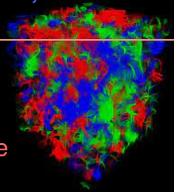
Lattice QCD with an inhomogeneous magnetic field background

Workshop on Electromagnetic Effects in Strongly Interacting Matter - ICTP-SAIFR, SP



Dean Valois
dvalois@physik.uni-bielefeld.de

Gergely Endrődi   Bastian Brandt   Gergely Marko   Francesca Cuteri

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Department of Physics
Bielefeld University

Outline

1. Strongly magnetized physical systems
2. Lattice QCD and magnetic fields
3. Lattice simulations
4. Summary & Conclusions

Strongly magnetized physical systems

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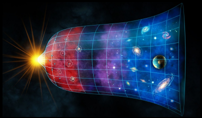
Early universe
$\sqrt{eB}$ 1.5 GeV



Heavy-ion collision
$\sqrt{eB}$ 0.5 GeV

Neutron stars
$\sqrt{eB}$ 1 MeV

QCD vacuum

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Magnetic fields in heavy-ion collisions

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- highly non-homogeneous background.
- A real E leads to sign problem.
- No Minkoswki time evolution from Euclidean simulations.

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$B(x)$ as background in lattice QCD!

Lattice QCD and magnetic elds

LATTICE QCD IN A NUTSHELL

Lattice QCD in a nutshell

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$$\langle O \rangle = \frac{1}{Z}\int D\psi\, D\bar{\psi}\, DA\, O e^{-S[\psi,\bar{\psi},A]} \rightarrow \frac{1}{Z}\int DA \det\left(\not{D}(A) + m\right) O e^{-S_g[A]}$$

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magnetic eld B ! $u = e^{iaqA} \in U(1)$ (BACKGROUND!)

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$$\boxed{qB = \frac{2 N_b}{L_x L_y}; \quad N_b \; 2 \; Z}$$

The magnetic ux is quantized inside a box!

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We can perform gauge transformations on the links

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$$u'_y(x) = \Omega(x)\, u_y\, \Omega^{\dagger}(x + a\hat{y})$$

a is the lattice spacing.

$$u_x = \begin{cases} e^{-iqBL_x y} & \text{if } x = L_x - a \\ 1 & \text{if } x \neq L_x - a \end{cases}$$

$$u_y = e^{iaqBx} \qquad 0 \leq x \leq L_x - a$$

$$u_z = 1$$

$$u_t = 1$$

INHOMOGENEOUS MAGNETIC FIELD ON THE LATTICE

$$B = \frac{B}{\cosh \frac{x \quad L_x = 2}{}^{\,2}} \hat{z}$$

Pro le motivated by heavy-ion collision scenarios Deng and Huang 2012, Cao 2018.

$$qB = \frac{N_b}{L_y \tanh \frac{L_x}{2}} \qquad N_b \; 2 \; Z$$

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$$u_y = e^{iaqB \left[\tanh\left(\frac{x \quad L_x = 2}{}\right) + \tanh\left(\frac{L_x}{2}\right)\right]}; \quad 0 \quad x \quad L_x \quad a$$

$$u_z = u_t = 1$$

Lattice simulations

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strength 0 GeV $\sqrt{eB}$ 1:2 GeV ! magnetars, HIC and early universe.

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- Temperature range 68 MeV T 300 MeV (crossover transition at T_c 155 MeV).

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$$P = \frac{1}{L_x L_y} \mathop{\mathrm{X}}_{y;z} \mathrm{Re}\,\mathrm{Tr} \mathop{\mathrm{Y}}_{n} U_t(x; y; z; n)$$

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$$P(x;T;0) \qquad P(\quad L=2;T;B)$$

- Local electric currents (u, d and s quarks!)

$$\langle J_i(x) \rangle = e \quad \frac{2}{3}u \quad {}^i u \quad \frac{1}{3}d \quad {}^i d \quad \frac{1}{3}s \quad {}^i s$$

Chiral condensate

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Chiral condensate in the T-B plane

What happens to the peak of the condensate as a function of T and B?

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CHIRAL CONDENSATE IN THE T-B PLANE

What happens to the peak of the condensate as a function of T and B ?

- Magnetic catalysis T away from T c
- Inverse catalysis for T around T_c

Endrodi et al. 2019

Valence effect vs Sea effect = (inverse) magnetic catalysis

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The interaction of the condensate with P causes the dips! (Local inverse magnetic catalysis)

INHOMOGENEOUS VS UNIFORM CASE

How different do the condensates behave in the two cases?

Inhomogeneous vs uniform case

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Electric currents

$$J \propto \nabla \times B \rightarrow J_y \propto \frac{\partial B_z}{\partial x} = -\frac{2B}{\cosh^2\left(\frac{x - L_x/2}{\;}\right)} \tanh\left(\frac{x - L_x/2}{\;}\right)$$

Figure 6: Lattice electric currents for RHIC-like ($\sqrt{eB} = 0.1$ GeV) and LHC-like ($\sqrt{eB} = 0.5$ GeV) magnetic fields, respectively.

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The susceptibility contains an additive divergence $\ _m \quad \log(a)$

(RENORMALIZED) MAGNETIC SUSCEPTIBILITY

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(Renormalized) magnetic susceptibility

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Material	$_{m}$
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Glass	$-1.13 \quad 10^{5}$

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Great agreement with the current-current method! Bali, Gergely Endrodi, and Piemonte 2020

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- Our $\ _m$ corroborates the picture of weak diamagnetism in QCD for $T < T_c$ and strong paramagnetism for $T > T_c$;
- The knowledge of these processes is important to capture the correct physics in heavy-ion collision studies (QCD models, hydrodynamics, etc.);

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References

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