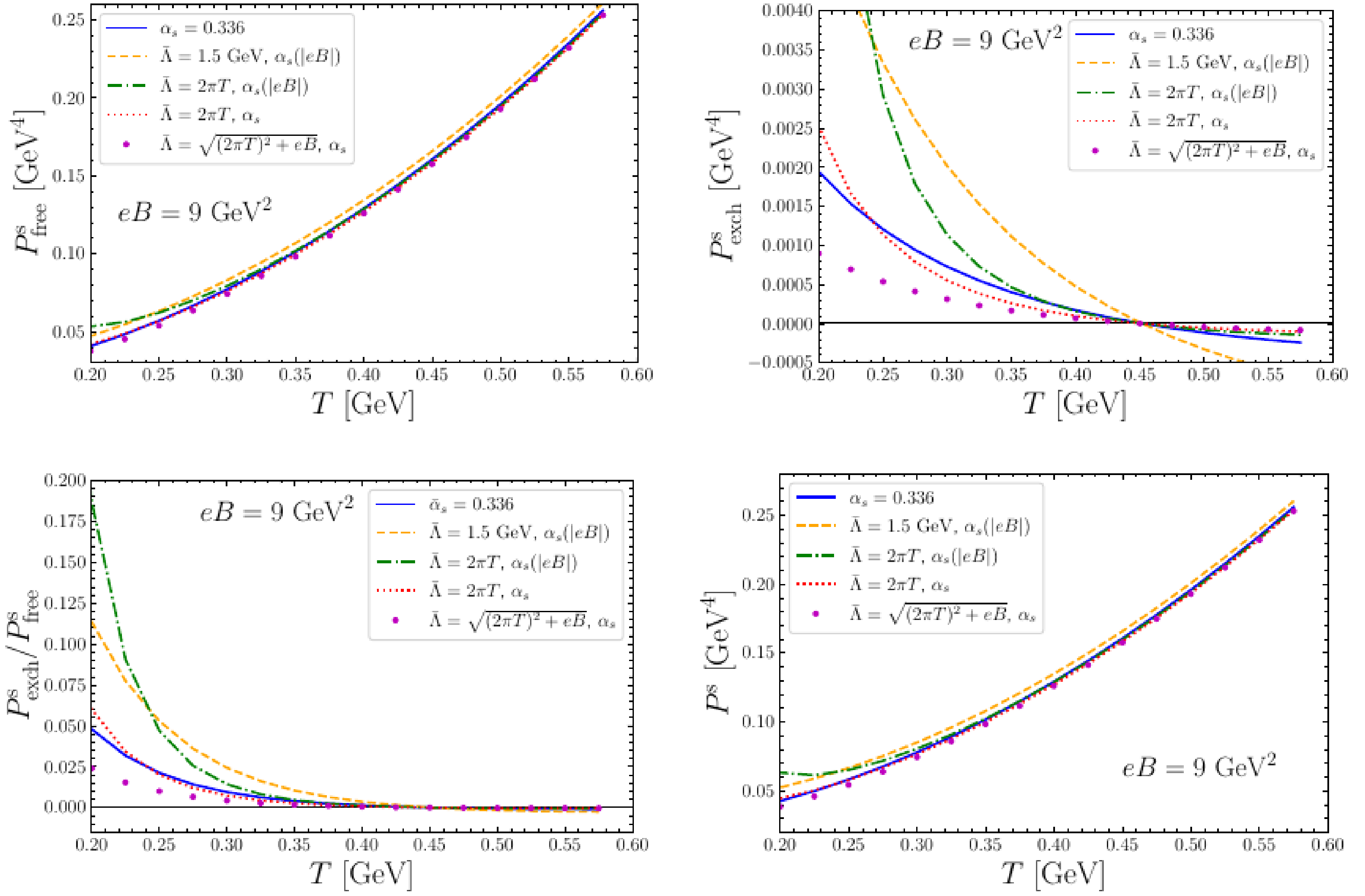
$$\alpha_s(\bar{\Lambda}) = \frac{4\pi}{\beta_0 L}\left(1 - \frac{2\beta_1}{\beta_0^2}\frac{\ln L}{L}\right) \qquad L = 2\ln\left(\bar{\Lambda}/\Lambda_{\overline{\rm MS}}\right)$$



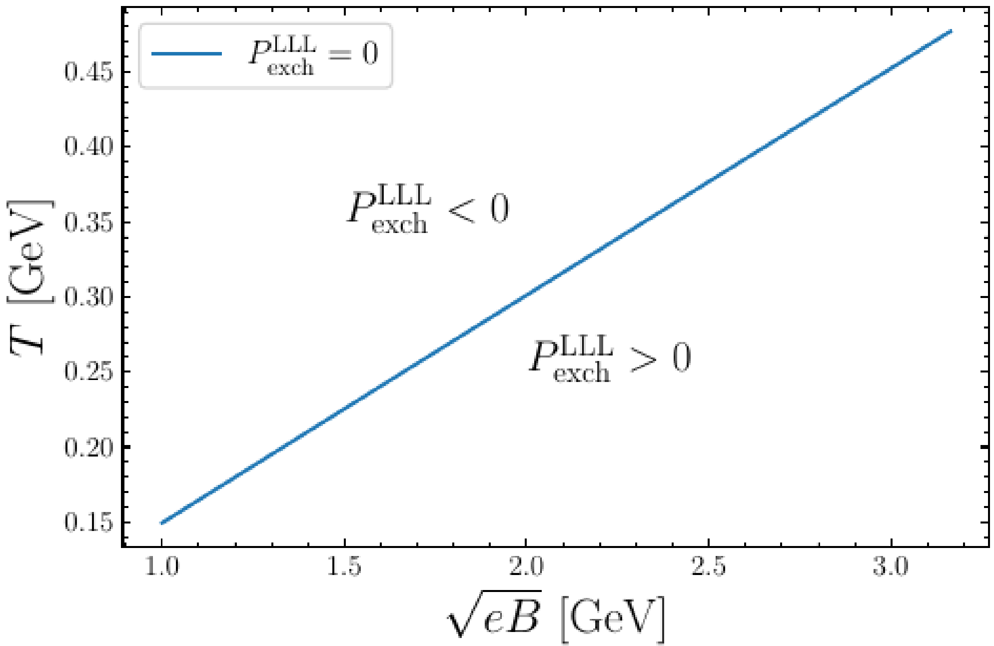
$$m_s(\bar{\Lambda}) = \hat{m}_s \left(\frac{\alpha_s}{\pi}\right)^{4/9} \left(1 + 0.895062 \left(\frac{\alpha_s}{\pi}\right)\right)$$



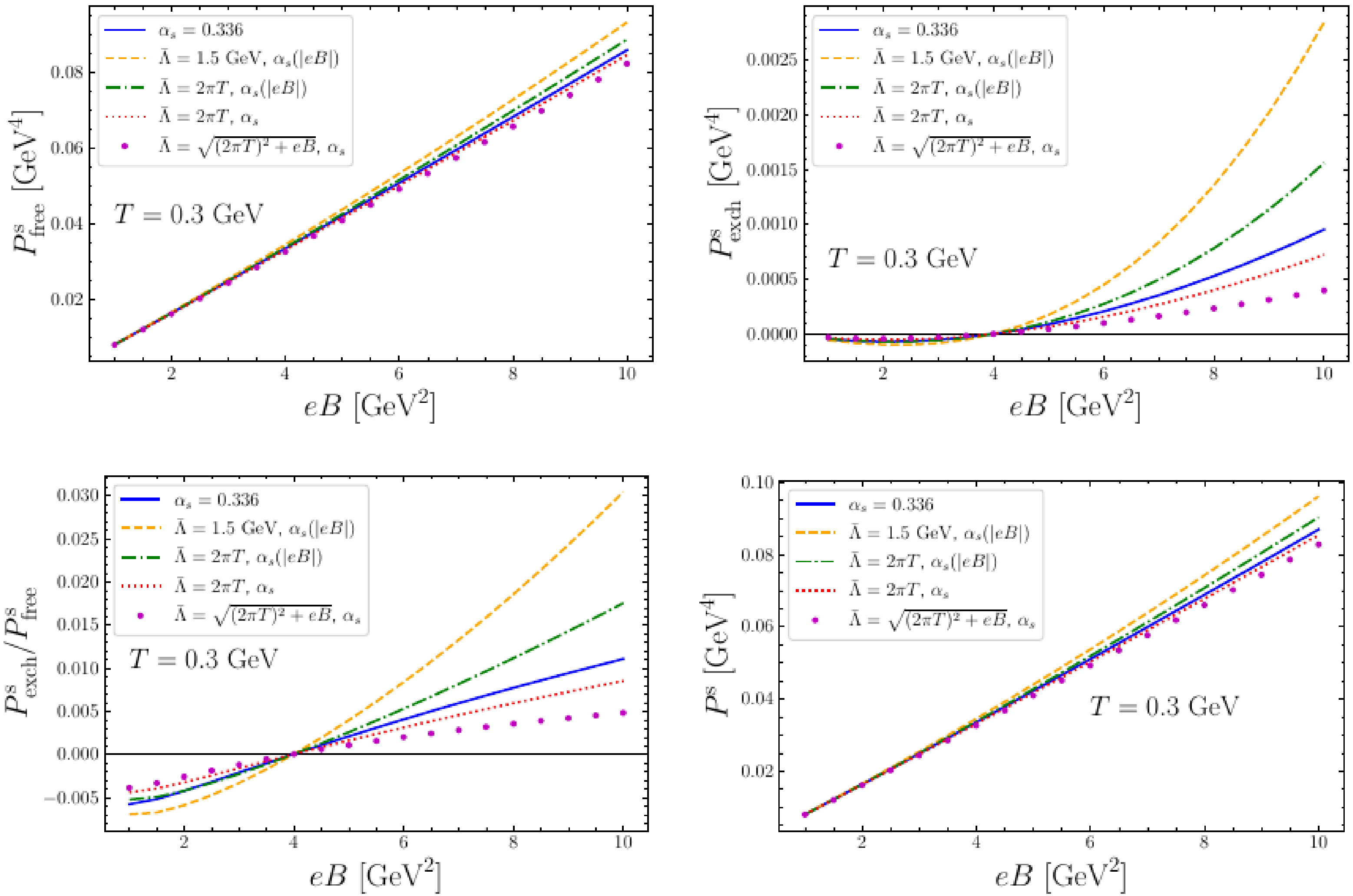
$\alpha_s = 0.336$
$\bar{\Lambda} = 1.5$ GeV, $\alpha_s(|eB|)$
$\bar{\Lambda} = 2\pi T$, $\alpha_s(|eB|)$
$\bar{\Lambda} = 2\pi T$, α_s
$\bar{\Lambda} = \sqrt{(2\pi T)^2 + eB}$, α_s
$eB = 2$ GeV^2
$P^s_{\rm free}$ [GeV^4]
$P^s_{\rm exch}$ [GeV^4]
$P^s_{\rm exch}/P^s_{\rm free}$
P^s [GeV^4]
T [GeV]

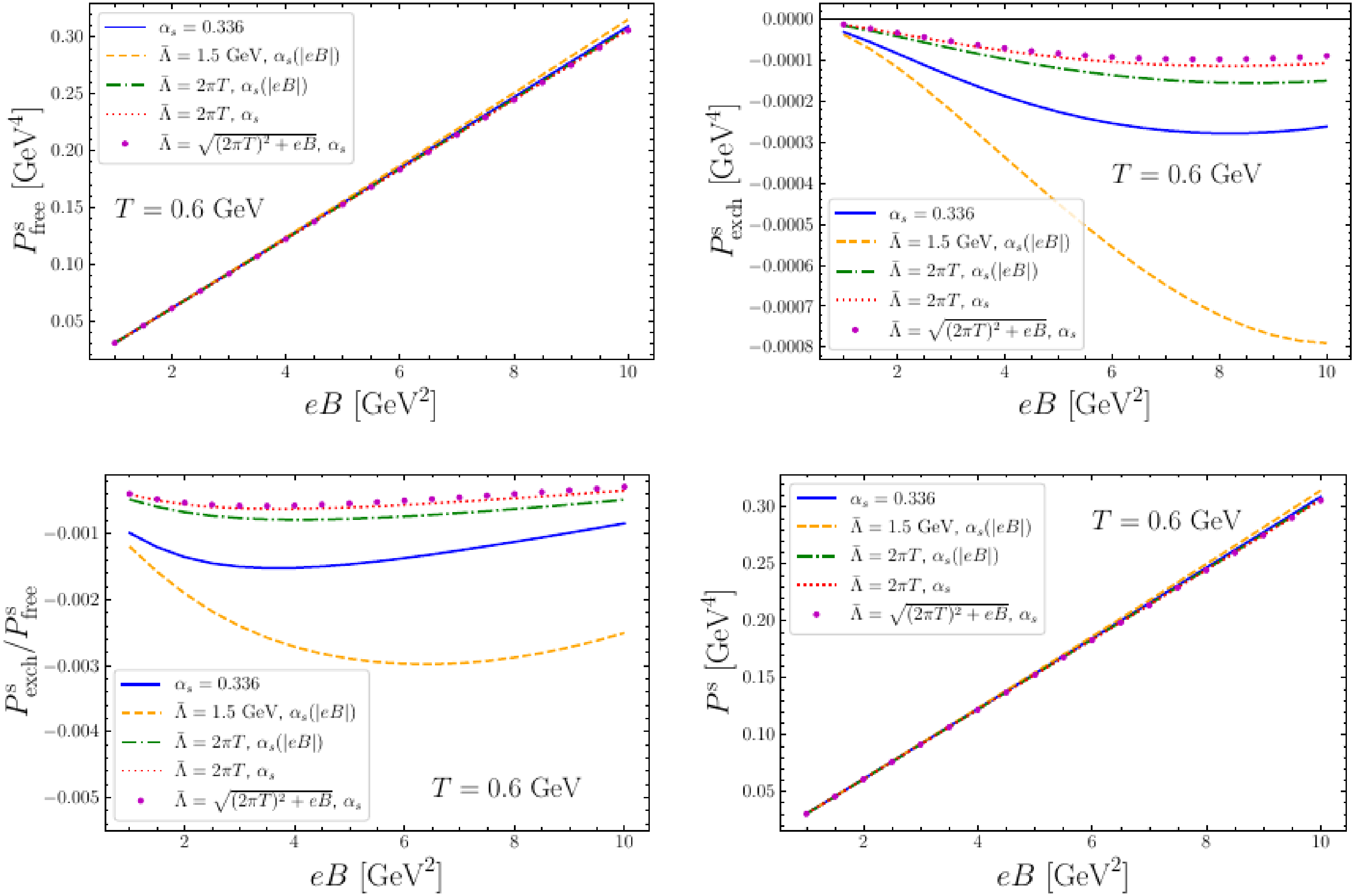


$\alpha_s = 0.336$
$\bar{\Lambda} = 1.5$ GeV, $\alpha_s(|eB|)$
$\bar{\Lambda} = 2\pi T$, $\alpha_s(|eB|)$
$\bar{\Lambda} = 2\pi T$, α_s
$\bar{\Lambda} = \sqrt{(2\pi T)^2 + eB}$, α_s
$eB = 9$ GeV^2
$P^{\rm s}_{\rm free}$ [GeV^4]
$P^{\rm s}_{\rm exch}$ [GeV^4]
$P^{\rm s}_{\rm exch}/P^{\rm s}_{\rm free}$
$P^{\rm s}$ [GeV^4]
T [GeV]



$P_{\rm exch}^{\rm LLL} = 0$
$P_{\rm exch}^{\rm LLL} < 0$
$P_{\rm exch}^{\rm LLL} > 0$
0.45
0.40
0.35
0.30
0.25
0.20
0.15
T [GeV]
1.0
1.5
2.0
2.5
3.0
$\sqrt{eB}$ [GeV]


$\alpha_s = 0.336$
$\bar{\Lambda} = 1.5$ GeV, $\alpha_s(|eB|)$
$\bar{\Lambda} = 2\pi T$, $\alpha_s(|eB|)$
$\bar{\Lambda} = 2\pi T$, α_s
$\bar{\Lambda} = \sqrt{(2\pi T)^2 + eB}$, α_s
$T = 0.3$ GeV
$P^{s}_{\rm free}$ [GeV^4]
$P^{s}_{\rm exch}$ [GeV^4]
$P^{s}_{\rm exch}/P^{s}_{\rm free}$
P^{s} [GeV^4]
eB [GeV^2]



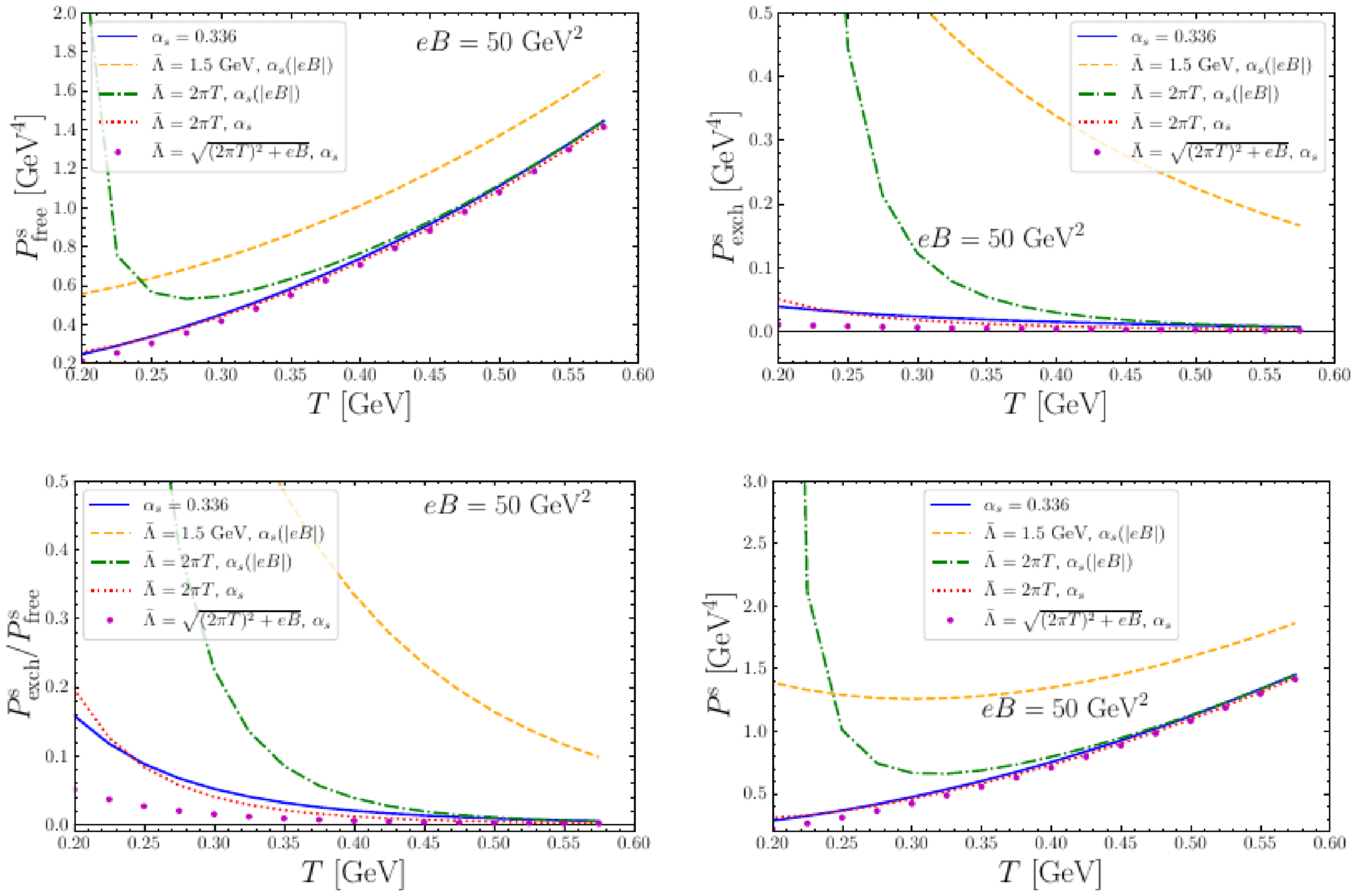
$\alpha_s = 0.336$
$\bar{\Lambda} = 1.5$ GeV, $\alpha_s(|eB|)$
$\bar{\Lambda} = 2\pi T$, $\alpha_s(|eB|)$
$\bar{\Lambda} = 2\pi T$, α_s
$\bar{\Lambda} = \sqrt{(2\pi T)^2 + eB}$, α_s
$T = 0.6$ GeV
$P^{\rm s}_{\rm free}$ [GeV^4]
$P^{\rm s}_{\rm exch}$ [GeV^4]
$P^{\rm s}_{\rm exch}/P^{\rm s}_{\rm free}$
$P^{\rm s}$ [GeV^4]
eB [GeV^2]

Conclusions

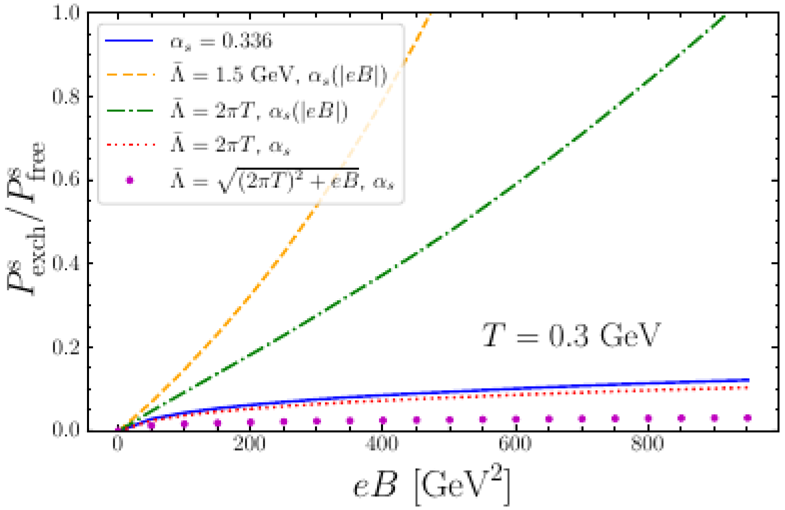
- We study the quark pressure at NLO for strong magnetic fields using different expressions of the coupling constant.
- In general, we found the same qualitative behavior for all α_s.
- The contribution of the exchange pressure is small compared to the free pressure, with the exception of high Bs.
- The different renormalization schemes and their impact on the pressure and the magnetization should be further studied in the context of perturbative QCD.
- It would be great to have LQCD pressure data for highs B in the near future!

Thanks!!!!!

Backup



$eB = 50$ GeV^2
$\alpha_s = 0.336$
$\bar{\Lambda} = 1.5$ GeV, $\alpha_s(|eB|)$
$\bar{\Lambda} = 2\pi T$, $\alpha_s(|eB|)$
$\bar{\Lambda} = 2\pi T$, α_s
$\bar{\Lambda} = \sqrt{(2\pi T)^2 + eB}$, α_s
$P^{s}_{\rm free}$ [GeV^4]
$P^{s}_{\rm exch}$ [GeV^4]
$P^{s}_{\rm exch}/P^{s}_{\rm free}$
P^{s} [GeV^4]
T [GeV]



$\alpha_s = 0.336$
$\bar{\Lambda} = 1.5$ GeV, $\alpha_s(|eB|)$
$\bar{\Lambda} = 2\pi T$, $\alpha_s(|eB|)$
$\bar{\Lambda} = 2\pi T$, α_s
$\bar{\Lambda} = \sqrt{(2\pi T)^2 + eB}$, α_s
$T = 0.3$ GeV
$P^s_{\rm exch}/P^s_{\rm free}$
eB [GeV^2]