

# **Molecular Dynamics Simulation of supercooled ZnSe: Structural relaxation and Crystal nucleation**

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II ICTP-SAIFR Condensed Matter Theory in the Metropolis  
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Temperature



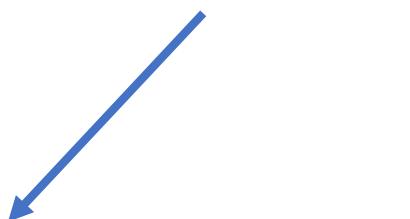
Liquid



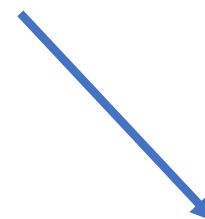
Super cooled Liquid

Melting Temperature

Ultimate fate



Amorphous/Glass



Crystal

The kinetics of crystal nucleation in SCLs are extremely important and play a crucial role in numerous natural processes such as

- snow precipitation
- crystallization of proteins and minerals

industrial technologies such as

- single crystals
- metal casting and solidification
- drug design and production
- glass-ceramics

- To properly understand the atomistic dynamics of SCL
  - ❖ Two characteristic times
    - ✓ intrinsic atomic diffusivity that leads to structural rearrangement of the SCL - RELAXATION TIME -  $\tau_R$
    - ✓ Average period need to spontaneously form the 1<sup>st</sup> crystalline critical nucleus – NUCLEATION TIME -  $\tau_N$

# Race between the two relaxation times

If  $\tau_N < \tau_R$   crystallization

If  $\tau_N > \tau_R$   liquid relax - glass

Temperature which  $\tau_N = \tau_R$   kinetic spinodal temperature  $T_{ks}$

OBS:  $T_{ks}$  is not the classical thermodynamics spinodal, where the thermodynamic barrier for the liquid/crystal transformation vanishes

# Race between the two relaxation times

The relationship between this characteristic time has been scarcely investigated.

Experimentally:  $\text{Li}_2\text{O} \cdot 2\text{B}_2\text{O}_3$  and  $\text{Li}_2\text{O} \cdot 2\text{SiO}_2$

- Simulation:
- Pressurized  $\text{SiO}_2$
  - Supercooled Ni, Cu,  $\text{Cu}_5\text{Zr}$
  - Binary L-J mixture
  - $\text{Cu}_x\text{Zr}_{1-x}$ ,  $0.15 \leq x \leq 0.645$
  - BaS

## Case studied: ZnSe

- Some properties:
  - direct wide band-gap
  - Low absorptivity at infrared wavelength
  - Preferred material for lenses, windows, output coupler, beam expander
  - Used laser-diodes
  - Green-blue LEDs
  - Crystallizes in both zinc-blende and wurtzite, under ambient conditions

# Our Approach

## Molecular Dynamics Simulation

- Interaction potential
- Spontaneous crystallization

# Interatomic Potential – Vashishta/Rahman

$$\Phi(r) = \sum_{i<j=1}^N V_{i,j}(r) + \sum_{i,j<k} V_{jik}^{(3)}(\vec{r}_{ij}, \vec{r}_{ik})$$

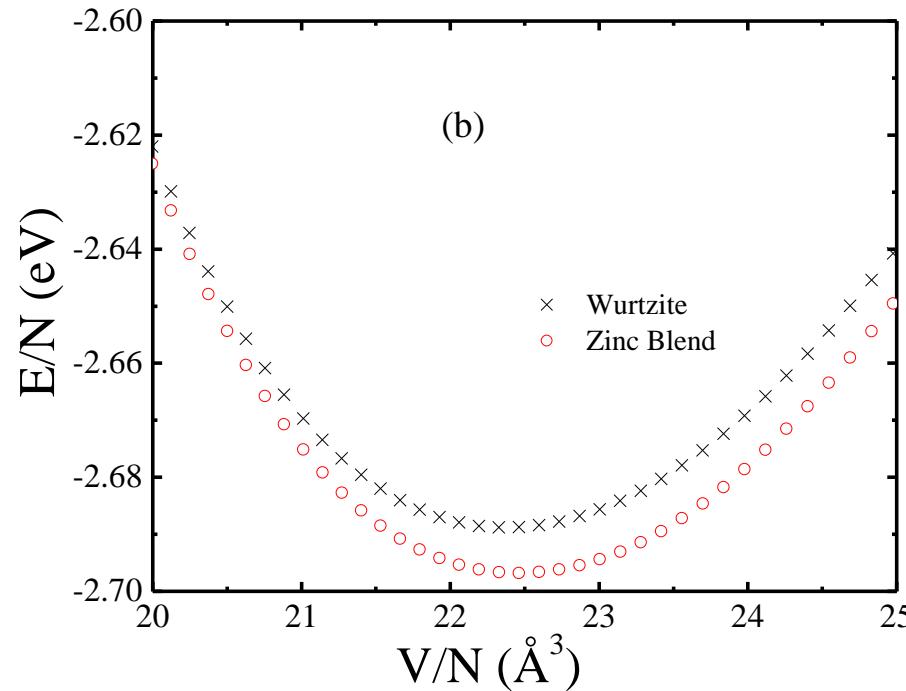
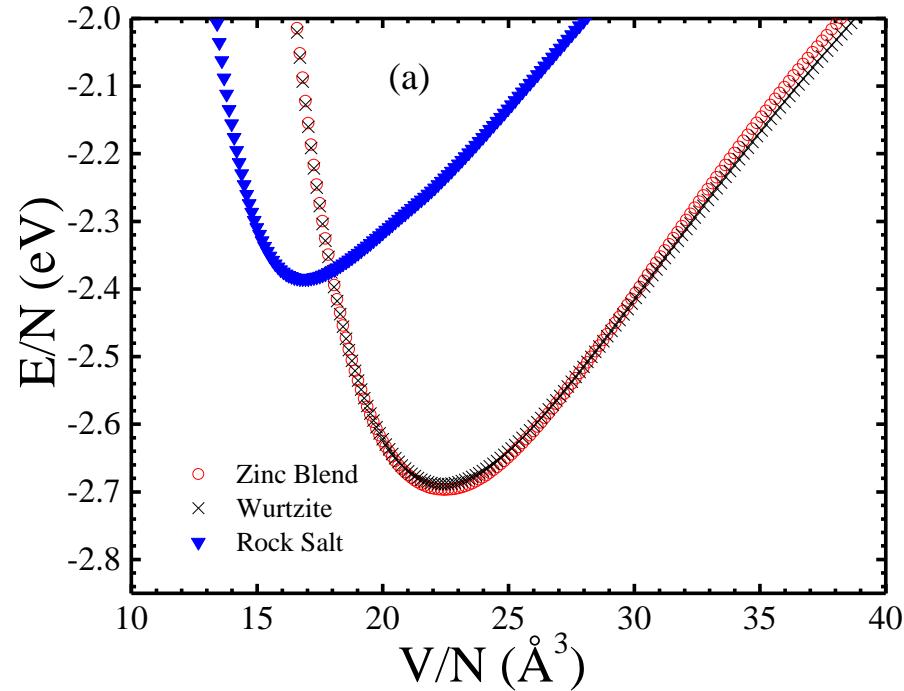
$$V_{ij}(r) = \frac{H_{i,j}}{r^{\eta_{i,j}}} + \frac{Z_i Z_j}{r} \cdot e^{-r/\lambda} - D_{i,j} \left( \frac{\alpha_i Z_j^2 + \alpha_j Z_i^2}{2r^4} \right) \cdot e^{-r/\zeta} - \frac{W_{i,j}}{r^6}$$

$$V_{jik}^{(3)}(\vec{r}_{ij}, \vec{r}_{ik}) = B_{jik} \exp\left(\left(\frac{1}{r_{ij}-r_o} + \frac{1}{r_{ik}-r_o}\right)\right) \frac{(\cos \theta_{jik} - \cos \bar{\theta}_{jik})^2}{1 + C_{jik} (\cos \theta_{jik} - \cos \bar{\theta}_{jik})^2}.$$

# Partial results

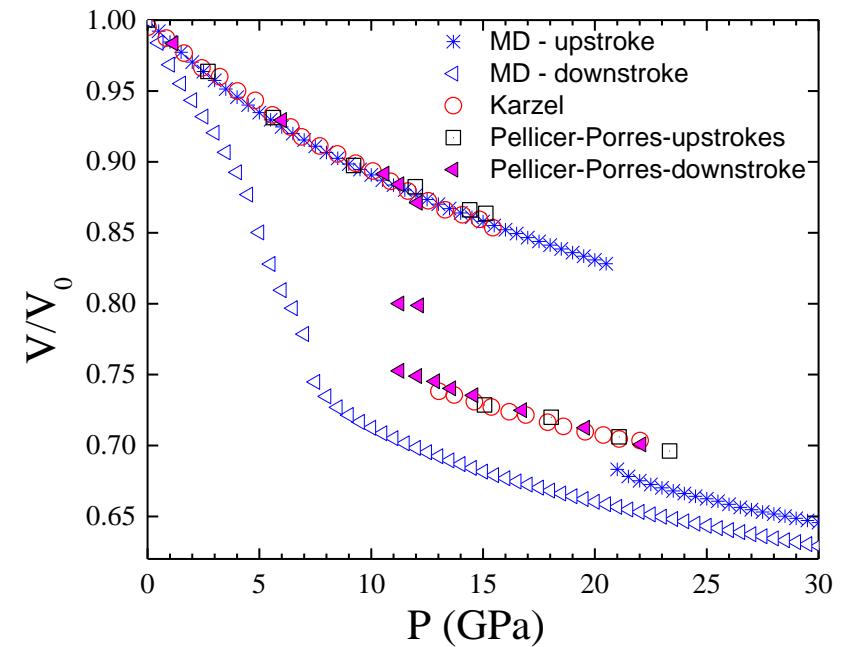
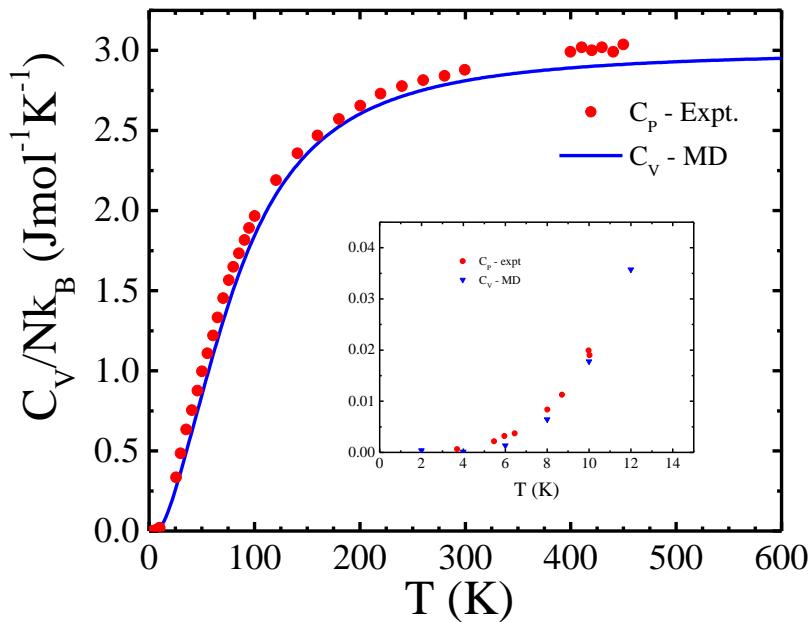
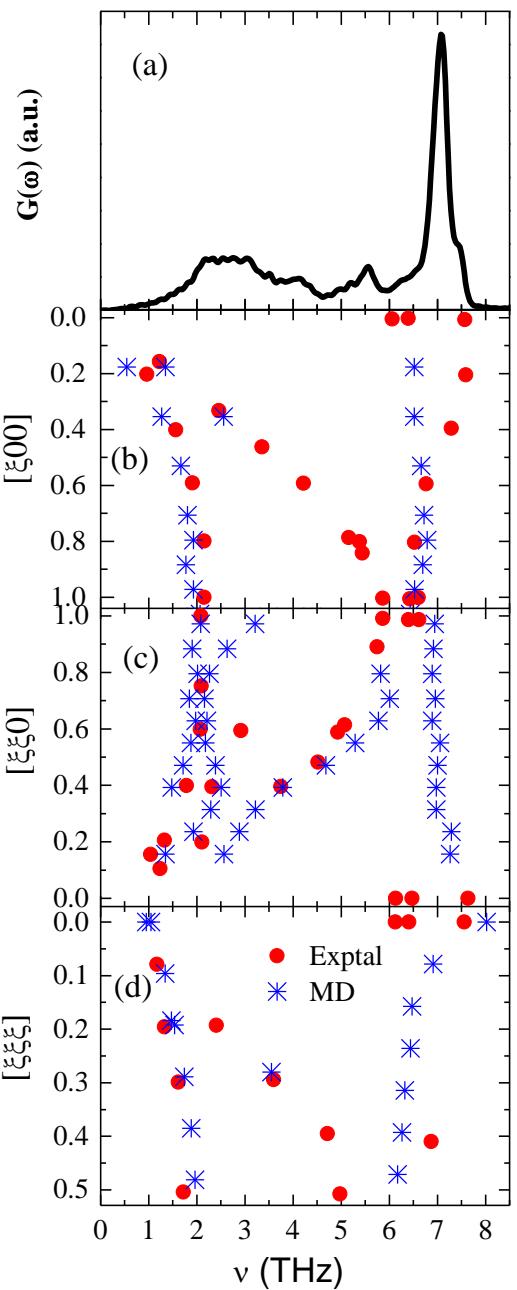
Quantities used to find parameterization	MD	Experimental
Cohesive energy (eV/N)	-2.697	-2.697
Lattice constant (Å)	5.643	5.643 (extrapolated to 0K), 5.667 at 300K
Bulk modulus (GPa)	61.85	59.53 - 78.4
Elastic Constant $C_{11}$ (GPa)	83.70	81 – 104.6
Elastic Constant $C_{12}$ (GPa)	50.91	48.8 – 65.3
Elastic Constant $C_{44}$ (GPa)	31.02	44.1 – 46.13
Melting Temperature (K)	1388	1800

# Partial results - Energetic



$$E_{ZB} - E_{WZ} = -8.5 \text{ meV}$$

# Partial results



	MD	Expt*
$\Theta_{\text{Debye}}$ (K)	$226 \pm 1$	$237 \pm 6$ at room temperature $224 \pm 2$ at nitrogen temperature

- N.P. Sharma, Indian Pure Appl. Phys. 10, 478 (1972)
- R. Passler, Phys. Status Solid B 248, 904 (2011)
- J. Pellicer-Porres et al. Phys.Rev.B 65, 012109 (2001)
- H. Karzel et al. Phys. Rev. 53, 11425 (1996)

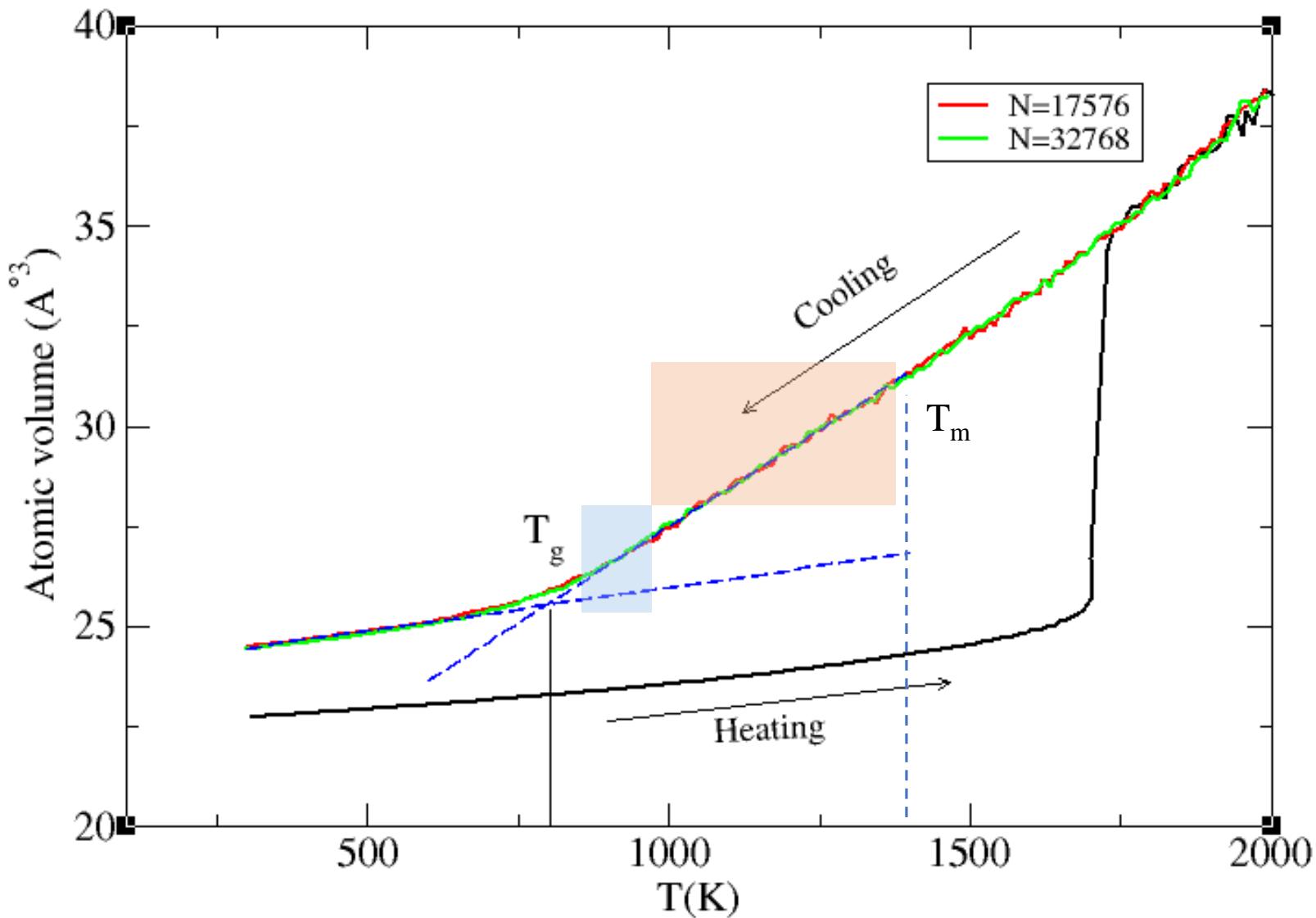
# Partial conclusions

- ✓ Very good interatomic potential
- ✓ The system spontaneously crystallize when cooled down

## Goal

- ✓ Nucleation rate, relaxation time
- ✓ Systems with 17,576 and 32,768 particles

# heating & cooling



# Spontaneous nucleation – T $\leq$ 1000K

Birth time, or nucleation time,  $\tau_N$

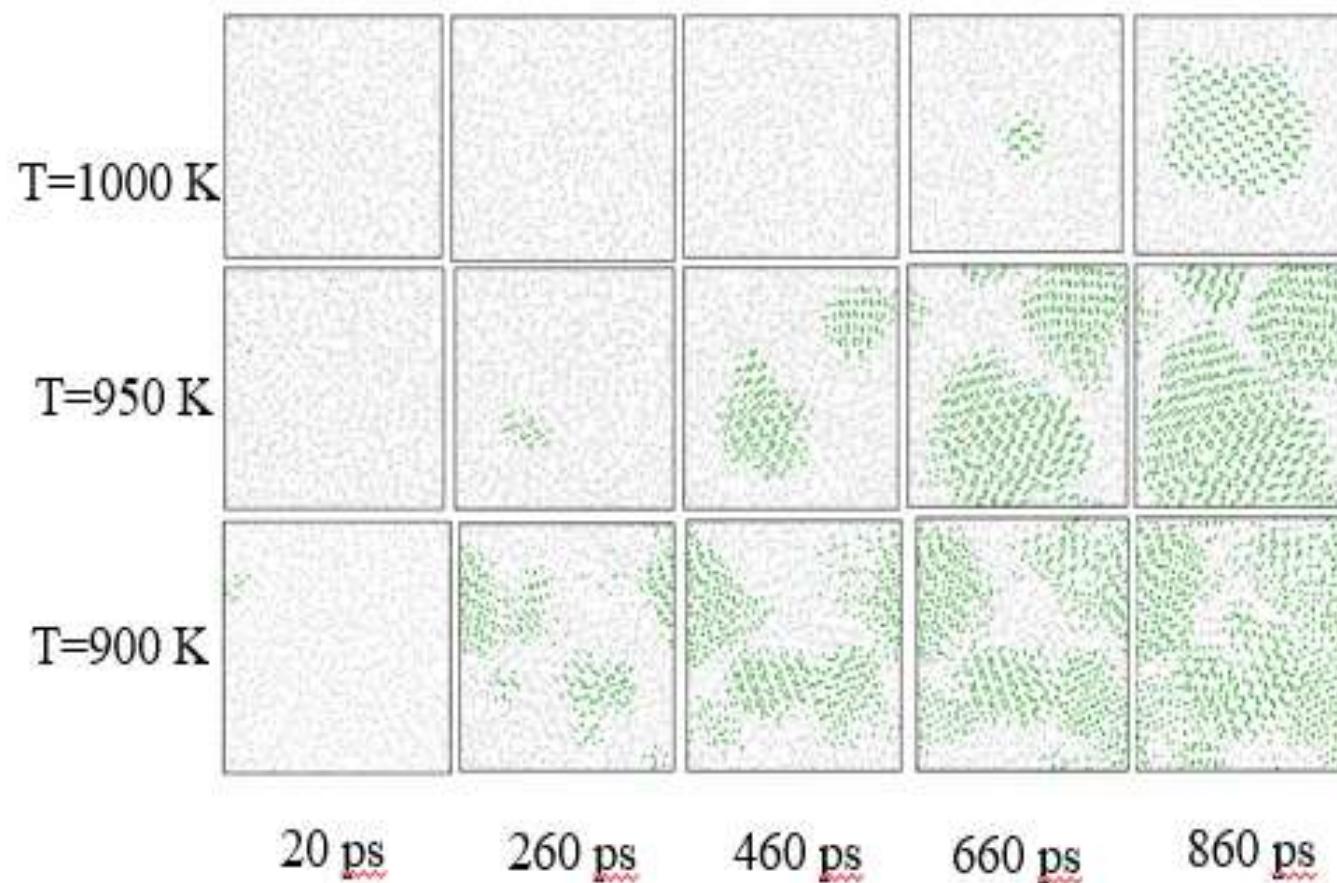
T = 1000K, 950K 900K

System sizes: 17,576 and 32,768 particles

Average over 15 samples

Solid like particles – Steinhart bond-order parameter

$$S_{ij} = \sum_{m=-6}^{m=+6} q_{6m}(i) \cdot q_{6m}^*(j); \quad q_{6m}(i) = \frac{1}{N_b(i)} \sum_{j=1}^{Nb(i)} Y_{lm}(\vec{r}_{in})$$



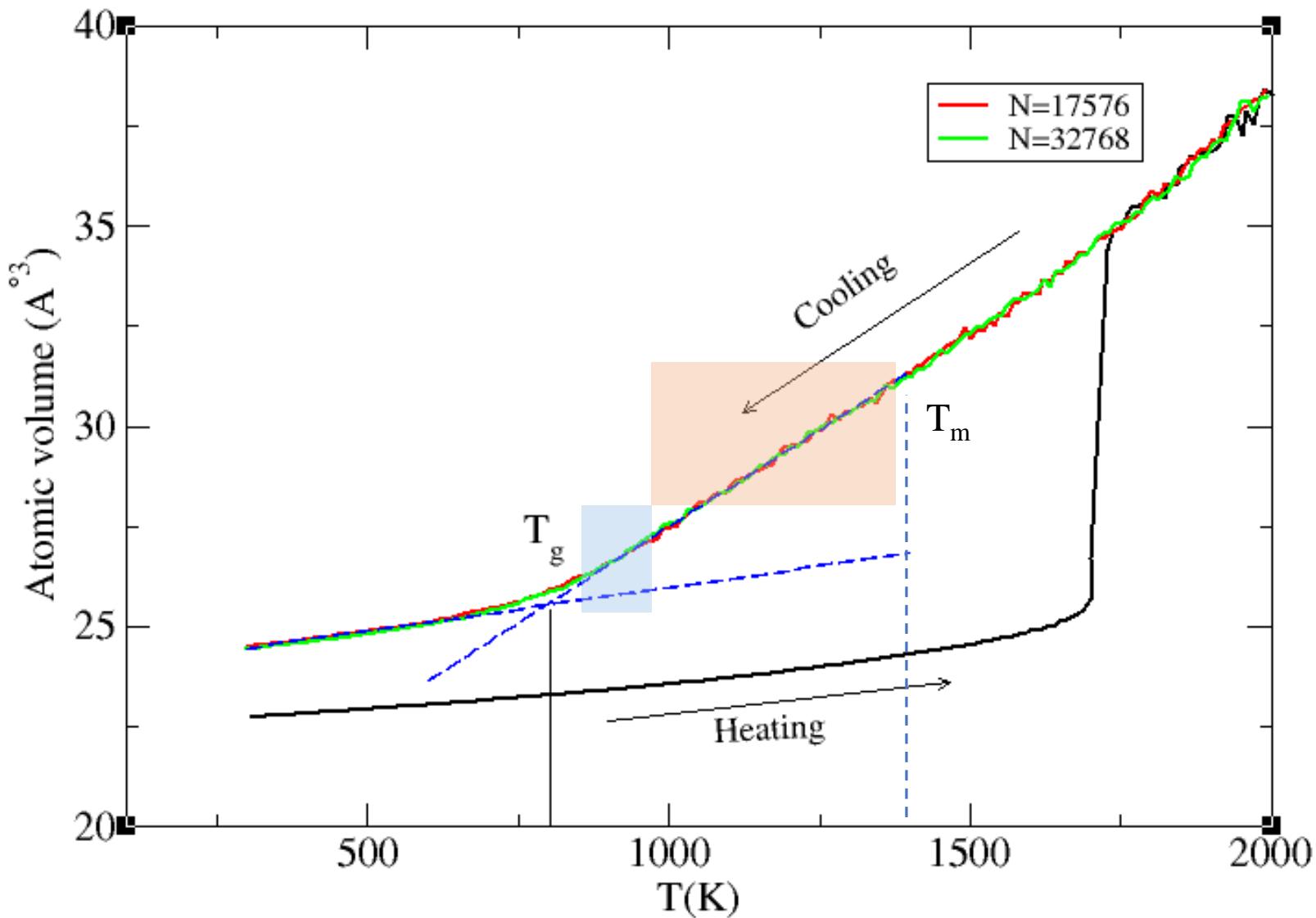
# Spontaneous nucleation – T $\leq$ 1000K

T (K)	N	$\tau_N$ (ps)	$J_{ss}$ (ps $^{-1}$ Å $^{-3}$ )
1000	17,576	803.8	2.5x10 $^{-9}$
	32,768	740	1.5x10 $^{-9}$
	3.7x10 $^{10}$	5.0x10 $^{-4}$	
950	17,576	224.7	0.9x10 $^{-8}$
	32,768	168.3	0.7x10 $^{-8}$
	3.7x10 $^{10}$	1.25x10 $^{-4}$	
900	17,576	161.2	1.3x10 $^{-8}$
	32,768	78.8	1.4x10 $^{-8}$
	3.7x10 $^{10}$	7.41x10 $^{-5}$	

$$J_{ss} = \frac{1}{\tau V}$$

N=3.7x10 $^{10}$  particles  $\longrightarrow$  V=1μm $^3$

# heating & cooling



# Structural Relaxation time

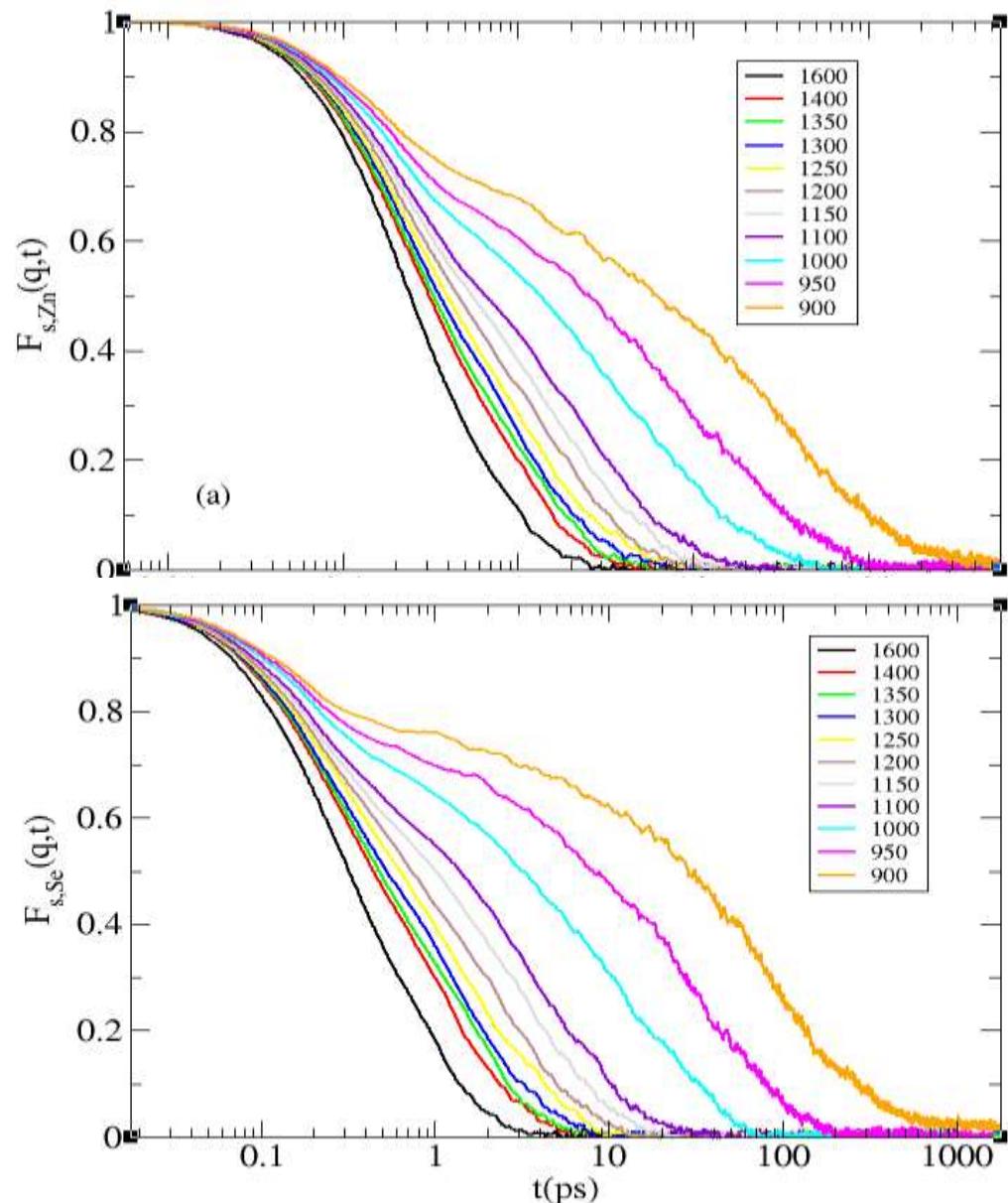
$$(1000 < T < T_m)$$

$$F_s(q, t) = N_\alpha^{-1} \sum_{j=1}^{N_\alpha} \langle \exp(i\vec{q} \cdot (\vec{r}_j^\alpha(t) - \vec{r}_j^\alpha(0))) \rangle$$

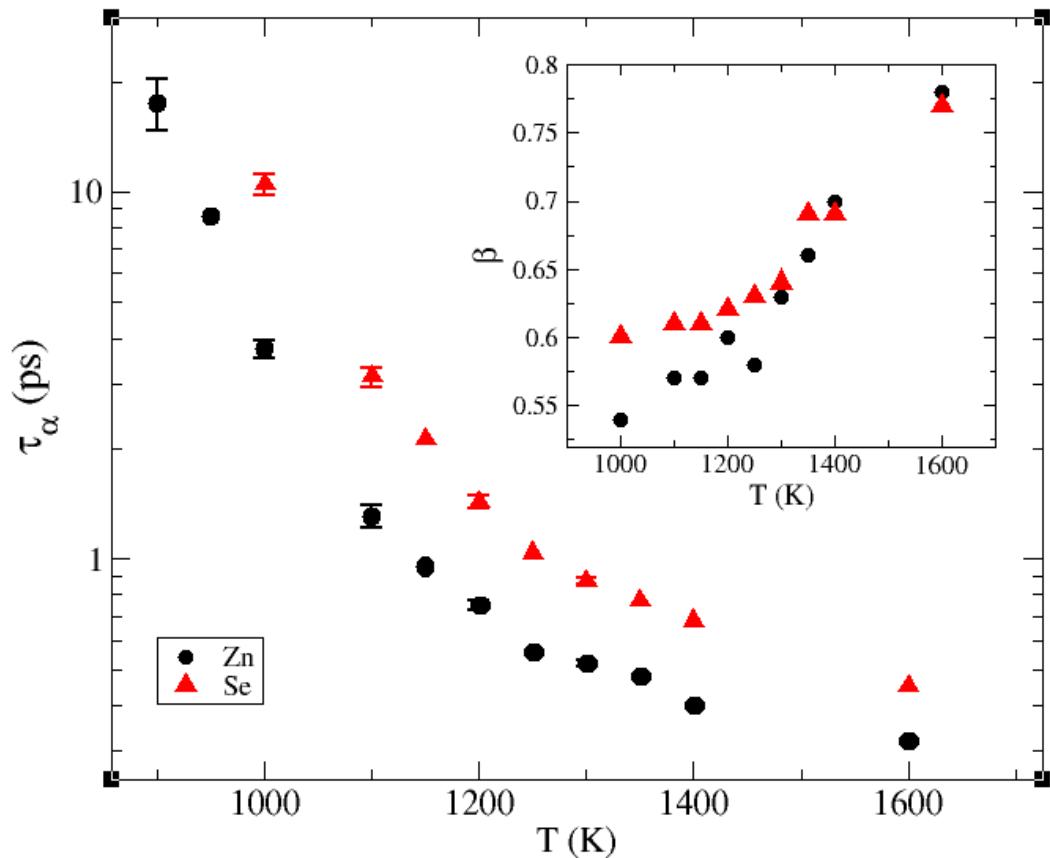
The tail (long time) is described by the KWW law  
(Kohlrausch-Williams-Watts)

$$F_s(q, t) = F_s(q) \exp[-(\frac{t}{\tau_\alpha})^\beta]$$

By decreasing the temperature, a plateau appears which is related to the time needed by particles to break out of the cage created by neighboring particles.



# Average relaxation time – structural relaxation time



$$\tau_\alpha(T) = (\langle \tau_{\alpha,Zn} \rangle + \langle \tau_{\alpha,Se} \rangle)/2$$

$$\beta(T) = (\langle \beta_{Zn} \rangle + \langle \beta_{Se} \rangle)/2$$

Relaxation time  $\tau_R$

$$\tau_R(T) = \frac{\tau_\alpha(T)}{\beta(T)} \Gamma \left( \frac{1}{\beta(T)} \right)^{(a),(b)}$$

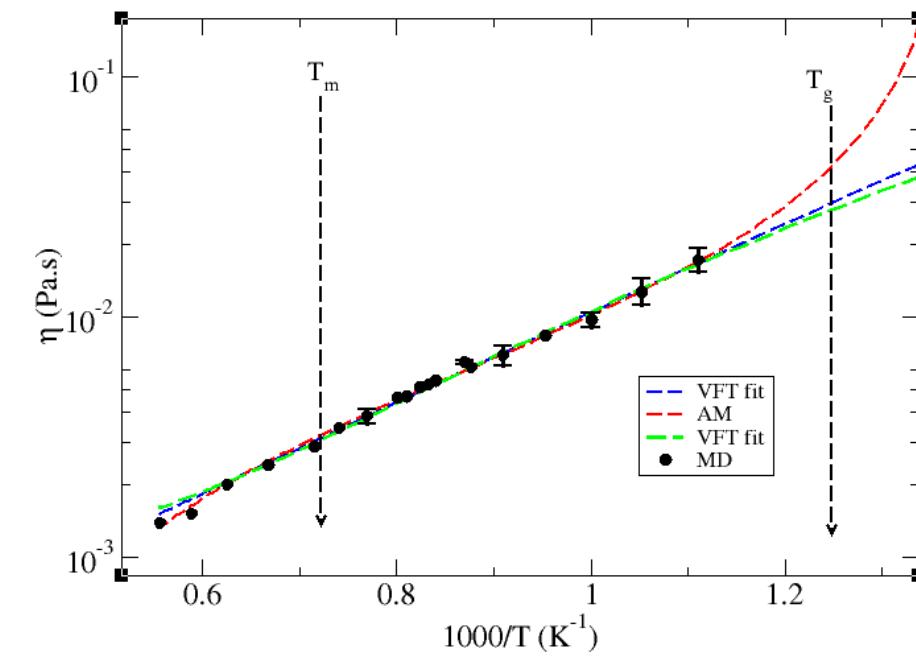
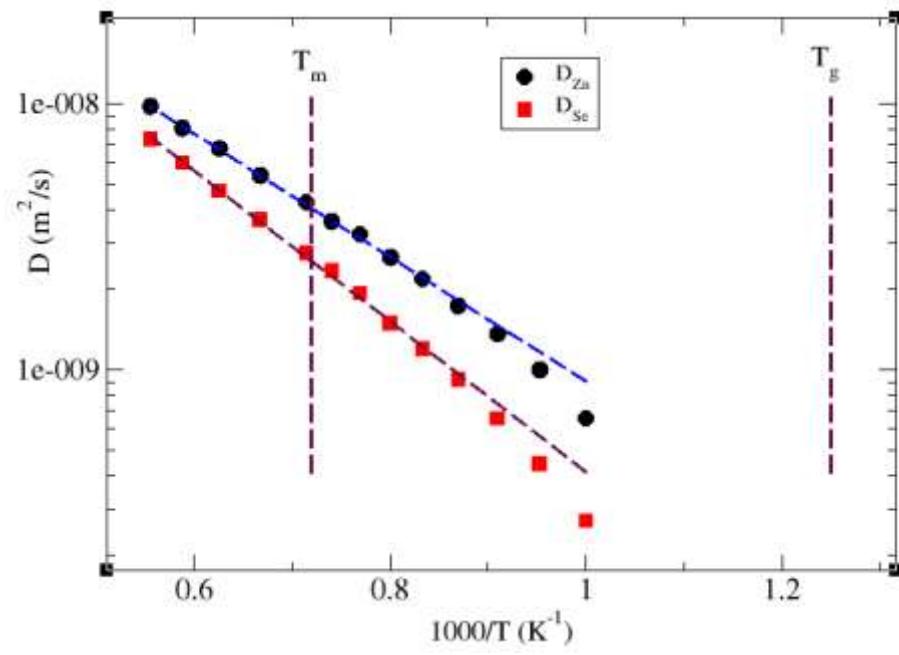
<sup>(a)</sup> Lancelotti, R.F. et al. J. Am. Ceram. Soc. 104 (2021) 2066.

<sup>(b)</sup> Doss, K. et al. J. Am. Ceram. Soc. 103 (2020) 23590.

# Diffusion and viscosity

$$\langle R^2(t) \rangle = 6Dt; D = D_o \exp\left(\frac{-E_A}{k_B T}\right)$$

$$\eta = \frac{V}{Nk_B} \int_0^\infty \langle P_{ij}(0)P_{ij}(t) \rangle dt$$

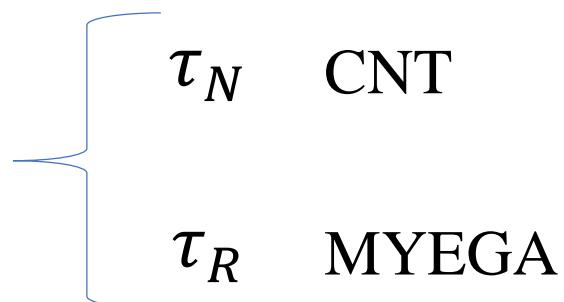


Deep supercooling : spontaneous crystallization  
(T: 1000K, 950K 900K)   $\tau_N$

Shallow supercooling : relaxation time  
( $900K \leq T \leq 1600 K$ )   $\tau_R$

How to compare ?

How to extrapolate to other temperatures?



## $\tau_N$ - Classical Nucleation Theory

Recall that   $J_{ss} = 1/\tau_N V$

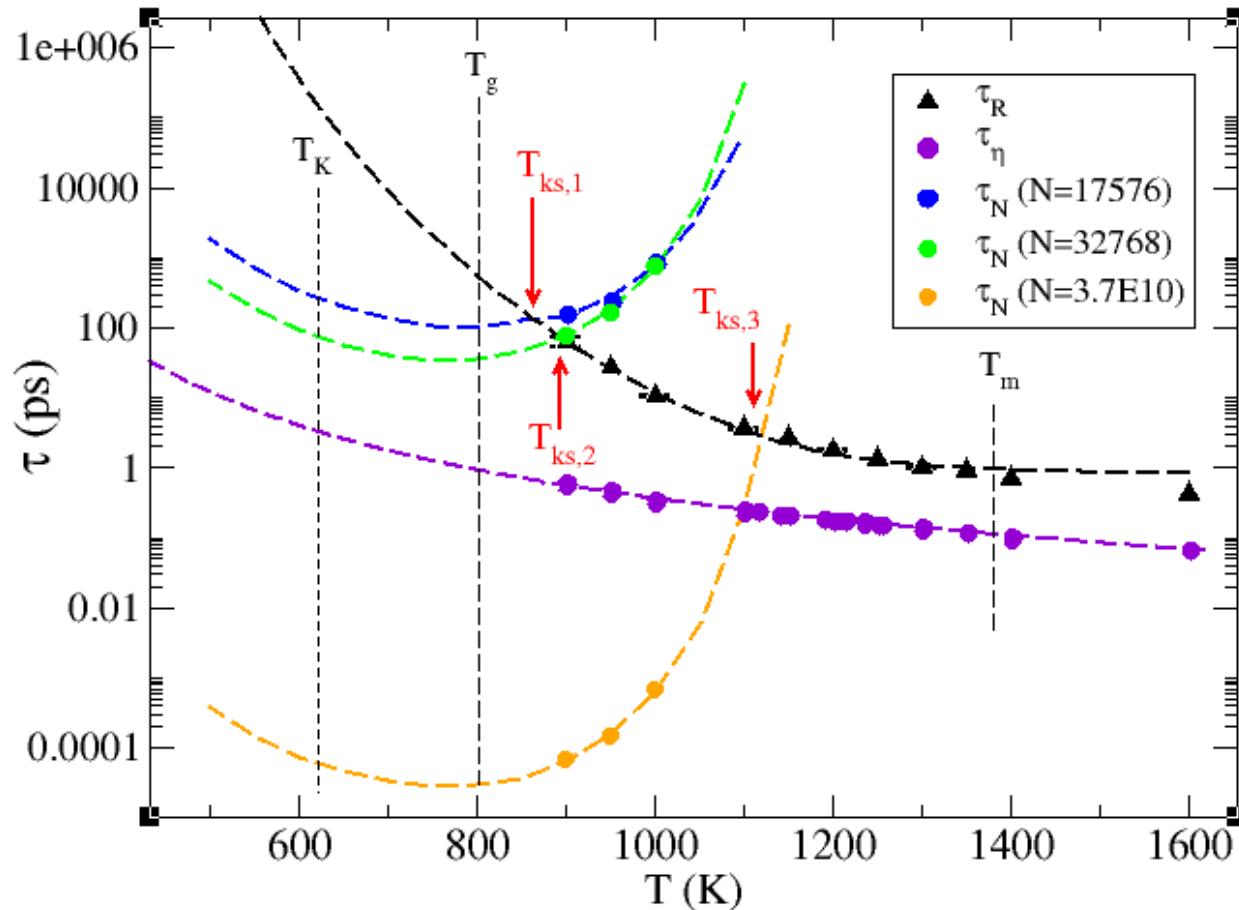
$$\frac{J_{ss}\sqrt{T}}{D} = A \exp\left(-\frac{B}{T}\right) \quad (\text{A and B fitted parameters})$$

## $\tau_R$ - MYEGA

Mauro-Yue-Ellison-Gupta-Allan - MYEGA

$$\log_{10}(\eta) = \log_{10}(\eta_\infty) + \frac{A}{T} \exp\left(\frac{B}{T}\right)$$

# The ultimate fate of supercooled ZnSe



Maxwell equation,

$$\tau_\eta = \frac{\eta}{G_\infty}$$

1.  $\tau_\eta < \tau_R$

2. above  $\sim 1100K$   $\tau_R < \tau_N$

3. Large system below  $\sim 1100K$   $\tau_N < \tau_R$

MYEGA:  $\log_{10}(\tau_R) = \log_{10}\tau_\infty + \frac{A}{T} \exp(B/T)$

## Critical cooling rate required to avoid nucleation

$$\frac{\Delta T}{\Delta t} = \frac{T_m - T_{nose}}{\tau_{nose}}$$

(Uhlmann D. R., J Non – Crystal Solids 7 (1972)337)

$\tau_{nose}$  related to  $T_{nose}$  (minimum point in the Time-Temperature transformation curve)

System size	Critical cooling rate (K/ps)
17576	5.5
32768	18.5

*cooling rate used 1K/ps*



*spontaneous nucleation already  
in the cooling procedure*

# Conclusions

- From MD we obtained both
  - nucleation time,  $\tau_N$  , (spontaneous crystallization)
  - relaxation time,  $\tau_R$  , from intermediate scattering function and viscosity
- Relaxation time using Maxwell relation,  $\tau_\eta$  , gives a lower boundary.  
(this confirms recent experimental results –  $\text{Li}_2\text{O} \cdot 2\text{B}_2\text{O}_3$  and  $\text{Li}_2\text{O} \cdot 2\text{SiO}_2$ )
- Shallow supercooling:  $\tau_R < \tau_N$  , relaxation occurs before nucleation.
- $T_{ks}$  increases as system size increases
- $T_{ks} > T_g$ , system crystallize (at least one critical nucleus) before become a glass.
- $T_g$ , refer to a residual supercooled liquid which already contain several crystalline nuclei formed on the cooling path.

# Collaborators



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Brazil