Strange magnetars admixed with fermionic dark matter

Eduardo S. Fraga
Cold dark matter & compact stars

★ DM is hard to probe, so one needs extreme gravitational interactions.

★ In principle, DM particles could collide with neutrons and other components of NS, loose energy, be gravitationally trapped, and accumulate in their cores.

★ NB: nucleon interactions (not ideal Fermi gas) + momentum dependence of the hadronic form factors → significant suppression of DM capture rate in NS [Bell et al (2021)].
★ For high enough central densities, one expects to find either hybrid stars, i.e., neutron stars with a quark matter core, or even more exotic objects, such as quark stars.

★ If quark stars are to be found in the universe, they have most likely accumulated some amount of dark matter over the course of their lives.

★ What is the effect of the presence of cold fermionic DM on:

→ the structure of quark stars (mass, radius, etc.)?

→ their stability w.r.t. radial oscillations?

→ quark magnetars with very high magnetic fields?

[F. Weber, 2000]
★ Which ingredients do we need?

- Equations of state for cold DM and cold QM.
- Stellar structure from TOV equations for two-fluid stars.

\[ \text{pressure}(T, \mu, B, \text{etc}) + \text{TOV} \]

- Stability equations & behavior of fundamental frequency.
- Incorporation of large magnetic fields in the EoS for QM.
Equations of state

Self-interacting CDM

[Narain, Schaffner-Bielich & Mishustin (2006); Mukhopadhyay & Schaffner-Bielich (2016)]

★ Fermi gas + two-body self-repulsion between fermions
★ Useful dimensionless quantities: \( z = k_F / m_D \); \( y = m_D / m_I \)
★ \( m_I \): interaction mass scale
★ \( m_D = 1, 10, 50, 100, 200, 500 \) GeV (dark fermion mass)
★ \( y = 0.1 \) (weak DM); \( y = 10^3 \) (strong DM)
★ Pressure:

\[
\rho_{DM} = \frac{1}{24\pi^2} \left[ (2z^3 - 3z)\sqrt{1 + z^2} + 3\sinh^{-1}(z) \right] + \left( \frac{1}{3\pi^2} \right)^2 y^2 z^6
\]
Effects from DM self-interaction

★ Self-interacting case corresponds to much larger masses and radii.

[Solving the TOV equations: Self-interacting case corresponds to larger masses and radii]

[Ferreira & ESF (in prep.)]
Cold QM

★ MIT bag model, perhaps the most popular approach to QM in NS.

Asymptotic freedom + confinement in the simplest and crudest fashion: bubbles (bags) of perturbative vacuum in a confining medium. + eventual corrections $\sim \alpha_s$

★ Asymptotic freedom: free quarks and gluons inside color singlet bags

★ Confinement: vector current vanishes on the boundary

★ $B = (145 \text{MeV})^4 \approx 57 \text{MeV/fm}^3$ is the bag constant chosen to surpass the two-solar mass limit.

★ Pressure:

\[
p_{\text{QM}} = \frac{3 \mu_q^4}{4\pi^2} - B
\]

($\mu_q$: quark chemical potential)
Dependence on the choice of the bag constant

[Weissenborn, Sagert, Pagliara, Hempel, Schaffner-Bielich (2011)]
Stellar structure of one-fluid stars

★ From the TOV equations
[Einstein's GR field equations + spherical symmetry + hydrostatic equilibrium]

\[
\frac{dp}{dr} = - \frac{GM(r)\epsilon(r)}{r^2 \left[ 1 - \frac{2GM(r)}{r} \right]} \left[ 1 + \frac{p(r)}{\epsilon(r)} \right] \left[ 1 + \frac{4\pi r^3 p(r)}{M(r)} \right]
\]

\[
\frac{dM}{dr} = 4\pi r^2 \epsilon(r) ; \quad M(R) = M
\]

<table>
<thead>
<tr>
<th>m_D</th>
<th>M_{max}(M_\odot)</th>
<th>R_{min}</th>
<th>Compact Star</th>
</tr>
</thead>
<tbody>
<tr>
<td>100 GeV</td>
<td>10^{-4}</td>
<td>1 m</td>
<td>neutralino star (cold DM)</td>
</tr>
<tr>
<td>1 GeV</td>
<td>1</td>
<td>10 km</td>
<td>neutron star</td>
</tr>
<tr>
<td>1 GeV/0.5 MeV</td>
<td>1</td>
<td>10^3 km</td>
<td>white dwarf</td>
</tr>
<tr>
<td>10 keV</td>
<td>10^{10}</td>
<td>10^{11} km</td>
<td>sterile neutrino star</td>
</tr>
<tr>
<td>1 keV</td>
<td>10^{12}</td>
<td>10^{13} km</td>
<td>axino star (warm DM)</td>
</tr>
<tr>
<td>1 eV</td>
<td>10^{18}</td>
<td>10^{19} km</td>
<td>neutrino star</td>
</tr>
<tr>
<td>10^{-2} eV</td>
<td>10^{22}</td>
<td>10^{23} km</td>
<td>gravitino star</td>
</tr>
</tbody>
</table>

[Mukhopadhyay & Schaffner-Bielich (2016)]
Quark stars admixed with DM

★ Three possible configurations for dark compact stars

[Karkevandi et al. (2021)]
Stellar structure of two-fluid stars

★ Two-fluid TOV equations

\[ \frac{dp_{QM}}{dr} = -\left( p_{QM} + \epsilon_{QM} \right) \frac{d\nu}{dr}, \quad \frac{dm_{QM}}{dr} = 4\pi r^2 \epsilon_{QM}, \]
\[ \frac{dp_{DM}}{dr} = -\left( p_{DM} + \epsilon_{DM} \right) \frac{d\nu}{dr}, \quad \frac{dm_{DM}}{dr} = 4\pi r^2 \epsilon_{DM}, \]
\[ \frac{d\nu}{dr} = 2 \left( m_{QM} + m_{DM} \right) + 4\pi r^3 \left( p_{QM} + p_{DM} \right) \frac{1}{r(r - 2(m_{QM} + m_{DM}))}, \]

★ Boundary conditions:

- \( m_{QM}(r \to 0) = m_{DM}(r \to 0) \to 0 \)
- \( R_{QM} > R_{DM} \): first \( p_{DM}(R_{DM}) \to 0 \); later \( p_{QM}(R_{QM}) \to 0 \)
- \( R_{DM} > R_{QM} \): first \( p_{QM}(R_{QM}) \to 0 \); later \( p_{DM}(R_{DM}) \to 0 \)
Radial oscillations

\* \( \Delta r/r \equiv \xi \) \& \( \Delta p \) are the independent variables ; \( \Gamma \): adiabatic index

[Jiménez & ESF (2022)]

\* For two-fluid stars one can write the total Lagrangian variables as 
\( \xi \equiv \xi_{QM} + \xi_{DM} \) \& \( \Delta p \equiv \Delta p_{QM} + \Delta p_{DM} \)

Two-fluid radial pulsating equations

\[
\frac{d\xi_{QM/DM}}{dr} \equiv -\frac{1}{r} \left( 3\xi_{QM/DM} + \Delta p_{QM} \right) - \frac{dp}{dr} \frac{\xi_{QM/DM}}{(p + \epsilon)},
\]

\[
\frac{d\Delta p_{QM/DM}}{dr} \equiv \xi_{QM/DM} \left\{ \omega^2 e^{\lambda-\nu}(p + \epsilon)r - 4\frac{dp}{dr} \right\} + \xi_{QM/DM} \left\{ \left( \frac{dp}{dr} \right)^2 \frac{r}{(p + \epsilon)} - 8\pi e^\lambda(p + \epsilon)pr \right\} + \Delta p_{QM/DM} \left\{ \frac{dp}{dr} \frac{1}{p + \epsilon} - 4\pi(p + \epsilon)re^\lambda \right\}
\]

\[
\lambda(r) = -\ln(1 - 2(m_{QM}(r) + m_{DM}(r))/r)
\]

\* \( \omega \): oscillation frequency ; \( \lambda(R_{QM}) = -\nu(R_{QM}) \) \& \( \lambda(R_{DM}) = -\nu(R_{DM}) \)

[Condek et al. (1997)]
The increase in DM central energy density does not change the maximum mass and radius very much, but shifts the curves towards higher central energy densities.

The range of stable configurations occurs at higher central energy densities.
Strange star admixed with dark matter

Self-interacting DM

- Slight decrease of maximum mass with the increase of DM central energy density.
Results for different values of $m_D$ – structure and stability of quark stars admixed with DM

$w_{DM}$: $y = 0.1$ ; $m_D = 1, 10$ GeV

★ Mass–radius visible modifications only for small $m_D$.

★ Higher QM energy densities to compensate for the extra gravitational pull from DM.

★ Stability window of ultra–light quark stars (surrounded by DM): $10^{-18} – 10^{-4}$ $M_\odot$, depending on $m_D$ → dark strange “planets” and strangelets.
Increasing DM central densities, the maximum QM central densities are increased by a factor of ~20 in some cases.

Results very sensitive to m_D.

sDM: y = 10^3 ; m_D = 1, 10 GeV

★ As for wDM, in most of the cases M, R and central energy densities of the QM core are not appreciably affected.

★ As we increase m_D, the fundamental frequency is strongly affected.
Effects from high magnetic fields

Soft gamma repeater (SGR) in 1979
(Mazets et al., 1979 [7])
(Cline et al, 1980 [8])

Anomalous X-ray pulsar (AXP)
(Mereghetti & Stella, 1995 [9])

Magnetars surface magnetic fields of the order of $10^{14}$ G - $10^{15}$ G.

★ Magnetic fields inside magnetars may reach values $B \sim 10^{18}$ G.

★ For quark magnetars:

![Graph 1](image1.png)

![Graph 2](image2.png)
Strange magnetars admixed with self-interacting dark matter

Again, the magnetic field produces a additional shift.

★ Magnetic fields tend to “pull” in the same direction as DM: larger central energy densities, smaller masses.

★ NB: red curves have no DM.

★ There was no concern in producing 2M☉ stars here (which is possible).
★ Compactness:

![Graphs showing the decrease in total mass as a function of the fraction of DM for different values of B.](image)

★ Decrease in total mass as a function of the fraction of DM for different values of B:

<table>
<thead>
<tr>
<th>B (G)</th>
<th>Slope of the linear fit ($\alpha$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-3.6</td>
</tr>
<tr>
<td>$5 \times 10^{17}$ G</td>
<td>-3.2</td>
</tr>
<tr>
<td>$1 \times 10^{18}$ G</td>
<td>-2.9</td>
</tr>
<tr>
<td>$2 \times 10^{18}$ G</td>
<td>-2.1</td>
</tr>
</tbody>
</table>
Other observables - preliminary

★ Radial profiles (no DM):

★ Magnetic fields tend to increase the central energy density.

★ Significant effect on the tidal deformability.

★ Tidal deformability:

★ More observables on the way…
Summary and outlook

★ We investigated effects of weakly ($y = 0.1$) and strongly ($y = 10^3$) self-interacting DM on the structure of quark stars for dark fermion masses $m_D = 1, 10, 50, 100, 200, 500$ GeV.

★ Results are very sensitive to $(m_D, y)$. In most situations, central QM densities are increased by the presence of DM (extra gravitational pull). Other effects are usually modest modifications.

★ Strong magnetic fields affect significantly density profiles and tidal deformability. Total mass, radius and compactness not so much.

★ Next steps: new observables, quark matter EoS from cold and dense pQCD, hybrid stars, include magnetic field effects on TOV.
Back up slides
Boundary conditions

★ Demanding:

→ smoothness at the QM or DM stellar center

→ Vanishing \( p_{QM/DM} \) at \( R_{QM/DM} \)

\[
\begin{align*}
\nu(R_{QM}) &= \ln \left( 1 - \frac{2(M_{QM} + m_{DM}(R_{QM}))}{R_{QM}} \right) \\
\nu(R_{DM}) &= \ln \left( 1 - \frac{2(m_{QM}(R_{DM}) + M_{DM})}{R_{DM}} \right)
\end{align*}
\]

\[\Delta p_{QM/DM}\]_center \equiv -3(\xi_{QM/DM} \Gamma p_{QM/DM})_center

\[\Delta p_{QM/DM}\]_surface \equiv 0

★ We define \( \omega^2 \to \omega^2_{QM/DM} \) if we are dealing with a QM/DM oscillating core in the admixed star.
$y = 0.1$

$\mathbf{m_D = 50, 100 \, \text{GeV}}$

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**Workshop on EM effects in strongly interacting matter, São Paulo, October/2022**

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\[ y = 10^3 \]
\[ m_D = 50, 100 \text{ GeV} \]
y = 10^3
m_D = 200, 500 GeV