The magnetic field independent regularization applied to light meson masses: the neutral ρ meson case

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Outline

- Brief review of the SU(2) Nambu–Jona-Lasinio model
- Regularization procedures: The MFIR and non-MFIR regularizations
- Regularization procedures applied to ρ^0 meson
- Results
- Summary and conclusions

Why magnetic fields?

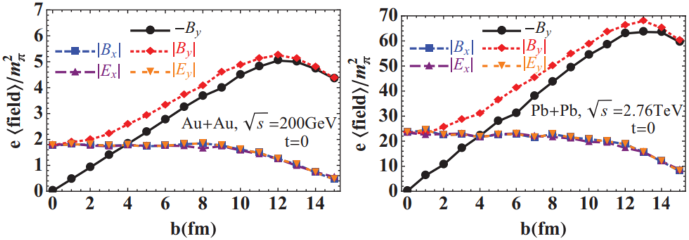


Figure: Wei-Tian Deng and Xu-Guang Huang, Phys. Rev. C 85, 044907 (2012)

- Peripheral Heavy Ion Collisions with $eB \sim 10^{19}$ G for ALICE/(LHC) and $eB \sim 10^{18}$ G for RHIC/(BNL)
- Magnetars: $eB \sim 10^{16} G$
- Primordial universe: Electroweak phase transition? $eB \sim 10^{20}$ to 10^{24}G

What does LQCD tell us?

G. S. Bali, et al. Phys. Rev. D 86, 071502(R), 2012

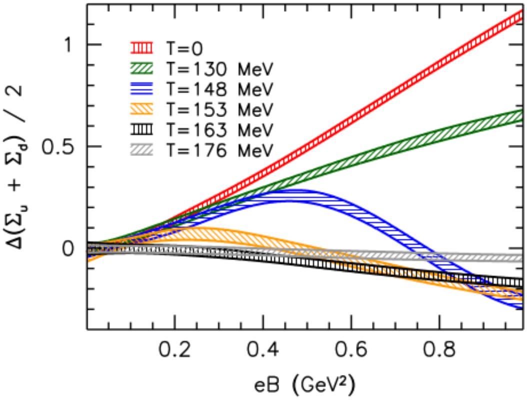


Figure: Average quark condensate as a function of the temperature for different values of temperature evaluated in LQCD.

SU(2) NJL model

The SU(2) NJL Lagrangian is given by:

$$\mathcal{L} = \overline{\psi}\left(i\not{D} - \tilde{m}\right)\psi + G\left[(\overline{\psi}\psi)^2 + (\overline{\psi}i\gamma_5\overrightarrow{\tau}\psi)^2\right] - \frac{1}{4}F^{\mu\nu}F_{\mu\nu}, \tag{1}$$

with the current quark masses matrix $\tilde{m}$ =diag(m_u, m_d) in the isospin symmetry approximation, $m_u = m_d = m_0$.

The covariant derivative is given by $\partial^\mu \rightarrow D^\mu = (i\partial^\mu - QA^\mu)$; the electromagnetic field tensor $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$; and the charge matrix Q_q =diag$(2/3, -1/3)e$. The gauge adopted is:

- $A_\mu = \delta_{\mu 2}x_1 B$, $(\overrightarrow{B} = B\hat{e}_z)$;

In the mean field approximation, the Lagrangian $\mathcal{L}$ is denoted by

$$\mathcal{L} = \overline{\psi}\left(i\not{D} - M\right)\psi - G\left\langle\overline{\psi}\psi\right\rangle^2 - \frac{1}{4}F^{\mu\nu}F_{\mu\nu}\,, \qquad M = m_0 - 2G\left\langle\overline{\psi}\psi\right\rangle \tag{2}$$

SU(2) NJL model

The quark condensate is given by

$$\langle \overline{\psi}\psi \rangle = -i \int \frac{d^4k}{(2\pi)^4} \,\mathrm{tr}\, \tilde{S}(k) \ , \tag{3}$$

The mean field quark propagator in coordinate space is given by $i\widetilde{S}(x,y)$=diag($i\widetilde{S}_u(x,y)$,$i\widetilde{S}_d(x,y)$) where

$$i\tilde{S}_f(x,y) = e^{i\Phi_f(x,y)} \ i\widetilde{S}_f(x-y) \ , \ f = (u,d) \ . \tag{4}$$

The translationally invariant part

J. S. Schwinger, Phys. Rev. 82 (1951) 664-679.

$$\begin{aligned} i\widetilde{S}_f(x-y) &= \int \frac{d^4k}{(2\pi)^4} e^{-ik\cdot(x-y)} \ i\tilde{S}_f(k), \\ i\tilde{S}_f(k) &= \int_0^\infty ds \exp\left[is\left(k_{\|}^2 - k_{\perp}^2 \frac{\tan(\beta_f s)}{\beta_f s} - M^2 + i\epsilon\right)\right] \\ &\times \left[(k_{\|}.\gamma_{\|} + M)\ \Pi_f(s) - k_{\perp}.\gamma_{\perp}\ g_f(s)\right] \ , \end{aligned} \tag{5}$$

SU(2) NJL model

The quark condensate at $eB \neq 0$ is given by

$$\langle\overline{\psi}\psi\rangle = -\frac{2MN_c}{8\pi^2}\sum_f \int_0^\infty ds \frac{e^{-sM^2}}{s}\beta_f \coth(\beta_f s), \quad \text{Schwinger Formalism}$$

$$\langle\overline{\psi}\psi\rangle = -\frac{2MN_c}{8\pi^2}\sum_f \beta_f \sum_{k=0}^{\infty}\alpha_k \int_{-\infty}^{\infty} \frac{dp_3}{\sqrt{p_3^2 + M^2 + 2\beta_f k}}, \quad \text{Landau Level basis}$$

where k are the Landau Levels and $\alpha_k = 2 - \delta_{k,0}$. Both representations are equivalent and ultraviolet divergent.

MFIR regularizations

The magnetic field independent regularization (MFIR) is based in the subtraction scheme of divergences from [Schwinger, 1951]. Only the first term of the $\coth(s) \sim 1/s$ when $s \to 0$ is divergent

$$\langle\overline{\psi}\psi\rangle \rightarrow \lim_{\epsilon\to 0} \frac{2MN_c}{8\pi^2}\left[\sum_f \int_0^\infty ds \frac{e^{-sM^2}}{s^{2+\epsilon}}(\beta_f s \coth(\beta_f s) - 1) + N_f \int_{\frac{1}{\Lambda}}^\infty ds \frac{e^{-sM^2}}{s}\right]$$
$$= -2MN_c[\sum_f I_f + I^0], \quad (6)$$

where $\beta_f = |q_f eB|$ and I^0 is the vacuum contribution that must be regularized. It is possible to show, analytically that

$$I_f = \frac{M^2}{8\pi^2}\left[\frac{\ln\Gamma[x_f]}{x_f} - \frac{\ln 2\pi}{2x_f} + 1 - \left(1 - \frac{1}{2x_f}\right)\ln x_f\right], \quad x_f = \frac{M^2}{2\beta_f} \quad (7)$$

non-MFIR regularizations

$$\langle\overline{\psi}\psi\rangle = -\frac{2MN_c}{8\pi^2}\sum_f \int_{\frac{1}{\Lambda^2}}^{\infty} ds \frac{e^{-sM^2}}{s}\beta_f \coth(\beta_f s),$$ Proper-Time regularization

$$\langle\overline{\psi}\psi\rangle = -\frac{2MN_c}{8\pi^2}\sum_f \beta_f \sum_{k=0}^{\infty}\alpha_k \int_{-\infty}^{\infty}\frac{U_\Lambda(p_3+2\beta_f k)dp_3}{\sqrt{p_3^2+M^2+2\beta_f k}},$$ Form-Factor Regularization

where $U_\Lambda(p_3+2\beta_f k)$ is a Form-Factor function. There are several options

$$U_\Lambda^{LorN} = \left[1+\left(\frac{x}{\Lambda}\right)^N\right]^{-1}, \quad U_\Lambda^{WS\alpha} = \left[1+e^{\frac{x/\Lambda-1}{\alpha}}\right]^{-1}, \tag{8}$$

$$U_\Lambda^{GR} = e^{-\frac{x^2}{\Lambda^2}}, \quad U_\Lambda^{FD\alpha} = \frac{1}{2}\left[1+\tanh\left(\frac{\frac{x}{\Lambda}-1}{\alpha}\right)\right] \tag{9}$$

Regularization at $eB \neq 0$ (MFIR) vs (non-MFIR)

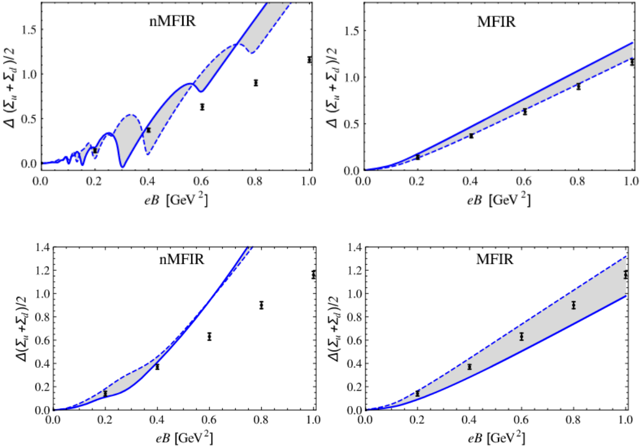


Figure: Average quark condensate as a function of eB with the Fermi-Dirac (top) and Lorentzian (bottom) regularizations. S. Avancini, R. Farias, N. Scoccola, **W. Tavares**, Phys. Rev. D 99, 116002 (2019)

SU(2) NJL model: MFIR Regularization

The gap equation in the MFIR scheme is given by the following expression

$$\frac{M-m_0}{2G} - 2MN_c I^0 - \frac{M^3 N_c}{4\pi^2} \sum_{f=u,d} \eta(x_f) = 0 \tag{10}$$

where the finite magnetic contribution is given by

$$\eta(x_f) = \left[\frac{\ln \Gamma(x_f)}{x_f} - \frac{1}{2x_f} \ln 2\pi + 1 - \left(1 - \frac{1}{2x_f}\right) \ln x_f \right], \tag{11}$$

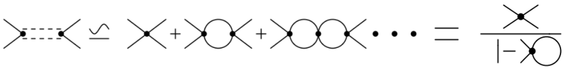
The usual divergent non-magnetic contribution is given by I_1

$$I_1 = N_f \int \frac{d^3k}{(2\pi)^3} \frac{1}{\sqrt{k^2+M^2}} \to N_f \int \frac{d^3k}{(2\pi)^3} \sum_i c_i \frac{1}{\sqrt{k^2+M_i^2}}, \quad M_i^2 = M^2 + a_i \Lambda^2. \tag{12}$$

where a_i and c_i are the Pauli-Villars regularization parameters.

SU(2) NJL model: Meson masses

The meson masses in the NJL can be evaluated in the random phase approximation (RPA)



$$\frac{-ig_{\pi qq}^2}{k^2 - m_\pi^2} \approx \frac{2iG}{1 - 2G\Pi_\pi(k^2)}, \tag{13}$$

where $g_{\pi qq}$ is the coupling between quarks and pions. The (pole)-mass for the π meson is given by

$$1 - 2G\Pi_\pi(k^2)|_{k^2=m_\pi^2} = 0, \tag{14}$$

$$\frac{1}{i}\Pi_\pi = -\int \frac{d^4p}{(2\pi)^4} Tr i\gamma_5\tau_k iS(p + \frac{1}{2}k) i\gamma_5\tau_j iS(p - \frac{1}{2}k), \tag{15}$$

where τ_j are the Pauli matrices.

SU(2) NJL model: Meson masses

In the RPA formulation, the polarization tensors for each channel, i.e, pseudo-scalar, scalar and vector channels, are defined in coordinate space by

$$\frac{1}{i}\Pi_\pi(x,y) = -\,\mathrm{tr}_{f,c,D}[i\gamma^5\tau^a i\tilde{S}(x,y)i\gamma^5\tau^b i\tilde{S}(y,x)], \tag{16}$$

$$\frac{1}{i}\Pi_\sigma(x,y) = -\,\mathrm{tr}_{f,c,D}[i\tilde{S}(x,y)i\tilde{S}(y,x)], \tag{17}$$

$$\frac{1}{i}\Pi_\rho^{\mu\nu,ab}(x,y) = -\,\mathrm{tr}_{f,c,D}[\gamma^\mu\tau^a i\tilde{S}(x,y)\gamma^\nu\tau^b i\tilde{S}(y,x)]\ . \tag{18}$$

The vector interaction in the lagrangian is given by including the following term

$$\mathcal{L} = \overline{\psi}\left(i\not{D} - \hat{m}\right)\psi + G\left[(\overline{\psi}\psi)^2 + (\overline{\psi}i\gamma_5\vec{\tau}\psi)^2\right] - G_v\left[(\overline{\psi}\gamma^\mu\vec{\tau}\psi)^2 + (\overline{\psi}\gamma^\mu\gamma_5\vec{\tau}\psi)^2\right] - \frac{1}{4}F^{\mu\nu}F_{\mu\nu}\ . \tag{19}$$

SU(2) NJL model: Meson masses

The general structure of the meson propagator in the vector channel, is given by the Schwinger-Dyson equation as

$$D_{ab}^{\mu\nu}(q^2) = -2G_v\delta_{ab}g^{\mu\nu} + (2G_v\delta_{ac}g^{\mu\lambda})(\Pi_{\lambda\sigma,cd})(D_{db}^{\sigma\nu}), \text{ General case}$$
$$D^{\mu\nu}(q^2) = -2G_v g^{\mu\nu} + 2G_v g^{\mu\lambda}\ \Pi_{\rho\ \lambda\sigma}\ D^{\sigma\nu}(q^2)\ . \text{ Neutral vector mesons, i.e., } a = b$$

$\rightarrow$ We consider the polarization tensor in the ρ rest frame, i.e., $q^\mu \equiv (q^0 = m_\rho, \vec{q} = \vec{0})$.
$\rightarrow$ The only non-null components of the polarization tensor in its rest frame are $\Pi_\rho^{11}(q_0^2) = \Pi_\rho^{22}(q_0^2)$ and $\Pi_\rho^{33}(q_0^2)$.

SU(2) NJL model: Meson masses

The polarization function and the generalized meson propagator can be decomposed as

$$\Pi_\rho^{\mu\nu} = \Pi_\rho^{11}\epsilon_1^\mu\epsilon_1^{*\nu} + \Pi_\rho^{22}\epsilon_2^\mu\epsilon_2^{*\nu} + \Pi_\rho^{33} b^\mu b^\nu,$$
$$D^{\mu\nu} = D^{11}\epsilon_1^\mu\epsilon_1^{*\nu} + D^{22}\epsilon_2^\mu\epsilon_2^{*\nu} + D^{33} b^\mu b^\nu,$$

$$\epsilon_1^\mu = \frac{1}{\sqrt{2}}(0,1,i,0),$$
$$\epsilon_2^\mu = \frac{1}{\sqrt{2}}(0,1,-i,0),$$
$$b^\mu = (0,0,0,1),$$
$$u^\mu = (1,0,0,0).$$

$$D^{11}(q_0^2) = \frac{-2G_v}{1-2G_v\Pi_\rho^{11}(q_0^2)},$$
$$D^{22}(q_0^2) = \frac{-2G_v}{1-2G_v\Pi_\rho^{22}(q_0^2)},$$
$$D^{33}(q_0^2) = \frac{-2G_v}{1-2G_v\Pi_\rho^{33}(q_0^2)}.$$

The neutral ρ mass for the three spin projections are given by

$$1-2G_v\Pi_\rho^{11}(q_0^2) = 0, (s_z = \pm 1),$$
$$1-2G_v\Pi_\rho^{33}(q_0^2) = 0, (s_z = 0).$$

NJL model: ρ^0 meson at $eB = 0$:vacuum contribution

The regularized polarization tensor in the Pauli-Villars scheme is given by

$$\frac{1}{i}\Pi_\rho^{\mu\nu}(q^2,0) = 4N_cN_f\sum_i c_i\int_0^1 dx\int\frac{d^4k}{(2\pi)^4}\frac{-2q^2x(1-x)}{(k^2-\overline{M}_i^2+i\epsilon)^2}\left(-g^{\mu\nu}+\frac{q^\mu q^\nu}{q^2}\right),$$
$$\overline{M}_i^2 = M^2 - x(1-x)q_0^2 + a_i\Lambda^2. \tag{20}$$

The integral in dk can be calculated analytically yielding the regularized expression for the polarization Π_ρ^{11} in the limit $q^\mu \to (q_0, 0)$

$$\Pi_\rho^{11}(q_0^2,0) = -\frac{N_cN_fq_0^2}{2\pi^2}\int_0^1 dx\, x(1-x)\left[\log\left(1+\frac{2\Lambda^2}{\overline{M}^2}\right) - 2\log\left(1+\frac{\Lambda^2}{\overline{M}^2}\right)\right]. \tag{21}$$

NJL model: ρ^0 meson at $eB \neq 0$: $s_z = \pm 1$

For the neutral ρ meson with $s_z = \pm 1$, we have

$$\Pi^{11}_{\rho}(q_0^2, \beta_f) = \frac{N_c \beta_f}{4\pi^2} \int_0^1 dx \int_0^{\infty} dv\, e^{-v \frac{\bar{\mathcal{M}}_+^2}{\beta_f}} \left(\frac{1}{v} + \frac{\bar{\mathcal{M}}_-^2}{\beta_f} \right) \Phi(v, x)$$
$$\Phi(v, x) = \left\{ \frac{1 - \tanh\left[v(1-x)\right]\ \tanh(vx)}{\tanh\left[v(1-x)\right] + \tanh(vx)} \right\} ,$$
$$\bar{\mathcal{M}}_{\pm}^2 = M^2 \pm q_4^2\, x(1-x) = M^2 \mp\, x(1-x)\, q_0^2. \tag{22}$$

For the regularization in the MFIR scheme we note that for $v = \beta_f y \sim 0$:

$$\Phi(\beta_f y, x) \approx \frac{1}{\beta_f y} + \left(\frac{1}{3} - 2x(1-x) \right) \beta_f y + \mathcal{O}((\beta_f y)^3)\ ,\ (v \ll 1)\ . \tag{23}$$

Just the first term of the expansion contributes with divergence in $\Pi^{11}_{\rho}(q_0^2, \beta_f)$.

NJL model: Meson Masses at $eB \neq 0 : s_z = \pm 1$

We obtain the finite pure magnetic term of Π_ρ^{11} in the MFIR scheme as

$$\Pi_{\rho,f}^{11}(q_0^2, \beta_f) = \frac{N_c \beta_f}{4\pi^2} \int_0^1 dx \int_0^\infty dv \, e^{-v\frac{\bar{\mathcal{M}}_+^2}{\beta_f}} \left(\frac{1}{v} + \frac{\bar{\mathcal{M}}_-^2}{\beta_f} \right) \Phi(v, x)_R \ , \tag{24}$$

where we define

$$\Phi_R(v, x) = \left\{ \frac{1 - \tanh\left[v(1-x)\right] \tanh\left(vx\right)}{\tanh\left[v(1-x)\right] + \tanh\left(vx\right)} - \frac{1}{v} \right\} . \tag{25}$$

The final regularized expression for Π_ρ^{11} is now given by

$$\Pi_{\rho,R}^{11}(q_0^2, eB) = \Pi_\rho^{11}(q_0^2, 0) + \sum_{f=u,d} \Pi_{\rho,f}^{11}(q_0^2, \beta_f), \tag{26}$$

In the last equation, the contribution of $eB = 0$ is

$$\Pi_\rho^{11}(q_0^2, \beta_f = 0) = \lim_{\beta_f \to 0} \Pi_\rho^{11}(q_0^2, \beta_f) = \frac{N_c}{4\pi^2} \int_0^1 dx \int_0^\infty dy \, e^{-y\bar{\mathcal{M}}_+^2} \left(\frac{1}{y} + \bar{\mathcal{M}}_-^2 \right) \frac{1}{y} . \tag{27}$$

NJL model: Meson Masses at $eB \neq 0 : s_z = 0$

We obtain the finite pure magnetic term of Π_ρ^{33} in the MFIR scheme as

$$\Pi_\rho^{33}(q_0^2, \beta_f) = \frac{N_c \beta_f^2}{4\pi^2} \int_0^1 dx \int_0^\infty dy\, e^{-y\bar{\mathcal{M}}_+^2} \left[\frac{\bar{\mathcal{M}}_-^2}{\beta_f} \coth(\beta_f y) + \sinh^{-2}(\beta_f y) \right] , \qquad (28)$$

where we identify

$$\coth v \sim \frac{1}{v} + \frac{v}{3} + \mathcal{O}(v^3) ,$$
$$\sinh^{-2} v \sim \frac{1}{v^2} - \frac{1}{3} + \mathcal{O}(v^2) \qquad (29)$$

in the final regularized expression for Π_ρ^{33} is now given by.

$$\Pi_{\rho,R}^{33}(q_0^2, eB) = \Pi_\rho^{33}(q_0^2, 0) + \sum_{f=u,d} \Pi_{\rho,f}^{33}(q_0^2, \beta_f) , \qquad (30)$$

Results

The parameter set is given by:

Λ	G	m	G_v	$\langle \bar{u}u \rangle^{1/3}$	f_π	m_π
761.22 MeV	$3.576/\Lambda^2$	6.565 MeV	1.3 G	−250 MeV	107 MeV	135 MeV

Table: Parameters of the PV Regularization.

The set of Pauli-Villars regularizartion coefficients are

c_0	c_1	c_2	a_0	a_1	a_2
1	−2	1	0	1	2

Table: Pauli-Villars coefficients.

Effective quark mass and ρ^0 meson: what did we see earlier?

From SU(2) NJL model with non-MFIR regularization:

Hao Liu, Lang Yu, Mei Huang, Phys.Rev.D 91 (2015) 1, 014017.

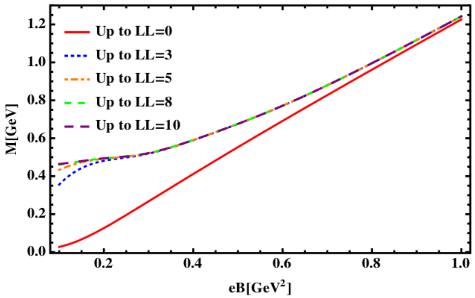


FIG. 3 (color online). Quark constitute mass M as a function of eB with different Landau levels included in the numerical calculations.

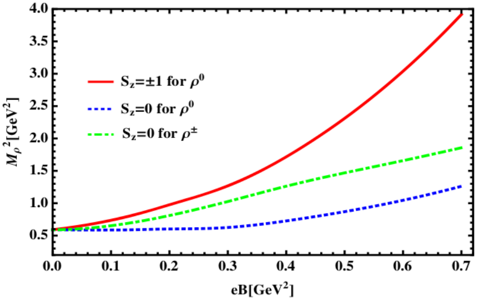
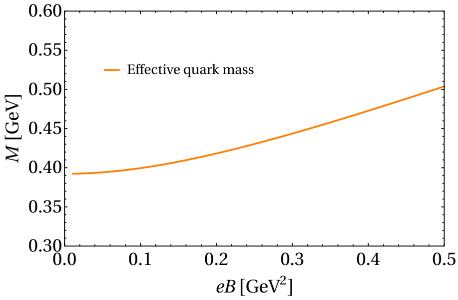


FIG. 5 (color online). Masses of the charged vector meson $\rho^\pm$ with $s_z = 0$ and neutral vector meson ρ_0 with $s_z = 0, \pm 1$ as functions of magnetic field eB.

Effective quark mass and ρ^0 meson: Our results.

From SU(2) NJL model with MFIR regularization: S. Avancini, R. Farias, **W. Tavares**, V. Timóteo, Nucl.Phys.B 981 (2022) 115862.



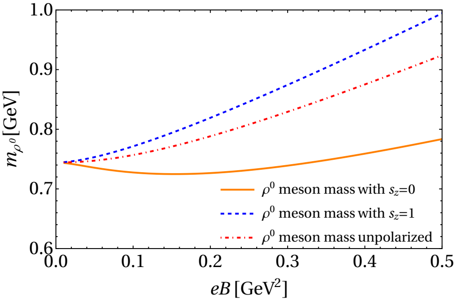


Figure: Left: Effective quark mass as function of the magnetic field. Right: ρ^0 meson as function of the magnetic field for different spin projections.

Obs: Unpolarized meson mass is evaluated with: $m_{\rho^0}^{Unp} = (m_{\rho^0_{s_z=0}} + m_{\rho^0_{s_z=+1}} + m_{\rho^0_{s_z=-1}})/3$.

Effective quark mass and ρ^0 meson: Our results.

From SU(2) NJL model with MFIR regularization: S. Avancini, R. Farias, **W. Tavares**, V. Timóteo, Nucl.Phys.B 981 (2022) 115862.

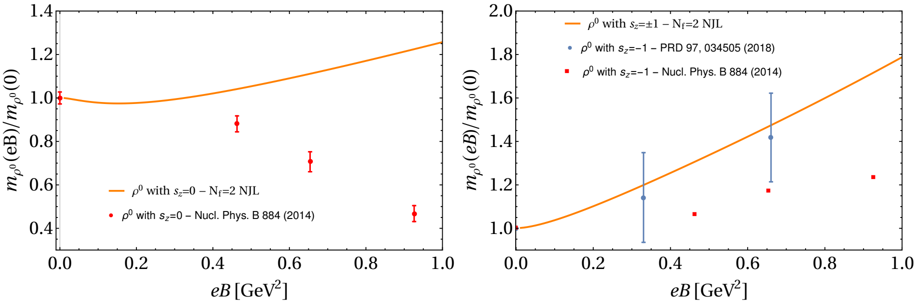


Figure: ρ^0 meson as function of the magnetic field with $s_z = 0$ (left) and $s_z = \pm 1$ (right).

Effective quark mass and ρ^0 meson: Our results.

From SU(2) NJL model with Hao Liu, Lang Yu, Mei Huang, Phys.Rev.D 91 (2015) 1, 014017.

MFIR regularization: S. Avancini, R. Farias, **W. Tavares**, V. Timóteo, Nucl.Phys.B 981 (2022) 115862.

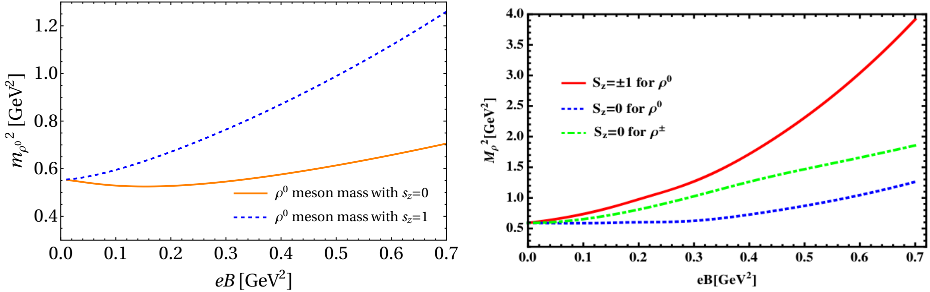


Figure: ρ^0 meson mass squared as function of the magnetic field with $s_z = 0$ (left) and $s_z = \pm 1$ (right).

Effective quark mass and ρ^0 meson: Our results.

From SU(2) NJL model with MFIR regularization: S. Avancini, R. Farias, **W. Tavares**, V. Timóteo, Nucl.Phys.B 981 (2022) 115862.

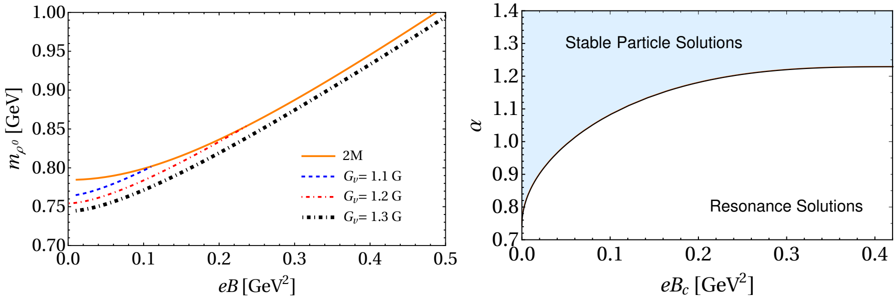


Figure: Left: The ρ^0 meson mas as function of the magnetic field with $s_z = \pm 1$ for different values of G_v. Right: Proportionality factor, $G_v = \alpha G$ as function of the magnetic field.

Final discussion

- At $T = 0$ and $eB \neq 0$, there are several different regularization prescriptions, but the ones based in the MFIR can avoid nonphysical results (e.g., chiral condensate and meson masses) : **Sidney S. Avancini et al., Phys. Rev. D 99, 116002 (2019)**;
- There is an alternative regularization method called vacuum magnetic regularization (VMR), based in the MFIR procedure, proper to study QCD phase phase diagram and its thermodynamics. See:**S. Avancini et al. Phys.Rev.D 103 (2021) 5, 056009**
- The MFIR procedure can clear the discussion in some cases: $\rho^{\pm}$ meson condensation ? see: **J.P. Carlomagno, D. Gomez Dumm, M.F. Izzo Villafañe, S. Noguera, N.N. Scoccola, arXiv:2209.10679.**
- There are several possibilities to explore: magnetized vector mesons at finite temperature and chemical potential, extensions beyond mean field approximation of NJL model and etc.

Thanks for your attention!

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