Quarkyonic matter: Excluded-Volume model and more…

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Workshop on Electromagnetic Effects in Strongly Interacting Matter

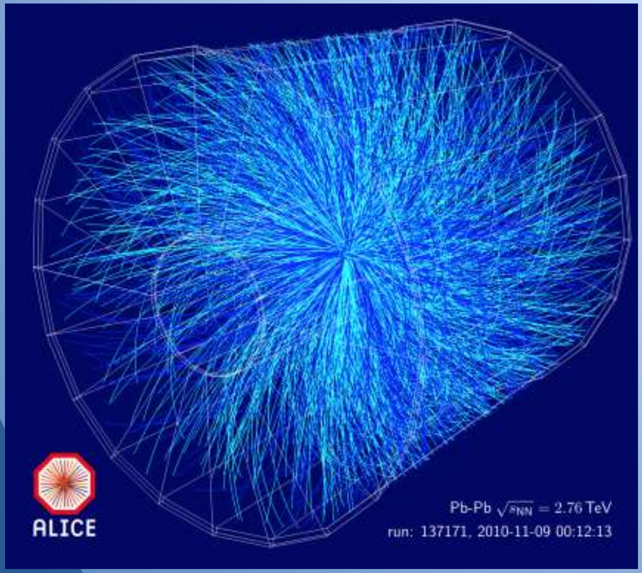
FAPESP ICTP unesp IFT - UNESP INSTITUTO DE FÍSICA TEÓRICA

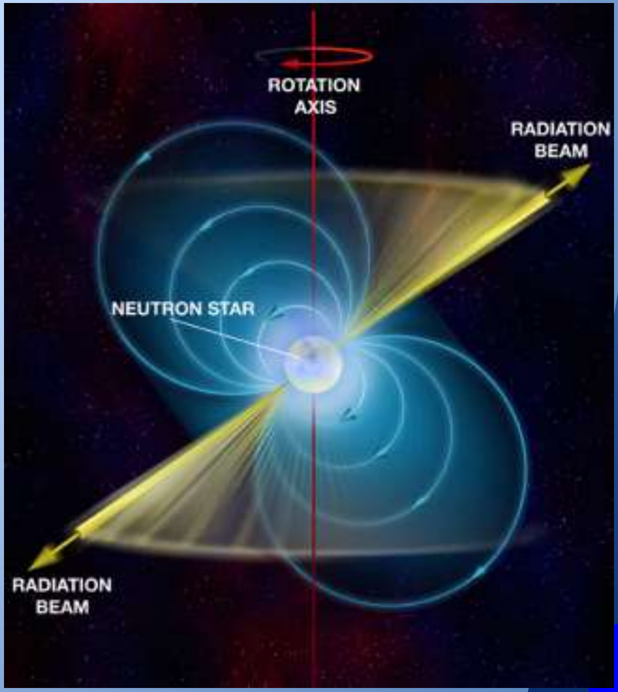
Outline

- Motivation
- Quarkyonic Matter
- Excluded Volume Model
- 3 Flavor Expansion
- Final Remarks

Motivation

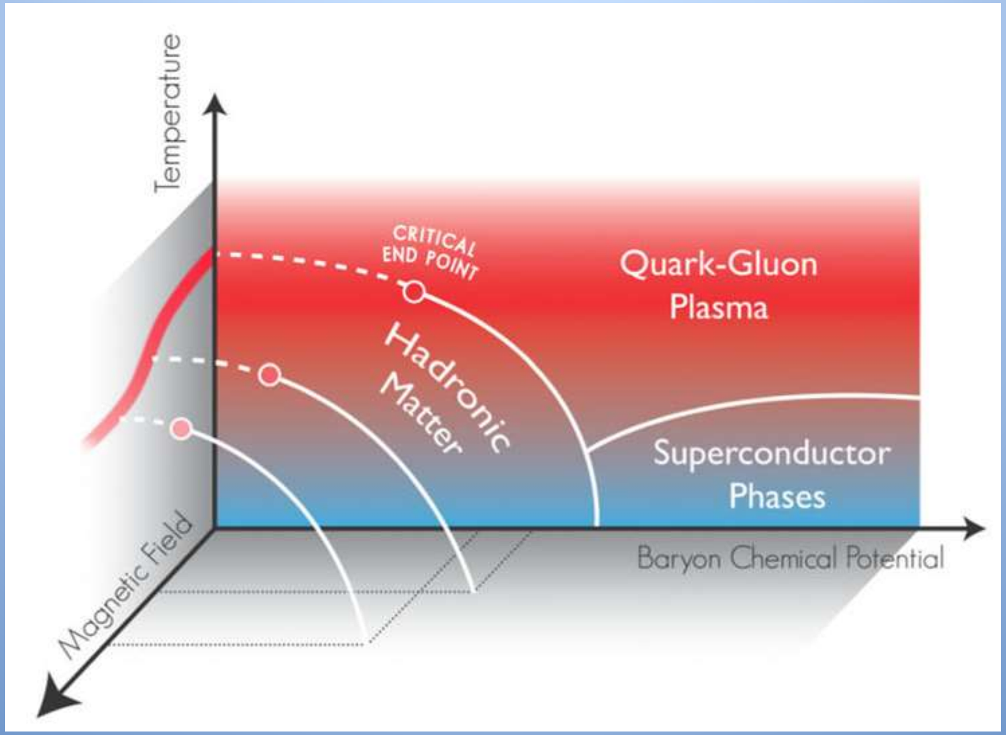
- QCD under extreme conditions (temperature, finite density and magnetic fields) plays an important role in understanding the transitions that took place in the early universe.





Motivation

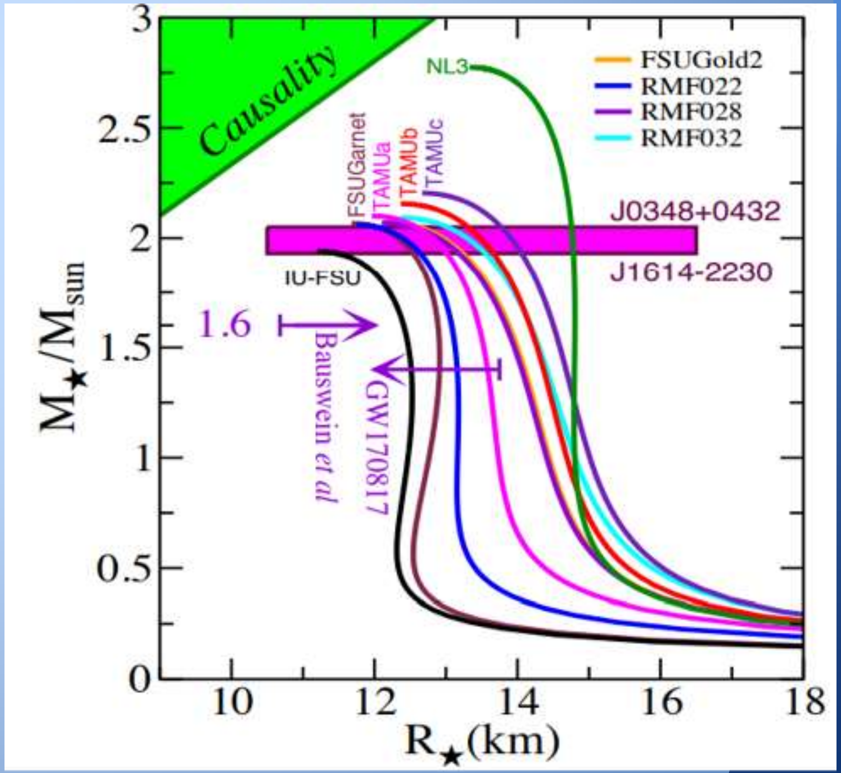
- One way to illustrate the different phases of strongly interacting matter we seek to study is by means of the phase diagram



Motivation

- Observation and analysis of GW170817: Important clues to understand cold and dense matter.

- The structure of a neutron star (NS) is determined by the Tolman-Oppenheimer-Volkoff Equation (TOV).

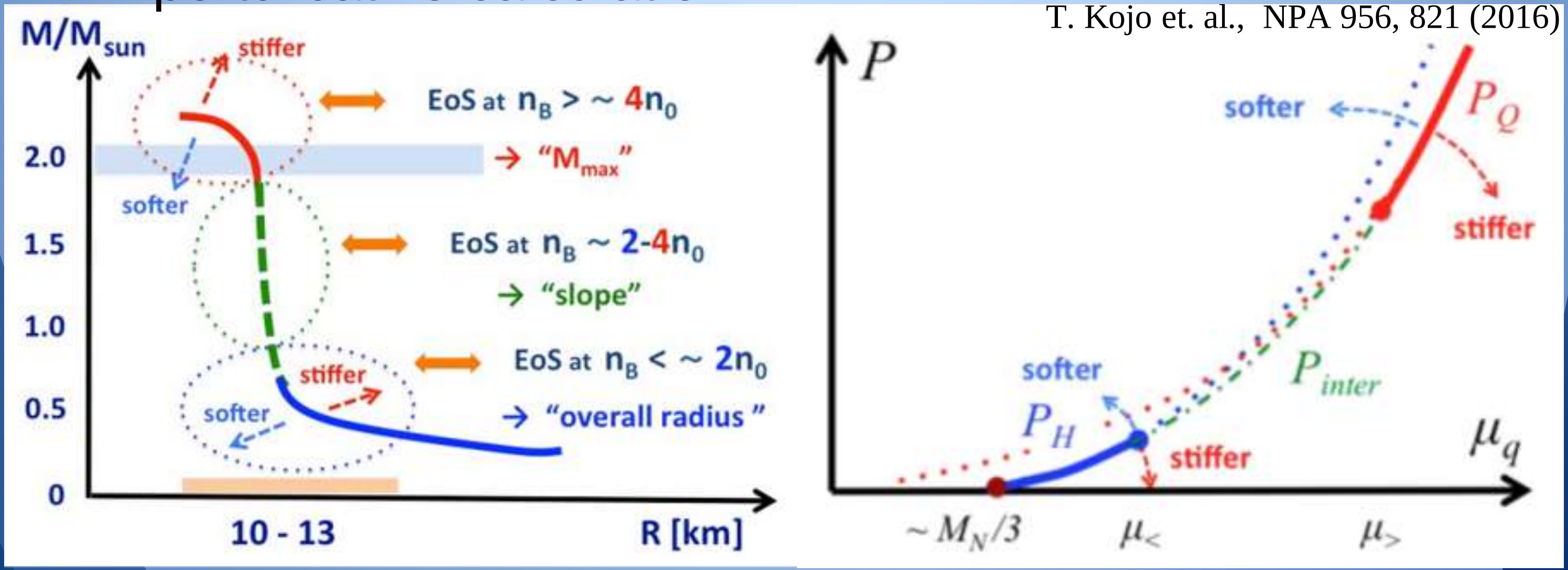


F. J. Fattoyev et. al, PRL 120, 172702 (2018)

Motivation

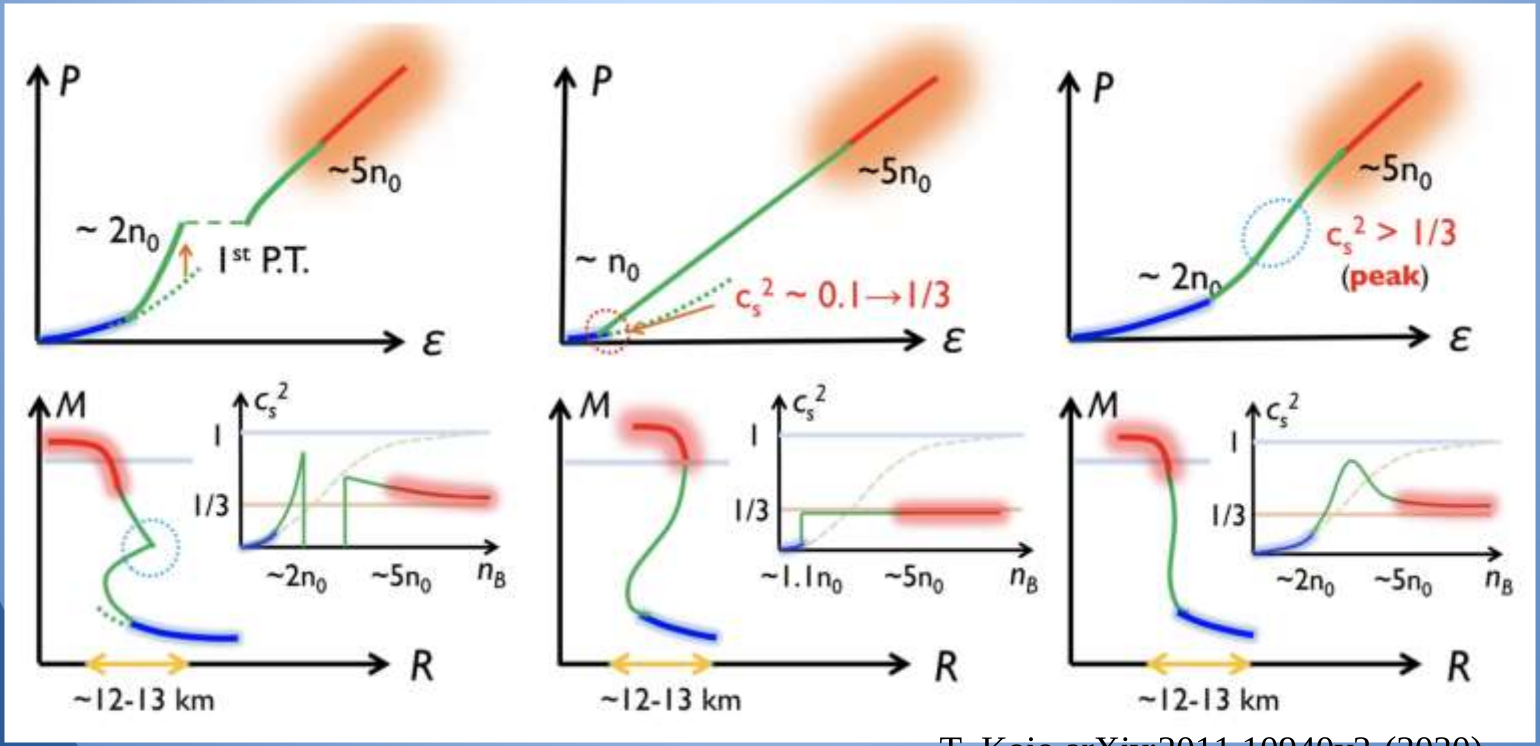
- Description of Equation of State (EoS) of dense QCD matter:
  - Around saturation density: Nuclear experiments.
  - Very high density limit: Asymptotic freedom allows perturbative calculation.

T. Kojo et. al., NPA 956, 821 (2016)



Motivation

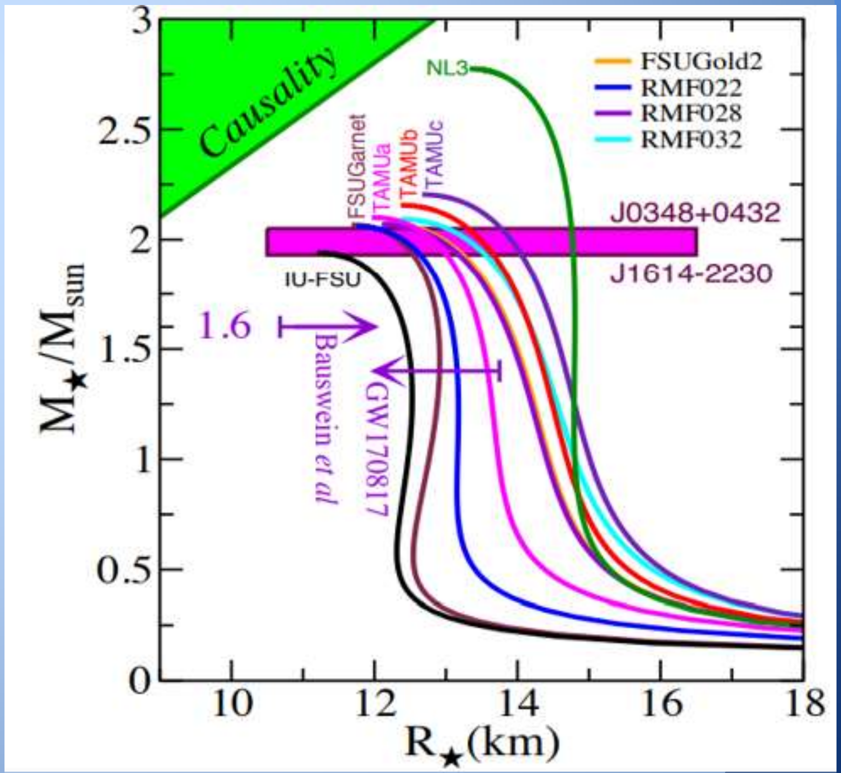
- TOV and EoS can give some insight about the transition quark-nucleon matter.



T. Kojo arXiv:2011.10940v2 (2020)

Motivation

- EoS should be hard enough to support $2M_{\odot}$ and soft enough to satisfy $R_{1.4} \leq 13.5$ km.
- This is also reflected in sound velocity, that should increase rapidly and can be greater then its conformal value $c_s^2 \geq 1/3$.

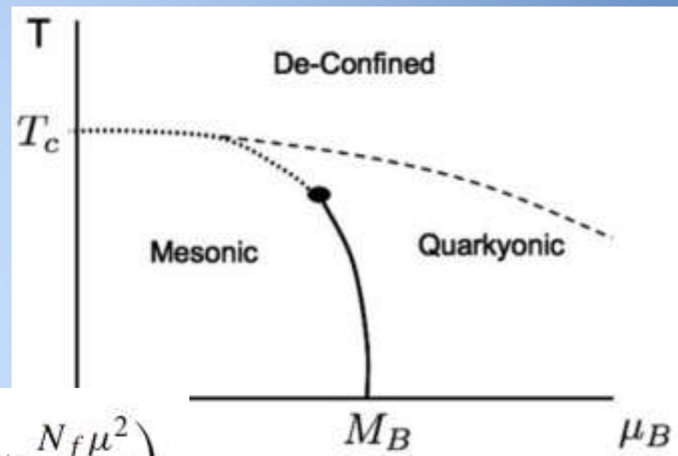


F. J. Fattoyev et. al, PRL 120, 172702 (2018)

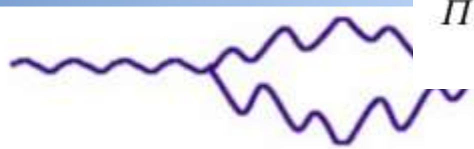
Any suggestions?

Quarkyonic Matter

Phase of dense matter, argued from large Nc approximation and model computations.



$$\Pi^{\mu\mu}(0) = g^2\left(\left(N_c + \frac{N_f}{2}\right)\frac{T^2}{3} + \frac{N_f \mu^2}{2\pi^2}\right).$$



Gluon loop

→$g^2 N_c T^2 \sim T^2$;

→Dynamics not affected by quarks;

→Debye screening at large distances.

Quark loop

→~ $\mu_Q^2 g^2 \Rightarrow$ Supressed by $1/N_c$ at large N_c.

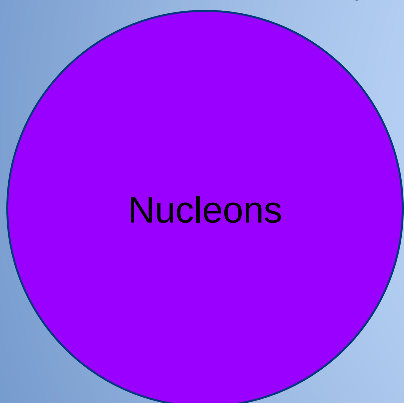
→High density limit: $\mu_Q \gg \Lambda_{QCD}$, so quarks are important when $\mu_Q \sim N_c^{1/2}\Lambda_{QCD}$.

→Debye screen mass $m_D \simeq g\,\mu_Q$

Quarkyonic Matter

Nuclear → Quarkyonic

(at few times ρ_0)

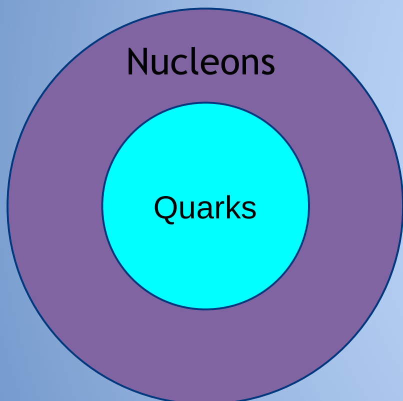


- For $k_F^B < \Lambda_{\mathrm{QCD}}$: Quarks confined in nucleons.

Quarkyonic Matter

Nuclear → Quarkyonic

(at few times ρ_0)

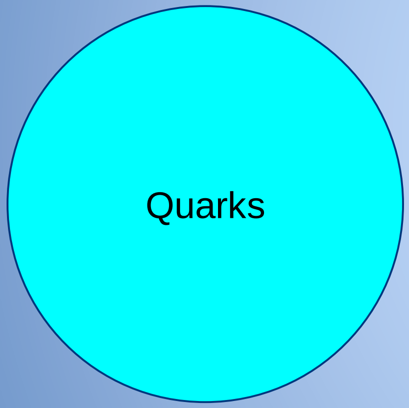


- For $k_F^B < \Lambda_{\mathrm{QCD}}$: Quarks confined in nucleons.
- For $\Lambda_{\mathrm{QCD}} \leq k_F^B \leq N_c \Lambda_{\mathrm{QCD}}$: Quarks starts to take low phase space, and a shell-like structure is formed.

Quarkyonic Matter

Nuclear → Quarkyonic

(at few times ρ_0)



- For $k_F^B < \Lambda_{\rm QCD}$: Quarks confined in nucleons.
- For $\Lambda_{\rm QCD} \leq k_F^B \leq N_c \Lambda_{\rm QCD}$: Quarks starts to take low phase space, and a shell-like structure is formed.
- For $k_F^B \simeq N_c^{3/2} \Lambda_{\rm QCD}$: Confinement disappears.

- Total baryon density has smooth behavior and chemical potential for confined states enhance suddenly, then pressure suddenly increases. *This is not an usual phase transition!*

Excluded Volume Model

- Nucleons with hard-core volume $v_0 = \frac{4}{3}\pi r_0^3$, where r_0 is the hard-core radius. Hard-core density: $n_0 = 1/v_0$
- In a system with baryon density n_N and volume V, the excluded volume (not occupied by baryon cores) is: $V_{ex} = V\left(1 - \frac{n_N}{n_0}\right)$

$$n_{ex} = \frac{n_N}{1 - n_N/n_0} = 2N_f \int^{k_F} \frac{d^3k}{(2\pi)^3} \qquad \mu_N = \frac{\partial \varepsilon}{\partial n_N}$$

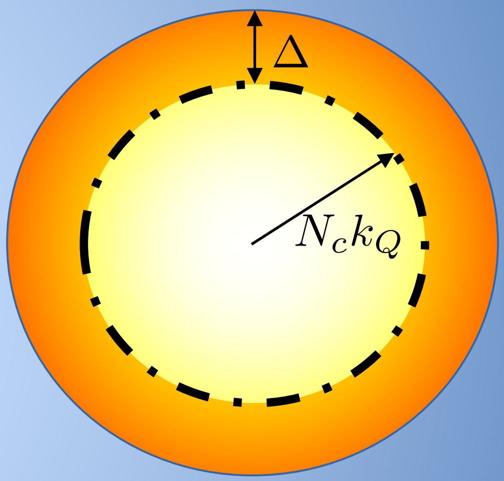
$$\Rightarrow$$

$$\varepsilon = 2N_f \left(1 - \frac{n_N}{n_0}\right) \int^{k_F} \frac{d^3k}{(2\pi)^3} \sqrt{k^2 + M^2} \qquad P = -\varepsilon + \mu_N n_N$$

Excluded Volume Model

$$n_{ex} = \frac{n_N}{1 - n_N/n_0} = \frac{N_f}{\pi^2} \int_{k_F}^{k_F+\Delta} dk\ k^2$$

$$k_Q = k_F/N_c \qquad m = M/N_c$$



$$\varepsilon = \frac{N_f}{\pi^2} \left(1 - \frac{n_N}{n_0}\right) \int_{k_F}^{k_F+\Delta} dk\ k^2 \sqrt{k^2 + M^2} + \varepsilon_Q$$

Free gas of quarks contribution

$$\begin{cases} n_Q = \dfrac{N_f}{\pi^2} \displaystyle\int_0^{k_Q} dk\ k^2 = \dfrac{N_f}{3\pi^2} k_Q^3 \\ \varepsilon_Q = \dfrac{N_c N_f}{\pi^2} \displaystyle\int_0^{k_Q} dk\ k^2 \sqrt{k^2 + m^2} \end{cases}$$

Modification in the low density Fermi distribution in a way that does not affect its behavior for large Fermi momenta:

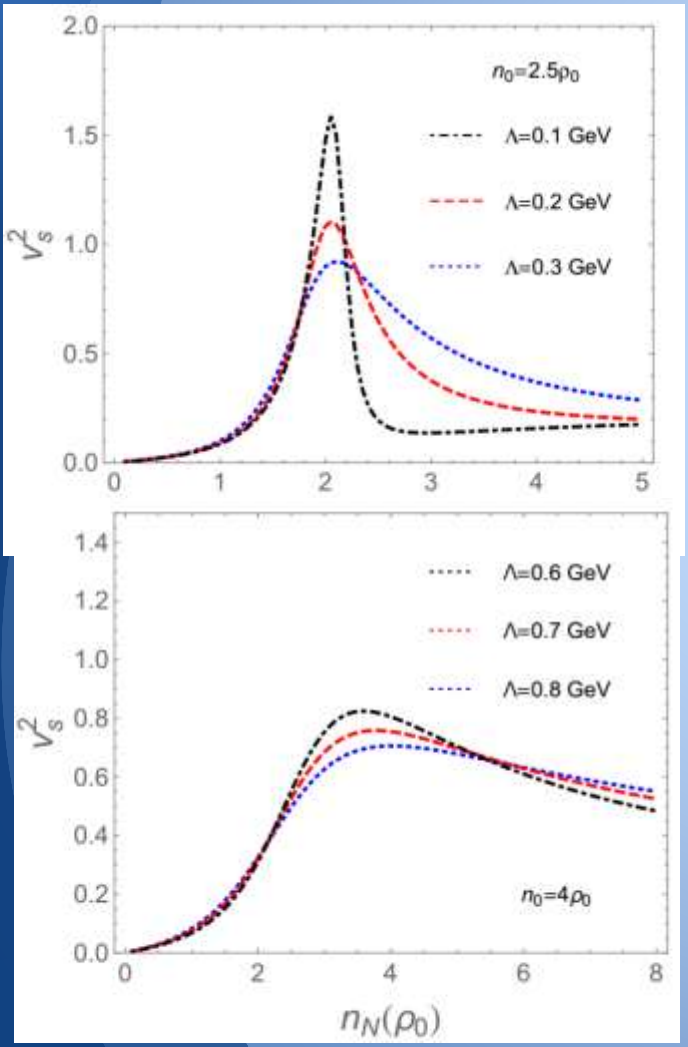
$$1 \rightarrow \frac{\sqrt{k_Q^2 + \Lambda^2}}{k_Q}$$

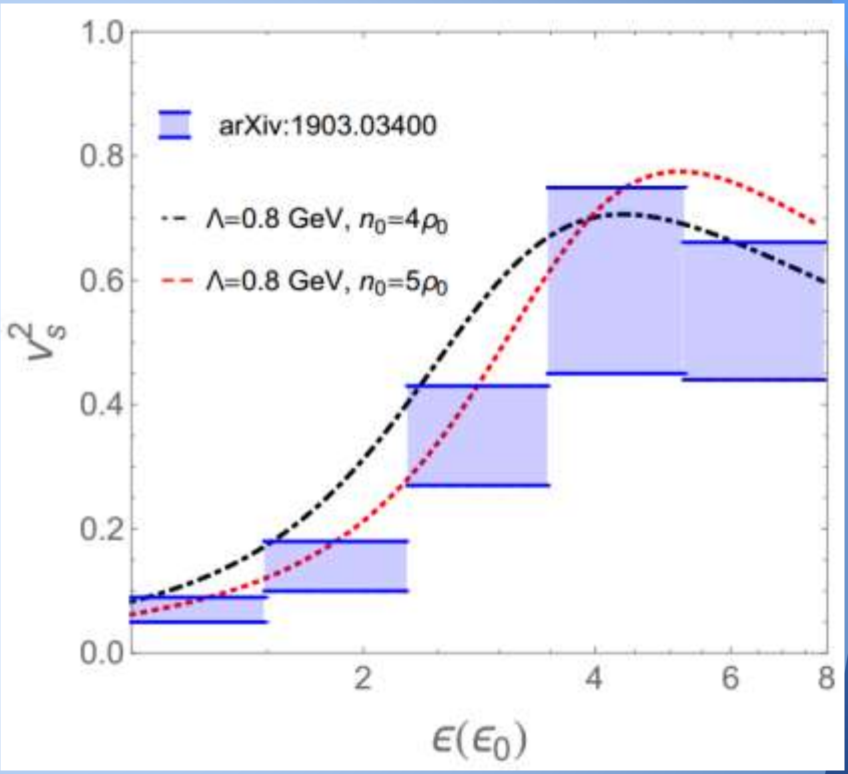
$$n_Q = \frac{N_f}{3\pi^2} \left[\left(k_Q^2 + \Lambda^2\right)^{3/2} - \Lambda^3 \right]$$

$$\varepsilon_Q = \frac{N_c N_f}{\pi^2} \int_0^{k_Q} dk\ k \sqrt{k^2 + \Lambda^2} \sqrt{k^2 + m^2}$$

K.S. Jeong, L. McLerran, S. Sen, Phys. Rev. C 101 035201 (2020)

Excluded Volume Model





Good agreement with sound velocity obtained from an equation of state extracted from neutron stars properties.

Excluded Volume Model

- Hard core repulsion: Scale can be measured by the effective size of the baryon.

- Protons + Neutrons + Hyperons in an excluded volume for the shell

$$n_N = n_p + n_n + n_\Lambda; \qquad n_{\tilde{N}} = n_p + n_n + (1+\alpha) n_\Lambda$$

Strength of Λ repulsion

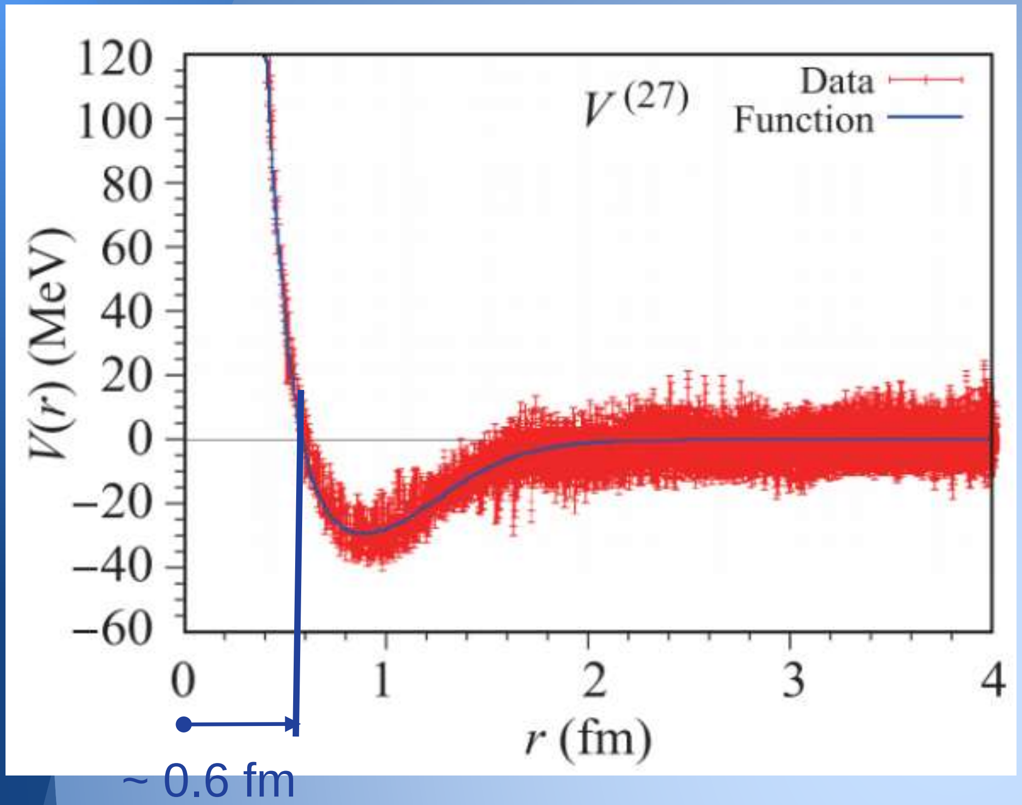
- For neutron stars phenomenology: β-equilibrium and charge neutrality must be imposed.

$$\varepsilon_{\mathrm{qy.}} = 2\left(1 - \frac{\tilde{n}_b}{n_0}\right) \sum_i^{\{n,p,\Lambda\}} \int_{k_F^{b_i}}^{[k_F+\Delta]_{b_i}} \frac{d^3k}{(2\pi)^3} \sqrt{k^2 + m_{b_i}^2}$$

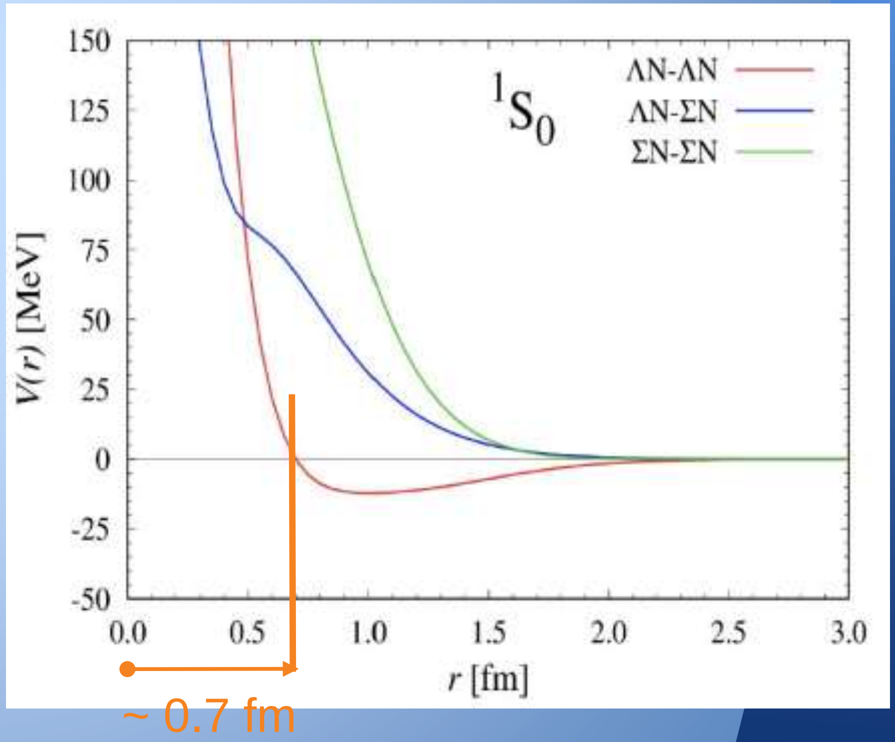
$$+ \frac{N_c}{\pi^2} \sum_j^{\{u,d,s\}} \int_0^{k_F^{Q_j}} dk \mathcal{M}_j(k^2) \sqrt{k^2 + m_{Q_j}^2} + \frac{(3\pi^2)^{\frac{4}{3}}}{4\pi^2} n_e^{\frac{4}{3}}$$

Lower boundary of baryon shell:

$$k_F^n = k_{\mathrm{conf}}^u + 2k_{\mathrm{conf}}^d$$
$$k_F^p = 2k_{\mathrm{conf}}^u + k_{\mathrm{conf}}^d$$
$$k_F^\Lambda = k_{\mathrm{conf}}^u + k_{\mathrm{conf}}^d + k_{\mathrm{conf}}^s$$



T. Hatsuda, Front. Phys. 13(6), 132105 (2018)



T. Hatsuda, private communication

- **Quark Density.**

$$n_{\tilde{Q}_i} \equiv \frac{1}{\pi^2}\int_0^{k_F^{Q_i}} dk\mathcal{M}_i(k^2) = \frac{1}{\pi^2}\int_0^{k_F^{Q_i}} dk(k^2+\Lambda_{Q_i}^2)$$

- **For the minimum of energy density:** $dn_B = dn_n + dn_Q = 0$ which results in $\mu_n = N_c\mu_{\tilde{d}} - \mu_e$

- **Electromagnetic charge neutrality**

$$n_e = n_p + 2n_{\tilde{u}} - n_{\tilde{d}} - n_{\tilde{s}}$$

- **Beta equilibrium conditions**

$$\mu_{\tilde{d}} = \mu_{\tilde{u}} + 3\mu_e$$

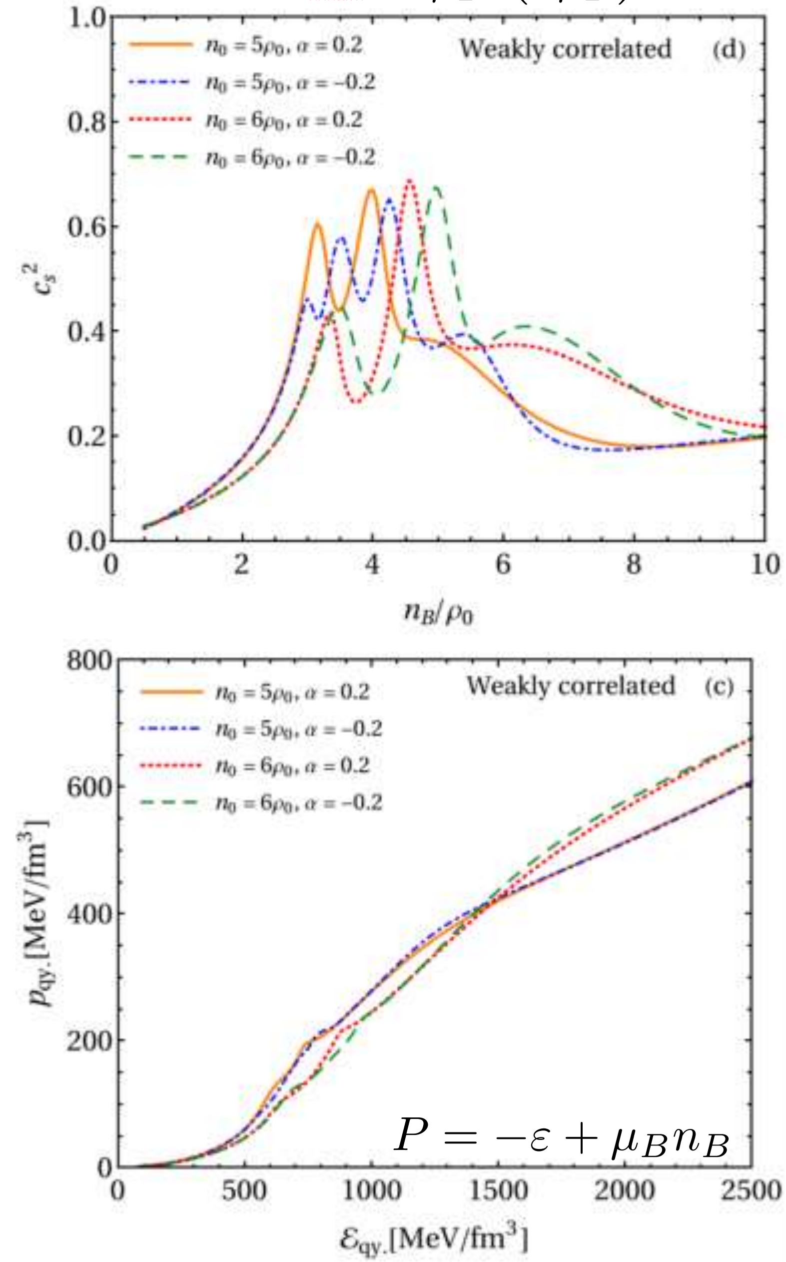
- **Existence of Λ hyperon**

$$\mu_\Lambda = \mu_n$$

- **Existence of s quark**

$$\mu_{\tilde{s}} = \mu_{\tilde{d}}$$

$$c_s^2 = \frac{\partial P}{\partial \varepsilon_{\rm mix}} = \frac{n_B}{\mu_B}\left(\frac{\partial n_B}{\partial \mu_B}\right)^{-1}$$



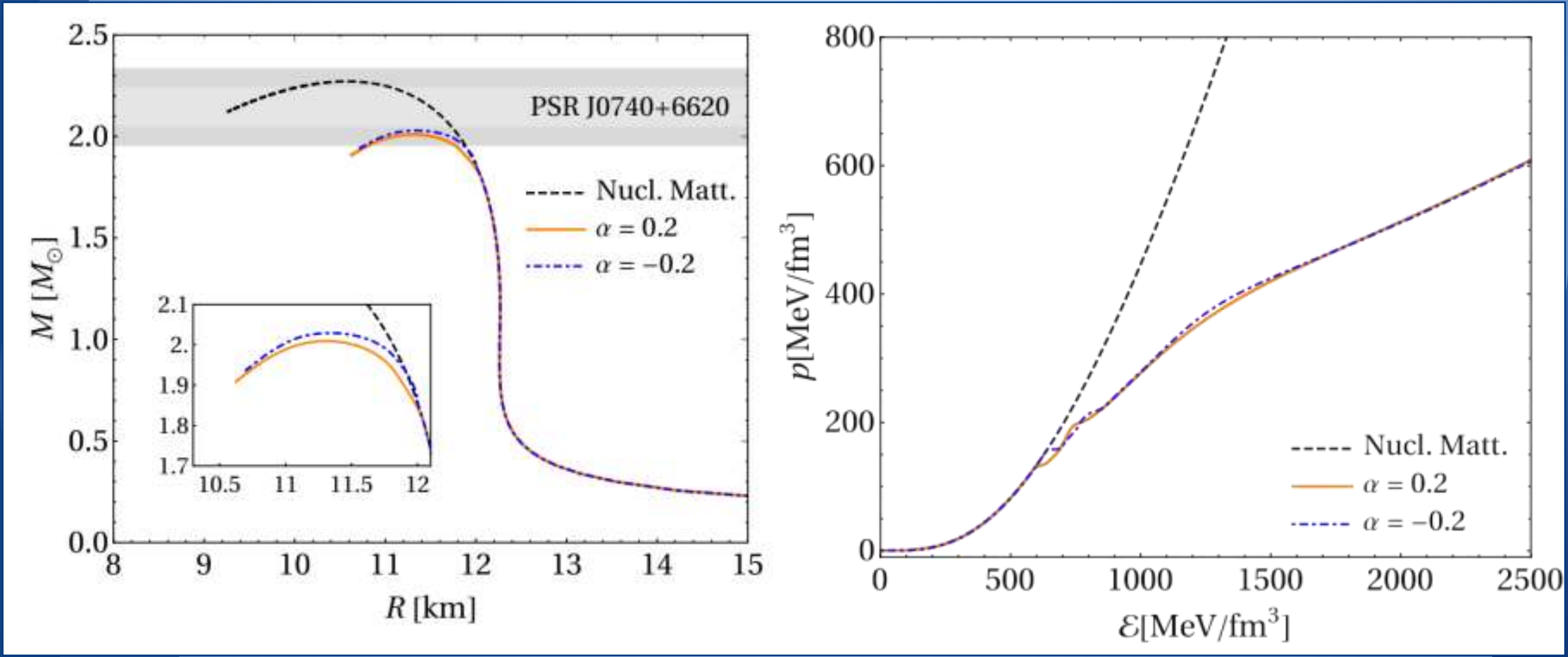
D. Duarte, K.S. Jeong, S.HO, PRC 102, 065202 (2020)

➡ Correction of low density regime EoS: use of a rich neutron matter EoS in the range $n_B < n_M$.

$$E/A = \sqrt{(p_F^n)^2 + M_n^2} - M_n + \tilde{a}\left(\frac{n_n}{\rho_0}\right) + \tilde{b}\left(\frac{n_n}{\rho_0}\right)^2$$

$$\tilde{a} = -28.3 \text{ MeV} \quad \text{and} \quad \tilde{b} = 10.7 \text{ MeV}$$

S. Gandolfi, et. al Eur. Phys. J. A 50, 10.



D. Duarte, K.S. Jeong, S.HO arXiv:2007.08098

Final Remarks

- Analysis of GW data have been providing very important insights about the properties of dense QCD matter.
- We extend the excluded volume model of isospin symmetric two-flavor dense Quarkyonic matter including strange baryons and quarks and address its implications for neutron stars.
- The extension to finite temperature is also an interesting problem, since future experiments in the NICA/FAIR facilities may provide more insights about the QCD phase diagram in the regime of high density and intermediate temperatures.

THANKS FOR WATCHING!

$$
\begin{aligned}
k_F^n = &\ k^u_{\text{conf.}} + 2k^d_{\text{conf.}} \\
= &\ \Theta(k_F^d - k_F^u)\left(3k_F^d + r_{qq}^{s/w} w_{s/w}\left(k_F^d - k_F^u\right)\right) + \Theta(k_F^u - k_F^d)\left(3k_F^u + 2r_{qq}^{s/w} w_{s/w}\left(k_F^u - k_F^d\right)\right), \\
k_F^p = &\ 2k^u_{\text{conf.}} + k^d_{\text{conf.}} \\
= &\ \Theta(k_F^d - k_F^u)\left(3k_F^d + 2r_{qq}^{s/w} w_{s/w}\left(k_F^d - k_F^u\right)\right) + \Theta(k_F^u - k_F^d)\left(3k_F^u + r_{qq}^{s/w} w_{s/w}\left(k_F^u - k_F^d\right)\right), \\
k_F^\Lambda = &\ k^u_{\text{conf.}} + k^d_{\text{conf.}} + k^s_{\text{conf.}} \\
= &\ \Theta(k_F^d - k_F^s)\left(3k_F^d + r_{qq}^{s/w} w_{s/w}\left(k_F^d - k_F^u\right) + r_{qs}^{s/w} w_{s/w}\left(k_F^d - k_F^s\right)\right) \\
&\ + \Theta(k_F^s - k_F^d)\left(3k_F^s + r_{qs}^{s/w} w_{s/w}\left(k_F^s - k_F^d\right) + r_{qs}^{s/w} w_{s/w}\left(k_F^s - k_F^u\right)\right),
\end{aligned}
$$