Building Quantum Machines with Light

LMCAL

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INCT-IQ

Measuring the field

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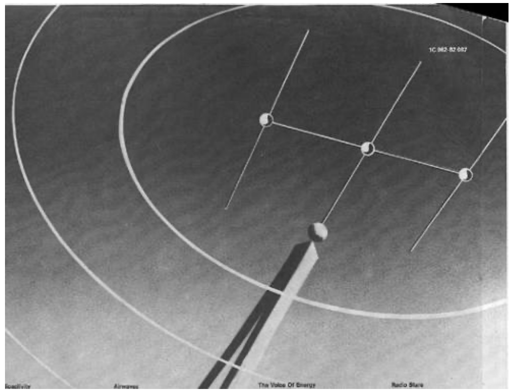
Quantum Optics – Measurement of the Field



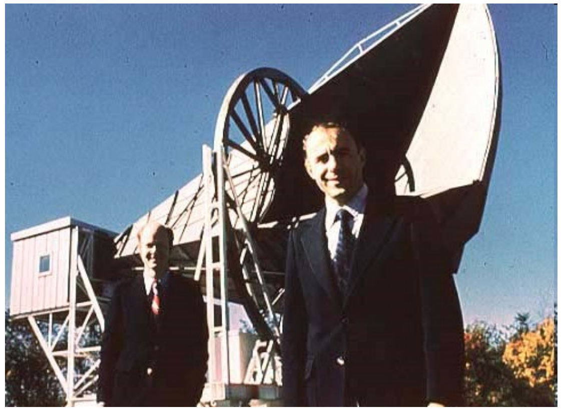
Slow varying EM Field can be detected by an antenna:

→ conversion of electric field in electronic displacement.

→amplification, recording, analysis of the signal.

→electronic readly available.

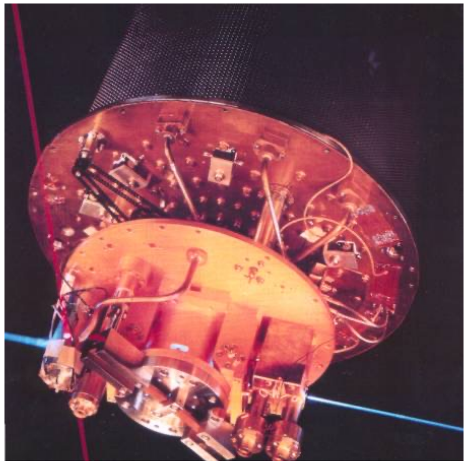
Example: 3 K cosmic background (Penzias & Wilson).

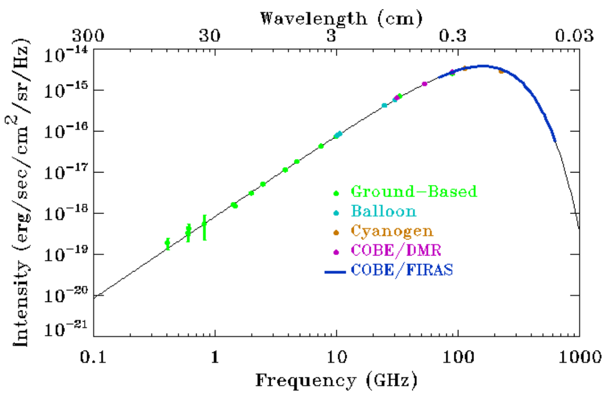


Problems:

→Even this tiny field accounts for a strong photon density.

→Every measurement needs to account for thermal background (e.g. Haroche *et al.*).





Quantum Optics – Measurement of the Field

Fast varying EM Field cannot be measured directly.

We often detect the mean value of the Poynting vector: $\mathbf{S} = \varepsilon_0 \mathbf{E} \times \mathbf{B}$

Photoelectric effect converts photons into ejected electrons

We measure photo-electrons

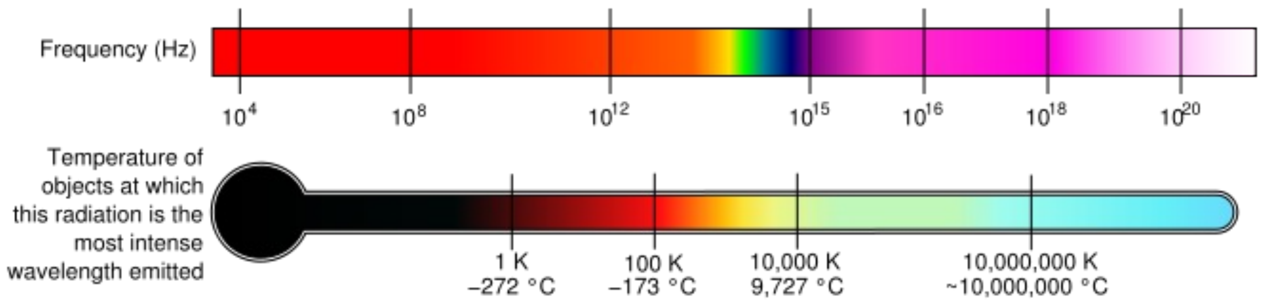
→individually with APDs or photomultipliers – a single electron is converted in a strong pulse – discrete variable domain,

→in a strong flux with photodiodes, where the photocurrent is converted into a voltage – continuous variable domain.

Advantages: in this domain, photons are energetic enough:

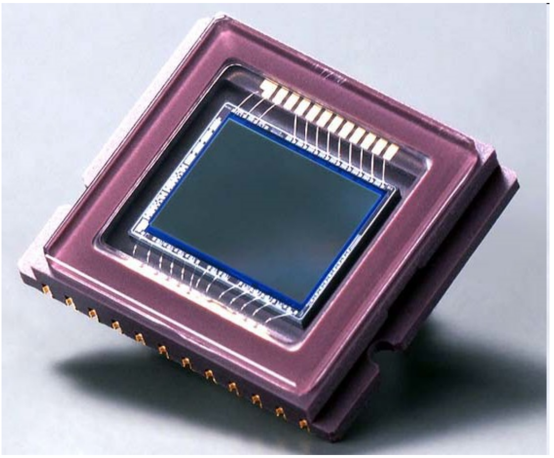
→in a small flux, every photon counts.

→for the eV region (visible and NIR), presence of background photons is negligible: measurements are nearly the same in L-He or at room temperature.



Quantum Optics – Measurement of the Field

And detectors are cheap!



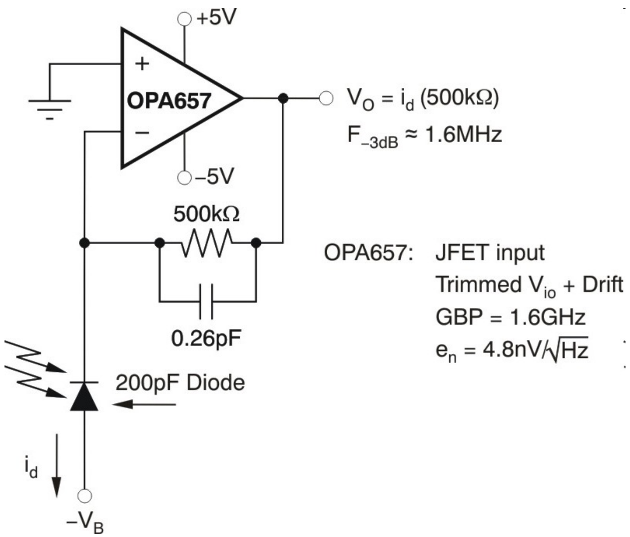
Quantum Optics – Measurement of the Field

Discrete output:

- Photomultipliers
- Avalanche photodiodes
- Superconducting nanowire single photon detectors
- Number resolvingn photodetectors

Continuous output:

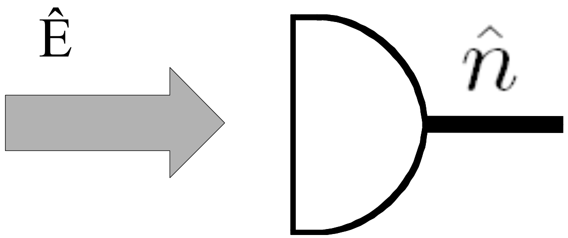
- PIN photodiodes



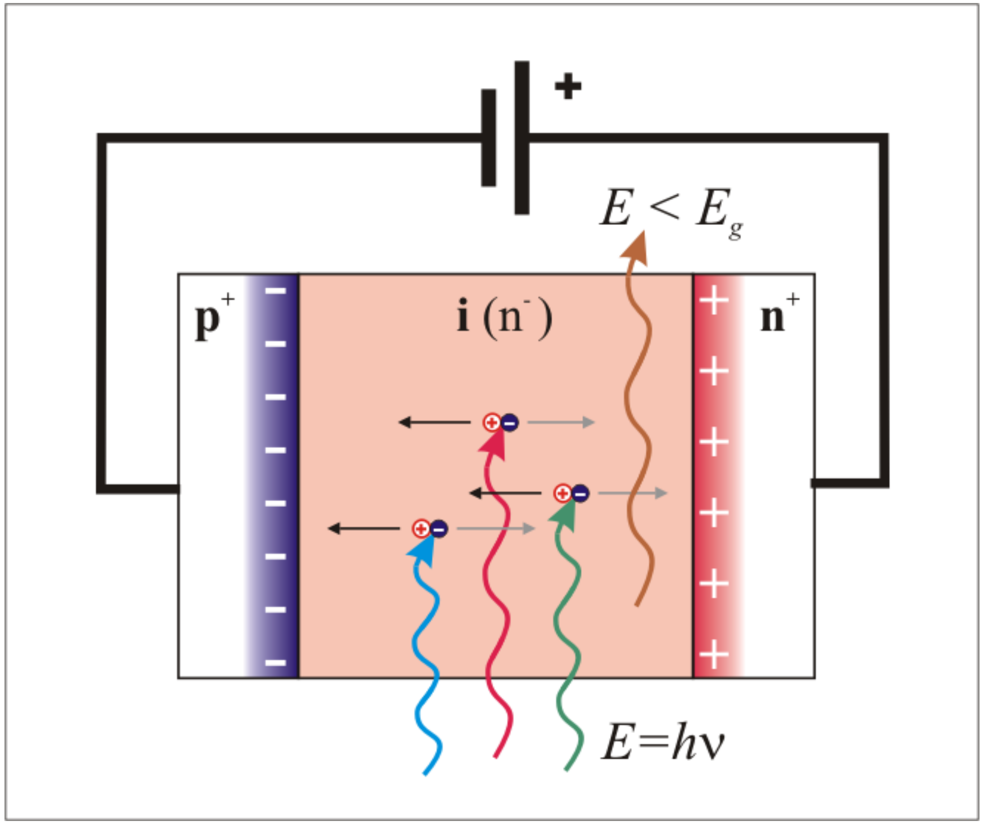
Quantum Optics – Measurement of the Field

We can easily measure photon flux: field intensity

(or more appropriate, optical power)



$$\boxed{I = \langle E^* \, E \rangle = \alpha^* \, \alpha}$$



Author: Kirnehkrib (under C. Commons)

$$\boxed{\hat{n} = \hat{a}^\dagger \hat{a}}$$

For a filtered input, it selects a mode of the field, and counts the photon number in that mode

→ discrete spectra

Quantum Optics – Coherence

JOURNAL OF THE OPTICAL SOCIETY OF AMERICA VOLUME 47, NUMBER 10 OCTOBER, 1957

Two-Beam Interference with Partially Coherent Light

B. J. Thompson, *Physics Department, The College of Science and Technology, University of Manchester, Manchester, England*

AND

E. Wolf, *The Physical Laboratories, University of Manchester, Manchester, England**

(Received December 17, 1956)

Two-beam interference with partially coherent light is discussed. A new proof of the general interference law for partially coherent fields is obtained and is illustrated by means of simple and direct experiments. Photographs are given which show the changes in the visibility of the fringes as the degree of coherence is varied and the results are compared with the predictions of the theory.

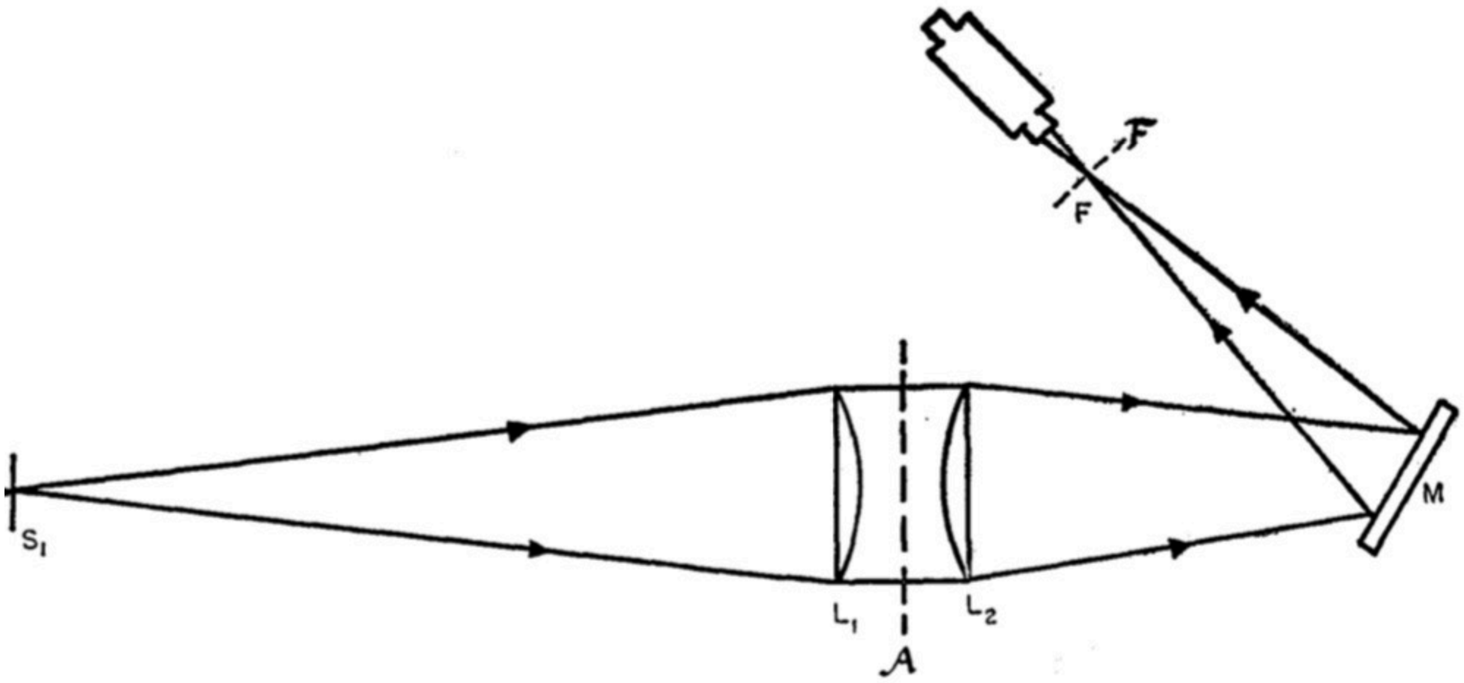
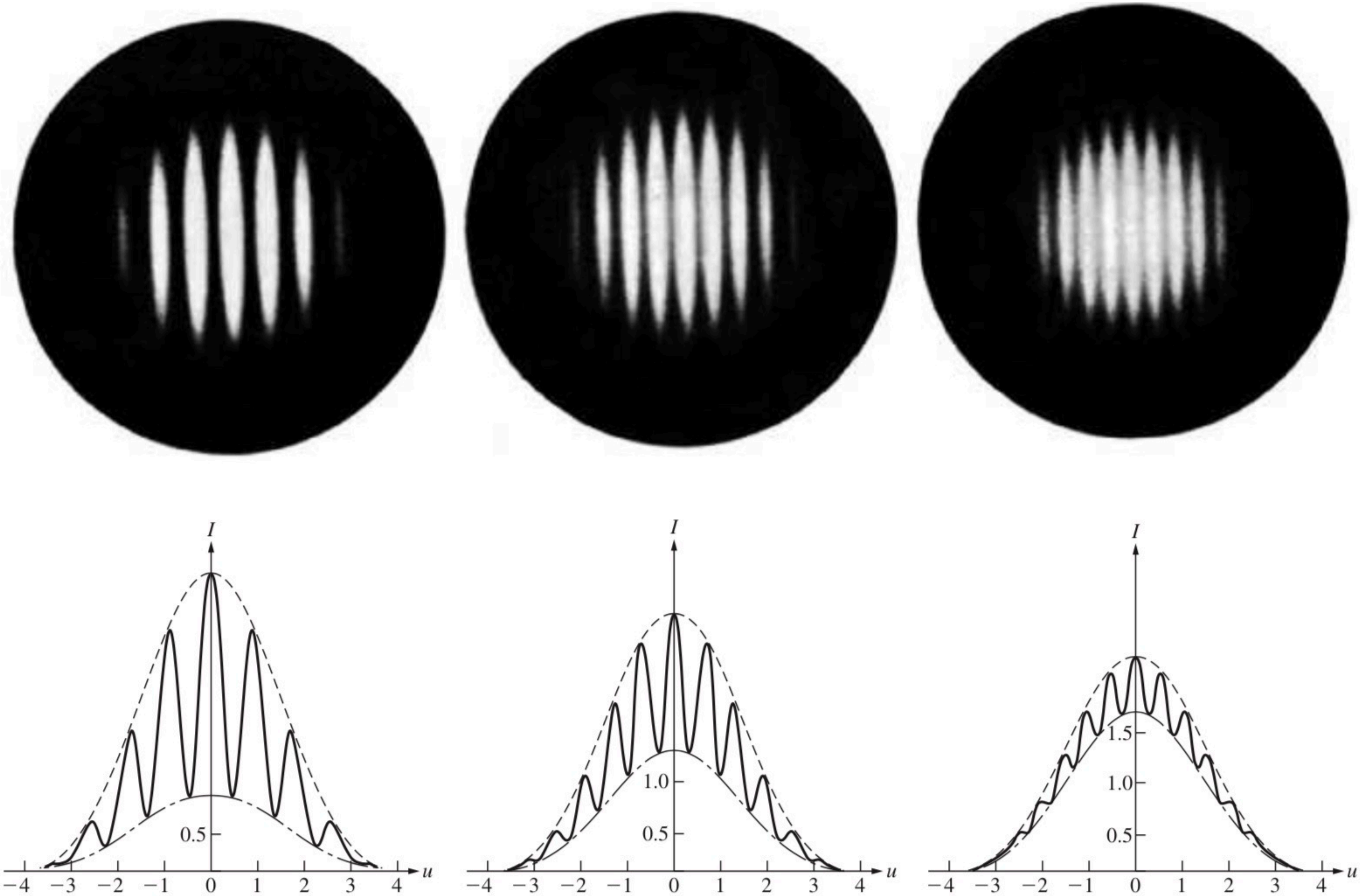


Fig. 3. The diffractometer.

Quantum Optics – Coherence



$a = 0.6$ cm

$\mathscr{V} = |\tilde{\gamma}_{12}| = 0.593,\ \alpha_{12} = 0$

(a)

$a = 0.8$ cm

$\mathscr{V} = |\tilde{\gamma}_{12}| = 0.361,\ \alpha_{12} = 0$

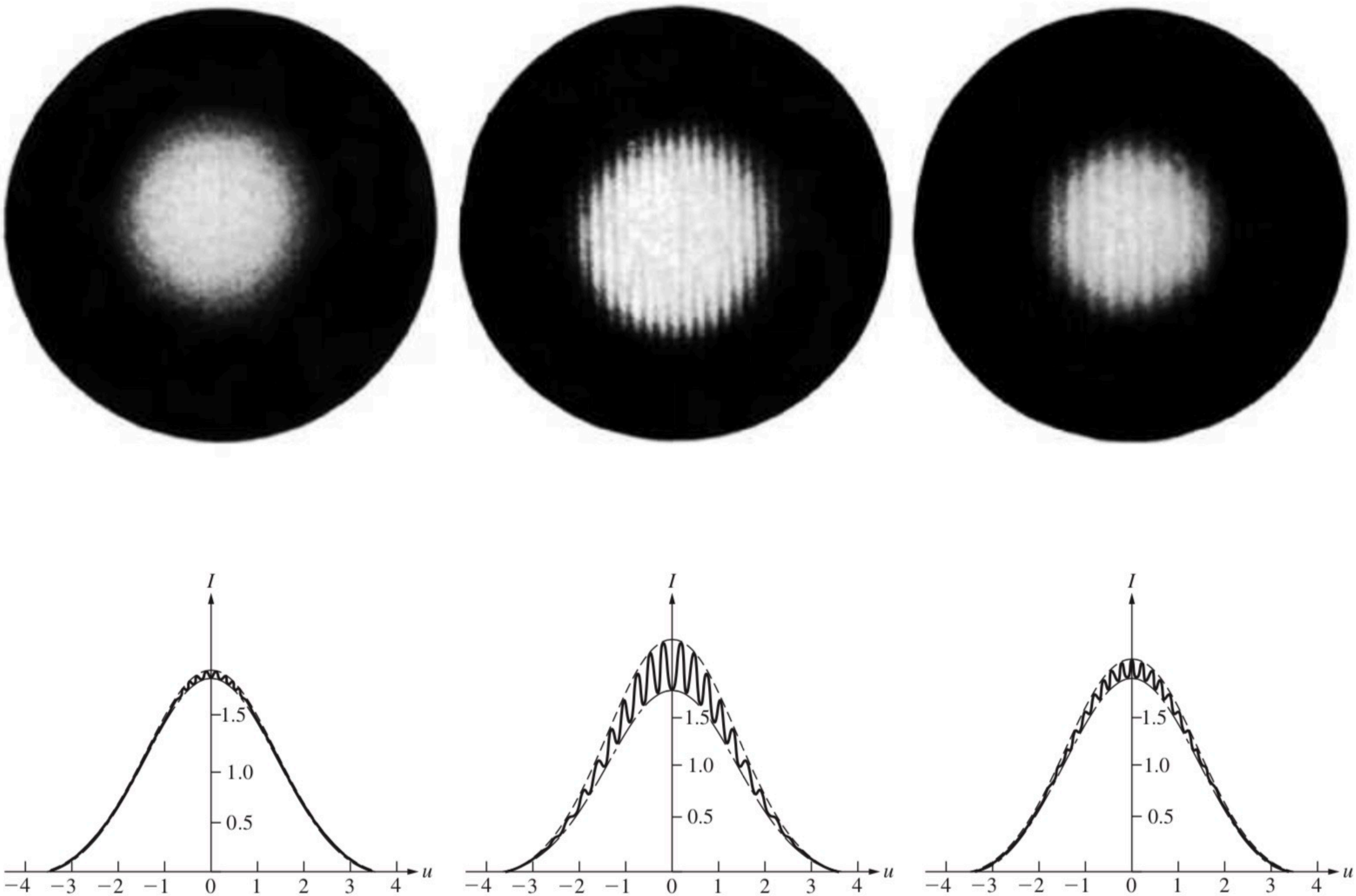
(b)

$a = 1$ cm

$\mathscr{V} = |\tilde{\gamma}_{12}| = 0.146,\ \alpha_{12} = 0$

(c)

Quantum Optics – Coherence



a = 1.2 cm
$\mathcal{V} = |\tilde{\gamma}_{12}| = 0.015, \alpha_{12} = \pi$
(d)

a = 1.7 cm
$\mathcal{V} = |\tilde{\gamma}_{12}| = 0.123, \alpha_{12} = \pi$
(e)

a = 2.3 cm
$\mathcal{V} = |\tilde{\gamma}_{12}| = 0.035, \alpha_{12} = 0$
(f)

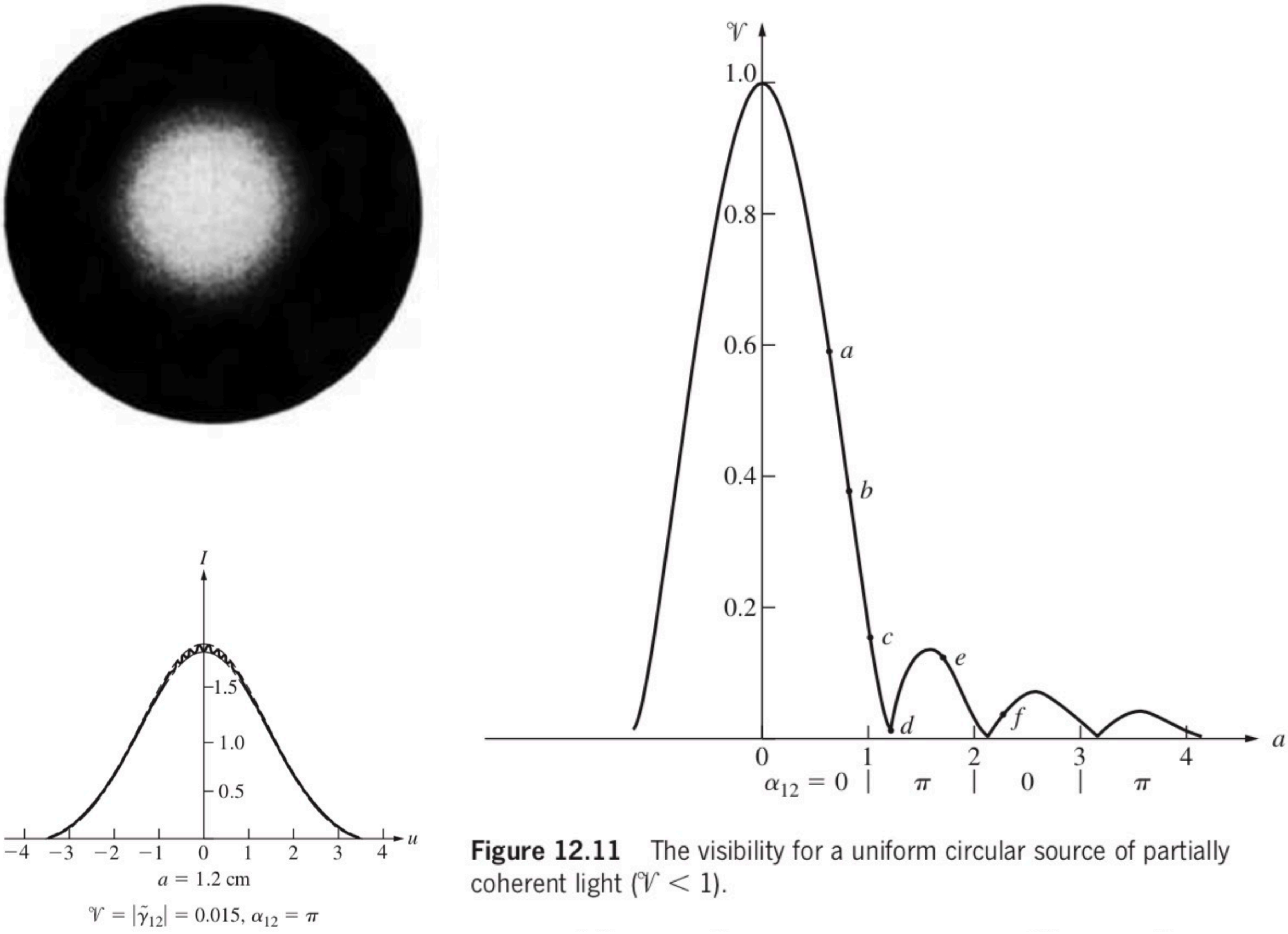
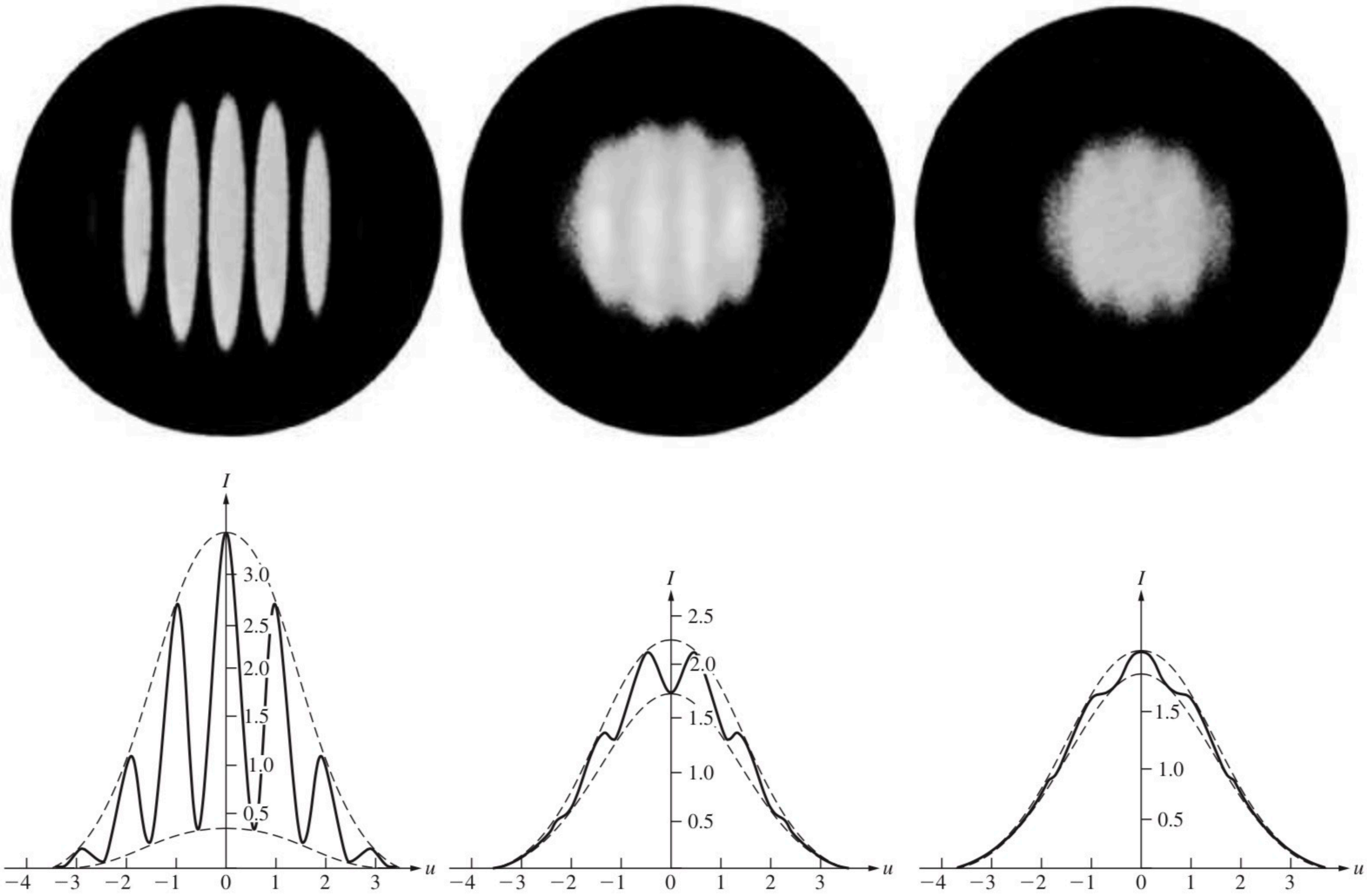


Figure 12.11 The visibility for a uniform circular source of partially coherent light ($\mathscr{V} < 1$).

(d) (e) (f)

Quantum Optics – Coherence



(a)	(b)	(c)
$\alpha_{12} = 0$	$\alpha_{12} = \pi$	$\alpha_{12} = 0$
$a = 0.5$ cm	$a = 0.5$ cm	$a = 0.5$ cm
$\mathcal{V} = \lvert\tilde{\gamma}_{12}\rvert = 0.703$	$\mathcal{V} = \lvert\tilde{\gamma}_{12}\rvert = 0.132$	$\mathcal{V} = \lvert\tilde{\gamma}_{12}\rvert = 0.062$

Figure 12.13 Double-beam interference patterns. Here the aperture separation was held constant, thereby yielding a constant number of fringes per unit displacement in each photo. The visibility was altered by varying the size of the primary incoherent source. (B.J. Thompson, *J. Soc. Photo. Inst. Engr.* 4, 7 [1965])

Quantum Optics – Coherence

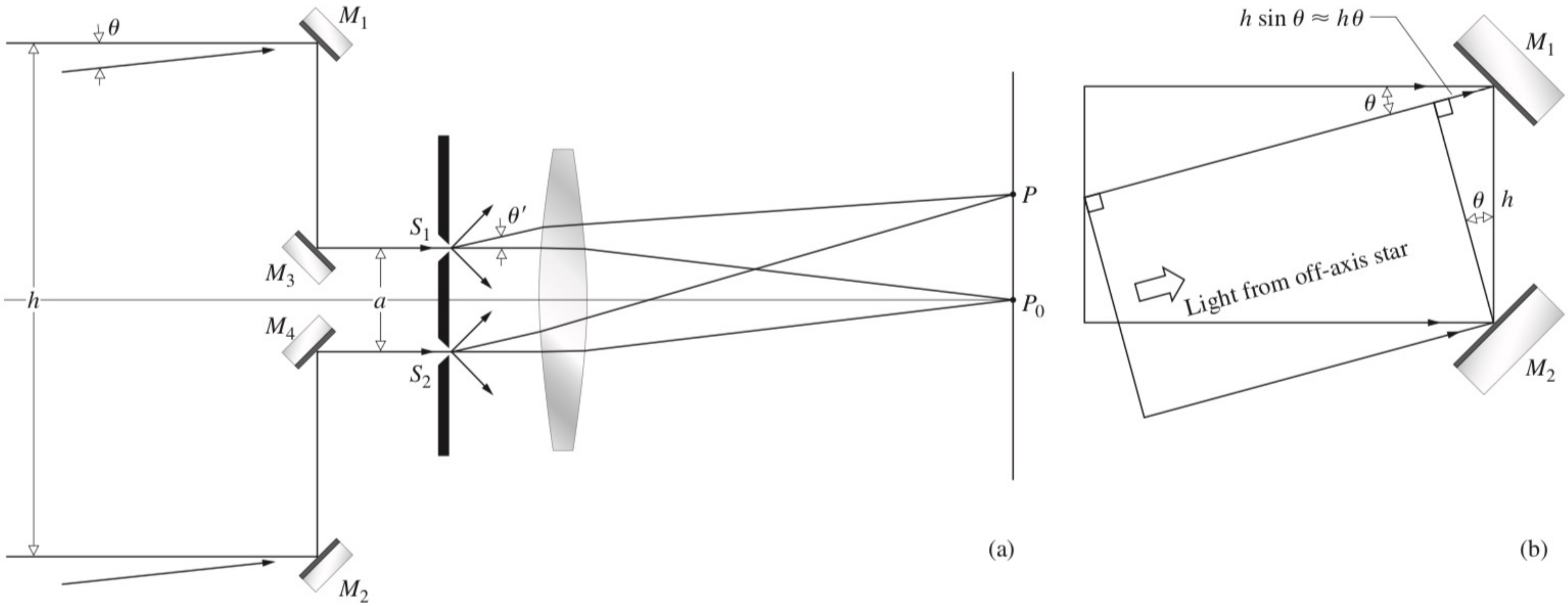


Figure 12.17 Michelson stellar interferometer.

Quantum Optics – Coherence

1046 NATURE November 10, 1956 Vol. 178

A TEST OF A NEW TYPE OF STELLAR INTERFEROMETER ON SIRIUS

By R. HANBURY BROWN
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Dr. R. Q. TWISS
Services Electronics Research Laboratory, Baldock

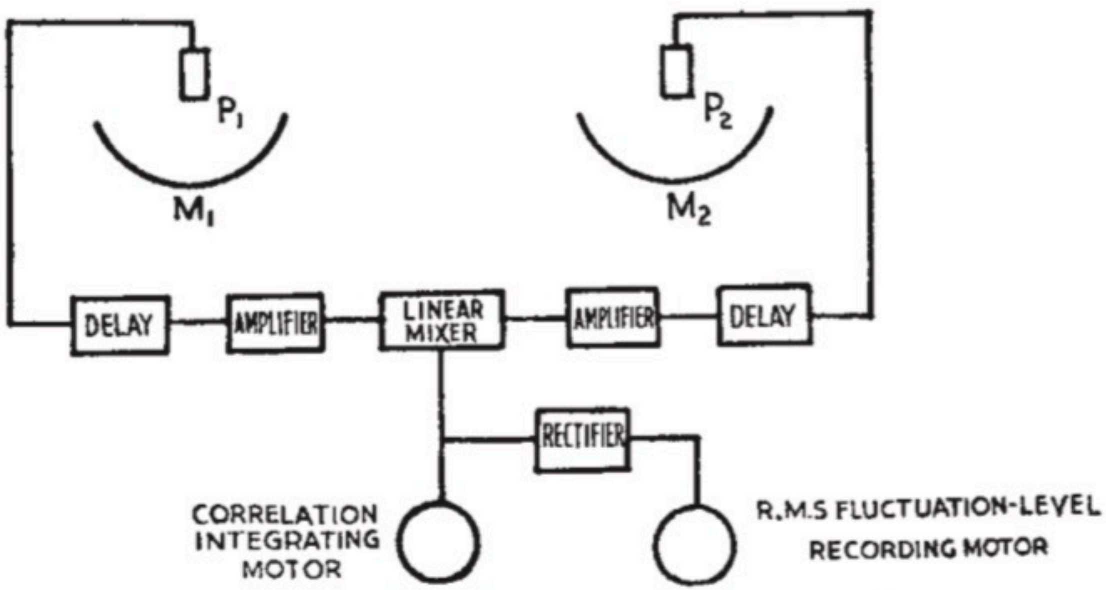

Fig. 1. Simplified diagram of the apparatus

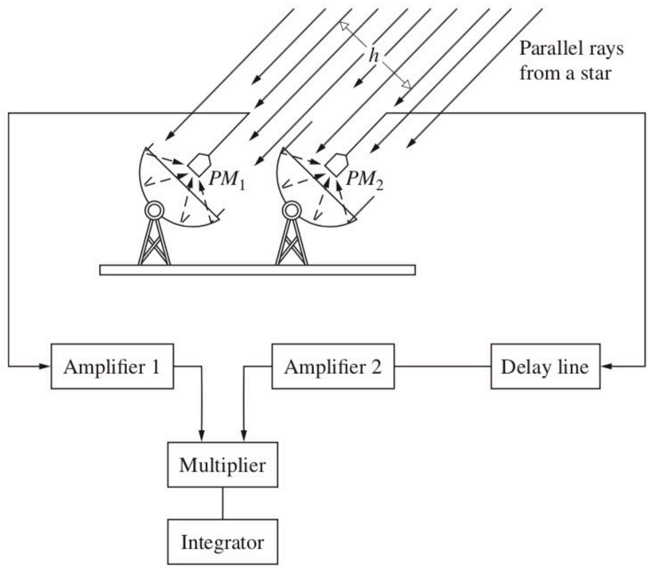

Figure 12.20 Stellar correlation interferometer.

Quantum Optics – Coherence

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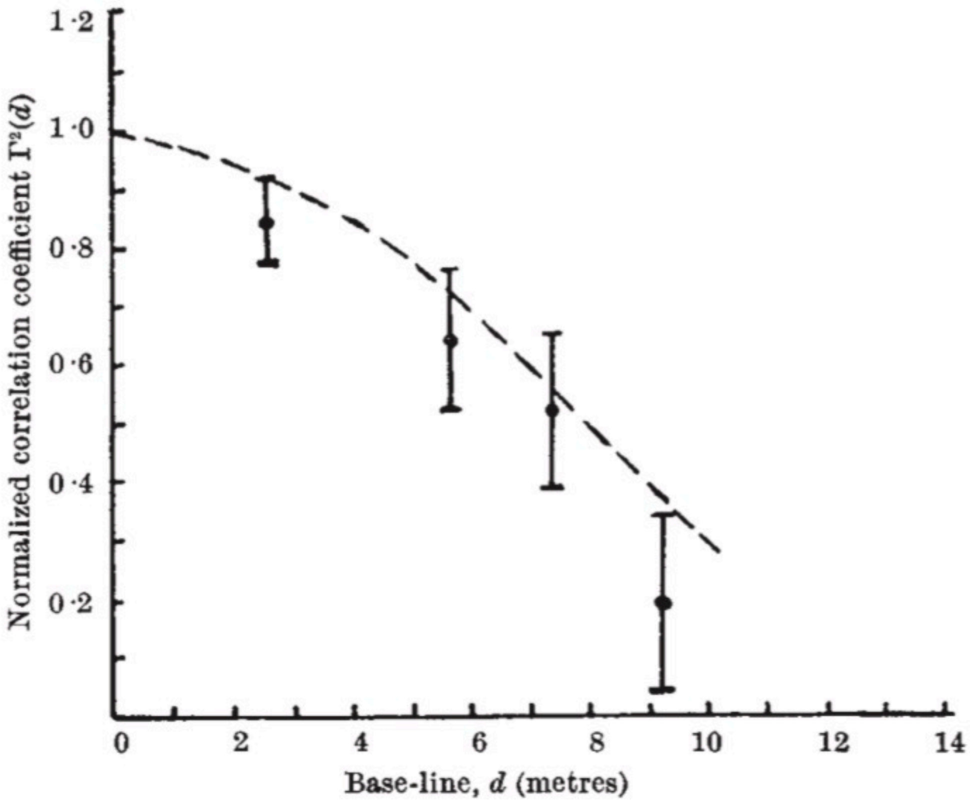

Fig. 2. Comparison between the values of the normalized correlation coefficient $\Gamma^2(d)$ observed from Sirius and the theoretical values for a star of angular diameter 0·0063″. The errors shown are the probable errors of the observations

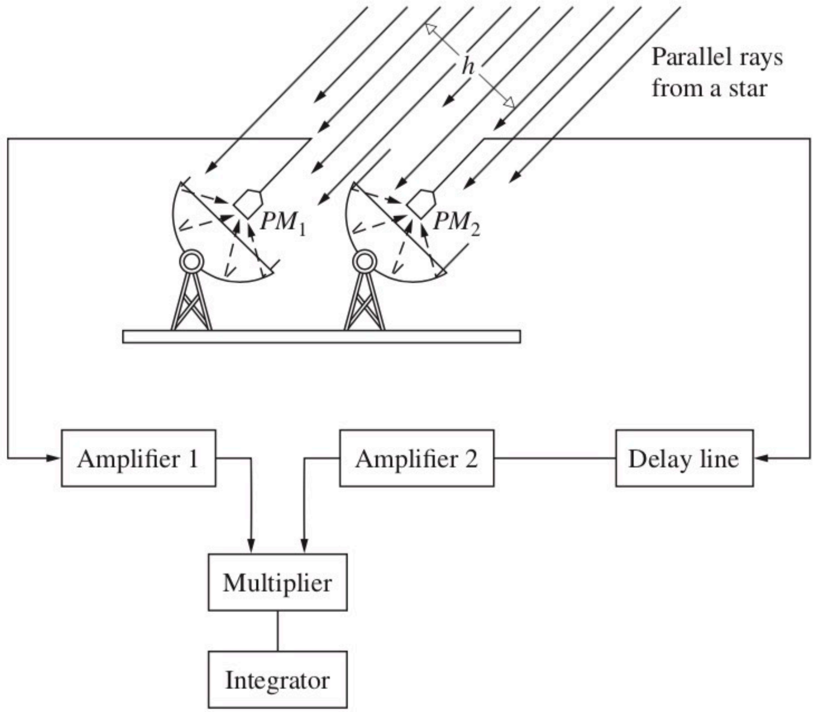

Figure 12.20 Stellar correlation interferometer.

Quantum Optics – Coherence

No. 4497 January 7, 1956 NATURE 27

CORRELATION BETWEEN PHOTONS IN TWO COHERENT BEAMS OF LIGHT

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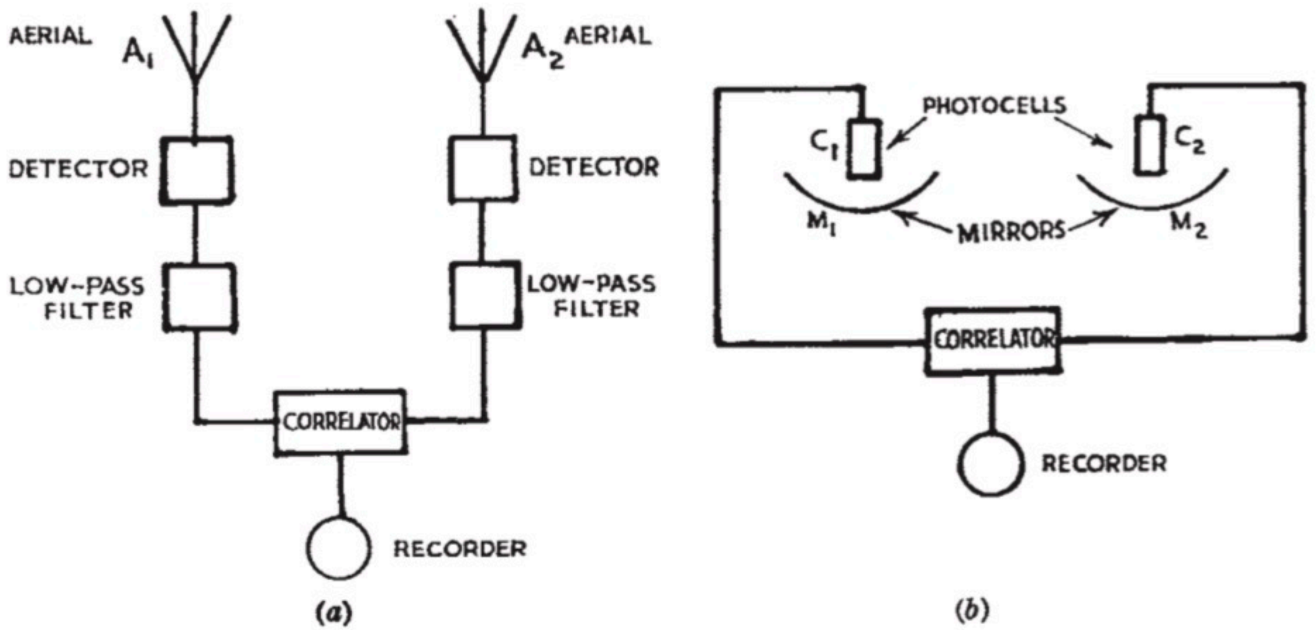

Fig. 1. A new type of radio interferometer (*a*), together with its analogue (*b*) at optical wave-lengths

Quantum Optics – Coherence

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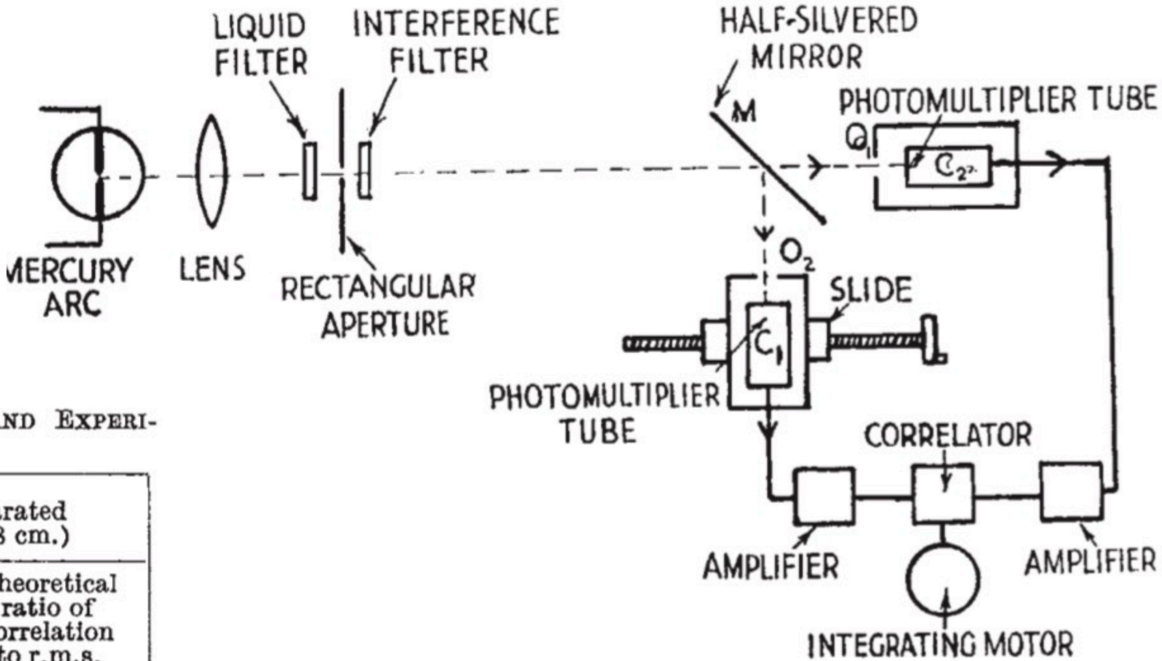


Fig. 2. Simplified diagram of the apparatus

Table 1. COMPARISON BETWEEN THE THEORETICAL AND EXPERIMENTAL VALUES OF THE CORRELATION

	Cathodes superimposed ($d = 0$)		Cathodes separated ($d = 2a = 1{\cdot}8$ cm.)	
	Experimental ratio of correlation to r.m.s. deviation $S_e(0)/N_e$	Theoretical ratio of correlation to r.m.s. deviation $S(0)/N$	Experimental ratio of correlation to r.m.s. deviation $S_e(d)/N_e$	Theoretical ratio of correlation to r.m.s. deviation $S(d)/N$
1	+ 7·4	+ 8·4	− 0·4	≃ 0
2	+ 6·6	+ 8·0	+ 0·5	≃ 0
3	+ 7·6	+ 8·4	+ 1·7	≃ 0
4	+ 4·2	+ 5·2	− 0·3	≃ 0

Correlation of fields

↓

Interference

$$C_E^{(2)}(\tau) = \overline{E_1(t)\,E_2(t+\tau)}$$

↓

Double-Slit

Correlation of Intensities

↱ 2^{nd} order

$$C_I^{(2)}(\tau) = \overline{I_1(t)\,I_2(t+\tau)}$$

↓

HB&T

$$C_E^{(4)}(\tau) = \overline{E_1(t)\,E_1^*(t)\,E_2(t+\tau)\,E_2^*(t+\tau)}$$

↳ 4^{th} order

PHYSICAL REVIEW VOLUME 130, NUMBER 6 15 JUNE 1963

The Quantum Theory of Optical Coherence*

ROY J. GLAUBER
Lyman Laboratory of Physics, Harvard University, Cambridge, Massachusetts
(Received 11 February 1963)

Correlation functions: $x_i \to (\vec{r}_i, t_i)$

$$G^{(n)}(x_1,\dots,x_n,x_{n+1},\dots,x_{2n}) \equiv$$

$$\mathrm{Tr}\{\rho E^-(x_1)\dots E^-(x_n)E^+(x_{n+1})\dots E^+(x_n)\}$$

with $E^+(x_i) = \sum_{k,s} \hat{a}_k u_k(\vec{r}_i) e^{-i\omega t} \hat{\varepsilon}_s$

<u>Application</u>

Interference: $G^{(1)}(x_1,x_2) = \mathrm{Tr}\{\rho E^-(x_1)E^+(x_2)\}$

Hanbury-Brown & Twiss

$$G^{(2)}(x_1,x_2,x_2,x_1) = \mathrm{Tr}\{\rho E^-(x_1)E^-(x_2)E^+(x_2)E^+(x_1)\}$$

Photon correlation

Normalized second order correlation:

$$g^{(2)}(\tau) \equiv \frac{G^{(2)}(\tau)}{|G^{(1)}(0)|^2}$$

Dealing with a single mode of the field

$$g^{(2)}(0) = \frac{\langle a^\dagger a^\dagger a a \rangle}{\langle a^\dagger a \rangle^2}$$

Dealing with a single mode of the field

$$g^{(2)}(0) = \frac{\langle a^\dagger a^\dagger a a\rangle}{\langle a^\dagger a\rangle^2}$$

using $a a^\dagger - a^\dagger a = 1$

$$g^{(2)}(0) = \frac{\langle a^\dagger a \cdot a^\dagger a\rangle}{\langle a^\dagger a\rangle^2} - \frac{\langle a^\dagger a\rangle}{\langle a^\dagger a\rangle^2} = \frac{\langle \hat{n}^2\rangle}{\langle \hat{n}\rangle^2} - \frac{\langle \hat{n}\rangle}{\langle \hat{n}\rangle^2}$$

Defining $\Delta^2 n = \langle \hat{n}^2\rangle - \langle \hat{n}\rangle^2$

$$g^{(2)}(0) = 1 + \frac{\Delta^2 n - \langle \hat{n}\rangle}{\langle \hat{n}\rangle^2} = 1 + \frac{Q}{\langle \hat{n}\rangle}$$

Mandel Q-factor $\quad Q = \frac{\Delta^2 n - \langle \hat{n}\rangle}{\langle \hat{n}\rangle}$

[PRL, 49, 136 (1982)]

(deviation from a Poissonian distribution)

Examples: Thermal state: $P(\alpha) = \dfrac{e^{-|\alpha|^2/\bar{n}}}{\pi \bar{n}}$

$$\langle a^\dagger a^\dagger a a \rangle = \int P(\alpha)\, \alpha^{*2} \alpha^2 \, d\alpha \quad \text{(using } P \text{ distribution)}$$

$$= \int_0^{2\pi}\int_0^{\infty} \frac{|\alpha|^4}{\pi} e^{-|\alpha|^2/\bar{n}} \, d|\alpha| \cdot |\alpha| \cdot d\theta =$$

$$x = \frac{|\alpha|}{\sqrt{\bar{n}}}$$

$$= 2\bar{n}^2 \int_0^{\infty} x^5 e^{-x^2} \, dx = 2\bar{n}^2$$

$$\langle a^\dagger a \rangle = \bar{n}$$

$$\underline{g^{(2)}(0) = 2} \quad \text{"bunching"}$$

$$Q = \frac{\Delta^2 n - \langle \hat{n} \rangle}{\langle \hat{n} \rangle}$$

$$= \frac{\bar{n}^2 - \bar{n}}{\bar{n}} =$$

$$\boxed{Q = \bar{n} - 1}$$

$\downarrow$

Thermal light

$$\langle \hat{n}^2 \rangle = 2\bar{n}^2$$

$$\Delta^2 n = \langle \hat{n}^2 \rangle - \langle \hat{n} \rangle^2 =$$

$$= 2\bar{n}^2 - \bar{n}^2 = \bar{n}^2$$

Poissonian $\boxed{\Delta^2 n = \bar{n}}$

$\Delta^2 n < \bar{n}$

$Q = 0$

$Q > 0 \Rightarrow$ superpoissonian

$Q < 0 \Rightarrow$ sub-poissonian

Coherent state: $\hat{\rho} = |\alpha\rangle\langle\alpha|$

$$\langle a^{\dagger 2} a^2 \rangle = \langle \alpha | \hat{a}^{\dagger 2} \hat{a}^2 | \alpha \rangle = |\alpha|^4$$

$$\langle a^\dagger a \rangle = |\alpha|^2$$

$$g^{(2)}(0) = 1$$

Number state: $|\psi\rangle = |n\rangle$

$$\Delta^2 n = 0$$

$$g^{(2)}(0) = 1 - \frac{1}{n} < 1$$

Anti-bunching

Coherent state: $\hat{\rho} = |\alpha\rangle\langle\alpha|$

$$\langle a^{\dagger 2} a^2 \rangle = \langle \alpha | a^{\dagger 2} \hat{a}^2 | \alpha \rangle = |\alpha|^4$$

$$\langle a^\dagger a \rangle = |\alpha|^2$$

$$\boxed{g^{(2)}(0) = 1} \qquad Q = 0$$

Number state: $|\psi\rangle = |n\rangle$

$$Q = \frac{\cancelto{0}{\Delta^2 n} - \bar{n}}{\bar{n}} = -1$$

$$\Delta^2 n = 0$$

$$g^{(2)}(0) = 1 - \frac{1}{n} < 1$$

$$n > 1$$

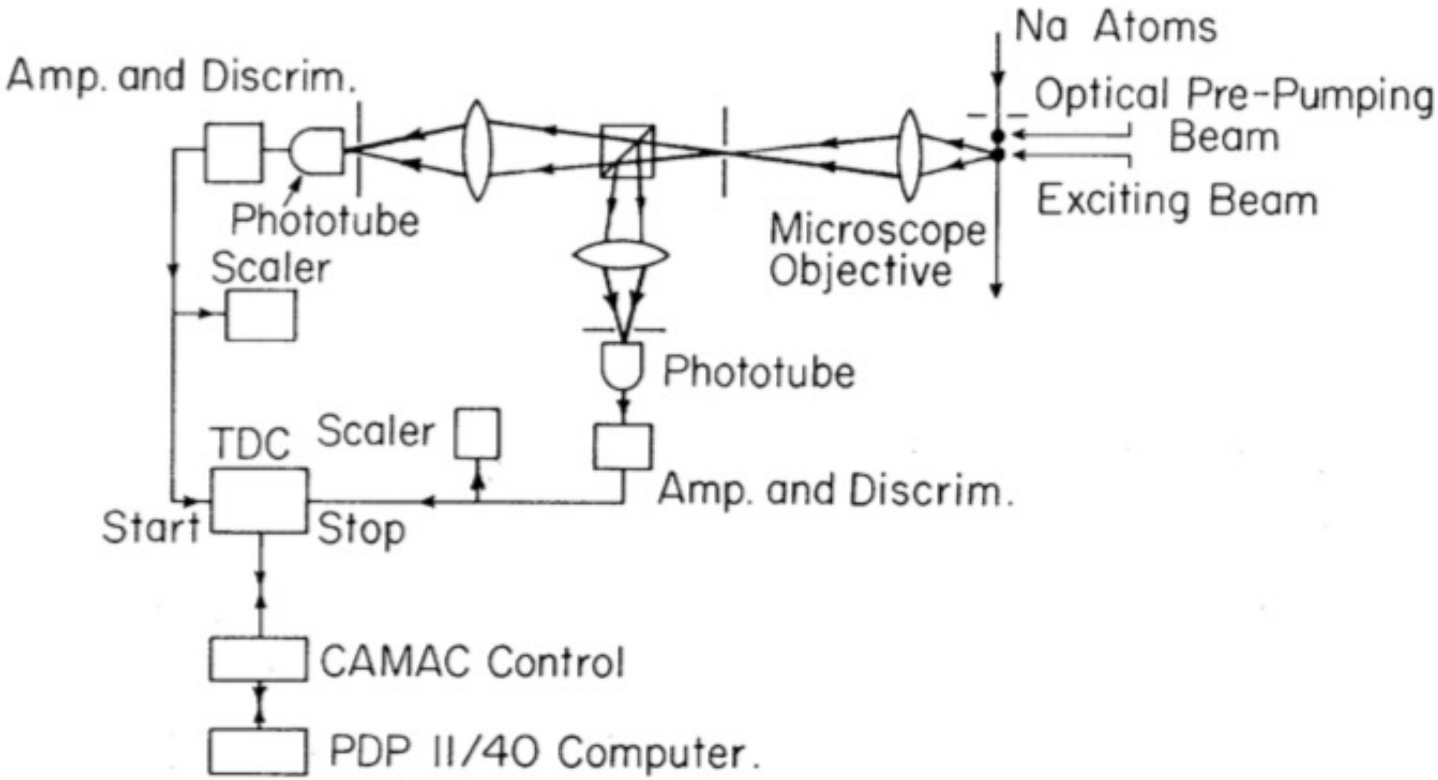
Anti-bunching

Squeezed State $|\alpha, \xi\rangle$?

Squeezed Vacuum $|0, \xi\rangle$?

Photon Antibunching in Resonance Fluorescence

H. J. Kimble,[a] M. Dagenais, and L. Mandel
Department of Physics and Astronomy, University of Rochester, Rochester, New York 14627
(Received 22 July 1977)



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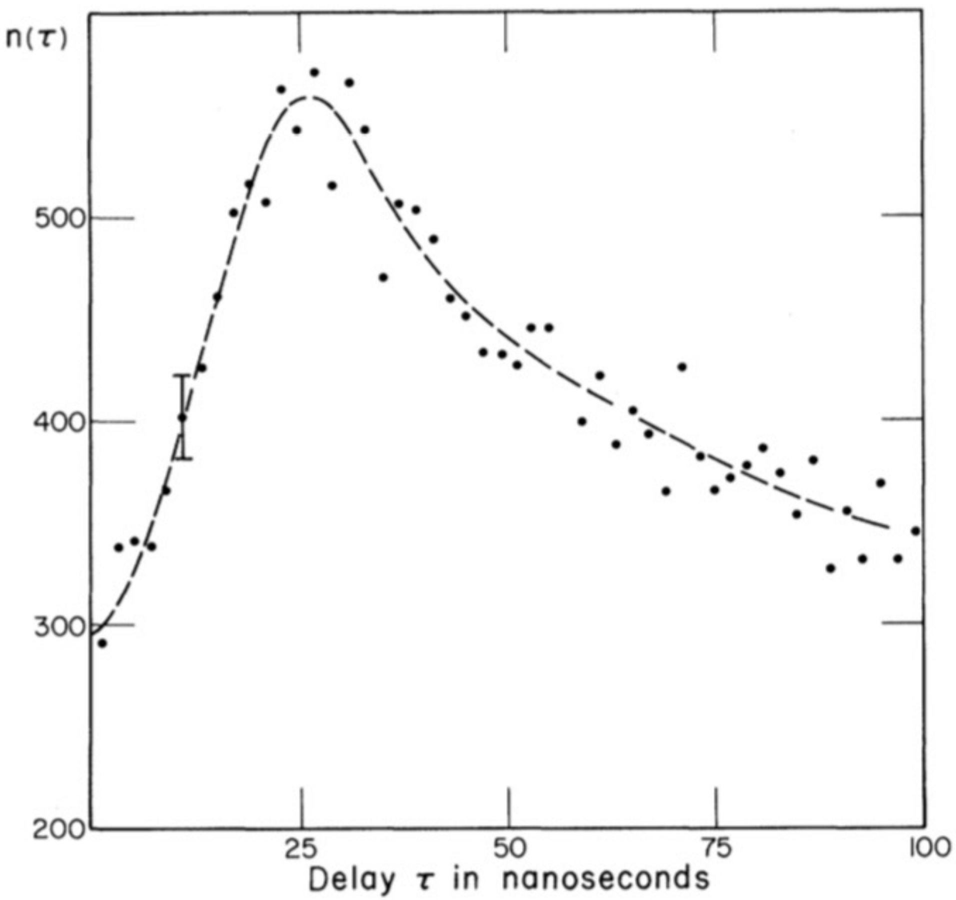


FIG. 2. The number of recorded pulse pairs $n(\tau)$ as a function of the time delay τ in nanoseconds. The growth of $n(\tau)$ from $\tau = 0$ shows antibunching. The bars on one point indicate statistical uncertainties corresponding to one standard deviation. The broken line just outlines the trend.

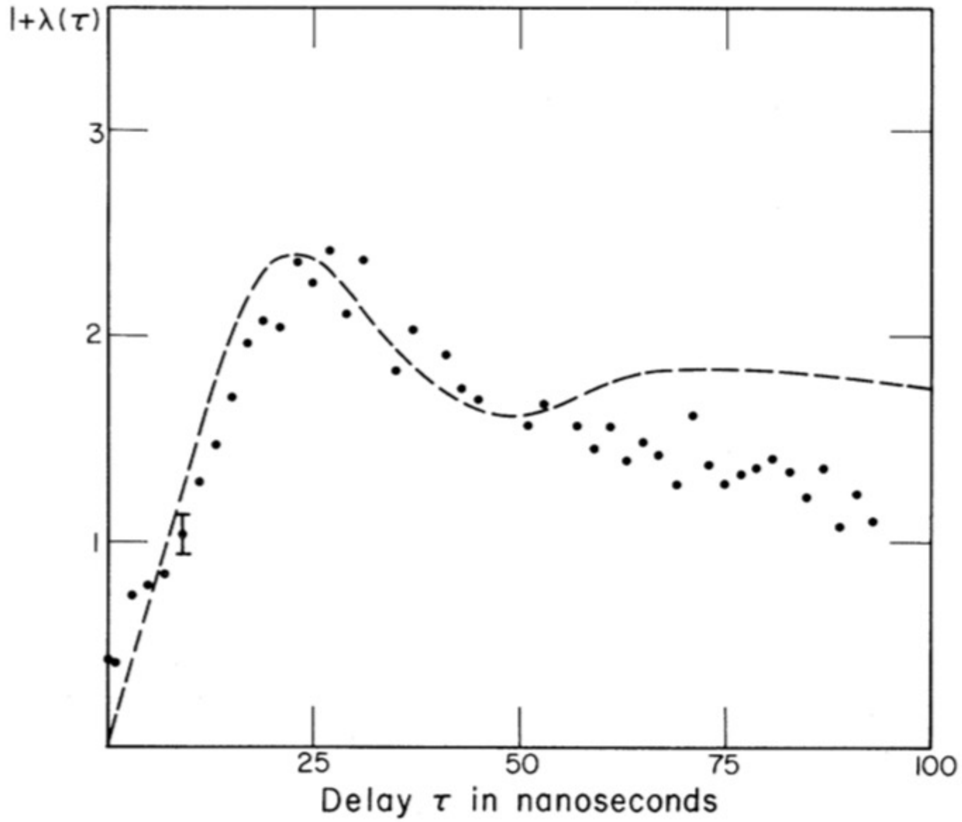
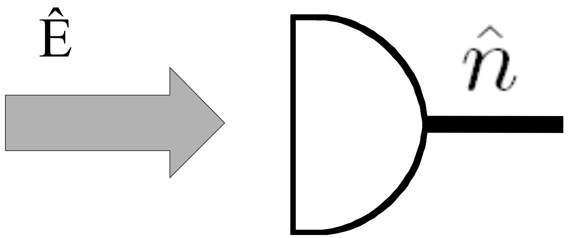


FIG. 3. Values of $1+\lambda(\tau)$ derived from the data. The broken curve shows the theoretically expected form of $\langle \hat{I}_G(\tau)\rangle$ (with $\Omega/\beta = 4$) for a single atom, arbitrarily normalized to the same peak.

Quantum Optics – Measurement of the Intense Field

We can easily measure photon flux: field intensity

(or more appropriate, optical power)



$$\boxed{I = \langle E^* \, E \rangle = \alpha^* \, \alpha}$$

$$\boxed{\hat{n} = \hat{a}^\dagger \hat{a}}$$

$$\boxed{\hat{a}^\dagger = \alpha + \delta \hat{a}^\dagger} \qquad \boxed{\alpha = |\alpha| exp(i\varphi)}$$

$$\boxed{\hat{n} = |\alpha|^2 + |\alpha| e^{i\varphi} \delta \hat{a}^\dagger + |\alpha| e^{-i\varphi} \delta \hat{a} + \delta \hat{a}^\dagger \delta \hat{a}}$$

$$\boxed{\hat{n} = |\alpha|^2 + |\alpha| \delta \hat{p} + O(2)}$$

Quantum Optics – Measurement of the Intense Field

We can easily measure photon flux: field intensity

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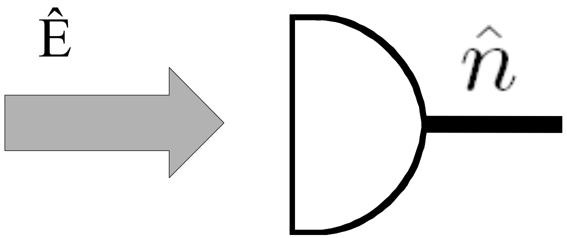


$$I = \langle E^* \; E \rangle = \alpha^* \; \alpha$$

$$\hat{n} = \hat{a}^\dagger \hat{a}$$

$$\hat{a} = \alpha + \delta\hat{a}$$

$$\alpha = |\alpha| exp(i\varphi)$$

$$\hat{n} = |\alpha|^2 + |\alpha| e^{i\varphi} \delta\hat{a}^\dagger + |\alpha| e^{-i\varphi} \delta\hat{a} + \delta\hat{a}^\dagger \delta\hat{a}$$

$$\hat{n} = |\alpha|^2 + |\alpha| \delta\hat{p} + O(2)$$

Quantum Optics – Measurement of the Intense Field

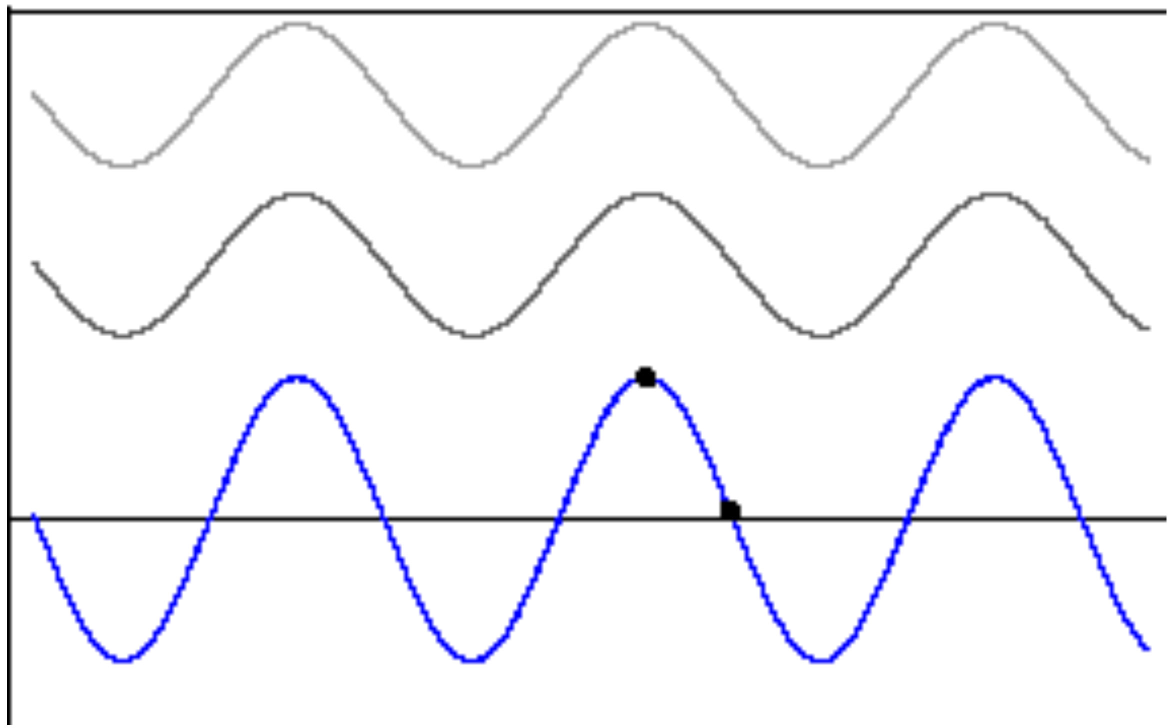
OK, we got the amplitude measurement, but that is only part of the history!

Amplitude is directly related to the measurement of the number of photon, (or the photon counting rate, if you wish).

This leaves an unmeasured quadrature, that can be related to the phase of the field.

But there is not such an evident "phase operator"!

Still, there is a way to convert phase into amplitude: interference and interferometers.



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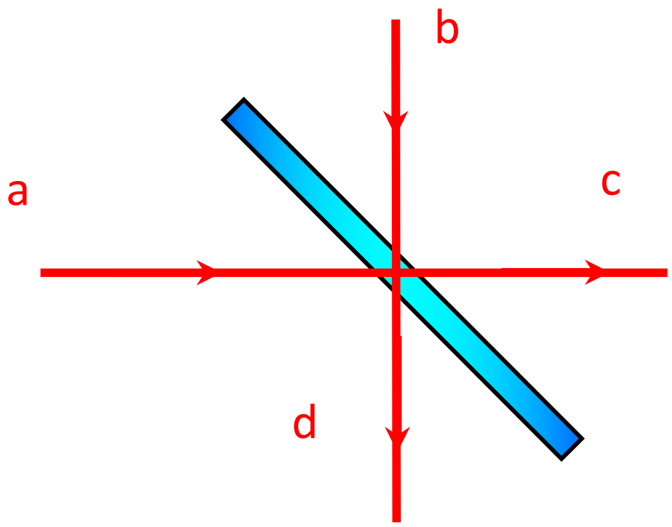
$$\boxed{\delta \hat{p} = e^{i\varphi}\,\delta\hat{a}^{\dagger} + e^{-i\varphi}\,\delta\hat{a}}$$

amplitude associated quadrature

$$\delta \hat{q} = -i\left(e^{i\varphi}\,\delta\hat{a}^{\dagger} - e^{-i\varphi}\,\delta a\right)$$

→ phase associated quadrature

Building an Interferometer – The Beam Splitter



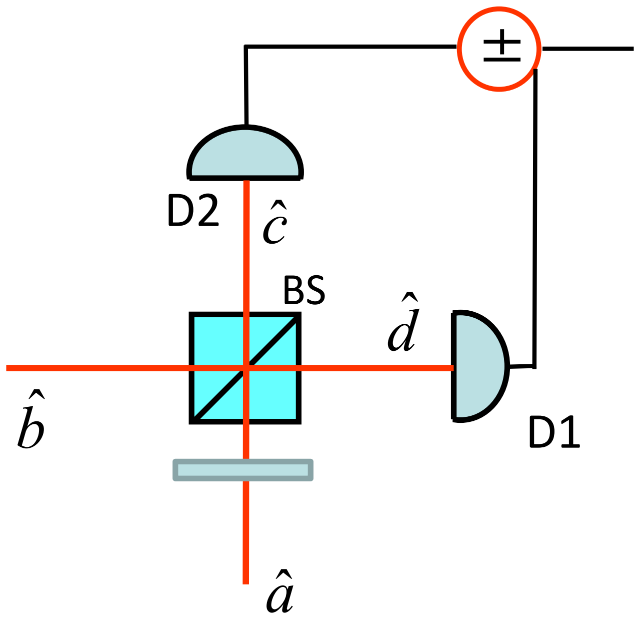
$$\hat{c} = \frac{1}{\sqrt{2}}\left(\hat{a} + \hat{b}\right)$$

$$\hat{d} = \frac{1}{\sqrt{2}}\left(\hat{b} - \hat{a}\right)$$

$$\hat{n}_c = \frac{\hat{n}_a + \hat{n}_b + \hat{a}^\dagger \hat{b} + \hat{b}^\dagger \hat{a}}{2}$$

$$\hat{n}_d = \frac{\hat{n}_a + \hat{n}_b - \hat{a}^\dagger \hat{b} - \hat{b}^\dagger \hat{a}}{2}$$

Building an Interferometer – The Beam Splitter



$$\hat{n}_c = \frac{\hat{n}_a + \hat{n}_b + \hat{a}^\dagger \hat{b} + \hat{b}^\dagger \hat{a}}{2}$$

$$\hat{n}_d = \frac{\hat{n}_a + \hat{n}_b - \hat{a}^\dagger \hat{b} - \hat{b}^\dagger \hat{a}}{2}$$

$$\boxed{\hat{n}_+ = \hat{n}_a + \hat{n}_b} \qquad \boxed{\hat{n}_- = \hat{a}^\dagger \hat{b} + \hat{b}^\dagger \hat{a}}$$

Homodyning

if $<|\hat{a}|> \ll <|\hat{b}|>$

$$\boxed{\hat{n}_-(t) = |\beta| \left[\hat{a}(t)e^{-i\theta} + \hat{a}^\dagger(t)e^{i\theta}\right] = |\beta|\hat{X}_a(\theta)}$$

Vacuum Homodyning

$$\boxed{\hat{n}_+ = \hat{n}_b} \qquad \boxed{\langle \hat{n}_- \rangle = 0} \qquad \boxed{\Delta^2 \hat{n}_- = \langle \hat{n}_b \rangle}$$

Calibration of the Standard Quantum Level

$$
\begin{aligned}
\Delta^2 \hat{n} &= \langle (\hat{n} - \langle \hat{n} \rangle)^2 \rangle = \langle \hat{n}^2 \rangle - \langle \hat{n} \rangle^2 \\
&= \langle \hat{a}^\dagger \hat{a} \hat{a}^\dagger \hat{a} \rangle - \langle \hat{a}^\dagger \hat{a} \rangle^2 = \langle \hat{a}^\dagger \hat{a}^\dagger \hat{a} \hat{a} \rangle - \langle \hat{a}^\dagger \hat{a} \rangle^2 + \langle \hat{a}^\dagger \hat{a} \rangle \\
&= \underbrace{\langle : \hat{n}^2 : \rangle - \langle \hat{n} \rangle^2}_{\text{“Classical” Variance}} + \langle \hat{n} \rangle
\end{aligned}
$$

Shot noise !

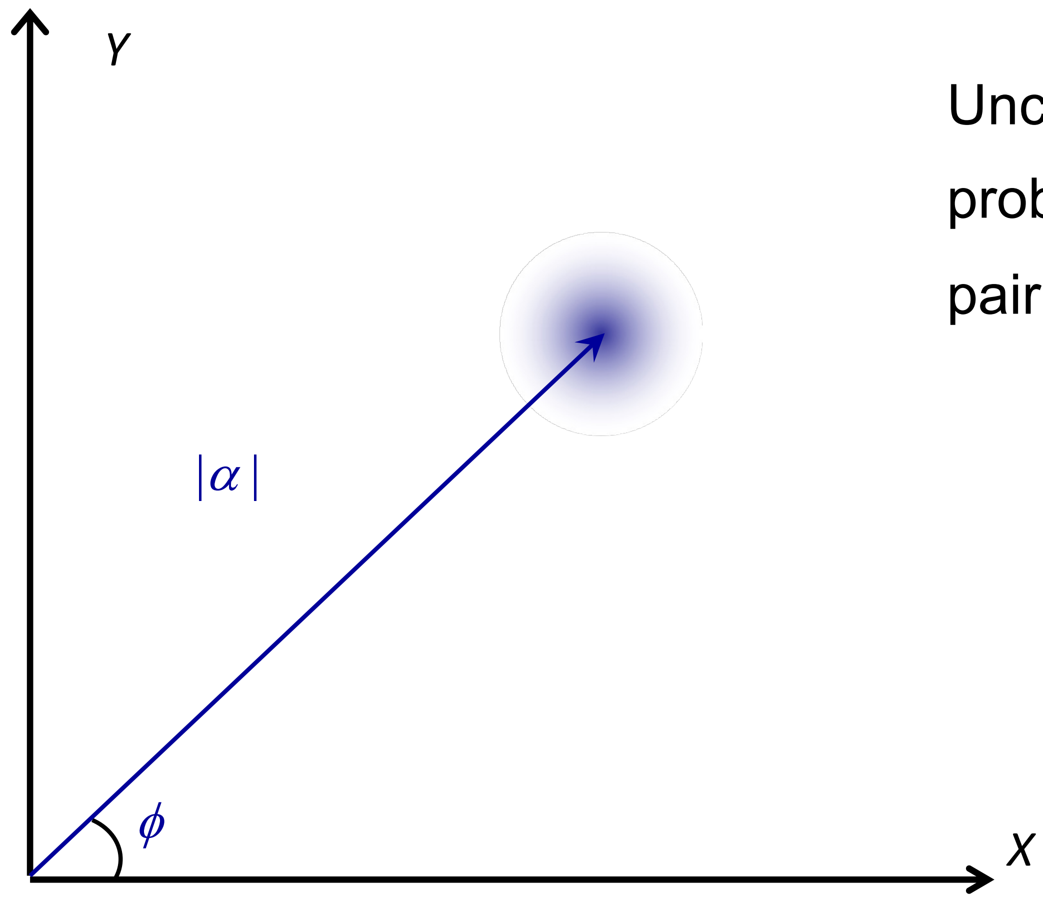
Vacuum Homodyning allows the calibration of the detection, producing a Poissonian distribution in the output (just like a coherent state).

$$
|\alpha\rangle = exp(-|\alpha|^2/2) \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}} |n\rangle
$$

$$
p_n = |\langle \hat{n} | \alpha \rangle|^2 = exp(-|\alpha|^2) \frac{|\alpha|^{2n}}{n!}
$$

Field Quadratures – Quantum Optics

$$\left[\hat{X}(\theta), \hat{X}\left(\theta + \frac{\pi}{2}\right)\right] = 2i \qquad \text{Thus,} \qquad \Delta X \Delta Y \geq 1$$

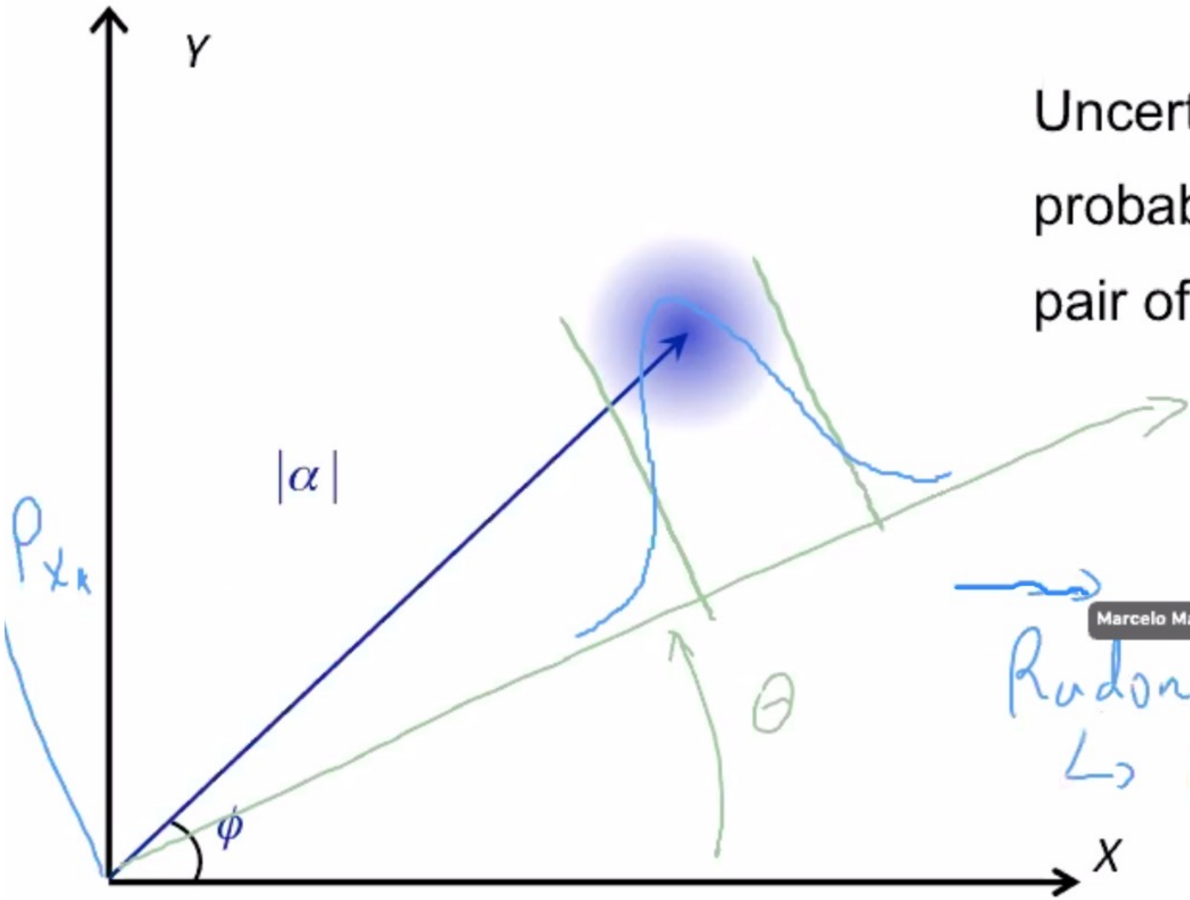


Uncertainty relation implies in a probability distribution for a given pair of quadrature measurements

Field quadratures behave just as position and momentum operators!

Field Quadratures – Quantum Optics

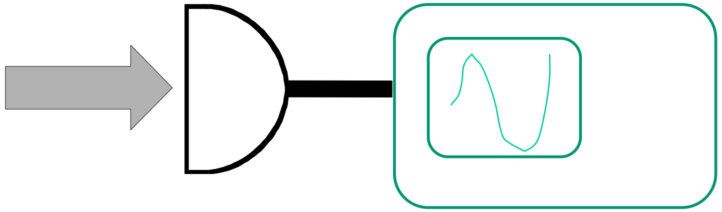
$$\left[\hat{X}(\theta), \hat{X}\left(\theta + \frac{\pi}{2}\right)\right] = 2i \qquad \text{Thus,} \qquad \Delta X \Delta Y \geq 1$$



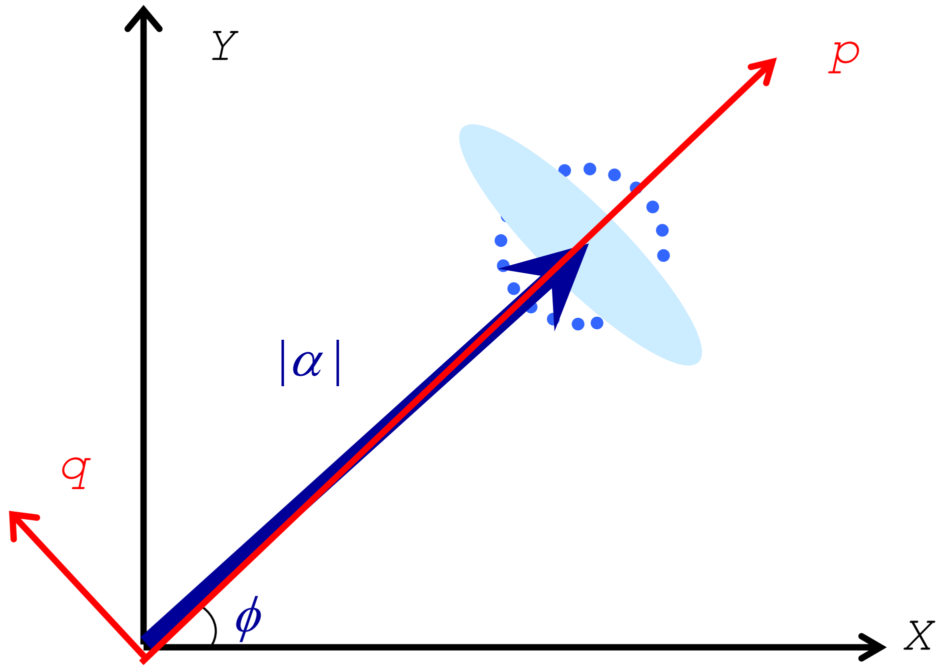
Uncertainty relation implies in a probability distribution for a given pair of quadrature measurements

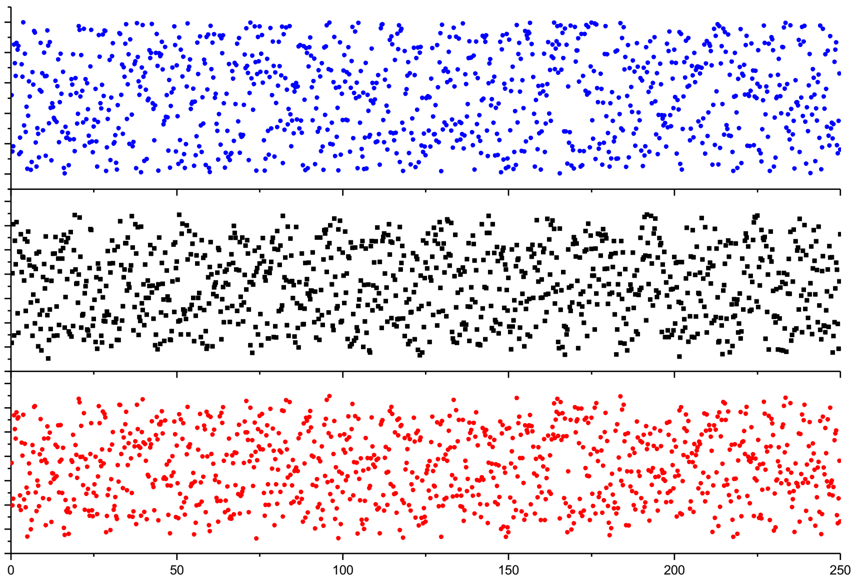
Radon transform
↳ Wigner function

Measurement of the Field in the time domain

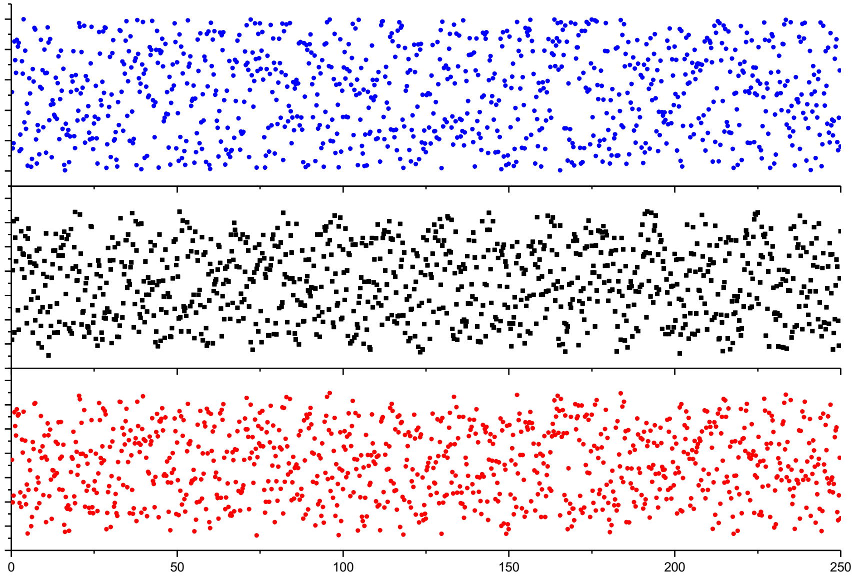


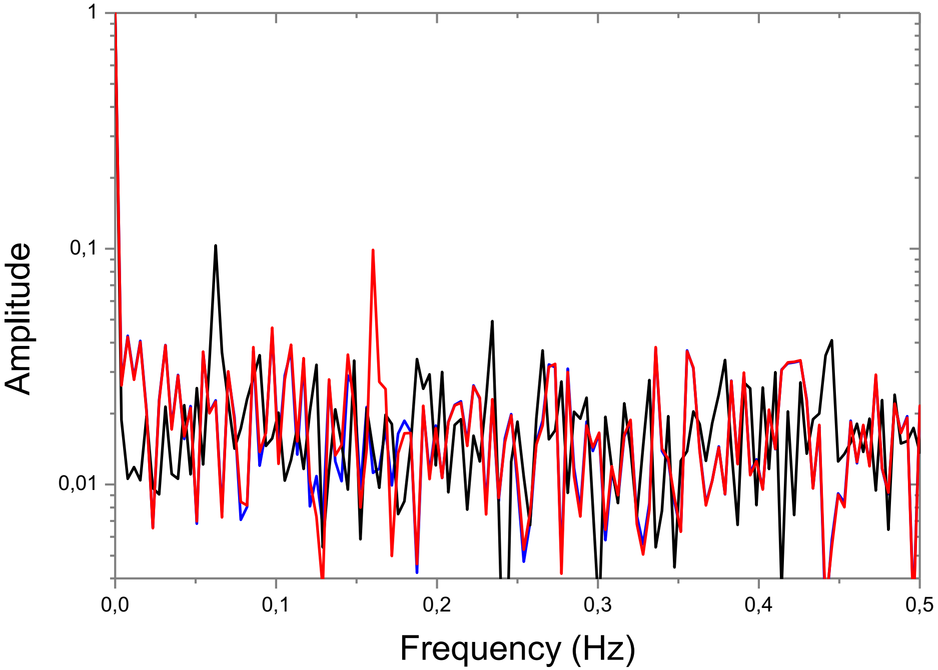
$$\hat{n} = |\alpha|^2 + |\alpha|\delta\hat{p}$$





Measurement of the Field in the frequency domain





Measurement of the Field in the frequency domain

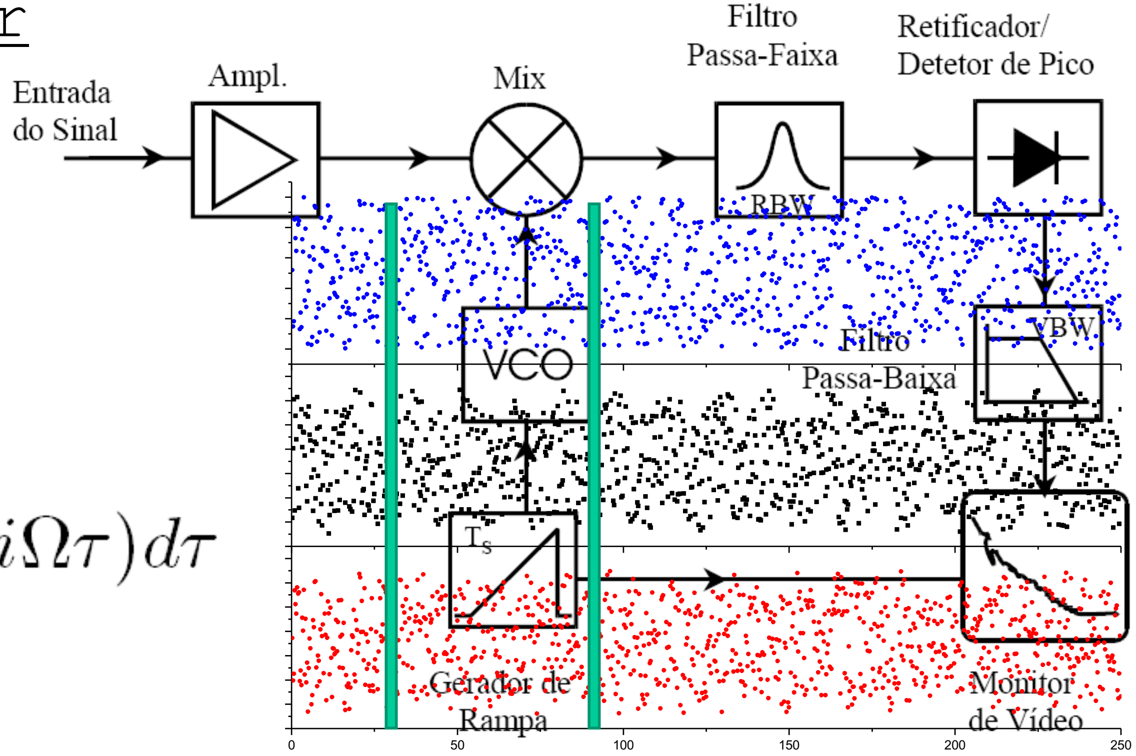
$$\hat{a}(t) = \int_{-\infty}^{\infty} \hat{a}(\Omega) \exp(-i\Omega t)\, d\Omega.$$

$$\hat{a}(\Omega) = \hat{x}(\Omega) + i\hat{y}(\Omega)$$

Spectrum Analyser

$$\langle \hat{X}(t)\hat{X}(t+\tau) \rangle =$$

$$\int \hat{X}(\Omega)\hat{X}(-\Omega) exp(i\Omega\tau) d\tau$$



Measurement of the Field in the frequency domain

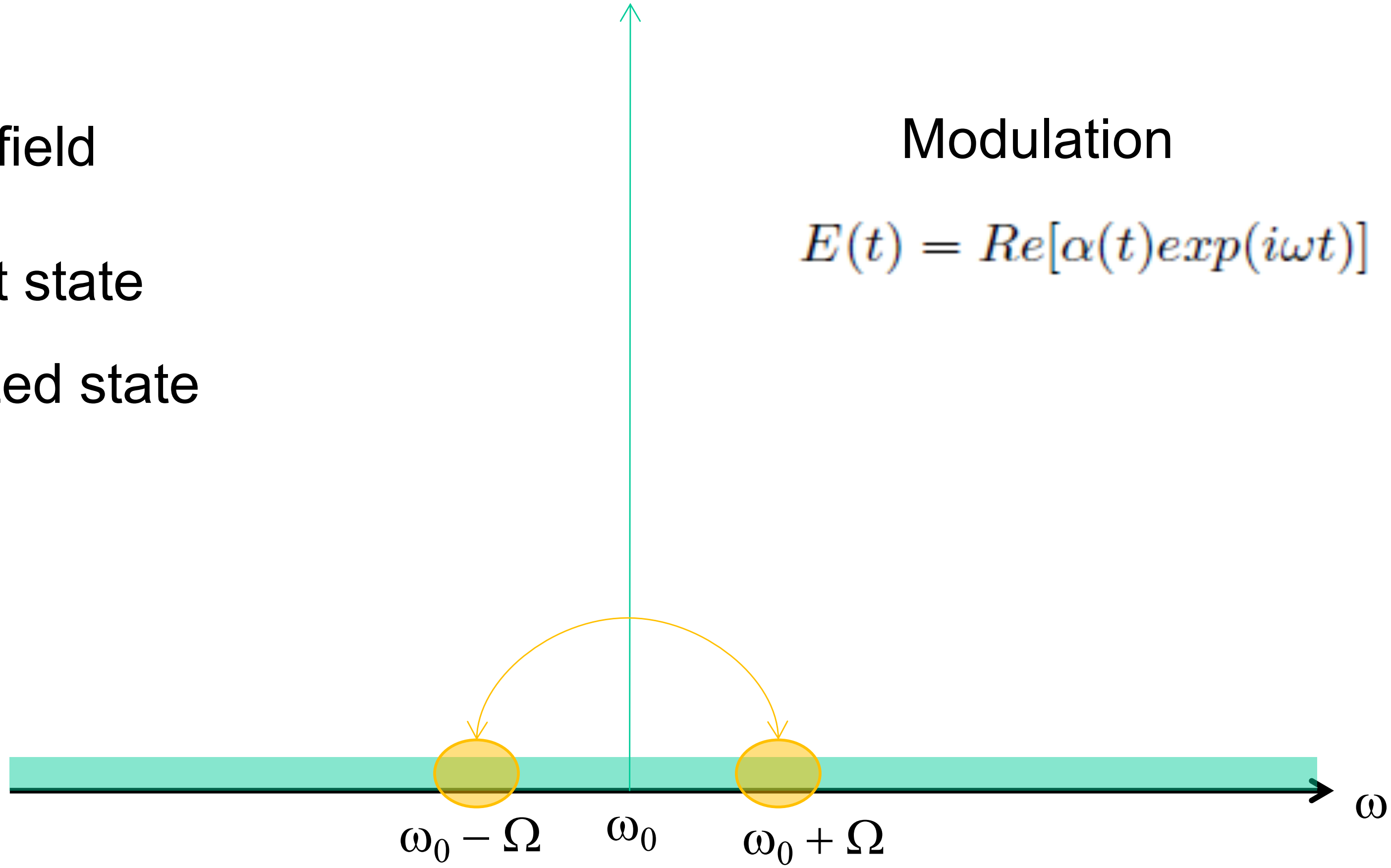
A classic field

Coherent state

Squeezed state

Modulation

$$E(t) = Re[\alpha(t)exp(i\omega t)]$$



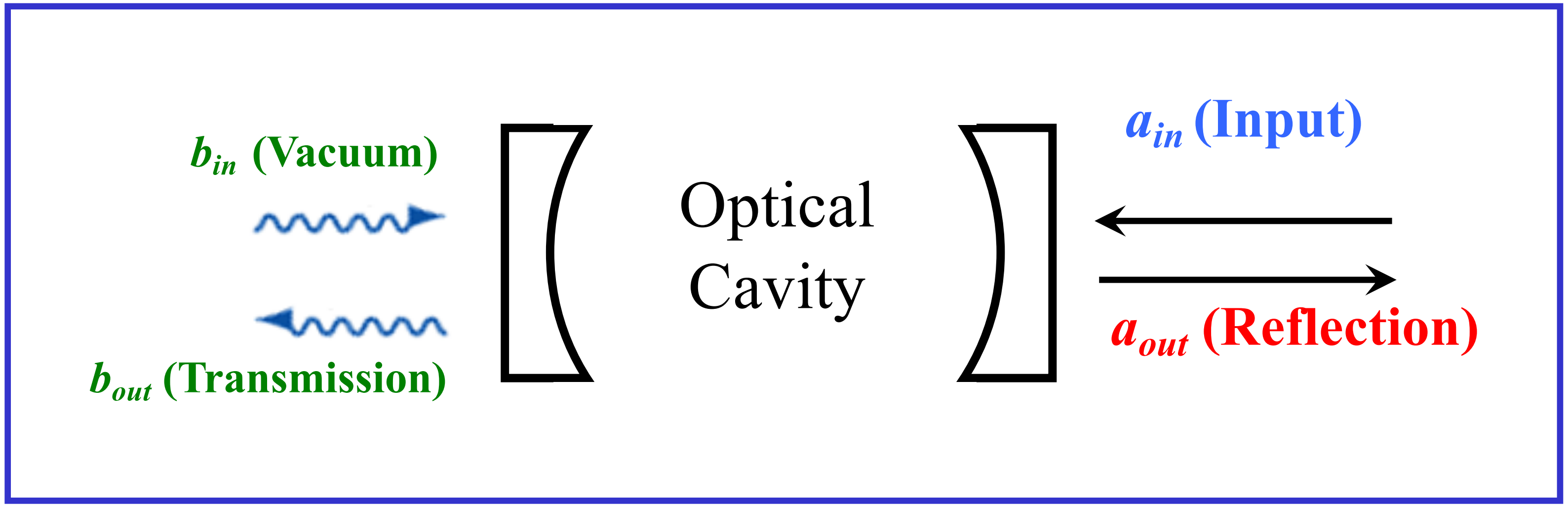
Amplitude $\alpha(t) = A[1 + 2\kappa cos(\Omega t)]$

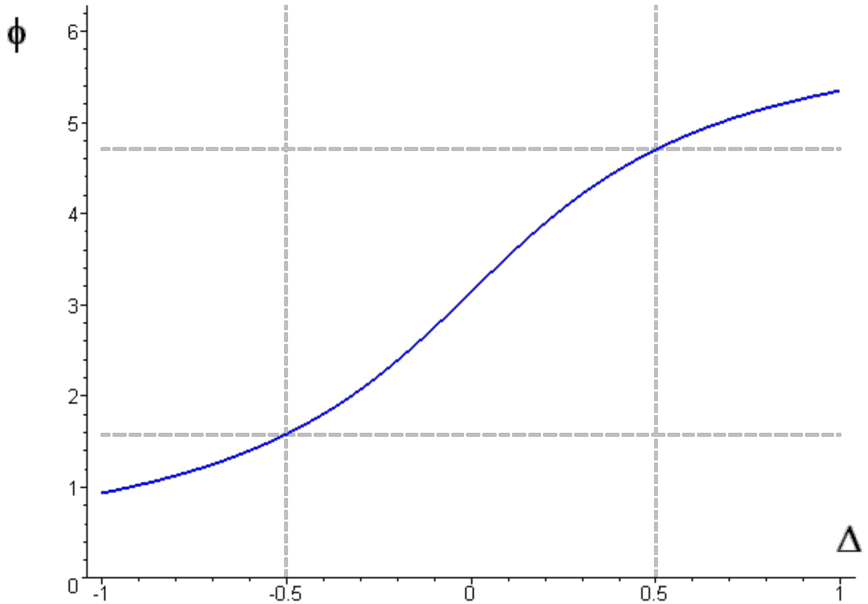
$$E(t) = A\, Re\{\kappa exp[i(\omega - \Omega)t] + exp(i\omega t) + \kappa exp[i(\omega + \Omega)t]\}$$

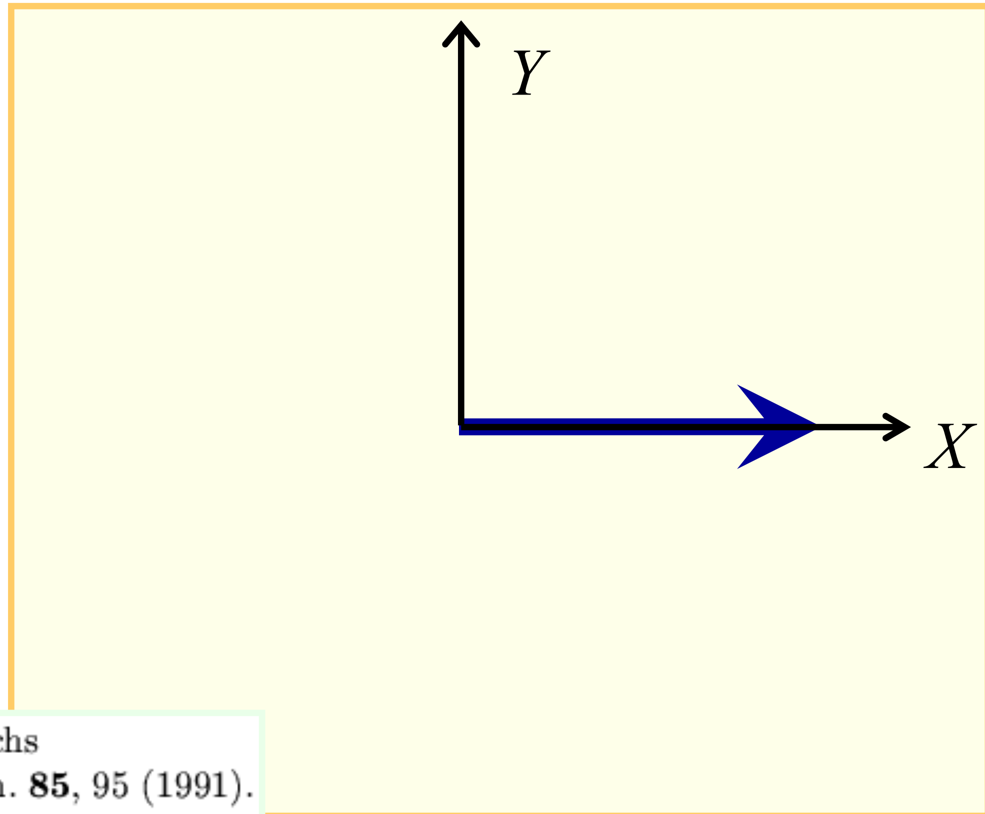
Phase $\alpha(t) = Aexp[2i\kappa cos(\Omega t)] \simeq A[1 + 2i\kappa cos(\Omega t)]$

$$E(t) = A\, Re\{i\kappa exp[i(\omega - \Omega)t] + exp(i\omega t) + i\kappa exp[i(\omega + \Omega)t]\}$$

Phase Rotation of Noise Ellipse





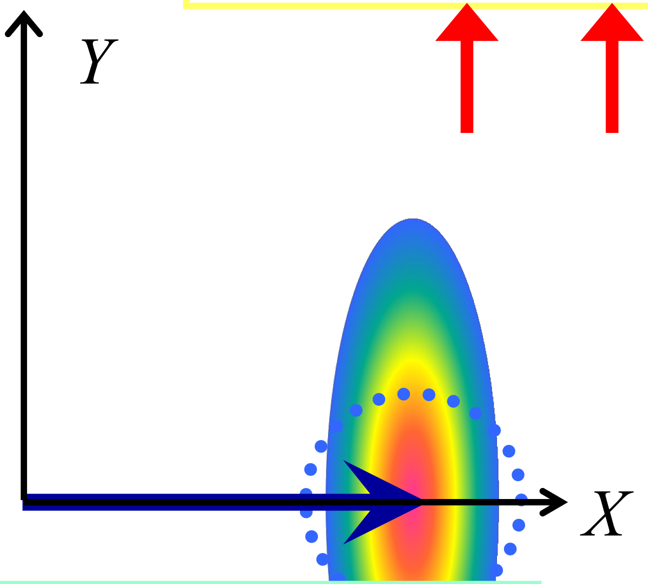


P. Galatola, L.A. Lugiato, M.G. Porreca, P. Tombesi e G. Leuchs
System control by variation of the squeezing phase, Opt. Comm. **85**, 95 (1991).

Alessandro S. Villar, *The conversion of phase to amplitude fluctuations of a light beam by an optical cavity*
American Journal of Physics **76**, pp. 922-929 (2008).

$$\delta p_{in} = \delta a_{in}(\Omega) + \delta a_{in}^{\dagger}(-\Omega)$$

$$\delta p_{out} = \cdots\cdots\; \delta a_{in}(\Omega) + \cdots\cdots\; \delta a_{in}^{\dagger}(-\Omega)$$



Reflected Beam
Amplitude Noise