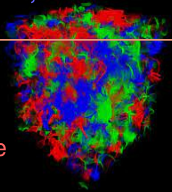
Lattice QCD with an inhomogeneous magnetic field background

Workshop on Electromagnetic Effects in Strongly Interacting Matter - ICTP-SAIFR, SP



Dean Valois
dvalois@physik.uni-bielefeld.de

Gergely Endrődi   Bastian Brandt   Gergely Marko   Francesca Cuteri

October 29, 2022

Department of Physics
Bielefeld University

Outline

Strongly magnetized physical systems

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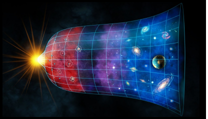
Early universe

$\sqrt{eB} \sim 1.5$ GeV



Heavy-ion collision

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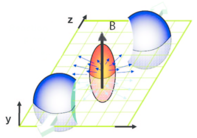

Neutron stars

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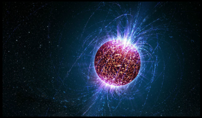
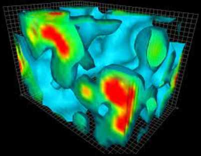


QCD vacuum



STRONGLY MAGNETIZED PHYSICAL SYSTEMS

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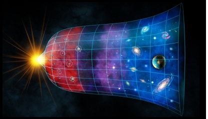

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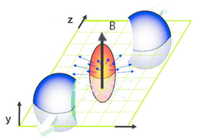

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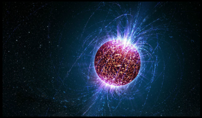

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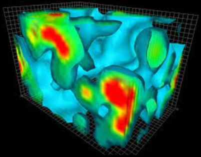

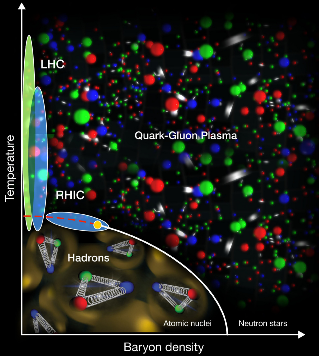

Magnetic fields in heavy-ion collisions

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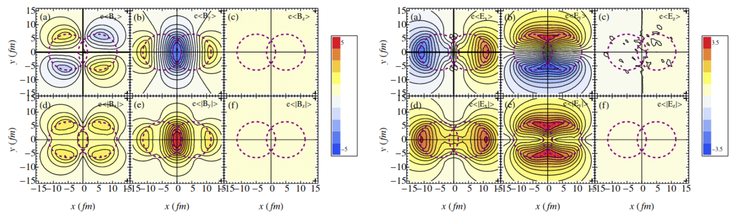
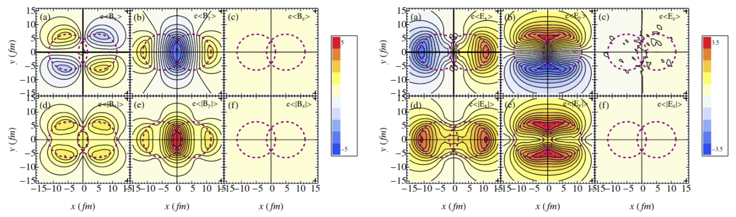


Figure 2: Spatial distributions of the electric (right) and magnetic (left) fields in the transverse plane for an impact parameter $b = 10$ fm Deng and Huang 2012.

MAGNETIC FIELDS IN HEAVY-ION COLLISIONS



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Caveats:

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- A real **E** leads to sign problem.
- No Minkoswki time evolution from Euclidean simulations.

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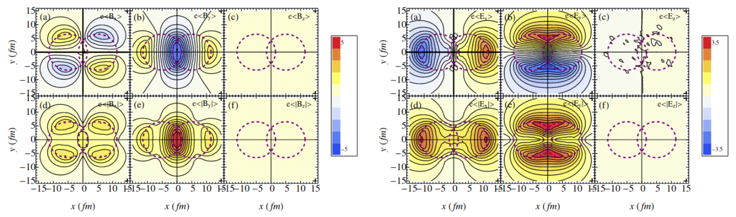

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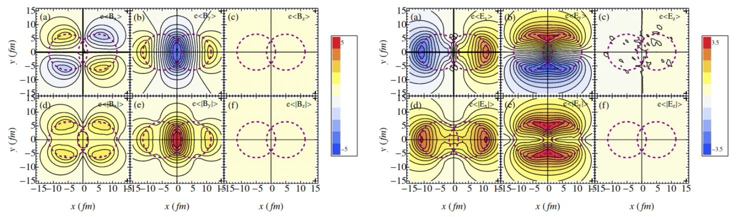

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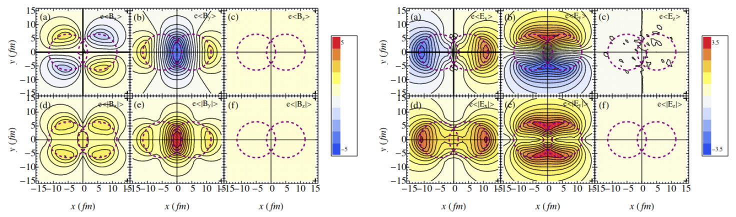

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$B(x)$ as background in lattice QCD!

Lattice QCD and magnetic fields

LATTICE QCD IN A NUTSHELL

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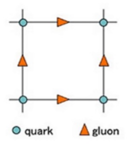

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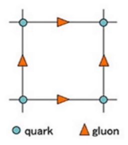

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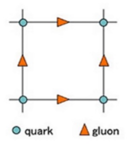

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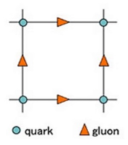

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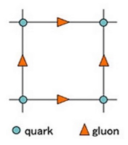

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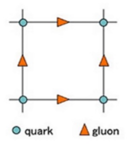

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- magnetic field $B \longrightarrow u_\mu = e^{iaqA_\mu} \in$ U(1) (BACKGROUND!)

MAGNETIC FIELD ON THE LATTICE

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$$\mathbf{B} = B\hat{z}$$

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$$\text{outer area:} \quad \oint \mathbf{A} \cdot d\mathbf{x} = (L_x L_y - S)B$$

$$e^{iqBS} = e^{-iqB(L_x L_y - S)}$$

$$qB = \frac{2\pi N_b}{L_x L_y}; \quad N_b \in \mathbb{Z}$$

The magnetic flux is quantized inside a box!

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We can perform gauge transformations on the links

$u^0(x) = (\ x)u \ (\ x + a\hat{\ })^y$

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Uniform magnetic field on the lattice

$B = \nabla \times A$

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We can perform gauge transformations on the links

$$u'_\mu(x) = \Omega(x) u_\mu \Omega(x + a\hat{\mu})^\dagger$$

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$$u_x = \begin{cases} e^{-iqBL_x y} & \text{if } x = L_x - a \\ 1 & \text{if } x \neq L_x - a \end{cases}$$

$$u_y = e^{iaqBx} \quad 0 \leq x \leq L_x - a$$

$$u_z = 1$$

$$u_t = 1$$

INHOMOGENEOUS MAGNETIC FIELD ON THE LATTICE

$$B = \frac{B}{\cosh\left(\frac{x \quad L_x = 2}{}\right)^2} \hat{z}$$

Pro le motivated by heavy-ion collision scenarios Deng and Huang 2012, Cao 2018.

$$qB = \frac{N_b}{L_y \tanh\frac{L_x}{2}} \qquad N_b \; 2 \; Z$$

$$u_x = \begin{cases} e^{\;2iqB\, y \quad \tanh\left(\frac{L_x}{2}\right)} & \text{if } x = L_x \quad a \\ 1 & \text{if } x \neq L_x \quad a \end{cases}$$

$$u_y = e^{iaqB \;\left[\tanh\left(\frac{x \quad L_x = 2}{}\right) + \tanh\left(\frac{L_x}{2}\right)\right]}; \quad 0 \quad x \quad L_x \quad a$$

$$u_z = u_t = 1$$

Lattice simulations

THE SIMULATION SET UP

The simulation set up

- $N_f = 2 + 1$ improved staggered fermions with physical masses;

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strength 0 GeV $\sqrt{eB}$ 1:2 GeV ! magnetars, HIC and early universe.

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- Temperature range 68 MeV T 300 MeV (crossover transition at T_c 155 MeV).

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Lattice observables

- Local chiral condensates (u and d quarks!)

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$$P = \frac{1}{L_x L_y} \sum_{y;z} \mathrm{Re\,Tr} \prod_{n} U_t(x; y; z; n)$$

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- Local electric currents (u, d and s quarks!)

$$h J_i(x) i = e \quad \frac{2}{3} u \; {}^{i} u \quad \frac{1}{3} d \; {}^{i} d \quad \frac{1}{3} s \; {}^{i} s$$

CHIRAL CONDENSATE

Chiral condensate

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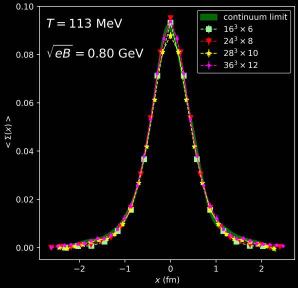

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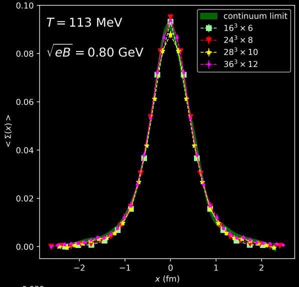

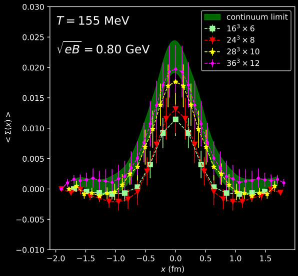

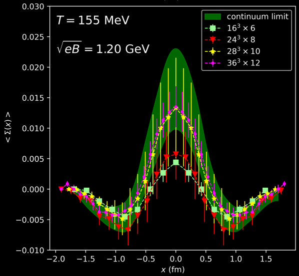

Chiral condensate in the T-B plane

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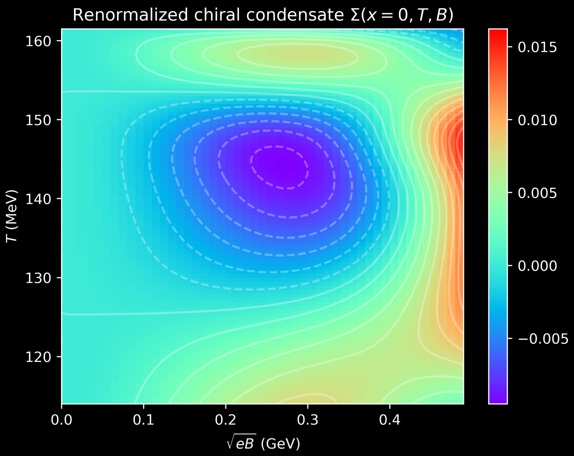

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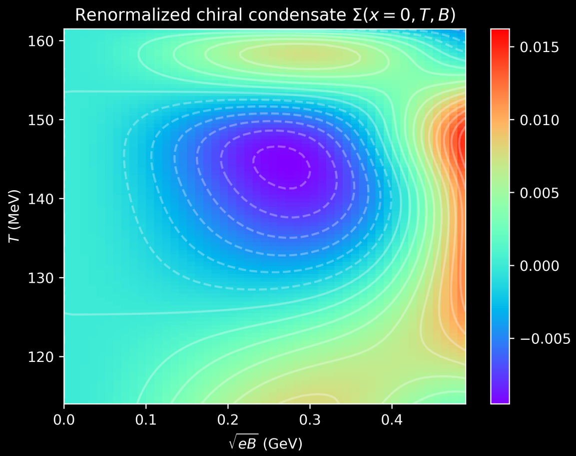


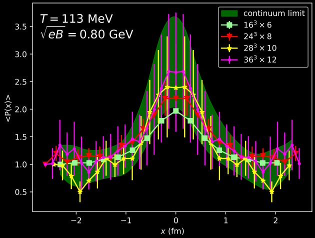
- Magnetic catalysis T away from Tc
- Inverse catalysis for T around T_c

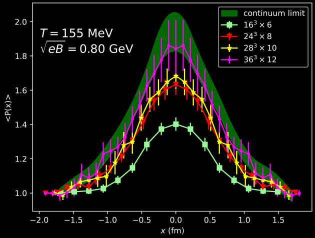
  Endrődi et al. 2019

Valence effect *vs* Sea effect $=$ (inverse) magnetic catalysis

Polyakov loop

POLYAKOV LOOP





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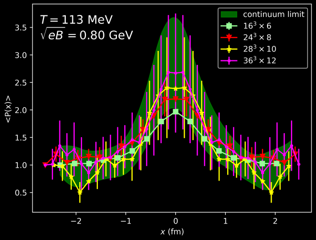

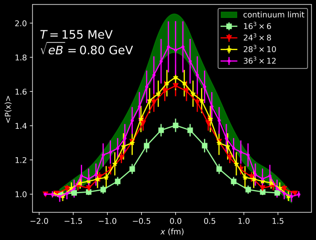

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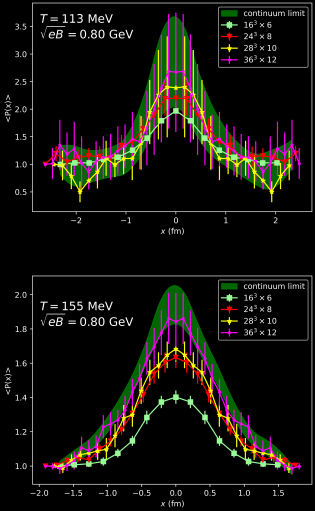

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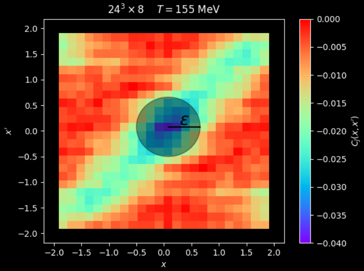

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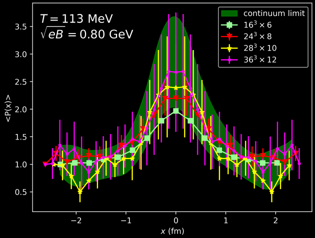

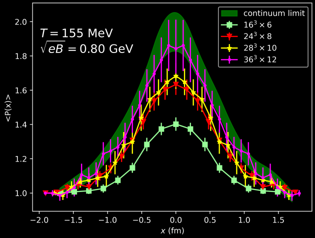

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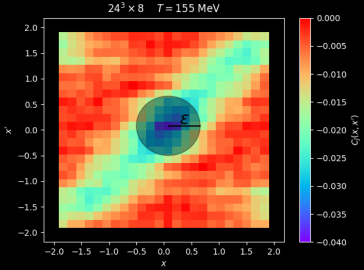

The interaction of the condensate with P causes the dips! (Local inverse magnetic catalysis)

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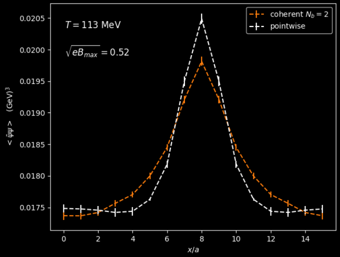

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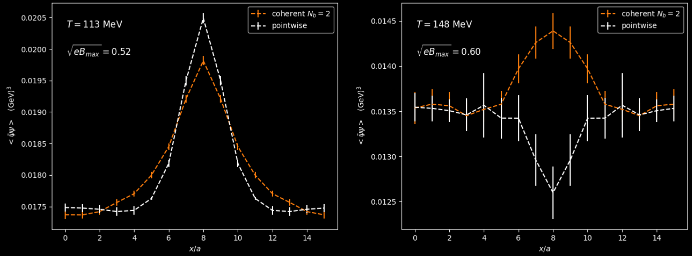

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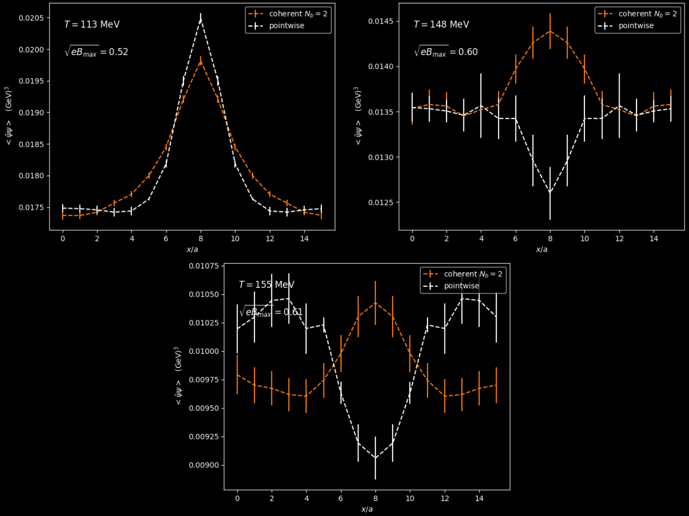

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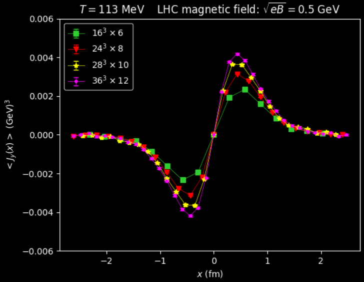

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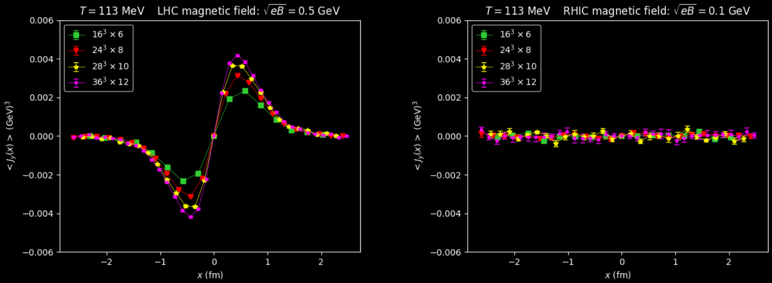

Figure 6: Lattice electric currents for RHIC-like ($\sqrt{eB} = 0.1$ GeV) and LHC-like ($\sqrt{eB} = 0.5$ GeV) magnetic fields, respectively.

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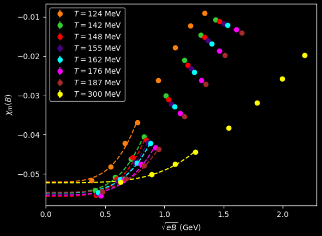

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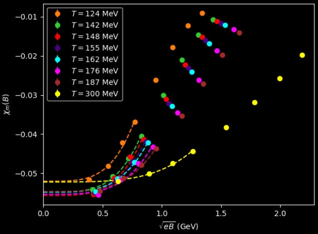

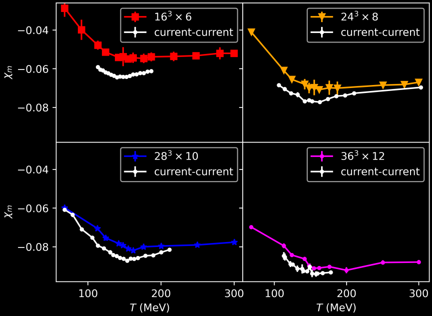

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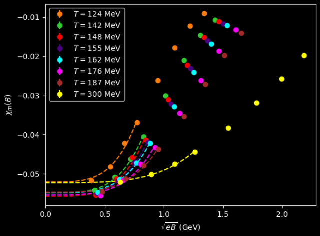

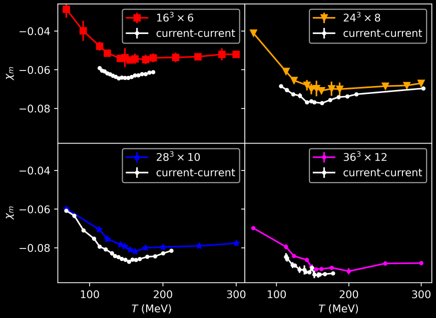

The susceptibility contains an additive divergence $\chi_m \sim \log(a)$

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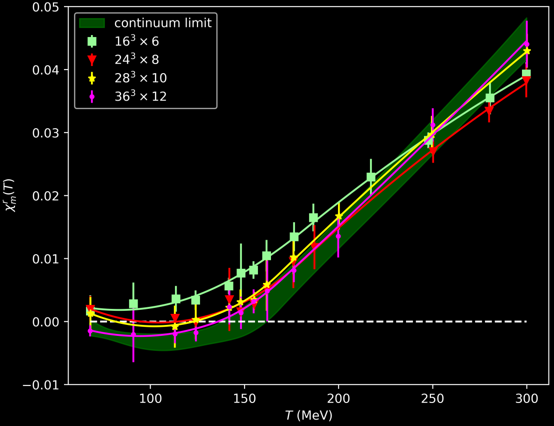

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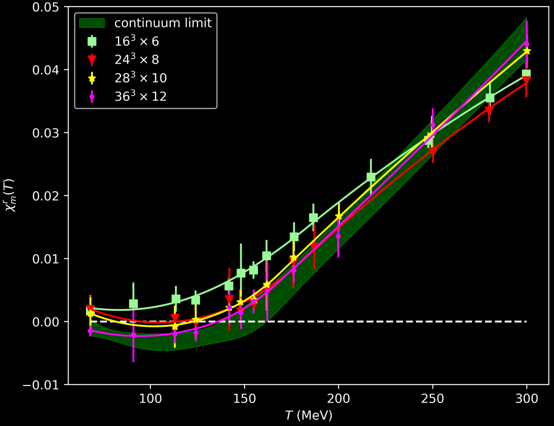

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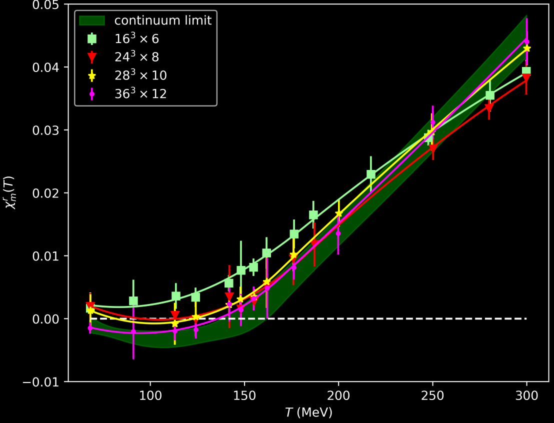

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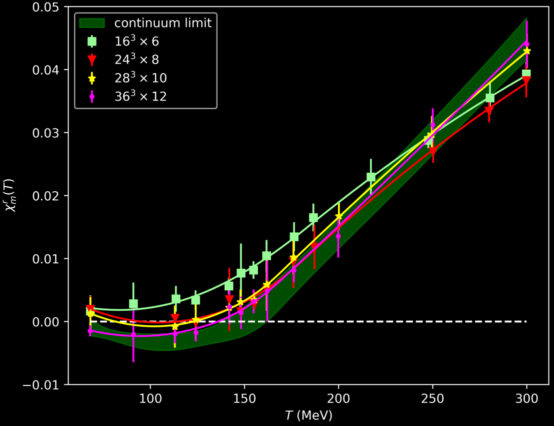

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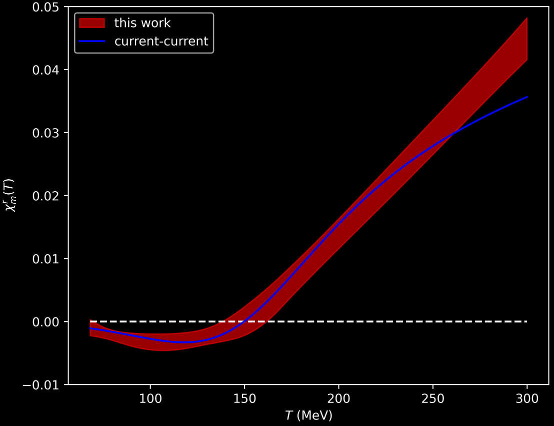

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Great agreement with the current-current method! Bali, Gergely Endrődi, and Piemonte 2020

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- The knowledge of these processes is important to capture the correct physics in heavy-ion collision studies (QCD models, hydrodynamics, etc.);

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