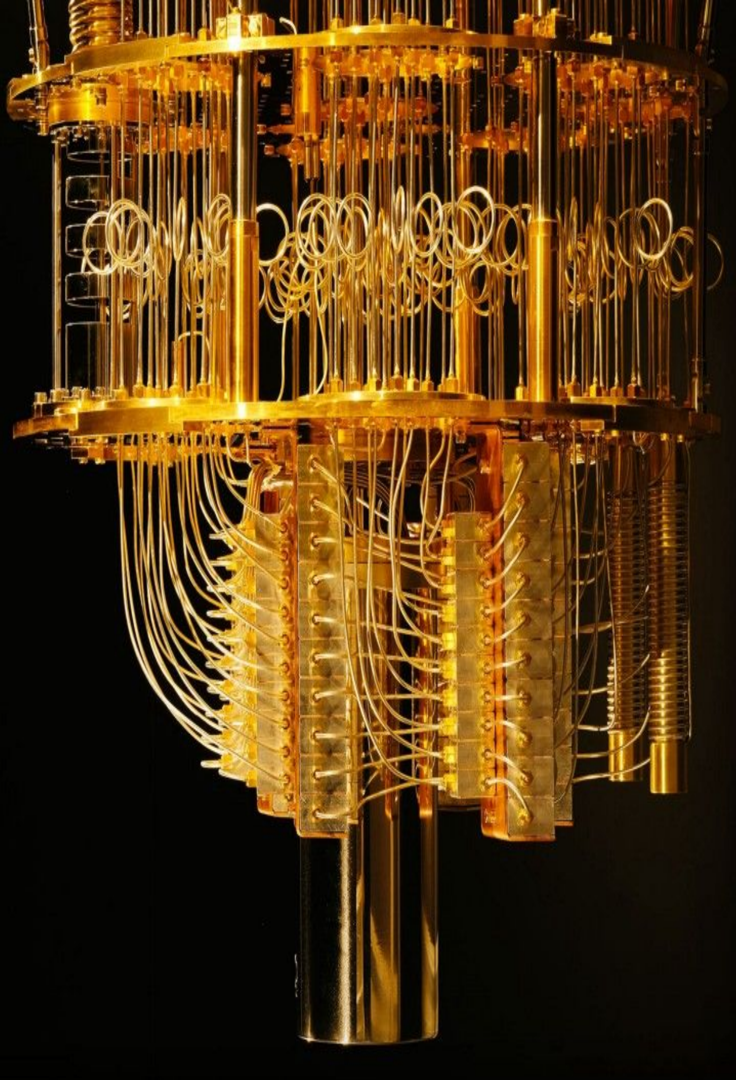
Quantum Finance

(Quantum Computing for Finance)



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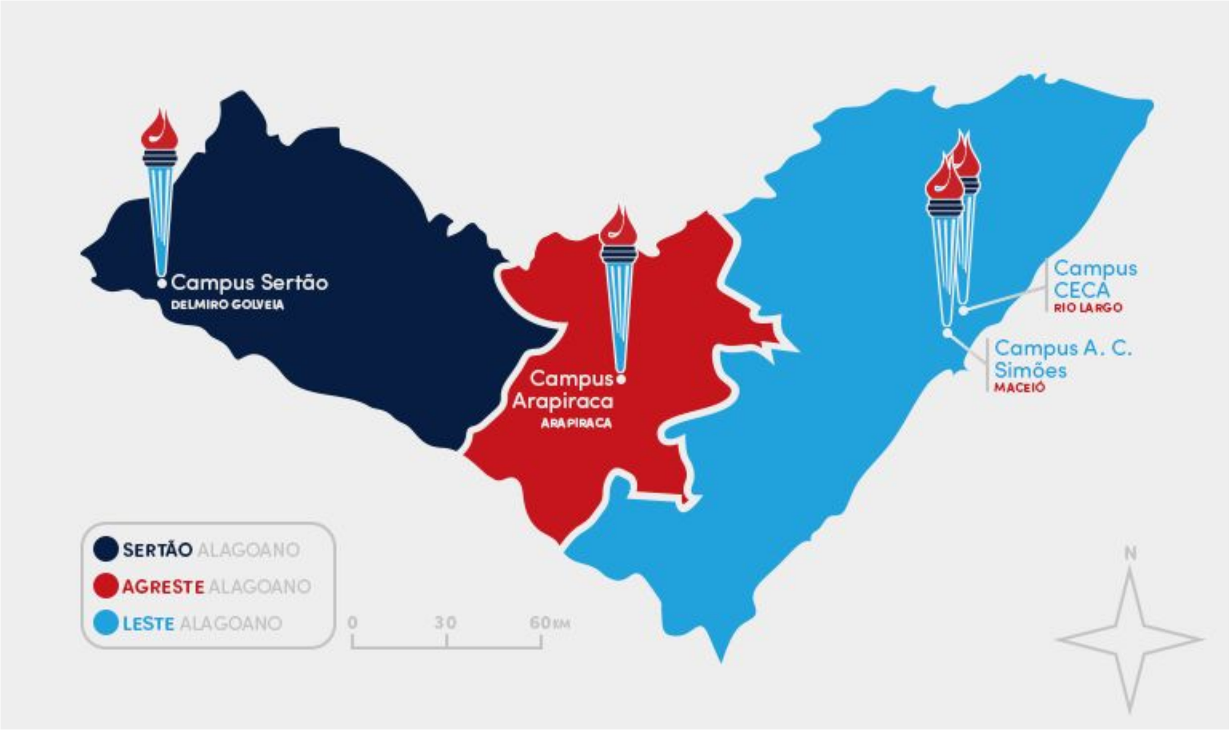
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Outline (Day 1)

1. Thoughts about the financial market
2. Some Definitions
   2.1 Price vs Value vs EMH
3. Portfolio and Portfolio Optimization
   3.0 Risk and Return
   3.1 Classical Approach
   3.2 Soccer Squad comparison (Quantum Approach)
4. Introduction to QAOA

Notice

This talk is **not** about:

- how to invest;
- where to invest;
- how much to invest...

Notice

This talk is about:

- applications of quantum algorithm (QAOA);
- (brief) introduction on the financial market;
- (brief) risk and risk management (Markowitz's Portfolio Theory, **1952**)...

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Quantum Finance: um tutorial de computação quântica aplicada ao mercado financeiro

Quantum Finance: a tutorial on quantum computing applied to the financial market

Askery Canabarro Taysa M. Mendonça Ranieri Nery George Moreno Anton S. Albino Gleydson F. de Jesus Rafael Chaves SOBRE OS AUTORES

Resumos

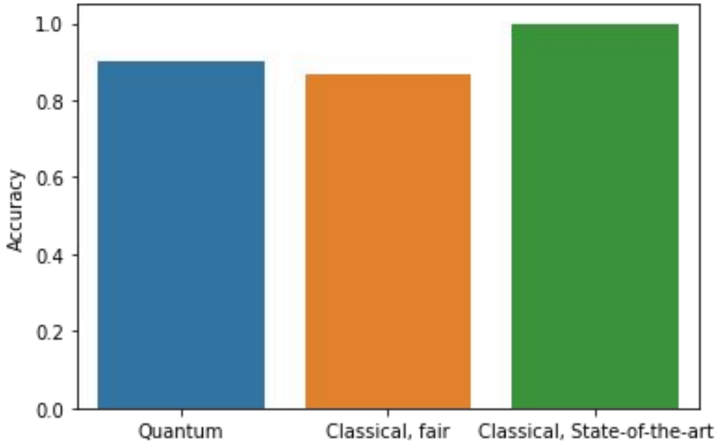
Antes restrita a uma área de fronteira da Física, a computação quântica é uma das áreas que mais tem crescido atualmente, justamente por suas aplicações tecnológicas em problemas de otimização, aprendizagem de máquina, segurança da informação e simulações. O objetivo deste artigo é introduzir os fundamentos da computação quântica, tendo como foco um algoritmo quântico promissor e sua aplicação a um problema do mercado financeiro. Mais especificamente, discutimos o problema da otimização de portfólio usando o *Quantum Approximate Optimization Algorithm* (QAOA). Não somente descrevemos os principais conceitos envolvidos, mas também consideramos exemplos simples práticos, envolvendo ativos financeiros disponíveis na bolsa brasileira, com códigos, tanto clássicos quanto quânticos, disponíveis livremente em um Jupyter Notebook. Também analisamos em detalhes a qualidade das soluções de otimização combinatória de portfólio por meio do QAOA usando o simulador quântico ATOS QLM do SENAI/CIMATEC.

Palavras-chave:
Computação Quântica; Mercado Financeiro; Finança Quântica

Main Ref. : https://doi.org/10.1590/1806-9126-RBEF-2022-0099

https://www.dualq.tech/tutoriais/quantum-finance/

Codes: https://github.com/askery/computacao-quantica-aplicada-ao-mercado-financeiro



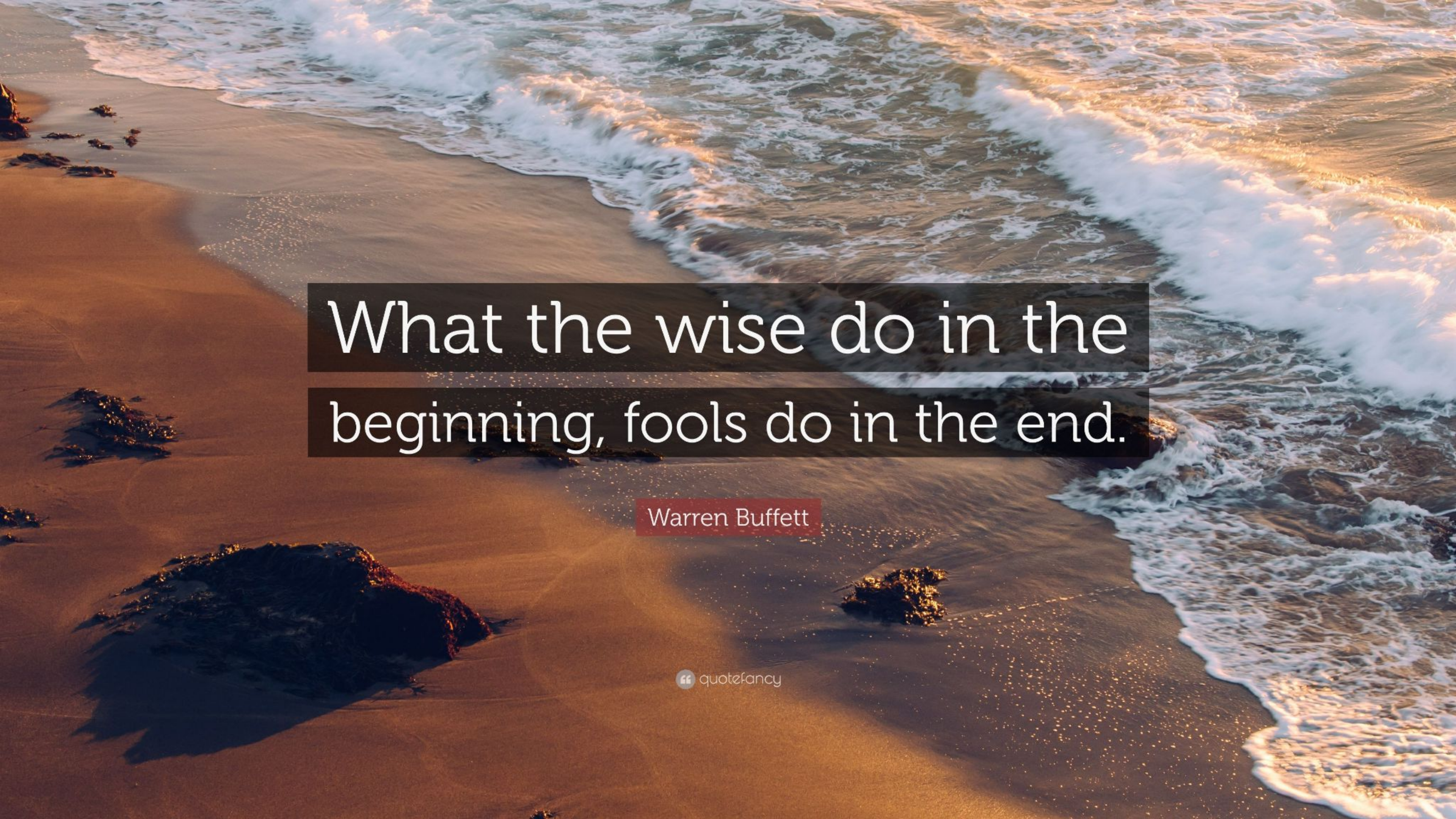
Proof of Concept chart showing the state-of-the-art classical accuracy, as well as the quantum solution accuracy

Exercise 0:

a) download and skim the paper;
b) run the related codes.

Disclaimer

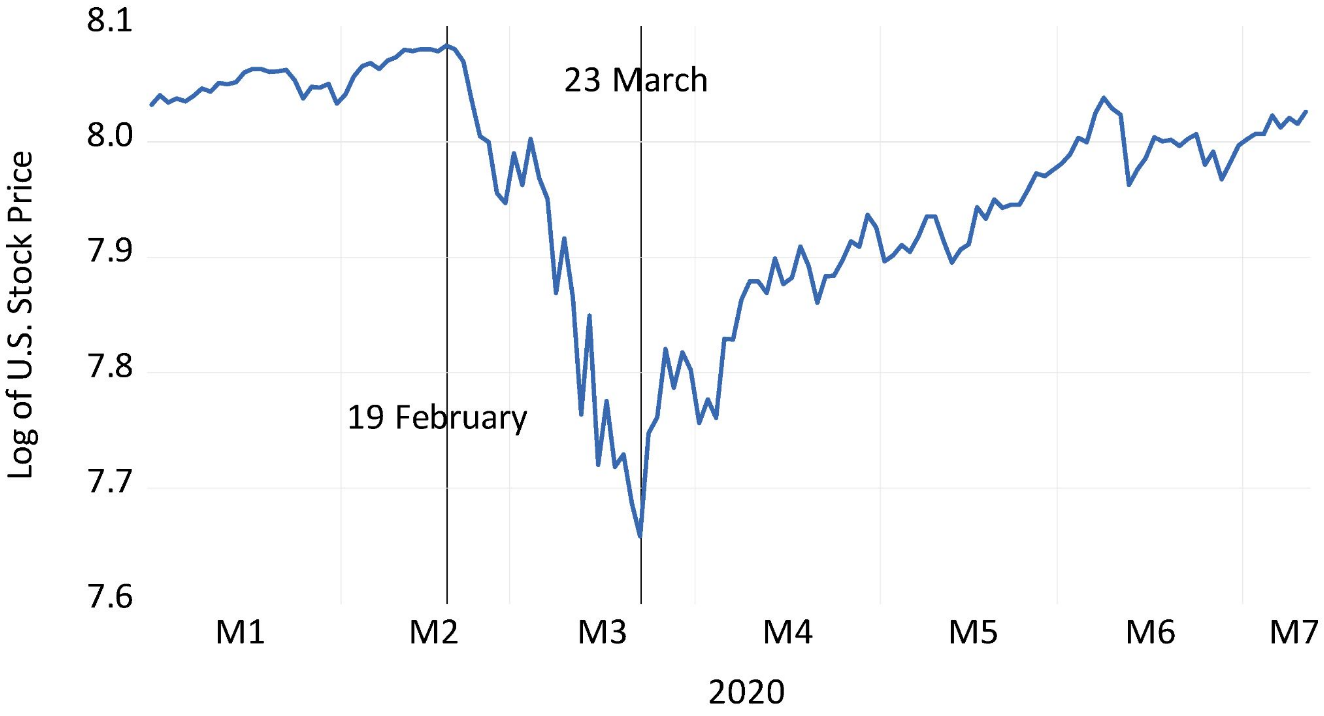
- I do participate in the financial market, but not using the whole ideas from this academic examples!!!
- **DO NOT** use the code to invest real money!!!
- Many interpretations are too personal...
- Non certified/amateur investitor...


What the wise do in the beginning, fools do in the end.
Warren Buffett
quotefancy

1. Thoughts about the financial market

Example: B3 (Bovespa), NYSE etc

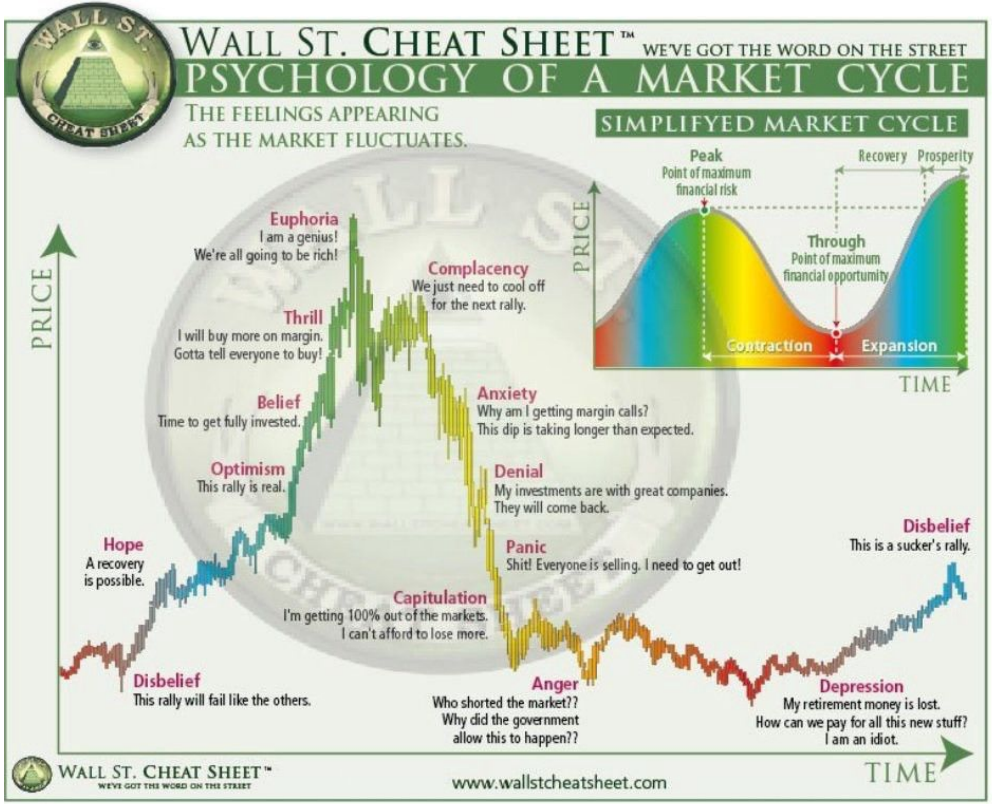
Provocation: What scientific field better explain this?



What scientific field better explain this?

What scientific field better explain this?

Do not be fooled by numbers!!!!



Notice

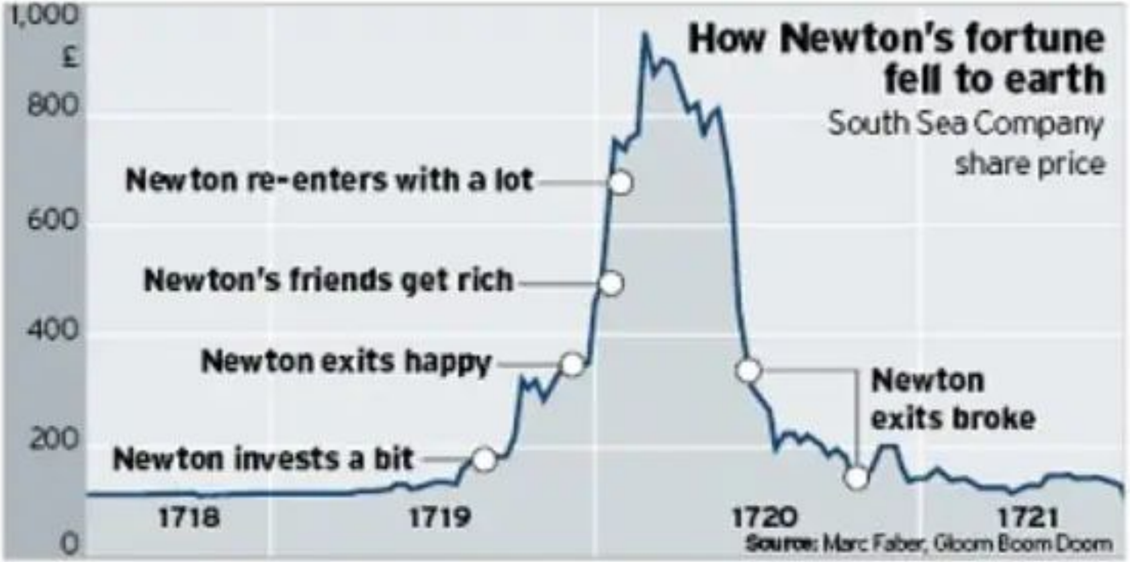
- Investing is dealing/reasoning about the future (a lot we do not know)
- Investing involves many emotions (a lot we can not control)

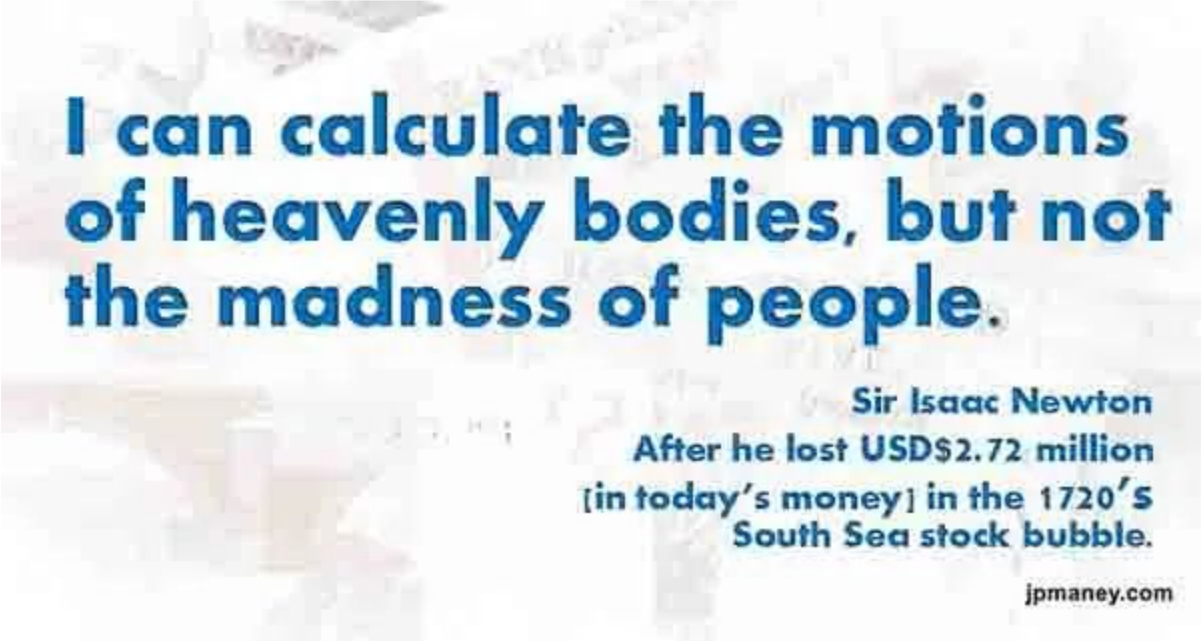
Notice

- Investing is reasoning about the future…
- Investing involves a lot of emotions…

⇩

Being genial is not enough (e.g., Sir Isaac Newton)


How Newton's fortune fell to earth
South Sea Company share price
1,000
£
800
600
400
200
0
Newton re-enters with a lot
Newton's friends get rich
Newton exits happy
Newton invests a bit
Newton exits broke
1718
1719
1720
1721
Source: Marc Faber, Gloom Boom Doom

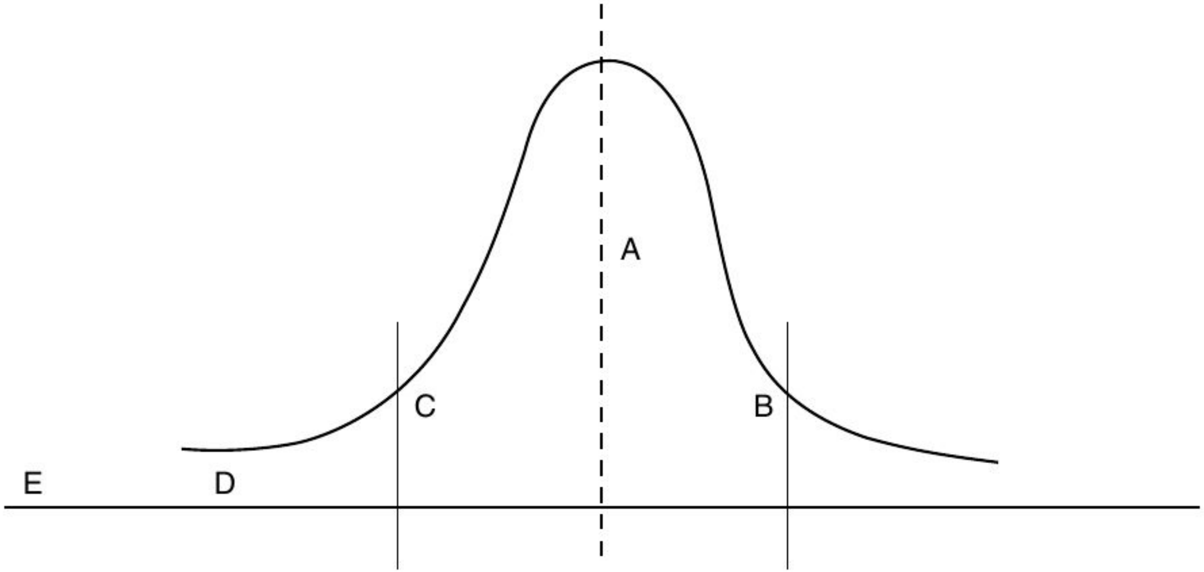


Market is not a rational, ordered place…

Analytical error vs Psychological error

Model error
Few or incomplete data
Strategic error
Emotional error (Greed, euphoria etc)

Future as a probability distribution



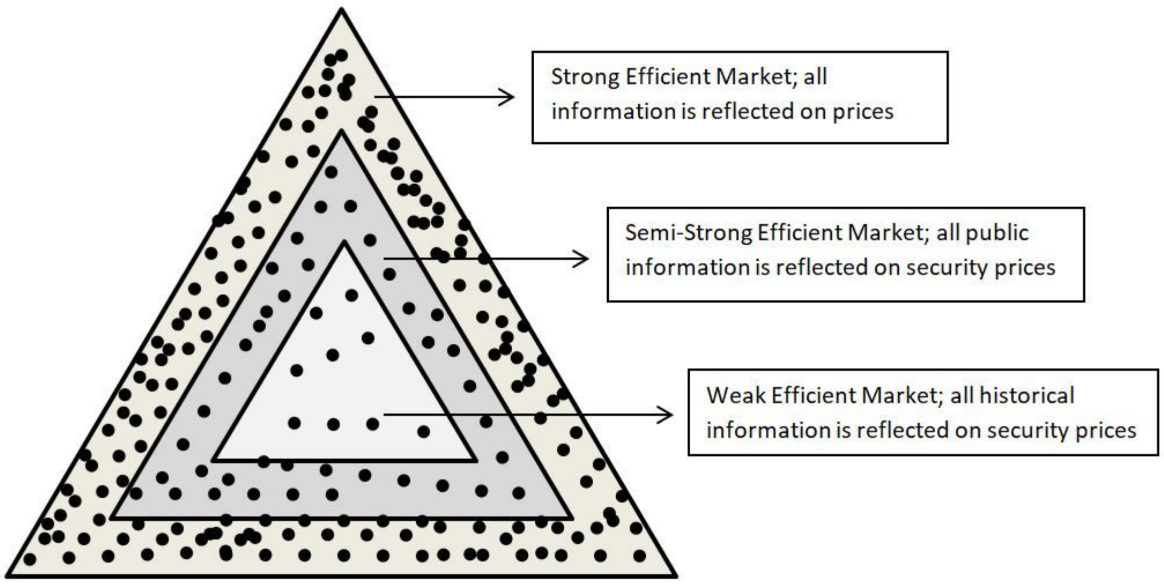
Risk means more things can happen than will happen (Elroy Dimson)
Source: H. Marks.

2. Some Definitions

Price vs Value vs EMH

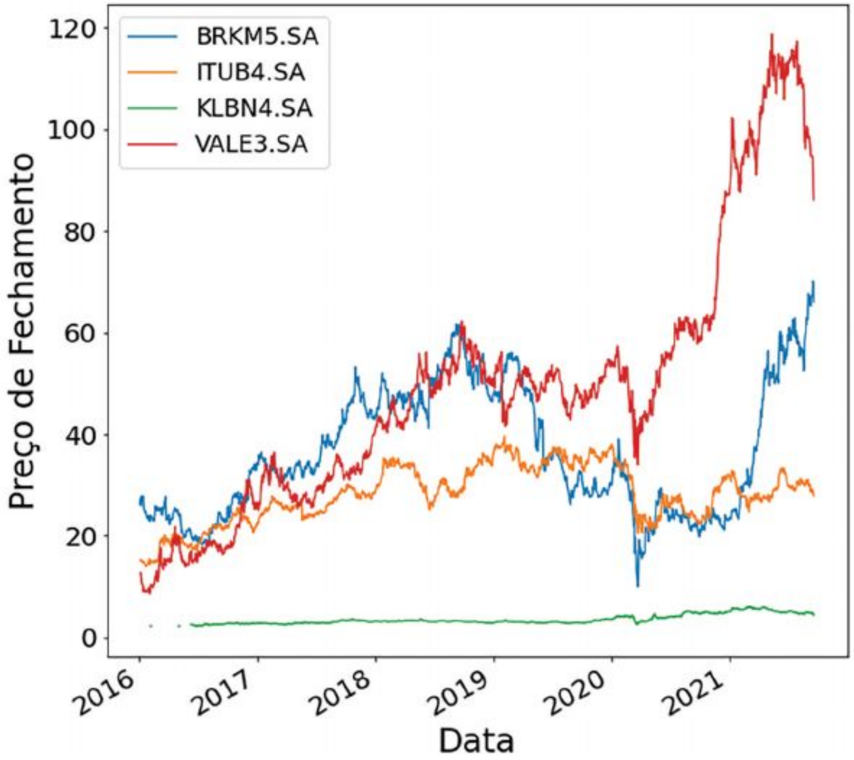
Would you buy a Ferrari F40 for 10000 USD?
Would you buy a Ford Focus for 4000 USD?

Share price
overpriced
Market Price
Intrinsic Value
underpriced
time


Strong Efficient Market; all information is reflected on prices
Semi-Strong Efficient Market; all public information is reflected on security prices
Weak Efficient Market; all historical information is reflected on security prices

3. Portfolio and Portfolio Risk Management

3. Portfolio



Portfolio: w = [0.25, 0.25, 0.25, 0.25]

3.0 Risk

$$\sigma(R_p) = \sqrt{\sum_{i=1}^{q} \sum_{j=1}^{q} w_i w_j \Sigma_{ij}},$$

where

$$\Sigma_{ij} = \frac{1}{M-1} \sum_{t=1}^{M} \left[\left(R_t^{(i)} - \mu^{(i)} \right) \left(R_t^{(j)} - \mu^{(j)} \right) \right]$$

number of days (steps)

and

$$\mu^{(i)} = \frac{1}{M} \sum_{t=1} R_t^{(i)},$$

daily price change

3.0 Return

$$R_p = w_1\mu^{(1)} + \ldots w_q\mu^{(q)}$$

where

$$\mu^{(i)} = \frac{1}{M}\sum_{t=1} R_t^{(i)},$$

daily price change

Exercise 1 (open the code and):

a) make sure you understand how to compute the risk (volatility) and return of a stock
b) make sure you understand how to compute the risk (volatility) and return of a portfolio

3.0 Example

Tabela 1: Retorno anual médio dos preços de fechamento das ações da Braskem (BRKM5), Itaú (ITUB4), Vale (VALE3) e Klabin (KLBN4) referentes ao período entre 01 de janeiro de 2016 até 20 de setembro de 2021.

Return

Ativos	BRKM5	ITUB4	KLBN4	VALE3
Retorno (μ)	0.32	0.06	0.11	0.30

Covariance

Ativos	BRKM5	ITUB4	KLBN4	VALE3
BRKM5	1.00	0.37	0.28	0.33
ITUB4	0.37	1.00	0.18	0.36
KLBN4	0.28	0.18	1.00	0.24
VALE3	0.33	0.36	0.24	1.00

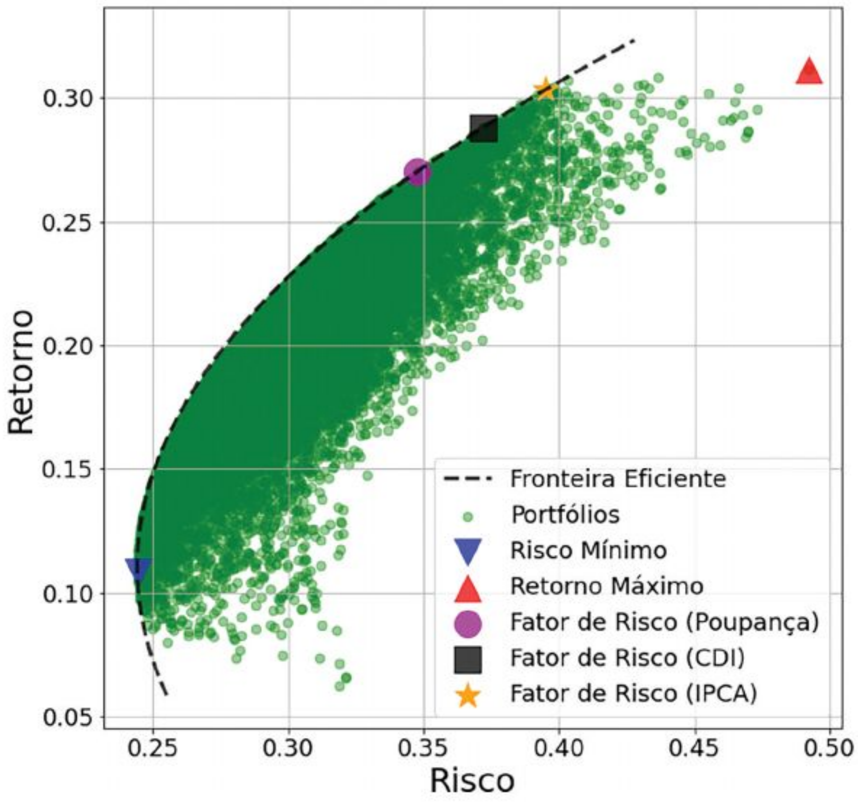
Comparing portfolios
(Sharpe Ratio)

Risk-free return

Portfolio return

$$SR = \frac{R_p - R_f}{\sigma},$$

Portfolio risk
(volatility)



Efficient Frontier

Descrição do Portfólio	BRKM5	ITUB4	KLBN4	VALE3	Risco	Retorno
Risco Mínimo	0.009	**0.404**	**0.503**	0.083	0.244	0.108
Retorno Máximo	**0.919**	0.006	0.009	**0.066**	0.492	0.311
Melhor SR ($R_f = 0.015$)	**0.370**	0.000	0.193	**0.436**	0.347	0.270
Melhor SR ($R_f = 0.028$)	**0.403**	0.002	0.099	**0.496**	0.372	0.287
Melhor SR ($R_f = 0.100$)	**0.451**	0.000	0.022	**0.526**	0.395	0.303

Markowitz approach after trying 30000 random weights

Exercise 2 (open the code and):

a) make sure you understand how to compute the efficient frontier
b) change the assets at you wish and compute the efficient frontier

4. Introduction to QAOA

Exercise (if you are physicist, if not drink water)
Suppose we want to find the ground state energy for the one-dimensional harmonic oscillator:

$$H = -\frac{\hbar^2}{2m}\frac{d^2}{dx^2} + \frac{1}{2}m\omega^2 x^2.$$

Assume a gaussian wave function:

$$\psi(x) = Ae^{-bx^2}$$

Estimate the ground state energy.

Exercise (if you are physicist, if not drink water)
Suppose we want to find the ground state energy for the one-dimensional harmonic oscillator:

$$H = -\frac{\hbar^2}{2m}\frac{d^2}{dx^2} + \frac{1}{2}m\omega^2 x^2.$$

Assume a gaussian wave function:

$$\psi(x) = Ae^{-bx^2}$$

Estimate the ground state energy.

Solution:

$$\boxed{E_{gs} \leq \langle\psi|H|\psi\rangle \equiv \langle H\rangle .}$$

$$1 = |A|^2 \int_{-\infty}^{\infty} e^{-2bx^2}\,dx = |A|^2 \sqrt{\frac{\pi}{2b}} \;\Rightarrow\; A = \left(\frac{2b}{\pi}\right)^{1/4}$$

Now

$$\langle H \rangle = \langle T \rangle + \langle V \rangle,$$

where, in this case,

$$\langle T \rangle = -\frac{\hbar^2}{2m}|A|^2 \int_{-\infty}^{\infty} e^{-bx^2} \frac{d^2}{dx^2}\left(e^{-bx^2}\right) dx = \frac{\hbar^2 b}{2m},$$

and

$$\langle V \rangle = \frac{1}{2} m\omega^2 |A|^2 \int_{-\infty}^{\infty} e^{-2bx^2} x^2\,dx = \frac{m\omega^2}{8b},$$

so

$$\langle H \rangle = \frac{\hbar^2 b}{2m} + \frac{m\omega^2}{8b}$$

let’s minimize <H>:

$$\frac{d}{db}\langle H \rangle = \frac{\hbar^2}{2m} - \frac{m\omega^2}{8b^2} = 0 \Rightarrow b = \frac{m\omega}{2\hbar}.$$

Putting this back into $\langle H \rangle$, we find

$$\langle H \rangle_{\min} = \frac{1}{2}\hbar\omega.$$

Take home message

- gaussian is a good trial function (even if not similar to the true ground state);
- gaussian is easy to manipulate (normalize etc);
- The **variational principle** (D. Griffiths) will give you an good upper bound for ground state energy;

QAOA

- **QAOA:**
  - Quantum Approximate Optimization Algorithm [1]

    or Quantum Alternating Operator Ansatz [2]

- It is a **variational** method used for optimization problems.
- It's **hybrid** – *i.e.* partly quantum, partly classical

  Suitable for use in NISQ devices

[1] arXiv:1411.4028
[2] Algorithms 2019, 12(2), 34

Variational algorithms

- They are heuristic methods for optimization that share the following elements:

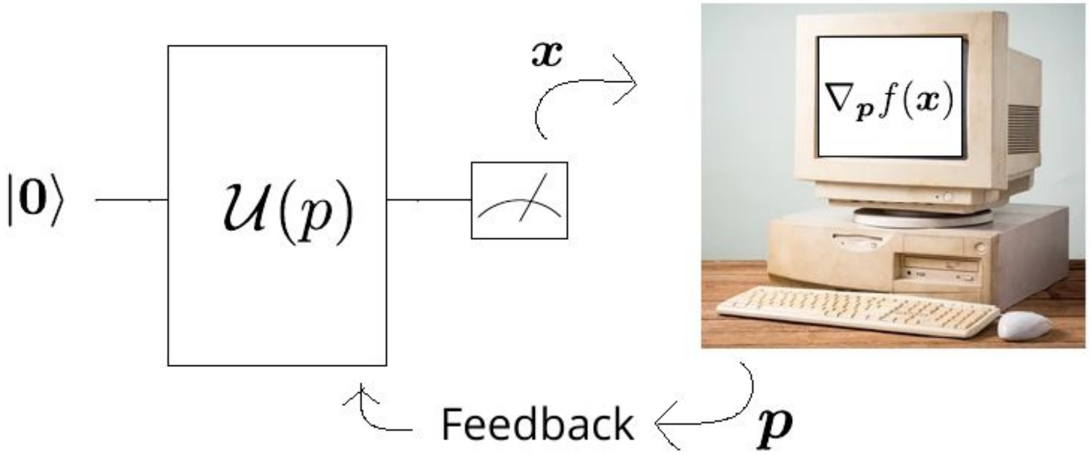
$\mathcal{O}$	An observable which encodes the solution to the problem in its maximum or minimum eigenvalue
$\|\psi(p_1, \ldots, p_K)\rangle$	A parametric family of states that *ideally* covers the relevant subspace of O, but such that K << dim(H)

- Optimize over **p** to get an estimate for the actual value of the problem:

$$f_{\text{est}} = \min_{\boldsymbol{p}} \langle \psi(\boldsymbol{p}) | \mathcal{O} | \psi(\boldsymbol{p}) \rangle$$

Variational algorithms

- The **hybrid** framework:
  - ***O*** and psi are produced and measured in a quantum device
  - Parameters **p** are optimized in a classical computer



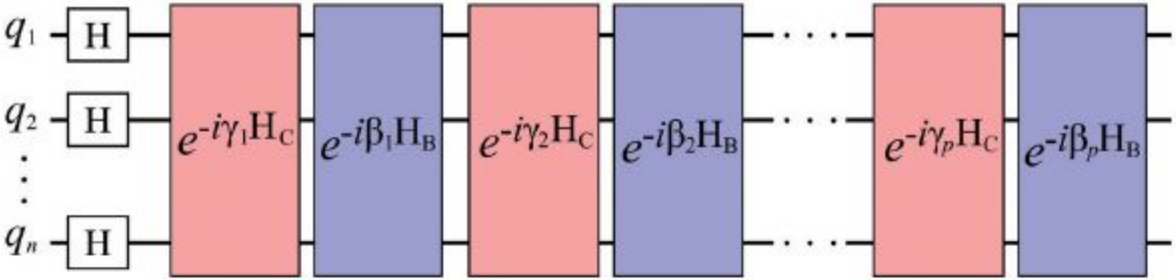


Figura 4: Circuito para implementação do QAOA.

$$U_C(\gamma) \coloneqq e^{-i\gamma H_C},$$
$$U_B(\beta) \coloneqq e^{-i\beta H_B}.$$

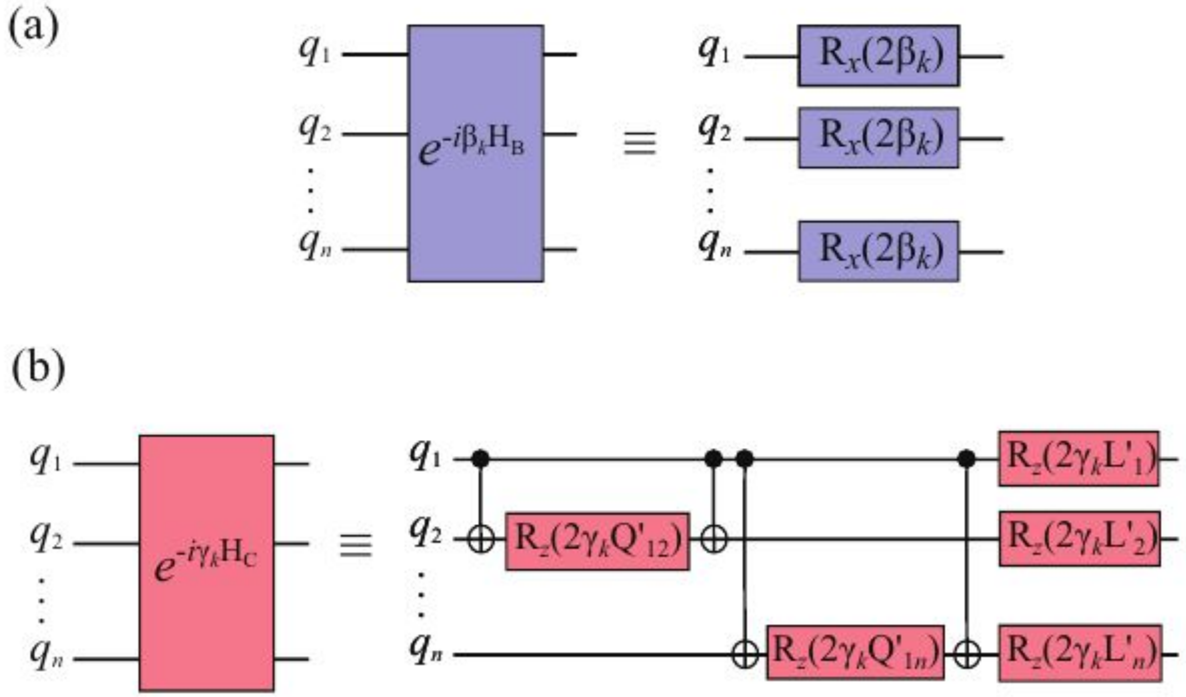


Figura 5: Circuitos implementando as unitárias (a) U_B e (b) U_C.

Conclusion

- QAOA is an interesting method for near-term noisy devices that could already provide some advantage with few resources
- Part of the family of Variational Quantum Algorithms with the benefit of being connected to Adiabatic Quantum Computing.
- Active area of research with potential for much experimentation (e.g. warm start, other ansatzes, optimization methods, etc.)
- Although simple, design considerations must be taken into account to avoid convergence problems (e.g. barren plateaus, NP-Hard classical optimization, ...)

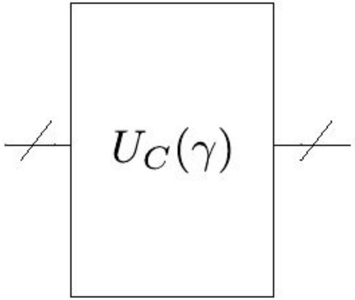
Useful References

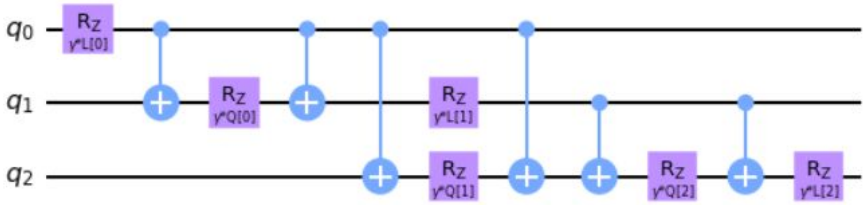
- ArXiv:1411.4028 – Original paper
- Algorithms 2019, 12(2), 34 – Other alternating ansatze
- ArXiv:1602.07674 – Supremacy via QAOA
- ArXiv:1911.05296 – Portifolio rebalancing
- Quantum 5, 479 (2021) – Warm start
- Phys. Rev. Lett. 127, 120502 – Classical optimization is NP-Hard
- ArXiv:2101.02138 – Controlability leads to barren plateaus
- ArXiv:2208.04382 – Our Quantum Finance tutorial (In Portuguese)

Exercise - QAOA implementation

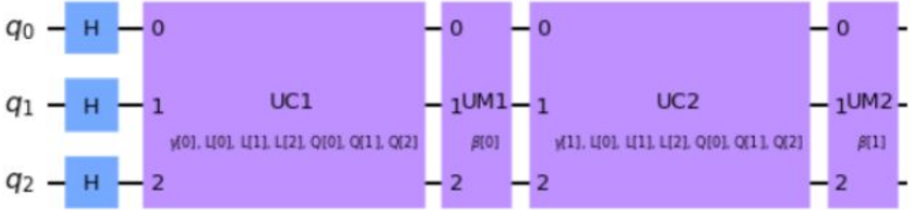
Qiskit

```python
nqbits = 3

qc = QuantumCircuit(nqbits)

Q = ParameterVector('Q', int(nqbits*(nqbits-1)/2))
L = ParameterVector('L',nqbits)
gamma = Parameter(r'$\gamma$')

k=0
for i in range(nqbits):

    qc.rz(L[i]*gamma, i)

    for j in range(i+1, nqbits):
        qc.cx(i, j)
        qc.rz(Q[k]*gamma, j)
        qc.cx(i, j)
        k += 1

qc.draw('mpl');
```





```python
nqbits = 3
layers = 2

Q = ParameterVector('Q', int(nqbits*(nqbits-1)/2))
L = ParameterVector('L',nqbits)

gamma = ParameterVector(r'$\gamma$', layers)
beta  = ParameterVector(r'$\beta$', layers)

qc = QuantumCircuit(nqbits)

qc.h(range(nqbits))

for p in range(layers):

    uc = Uc(nqbits, gamma[p], Q, L).to_gate()
    uc.label = "UC" + str(p+1)

    qc.append(uc, qargs=[0,1,2])

    um = Um(nqbits, beta[p]).to_gate()
    um.label = "UM" + str(p+1)

    qc.append(um, qargs=[0,1,2])

qc.draw('mpl');
```



May try also

Qiskit

IBM Q™

Cirq

TensorFlow Quantum

PENNYLANE

XANADU

Ocean

D:Wave

The Quantum Computing Company™