Mesons under strong magnetic field in the NJL model

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PLAN OF THE TALK

- Introduction
- Extended Nambu-Jona-Lasinio (NJL) model under strong magnetic fields
- Results for neutral mesons
- Results for charged mesons
- Summary & Conclusions

Refs: Carlomagno, Gomez Dumm, Noguera & NNS: Phys.Rev.D 106 (2022) 074002,
Carlomagno, Gomez Dumm, I. Villafañe, Noguera & NNS: arXiv:2209.10679

For related work see posters by M. Coppola and J. Sodré

Introduction

In the last decades there has been quite a lot of interest in investigating how the hadronic properties are affected by the presence of strong magnetic fields.

Motivation: their possible existence in physically relevant situations:

- High magnetic fields in non-central relativistic heavy ion collisions
- Compact Stellar Objects: magnetars
- Early Universe

Features of strongly interacting matter under intense magnetic fields have been investigated in a variety of approaches: NJL, LQCD, χPT, quark models, etc

Recent reviews:

Kharzeev, Landsteiner, Schmitt,Yee, Lect. Notes Phys. 871, 1 (2013).

Miransky, Shovkovy, Phys. Rept. 576, 1 (2015).

Andersen, W. R. Naylor, A. Tranberg, Rev. Mod. Phys. 88, 025001 (2016)

Generalized NJL model at finite B

We start from the Euclidean lagrangian of the NJL model for 2 flavors in the presence of an external e.m. field

$$\mathcal{L} = \bar\psi(x)\left(-i\not{D} + m_c\right)\psi(x) - g_s \sum_{a=0}^{3}\left\{\left[\bar\psi(x)\tau_a\psi(x)\right]^2 + \left[\bar\psi(x)i\gamma_5\tau_a\psi(x)\right]^2\right\}$$ S / P

$$-g_{v3}\left[\bar\psi(x)\,\gamma_\mu\vec\tau\,\psi(x)\right]^2 - g_{v0}\left[\bar\psi(x)\,\gamma_\mu\,\psi(x)\right]^2$$ V

$$+2g_d \sum_{\varepsilon=\pm1} \det[\bar\psi(x)(1+\varepsilon\,\gamma_5)\psi(x)]\,,$$ t'Hooft

where $D_\mu = \partial_\mu - i\hat{Q}\mathcal{A}_\mu \;,\; \hat{Q} = \mathrm{diag}(Q_u, Q_d) \;,\; Q_u = -2Q_d = 2e/3 \;,\; \tau_a = (1, \vec\tau) \;,\; \psi = \begin{pmatrix}\psi_u \\ \psi_d\end{pmatrix}$

We consider a constant and uniform magnetic field along the z-axis and choose the Landau gauge

$$\vec{B} = B\,\hat{x}_3 \qquad\qquad \mathcal{A}_4 = 0\,,\, \vec{\mathcal{A}} = (0, B\,x_1, 0)$$

We bosonize the fermionic theory, introducing $\sigma_a(x), \pi_a(x), \rho_{a\mu}(x)$ and integrating out the fermion fields. The bosonized Euclidean action reads

$$
\begin{aligned}
S_{\text{bos}} = & -\ln\det\mathcal{D} + \frac{1}{4\text{g}}\int d^4x\left[\sigma_0(x)\sigma_0(x) + \vec{\pi}(x)\cdot\vec{\pi}(x)\right] \\
& + \frac{1}{4\text{g}(1-2\alpha)}\int d^4x\left[\vec{\sigma}(x)\cdot\vec{\sigma}(x) + \pi_0(x)\pi_0(x)\right] \\
& + \frac{1}{4\text{g}_{v3}}\int d^4x\, \vec{\rho}_\mu(x)\cdot\vec{\rho}_\mu(x) + \frac{1}{4\text{g}_{v0}}\int d^4x\, \rho_{0\mu}(x)\rho_{0\mu}(x)
\end{aligned}
$$

where

$$
\mathcal{D}_{x,x'} = \delta^{(4)}(x-x')\left\{-i\not{\partial} + m_c + \sum_{a=0}^{3}\tau_a\left[\sigma_a(x) + i\gamma_5\pi_a(x) + \gamma_\mu\rho_{a\mu}(x)\right]\right\}
$$

$$
g = g_s + g_d
$$

$$
\alpha = g_d/(g_s + g_d)
$$

$\alpha = 0$	Flavors decoupled
$\alpha = 1/2$	Maximum flavor mixing
$\alpha \approx 0.1-0.2$	“empirical”

We proceed by expanding the bosonized action in powers of the fluctuations $\delta\sigma_a(x)$, $\delta\pi_a(x)$, $\delta\rho_{a\mu}(x)$ around the corresponding mean field (MF) values. We assume that only $\tau_a\bar{\sigma}_a = diag(\bar{\sigma}_u , \bar{\sigma}_d)$ is non-vanishing. Thus we write

$$\boxed{\mathcal{D}_{x,x'} = \mathrm{diag}\left(\mathcal{D}^{MF,u}_{x,x'}, \mathcal{D}^{MF,d}_{x,x'}\right) + \delta\mathcal{D}_{x,x'} \,.}$$

where $\mathcal{D}^{MF,f}_{x,x'} = \delta^{(4)}(x-x')\left(-i\,\not{\partial} - Q_f\,Bx_1\gamma_2 + M_f\right)$ with $M_f = m_c + \bar{\sigma}_f$

The effective action is $S_{\mathrm{bos}} = S^{MF}_{\mathrm{bos}} + S^{quad}_{\mathrm{bos}} + \ldots$

At MF level we have

$$\boxed{\begin{aligned}\frac{S^{MF}_{\mathrm{bos}}}{V^{(4)}} &= \frac{(1-\alpha)(\bar{\sigma}_u{}^2 + \bar{\sigma}_d{}^2) - 2\alpha\,\bar{\sigma}_u\bar{\sigma}_d}{8g(1-2\alpha)} \\ &\quad - \frac{N_c}{V^{(4)}} \sum_{f=u,d} \int d^4x\,d^4x'\; tr_D \ln\left(\mathcal{S}^{MF,f}_{x,x'}\right)^{-1},\end{aligned}}$$

MF quark propagator in presence of mag. field

For MF quark propagator we use

$$S^{MF,f}_{x,x'} = \exp\left[i\Phi_f(x,x') \right] \tilde{S}^{MF,f}(x-x')$$

Schwinger phase (SP)

$$\Phi_f(x,x') = s_f B_f (x_1 + x_1')(x_2 - x_2')/2$$

$$s_f = sign(Q_f B) \quad , \quad B_f = |Q_f B|$$

$$p_\perp = (p_1, p_2) \quad , \quad p_\parallel = (p_3, p_4)$$

Translational invariant part

$$\tilde{S}^{MF,f}(x-x') = \int_p e^{ip(x-x')}\, \tilde{S}^f_p$$

In proper time

$$\tilde{S}^f_p = \int_0^\infty d\tau\, \exp\left[-\tau \left(M^2 + p_\parallel^2 + \frac{\tanh(\tau B_f)}{\tau B_f}\, p_\perp^2 \right) \right]$$

$$\times \left[\left(M - p_\parallel \gamma_\parallel \right)\left(1 + i s_f \gamma_1 \gamma_2 \tanh(\tau B_f) \right) - \frac{p_\perp \gamma_\perp}{\cosh^2(\tau B_f)} \right]$$

To regularize the MF-action we sum and subtract the B=0 contribution. The B-dependent piece turns out to be finite. The B=0 one is regularized introducing 3D-cutoff Λ (Magnetic Field Independent Regularization – MFIR)

The MF effective masses M_f are obtained from the coupled set of gap equations

$$\partial S^{MF,reg}_{bos} \big/ \partial M_u = \partial S^{MF,reg}_{bos} \big/ \partial M_d = 0$$

NEUTRAL MESON MASSES IN NJL AT FINITE MAGNETIC FIELD

At quadratic level the neutral meson contribution is

$$S_{\rm bos}^{\rm quad,neutral} = \frac{1}{2}\int d^4x\, d^4x' \sum_{M,M'} \delta M(x)\; \mathcal{G}_{MM'}(x,x')\; \delta M'(x')$$

$M = \sigma_a, \pi_a\, \rho_{a,\mu}$

with $a = 0{,}3$

Note that for B=0 $(\sigma_0,\pi_0,\rho_0) \Leftrightarrow (\sigma, "\eta", \omega)$ $(\sigma_3,\pi_3,\rho_3) \Leftrightarrow (a_1^0, \pi^0, \rho^0)$

The inverse meson propagator $\mathcal{G}_{MM'}(x,x')$ is

$$\mathcal{G}_{MM'}(x,x') = \frac{1}{2g_M}\, \delta_{MM'}\, \delta^{(4)}(x-x') + \mathcal{J}_{MM'}(x,x')\,,$$

$$g_M = \begin{cases} g & \text{for } M = \sigma_0,\ \sigma_3 \\ g(1-2\alpha) & \text{for } M = \pi_0,\ \pi_3 \\ g_{v3} & \text{for } M = \rho_{3\mu} \\ g_{v0} & \text{for } M = \rho_{0\mu} \end{cases}$$

where

$$\mathcal{J}_{MM'}(x,x') = \mathcal{F}^u_{MM'}(x',x) + \varepsilon_M\, \varepsilon_{M'}\, \mathcal{F}^d_{MM'}(x',x)$$

$\varepsilon_M = +1(-1)$ for a=0 (3)

with

$$\mathcal{F}^f_{MM'}(x',x) = N_c\ \mathrm{tr}_D\left[\mathcal{S}^{MF,f}_{x,x'}\, \Gamma^{M'}\, \mathcal{S}^{MF,f}_{x',x}\, \Gamma^M\right]$$

$$\Gamma^M = \begin{cases} 1 & \text{for } M = \sigma_0,\ \sigma_3 \\ i\gamma_5 & \text{for } M = \pi_0,\ \pi_3 \\ \gamma_\mu & \text{for } M = \rho_{0\mu},\ \rho_{3\mu} \end{cases}$$

Contributions of Schwinger phases from quark propagators cancel out. The polarization functions are translational invariant. Taking Fourier transform of $M(x)$ polarization functions are diagonal in momentum space

$$\boxed{S_{\pi^0}^{quad} = \frac{1}{2}\sum_{M,M'}\int_q \left(\delta M(q)\right)^* G_{MM'}(q)\,\delta M(q)}$$

where
$$G_{MM'}(q) = \left[\frac{\delta_{MM'}}{2g_M} - \left(F^u_{MM'}(q) + \varepsilon_M \varepsilon_{M'} F^d_{MM'}(q)\right)\right]$$

with
$$F^f_{MM'}(q) = N_c \int_p \mathrm{tr}_D\left[\tilde{S}^f_{p+q/2}\,\Gamma^{M'}\,\mathcal{S}^{MF,f}_{x',x}\,\tilde{S}^f_{p-q/2}\right]$$

The expressions for $F^f_{MM'}(q)$ are divergent. As in MF-action we sum and subtract the B=0 contributions. B-dependent piece turns out to be finite. The B=0 ones are regularized introducing 3D-cutoff Λ (MFIR)

$$F^{f,mag}_{MM'}(q) = F^f_{MM'}(q) - F^{f,B=0}_{MM'}(q)$$

To calculate the meson masses we set $\vec{q}=0$ and $q_4 = i\,m$. In this case scalar meson decouples. Only non-vanishing polarization functions are

$$\boxed{\begin{aligned} F^{f,mag}_{\pi_a\pi_b} &= -N_c\left[I^{mag}_{1f} - m^2 I^{mag}_{2f}\right] \\ F^{f,mag}_{\pi_a\rho_{b\mu}} &= iN_c\, I^{mag}_{3f}\ \delta_{\mu 3} \\ F^{f,mag}_{\rho_{a\mu}\rho_{b\nu}} &= N_c\left[I^{mag}_{4f}\left(\delta_{\mu 1}\delta_{\nu 1}+\delta_{\mu 2}\delta_{\nu 2}\right)+m^2 I^{mag}_{5f}\delta_{\mu 3}\delta_{\nu 3}\right]\end{aligned}}$$

I^{mag}_{nf} finite integrals that depend on $(M_{f,}m, B)$

The ρ meson polarization vectors $\epsilon^{(Sz)}_{\mu}$ (Sz=±1,0 spin projection) are (at rest)

$$\epsilon^{(\pm 1)}_{\mu} = \frac{1}{\sqrt{2}}\left(1,\pm i,0,0\right) \quad \left[\rho_{\parallel}\right] \qquad \epsilon^{(0)}_{\mu} = \left(0,0,1,0\right) \quad \left[\rho_{\perp}\right]$$

Then, in the pseudocalar-vector sector we have

$$G = G_{\perp} \oplus 2G_{\parallel} \ \text{ where } \begin{cases} G_{\perp} \text{ corresponds to } \pi_0 - \pi_3 - \rho_{0\perp} - \rho_{3\perp} \text{ system (Sz=0)} \\ G_{\parallel} \text{ corresponds to } \rho_{0\parallel} - \rho_{3\parallel} \text{ system } \ (\text{Sz}=\pm 1)\end{cases}$$

Demanding $\det G_{\parallel} = 0$ and $\det G_{\perp} = 0$ masses and compositions obtained

CHARGED MESON MASSES IN NJL AT FINITE MAGNETIC FIELD

At quadratic level the $Q=+e$ meson contribution is (similar for $Q=-e$)

$$\boxed{S_{\rm bos}^{\rm quad,+} = \frac{1}{2}\int d^4x\, d^4x' \sum_{M,M'} \delta M(x)^\dagger \;\; \mathcal{G}_{MM'}(x,x') \;\; \delta M'(x')} \qquad \boxed{M = \pi^+, \rho_\mu^+}$$

The inverse meson propagator $\mathcal{G}_{MM'}(x,x')$ is

$$\boxed{\mathcal{G}_{MM'}(x,x') = \frac{1}{2g_M}\, \delta_{MM'}\, \delta^{(4)}(x-x') + \mathcal{J}_{MM'}(x,x')\,,}$$

where

$$\boxed{\mathcal{J}_{MM'}^{ud}(x',x) = \; N_c \exp\left[i\left(\Phi_u(x,x') + \Phi_d(x',x) \right) \right] \; {\rm tr}_D\left[\tilde{\mathcal{S}}^{MF,u}(x-x')\Gamma^{M'} \tilde{\mathcal{S}}^{MF,d}(x'-x)\Gamma^{M} \right]}$$

Note that using $Q_{\pi^+} = Q_{\rho^+} = Q_u - Q_d \equiv Q_+$ we have

$$\boxed{\Phi_+(x,x') = \Phi_u(x,x') + \Phi_d(x',x)}$$

SP does not vanish. Needed to keep gauge invariance of action. Translational invariance is broken. We cannot use plane waves (Fourier transform) to “diagonalize” space dependence of $\mathcal{G}_{MM'}(x,x')$

Charged pion

$$\boxed{\pi^+(x) = \sum_{\bar{q}} \mathbb{F}_{\bar{q}}(x)\ \pi^+(\bar{q})}$$

where

Solution of Klein-Gordon eq. in constant B

$$\boxed{\mathbb{F}_{\bar{q}}(x) = N_k\, e^{i(q_2x_2+q_3x_3+q_4x_4)}\, D_k(r)}$$

$$\bar{q} \equiv (k, q_2, q_3, q_4)\,, \quad \sum_{\bar{q}} \equiv \frac{1}{2\pi}\sum_{k=0}^{\infty}\int_{q_2 q_3 q_4}$$

$k = 0,1,\ldots$ Landau level

$$r = \sqrt{2B_+}\, x_1 - s\ \sqrt{2/B_+}\, q_2 \quad ; \quad N_k = (4\pi B_+)^{1/4}/\sqrt{k!}$$

$$B_+ = |Q_+B| \quad ; \quad s = \mathrm{sign}(Q_+B)$$

$$D_k(x) = 2^{-\ell/2} e^{-x^2/4} H_k\left(x/\sqrt{2}\right)$$ where $H_k(x)$ are Hermite polynomials

Charged rho

$$\boxed{\rho_\mu{}^+(x) = \sum_{\bar{q}} \mathbb{R}_{\bar{q}}^{\mu\nu}(x)\ \rho_\nu^+(\bar{q})}$$

where

Solution of Proca eq. in constant B

$$\boxed{\mathbb{R}_{\bar{q}}^{\mu\nu}(x) = \sum_{\lambda=-1,0,1} \mathbb{F}_{\bar{q}_\lambda}(x)\, \Delta_\lambda^{\mu\nu}}$$

$k = -1,0,1,\ldots$ Landau level

$$\bar{q}_\lambda \equiv (k - s\lambda, q_2, q_3, q_4)$$

For $k \geq 1$, $\rho_\mu^+(\bar{q})$ can be expressed in terms of 3 independent (transverse) polarization vectors. For $k = -1$ there is only one polarization vector while for $k = 0$ there are two. This way of writing the solutions is similar to that of Ritus for ½ fermions.

There is some freedom in the election of the 3 matrices Δ_λ which is compensated by the choice of the polarization vectors $\epsilon_\mu^+(\bar{q})$.

In this basis

$$S_{\text{bos}}^{\text{quad},+} = = \frac{1}{2} \sum_{M,M'} \sum_{\bar{q},\bar{q}'} \left[\delta M(\bar{q}) \right]^{\dagger} \left(\frac{\delta_{MM'} \hat{\delta}_{\bar{q}\bar{q}'}}{2 g_M} - \mathcal{J}_{MM'}(\bar{q},\bar{q}') \right) \delta M'(\bar{q}')$$

$$\hat{\delta}_{\bar{q}\bar{q}'} = (2\pi)^4 \delta_{kk'} \delta(q_2 - q_{2'}) \delta(q_3 - q_{3'}) \delta(q_4 - q_{4'}) .$$

For example

$$\mathcal{J}_{\pi^+\pi^+}(\bar{q},\bar{q}') = 2N_c \int_{p\nu} tr_D \left[\tilde{S}^d_{p+\nu/2} i\gamma_5 \tilde{S}^u_{p-\nu/2} i\gamma_5 \right] \int d^4x d^4x' \, e^{i\nu(x-x')} \, \mathbb{F}_{\bar{q}}(x)^* \mathbb{F}_{\bar{q}'}(x') \;\; e^{i\Phi_d(x,x')} e^{i\Phi_u(x',x)}$$

Explicit calculation shows

$$\mathcal{J}_{MM'}(\bar{q},\bar{q}') = \hat{\delta}_{\bar{q}\bar{q}'} \; J_{MM'}(k,\Pi^2)$$

$$\Pi^2 = (2k+1)B_+ + q_3^2 + q_4^2$$

Here, $J_{MM'}(k,\Pi^2)$ are integrals which also depend on M_f and B. They are regularized using the MFIR scheme.

We are interested in the lowest pion and rho meson states (thus we set q_3=0)

- ρ^+ it corresponds to $k = -1$. No coupling to π^+.

$$\frac{1}{2g_v} - J^{(\text{reg})}_{\rho^+\rho^+}(k, -m^2_{\rho^+}) = 0 .$$

$$E_{\rho^+}(eB) = \sqrt{m^2_{\rho^+} - eB}$$

- π^+ it corresponds to $k = 0$. Only couples with ρ^+ associated to S_z=0

$$\det\left[G^{(\text{reg})}(k, -m_M^2) \right] = 0 .$$

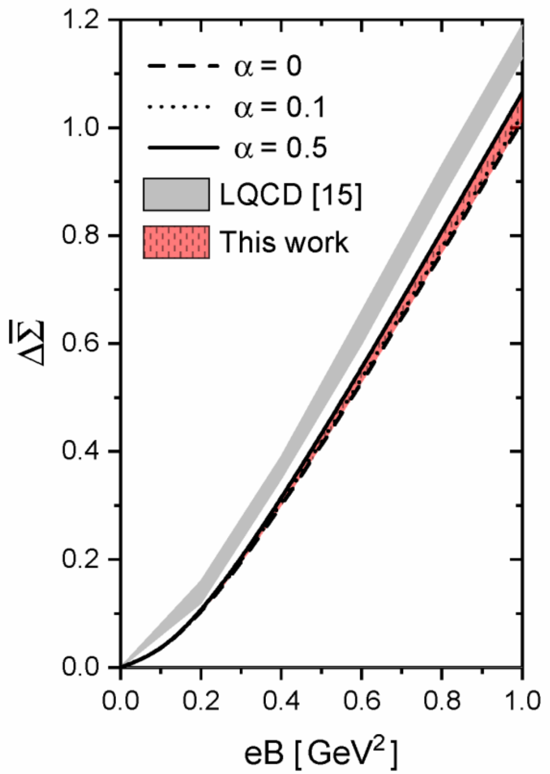
$$E_M(eB) = \sqrt{m_M^2 + eB}$$

2 x 2 matrix of $\pi^+\rho^+_\perp$ (k=0) sector

Results

In our numerical calculations the values of g, $g_{v0} = g_{v3}$, Λ and m_c are fixed so as to reproduce the B=0 values of f_π=92.4 MeV, m_π=138 MeV and $m_\rho = m_\omega$= 770 MeV together with M=400 MeV. For α = 0.1 we get $m_{"\eta"}$=520 MeV

Magnetic catalysis



$$\boxed{\Delta\bar{\Sigma}_B^f = (\Delta\Sigma_B^u + \Delta\Sigma_B^d)/2}$$

$$\boxed{\Sigma_B^f = \frac{2m_c}{S^4}\left[\langle\bar{q}_f q_f\rangle_B - \langle\bar{q}_f q_f\rangle_0\right] + 1}$$

$$S = (135\times 86)^{1/2}\,\mathrm{MeV}$$

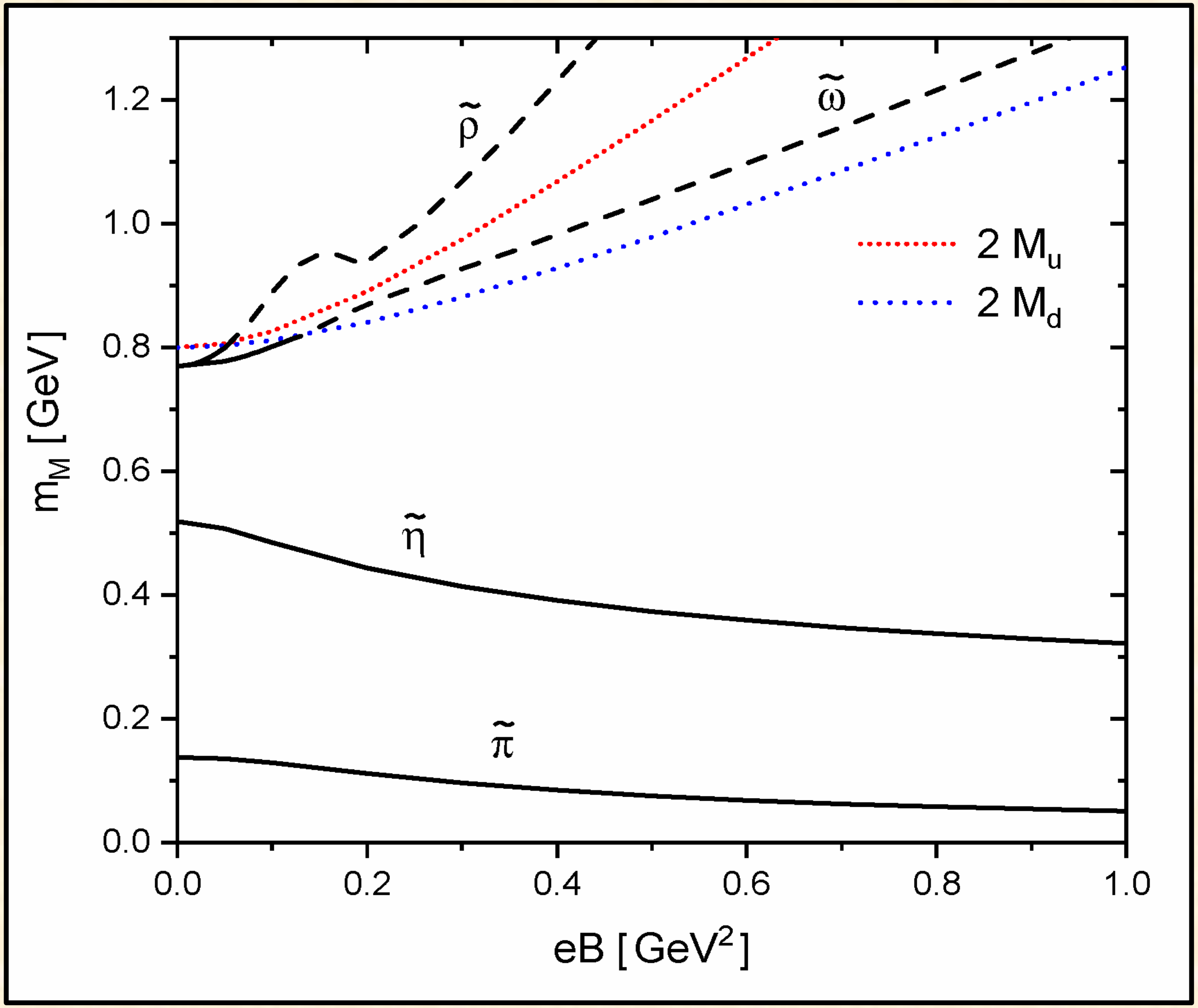
$$\boxed{<\bar{q}_f q_f>_{B,T} = \partial S_{bos}^{MF} / \partial m_c^f}$$

$$\boxed{<\bar{q}_f q_f>_{0,0} = (-241\ \mathrm{MeV})^3}$$

Good agreement with LQCD results (Bali et al, '12)

NEUTRAL MESON SECTOR

$S_z = \pm 1$ states



Masses of $\tilde{\rho}_\perp$ and $\tilde{\omega}_\perp$ increase with B

Masses of $\tilde{\pi}$ and $\tilde{\eta}$ decrease with B

States denoted $\tilde{M}$ to indicate meson with largest weight in spin-isospin decomposition

S_z=0 states compositions

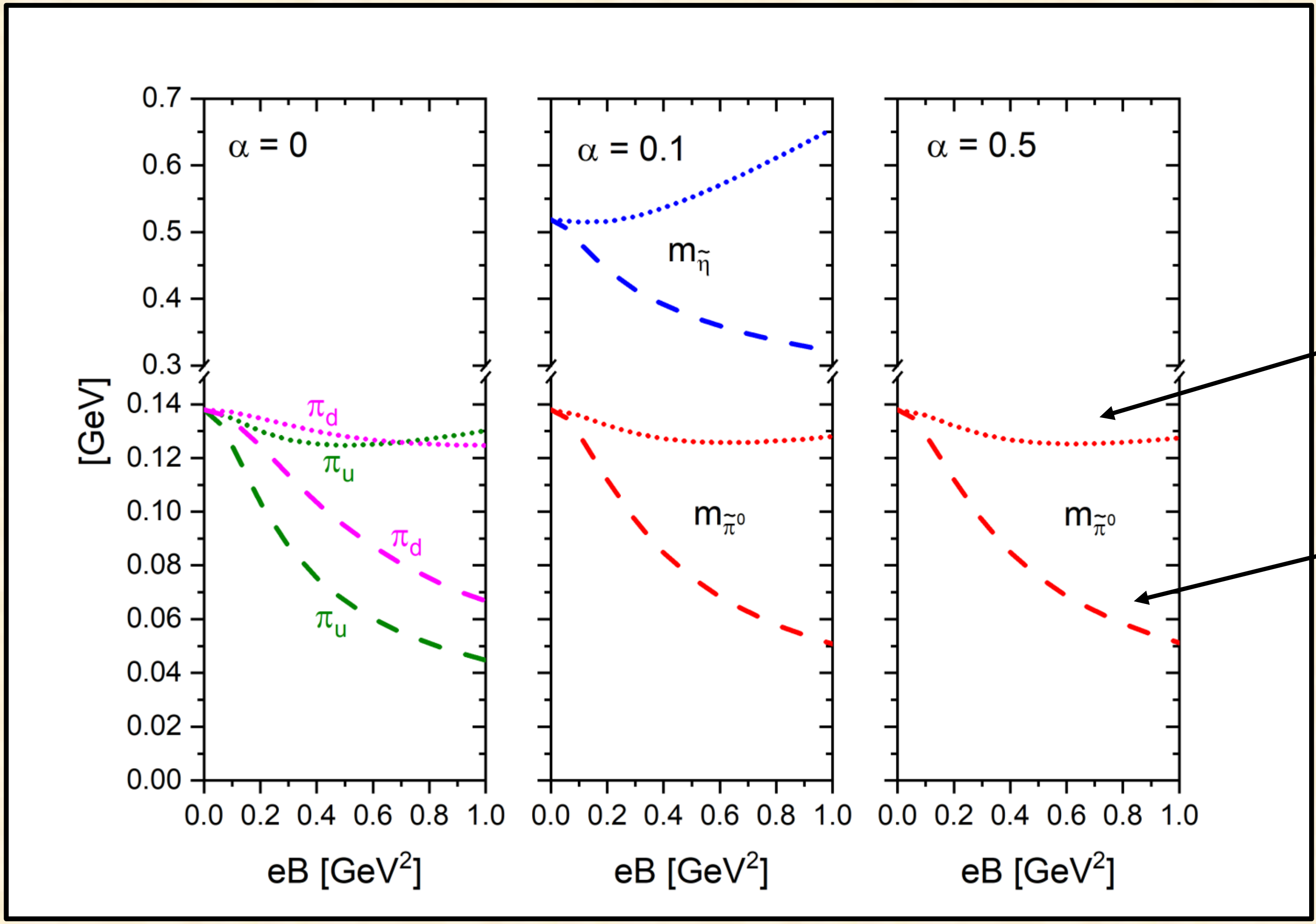
$$|k\rangle = c_{\pi_0}^{(k)}|\pi_0\rangle + c_{\pi_3}^{(k)}|\pi_0\rangle + ic_{\rho_0}^{(k)}|\rho_{0\perp}\rangle + ic_{\rho_3}^{(k)}|\rho_{3\perp}\rangle$$ for spin/isospin decomposition

$$|k\rangle = c_{\pi_u}^{(k)}|\pi_u\rangle + c_{\pi_d}^{(k)}|\pi_d\rangle + ic_{\rho_u}^{(k)}|\rho_{u\perp}\rangle + ic_{\rho_d}^{(k)}|\rho_{d\perp}\rangle$$ for spin/flavor decomposition .

State	eB [GeV^2]	Spin-isospin composition				Spin-flavor composition			
		$c_{\pi_0}^{(k)}$	$c_{\pi_3}^{(k)}$	$c_{\rho_{0\perp}}^{(k)}$	$c_{\rho_{3\perp}}^{(k)}$	$c_{\pi_u}^{(k)}$	$c_{\pi_d}^{(k)}$	$c_{\rho_{u\perp}}^{(k)}$	$c_{\rho_{d\perp}}^{(k)}$
$\tilde{\pi}$ $(k=1)$	0.05	0.00	0.99	-0.02	-0.01	0.71	-0.70	-0.02	-0.01
	0.5	0.10	0.99	-0.08	-0.03	0.77	-0.63	-0.08	-0.04
	1.0	0.16	0.98	-0.08	-0.03	0.81	-0.59	-0.08	-0.04
$\tilde{\eta}$ $(k=2)$	0.05	0.99	-0.04	-0.04	-0.13	0.67	0.73	-0.12	0.07
	0.5	0.87	-0.32	0.06	-0.38	0.38	0.84	-0.22	0.31
	1.0	0.83	-0.34	0.10	-0.42	0.35	0.83	-0.22	0.37
$\tilde{\omega}$ $(k=3)$	0.05	-0.20	0.27	0.76	-0.56	0.05	-0.33	0.14	0.93
$\tilde{\rho}$ $(k=4)$	0.05	0.49	0.33	0.47	0.65	0.58	0.11	0.79	-0.13

Table I. Composition of the $S_z = 0$ meson mass eigenstates for some selected values of eB. Results correspond to $\alpha = 0.1$. Relative signs hold for the choice $B > 0$.

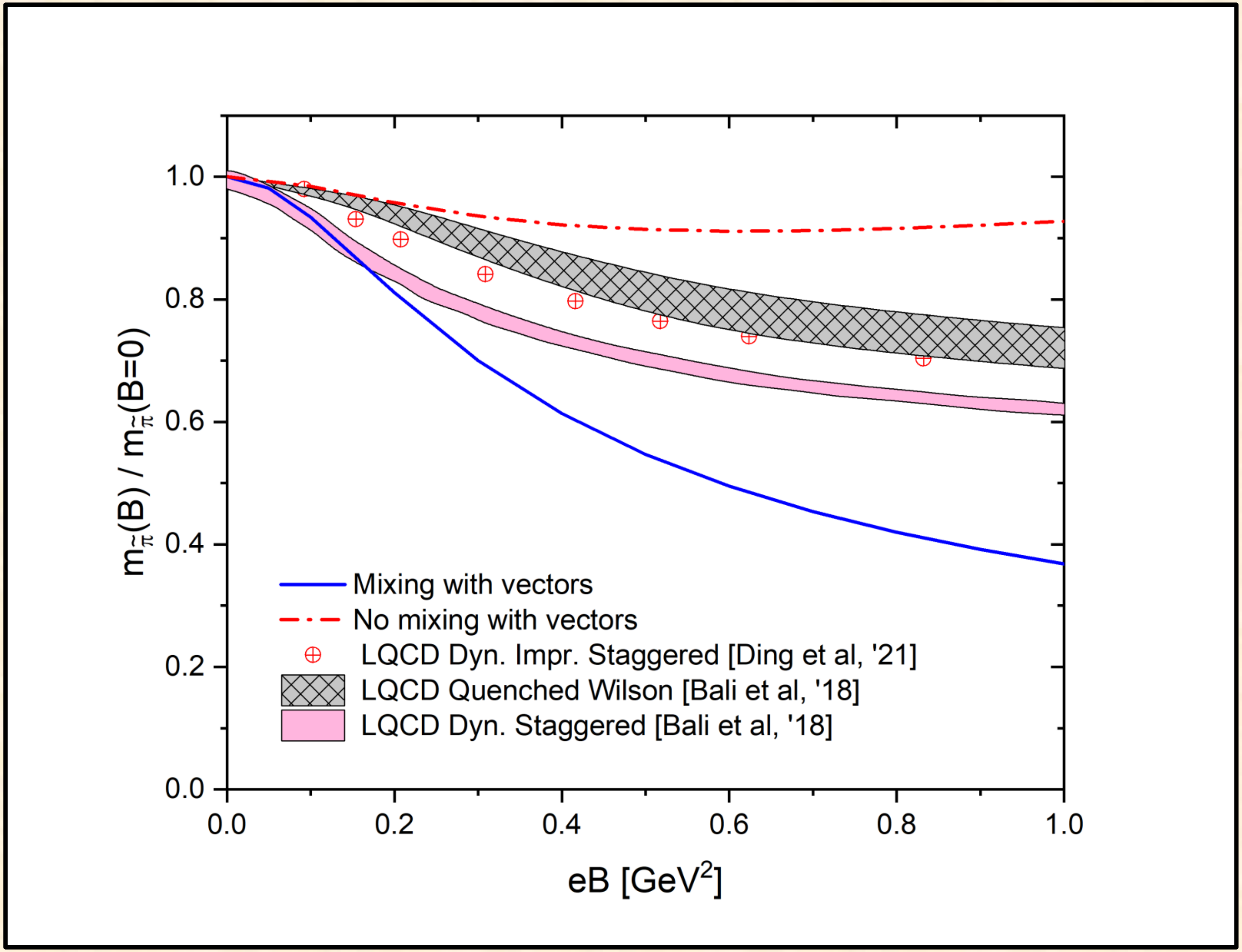
Effect of flavor mixing and the mixing with vectors on the pseudoscalar masses



No mixing with vectors

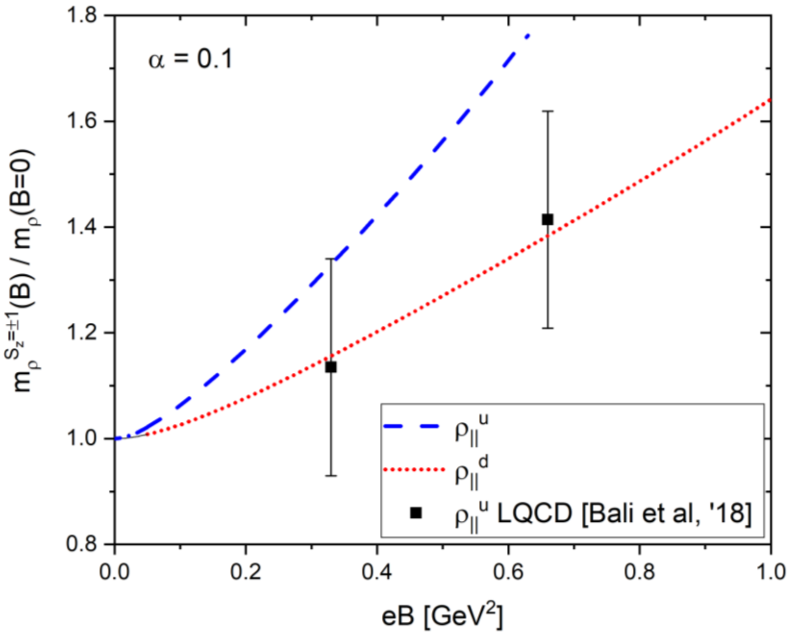
Mixing with vectors

Comparison of mass of lowest S_z=0 state with LQCD results

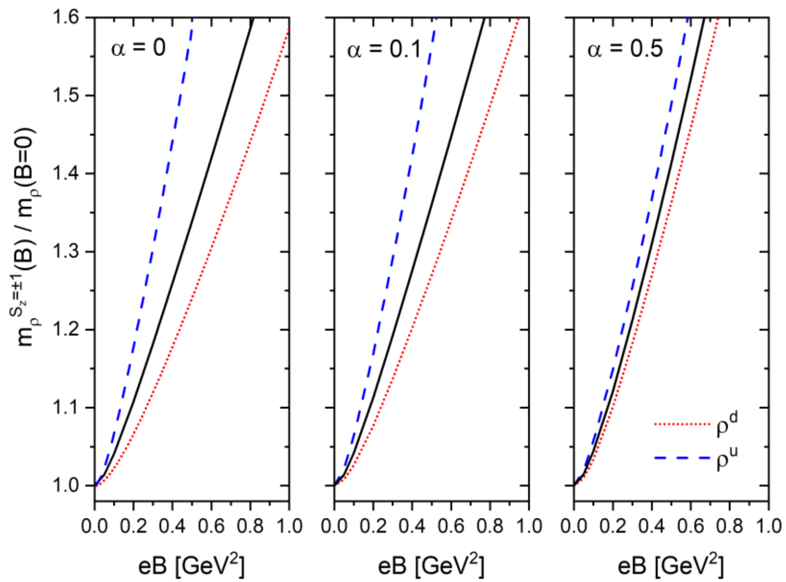


Masses of vector $S_z=\pm1$ states

Comparison with LQCD results



Effect of flavor mixing and $\rho\omega$ the mixing on masses of Sz=±1 vectors



Black line NO $\rho\omega$ mixing

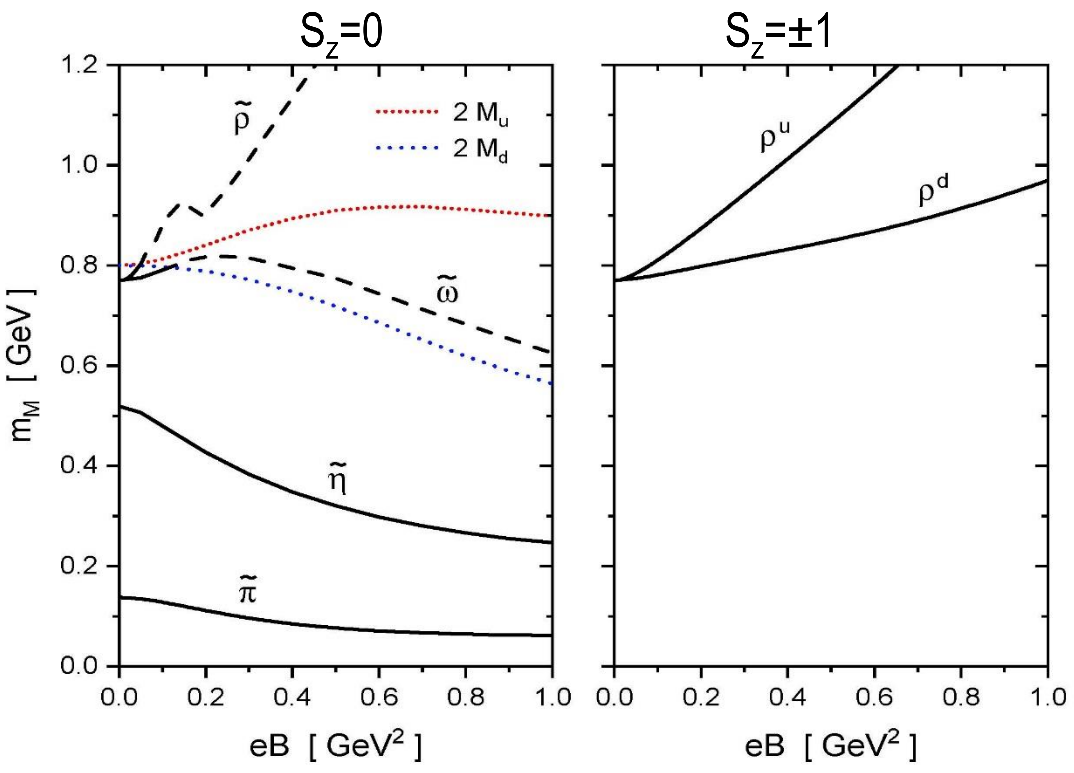
Effect of B-dependent couplings

We take

$$g(eB)/g = g_v(eB)/g_v = \kappa_1 + (1-\kappa_1)\exp\left[-\kappa_2 (eB)^2\right]$$

$$\begin{cases} \kappa_1 = 0.321 \\ \kappa_2 = 1.31\ \mathrm{GeV}^2 \end{cases}$$

This leads to behavior of quark masses similar to e.g. Endrodi-Marko, '18, etc

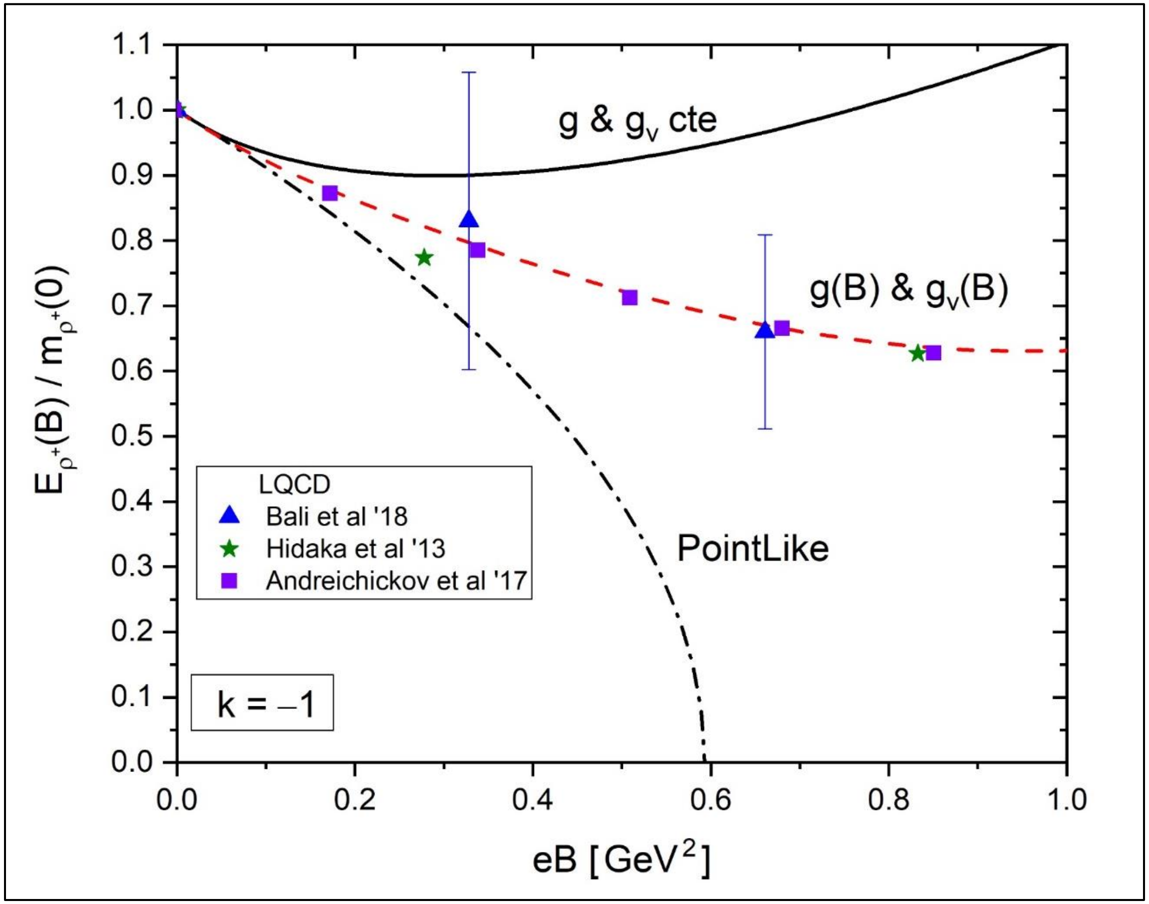


Results do not change much except for S_z=0 vectors (which lie above threshold)

CHARGED MESON SECTOR

We use $\alpha = 1/2$ for simplicity

Lowest $\rho^{\pm}$ state (k = -1)

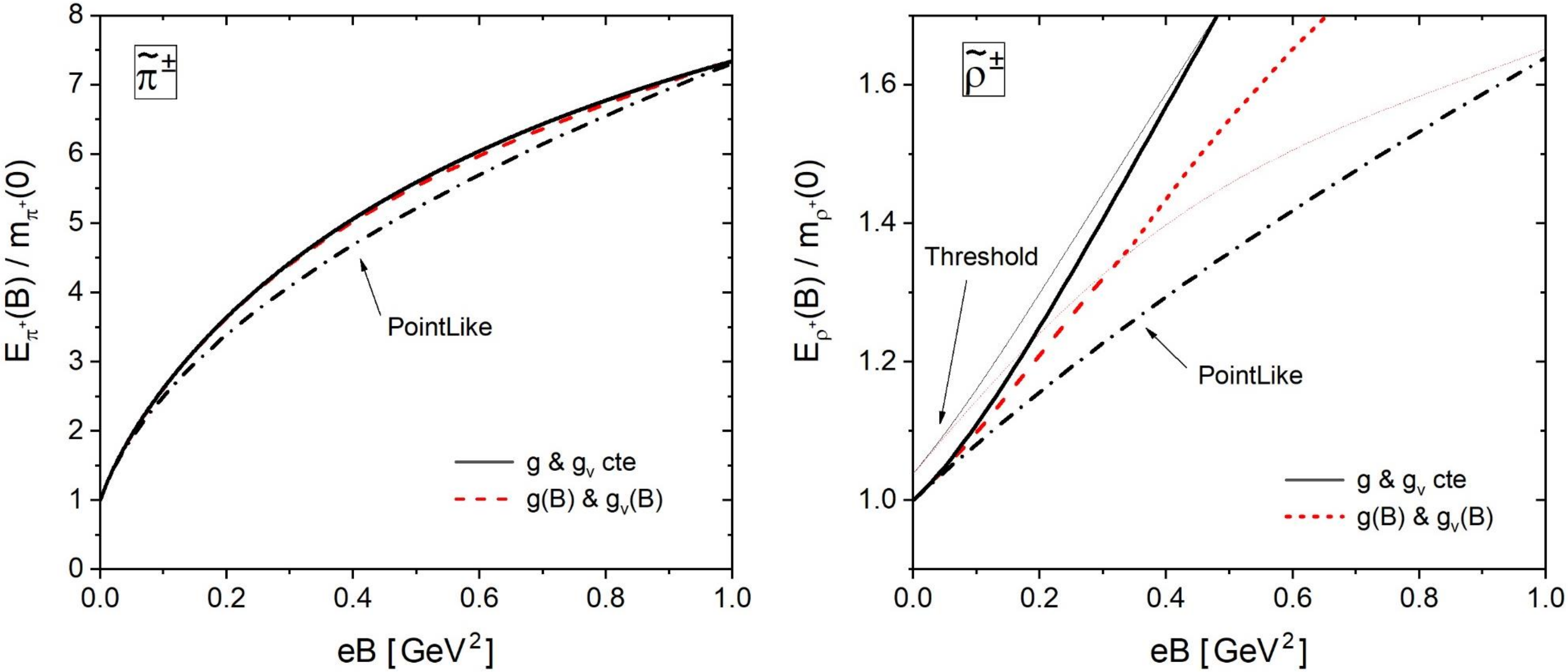


PointLike

$$E_\rho = \sqrt{\left(m_\rho^{B=0}\right)^2 - eB}$$

Approximation used by Chernodub, '10 to suggest existence of charged vector condensation, i.e. Vacuum Super Conductor (VSC)

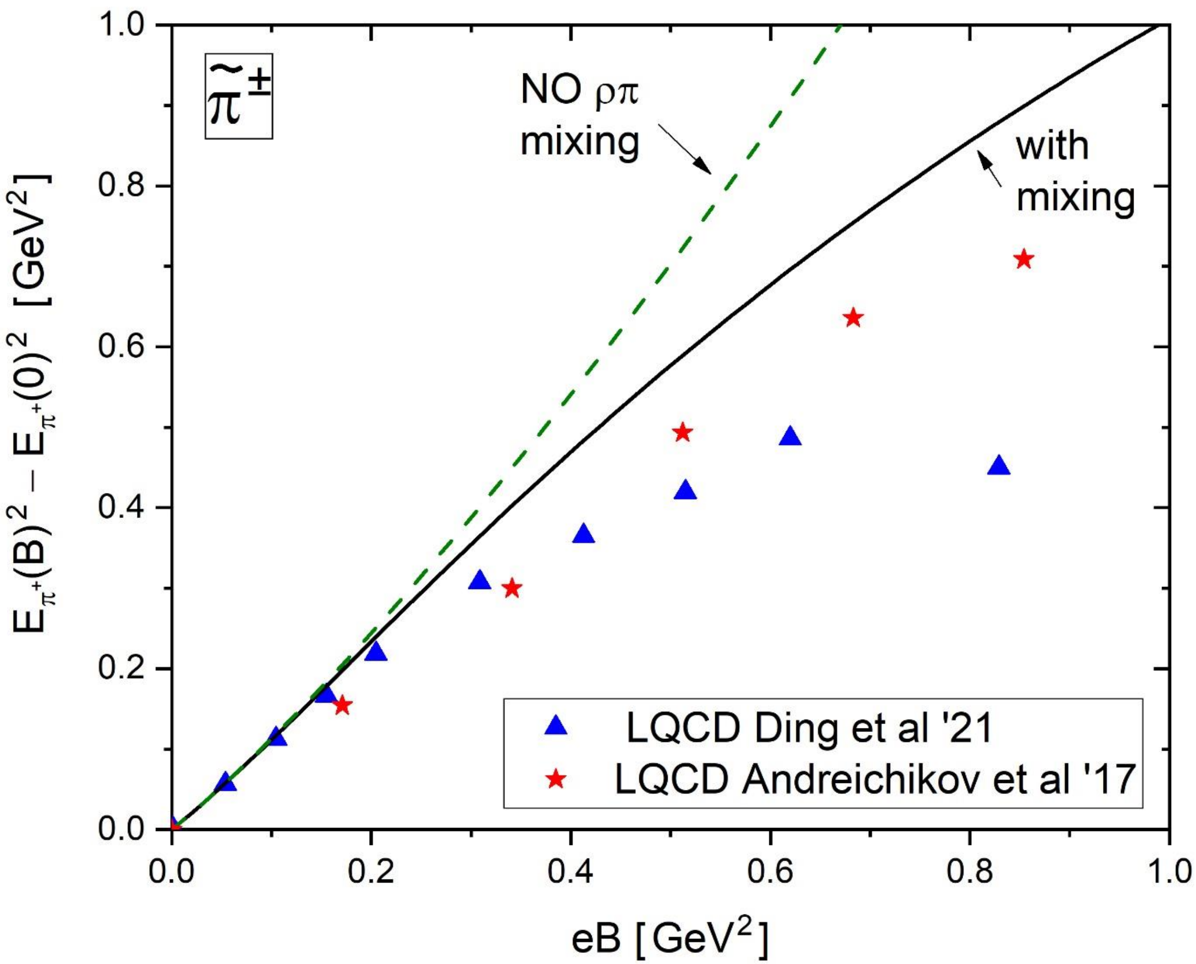
Our result differs from previous SU2 NJL calculations (Liu et al '15; Cao '19) which find VCS. They set ρ meson at rest ($\vec{q}$ =0) and neglect Schwinger Phase [It might be called ``Plane Wave Approximation''] k=0 states (S_z=0)



eB [GeV^2]	$\tilde{\pi}^+$ $c_{\pi^+}^{\tilde{\pi}^+}$	$c_{\rho^+}^{\tilde{\pi}^+}$	$\tilde{\rho}^+$ $c_{\pi^+}^{\tilde{\rho}^+}$	$c_{\rho^+}^{\tilde{\rho}^+}$
0.05	0.999	0.013	-0.156	0.988
0.5	0.960	0.281	-0.702	0.713
1.0	0.892	0.453		

Composition for selected values of eB (g and g_v ctes)

Comparison mass of lowest $\tilde{\pi}$ meson with LQCD results



(g and g_v ctes)

Summary and Conclusions

- We have considered the effect of a strong magnetic field on the masses of pseudoscalar and vector mesons in the context of an extended NJL model taking into account the mixings induced by the magnetic field

- For neutral mesons with Sz=0 we find that while the pseudocalars masses tend to decrease as the magnetic field increases those of the vectors behave in the opposite way.

- The effect of the mixing on mass of the lowest state (the \`pion´) is non-negligible. Mass somewhat small as compared with LQCD results. Axials ?

- Masses of neutral mesons with Sz=±1 tend to increase with B

- Effect of B-dependent couplings on neutral meson masses rather small

- The mass of lowest ρ^+ decreases at low B but stabilizes at a non-zero value. NO VSC. Different from other NJL calculations that neglect SP

- Effect of $\rho\pi$ mixing on mass of lowest charged \`pion´ non-negligible. Gets NJL results closer to those of LQCD