Building Quantum Machines with Light

Marcelo Martinelli
Laboratório de Manipulação Coerente de Átomos e Luz

Manipulating the field

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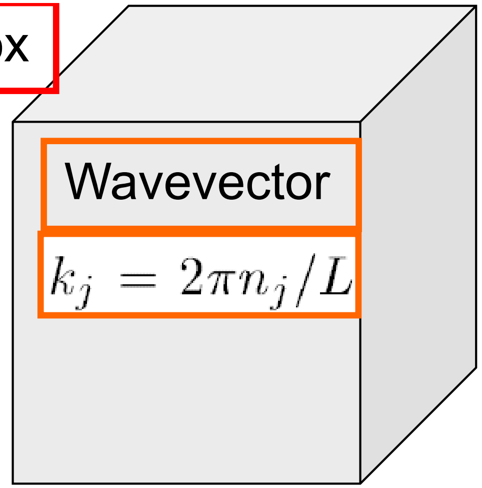
Energy of the EM Field

$$\hat{\mathcal{H}} = \sum_{\mathbf{k}} \sum_{s} \hbar\omega_{\mathbf{k}} \left(\hat{a}^\dagger_{\mathbf{k}s} \hat{a}_{\mathbf{k}s} + \frac{1}{2} \right)$$

$$[\hat{a}_{\mathbf{k}s}, \hat{a}^\dagger_{\mathbf{k}'s'}] = \delta^3_{\mathbf{k}\mathbf{k}'} \delta_{ss'} \qquad [\hat{a}_{\mathbf{k}s}, \hat{a}_{\mathbf{k}'s'}] = 0$$

Solution in a Box



Angular Frequency

$\omega = c|\mathbf{k}|$

Polarization

$$\boldsymbol{\epsilon}^*_{\mathbf{k}s} \cdot \boldsymbol{\epsilon}_{\mathbf{k}s'} = \delta_{ss'}$$

$$\boldsymbol{\epsilon}^*_{\mathbf{k}1} \times \boldsymbol{\epsilon}_{\mathbf{k}2} = \mathbf{k}/k$$

Field Quadratures – Quantum Optics

The electric field can be decomposed as $\hat{\mathbf{E}} = \hat{\mathbf{E}}^{(+)} + \hat{\mathbf{E}}^{(-)}$

$$\hat{\mathrm{E}}^{(+)} = \frac{i}{L^{3/2}} \sum_{\mathbf{k}} \sum_{s} \sqrt{\frac{\hbar\omega}{2\epsilon_0}} \left[\hat{a}_{\mathbf{k}s} \mathbf{u}_{\mathbf{k}s}(\mathbf{r}) e^{-i\omega t}\right] \quad ; \quad \hat{\mathrm{E}}^{(-)} = \left[\hat{\mathrm{E}}^{(+)}\right]^{\dagger}$$

“Photon Field” $\boxed{\mathcal{E} = \sqrt{\frac{\hbar\omega}{2\varepsilon_0 V}}}$

And also as

$$\hat{\mathbf{E}} = \sum_{\mathbf{k}} \sum_{s} \mathcal{E}(\omega) \epsilon_s \left[\hat{X}_{\mathbf{k}s} \cos(\mathbf{k}\cdot\mathbf{r} - \omega t) + \hat{Y}_{\mathbf{k}s} \sin(\mathbf{k}\cdot\mathbf{r} - \omega t)\right]$$

X and Y are the field quadrature operators, satisfying

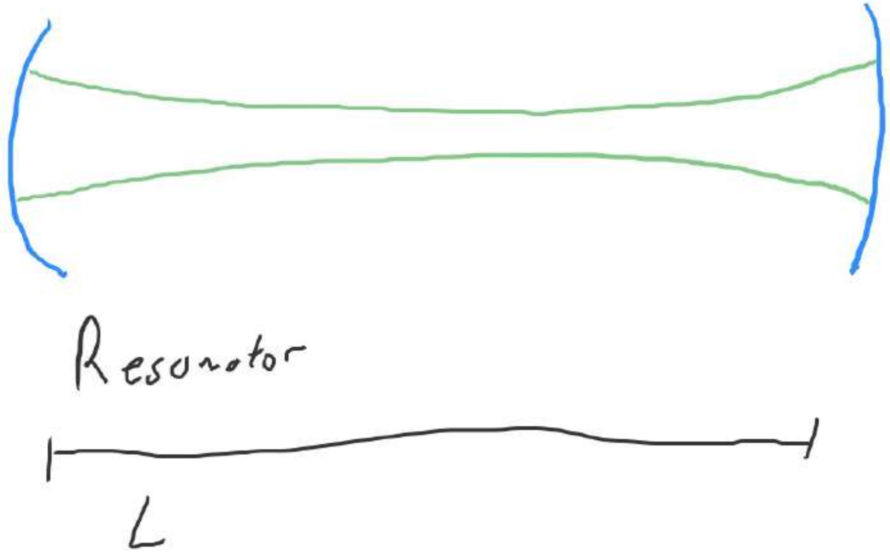
$$\hat{X}_\theta(t) = \mathrm{e}^{-i\theta}\,\hat{a}(t) + \mathrm{e}^{i\theta}\,\hat{a}^{\dagger}(t)\ , \qquad \hat{Y}_\theta(t) = -i\left[\mathrm{e}^{-i\theta}\,\hat{a}(t) - \mathrm{e}^{i\theta}\,\hat{a}^{\dagger}(t)\right]$$

(W&D, Ch. 5)

$$H = H_0 + V$$

H_0: $\hbar\omega(\hat{a}^\dagger\hat{a} + \frac{1}{2})$ — empty box

V: ? Interaction Hamiltonian



$$H = \sum_i H_0^{(i)}$$

$$H_0^{(i)} = \hbar\omega_i\left(\hat{a}_i^\dagger\hat{a}_i + \frac{1}{2}\right)$$

Following Walls & Drummond, Chap. 5 (with adjustments)

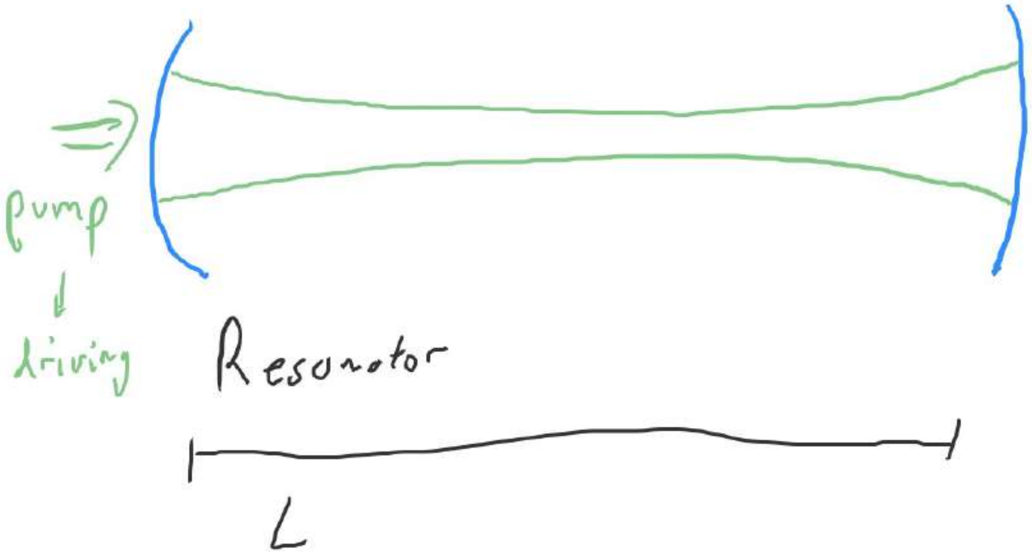
(W&D, Ch. 5)

$$H = H_0 + V$$

↳ ? Interaction Hamiltonian

$$H_0 \longrightarrow \hbar\omega\left(\hat{a}^\dagger \hat{a} + \frac{1}{2}\right)$$

↳ empty box



$$H = \sum_i H_0^{(i)}$$

$$H_0^{(i)} = \hbar\omega_i\left(\hat{a}_i^\dagger \hat{a}_i + \frac{1}{2}\right)$$

(W&D, Ch. 5)

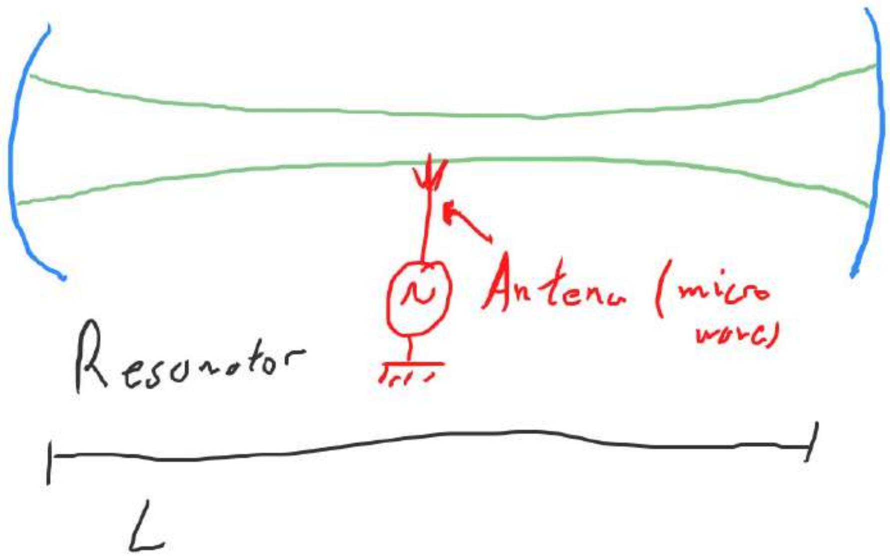
$$H = H_0 + V$$

$\hookrightarrow$? Interaction Hamiltonian

$\downarrow$

$$\hbar\omega\left(\hat{a}^\dagger\hat{a} + \frac{1}{2}\right)$$

$\downarrow$ empty box



$$H = \sum_i H_0^{(i)}$$

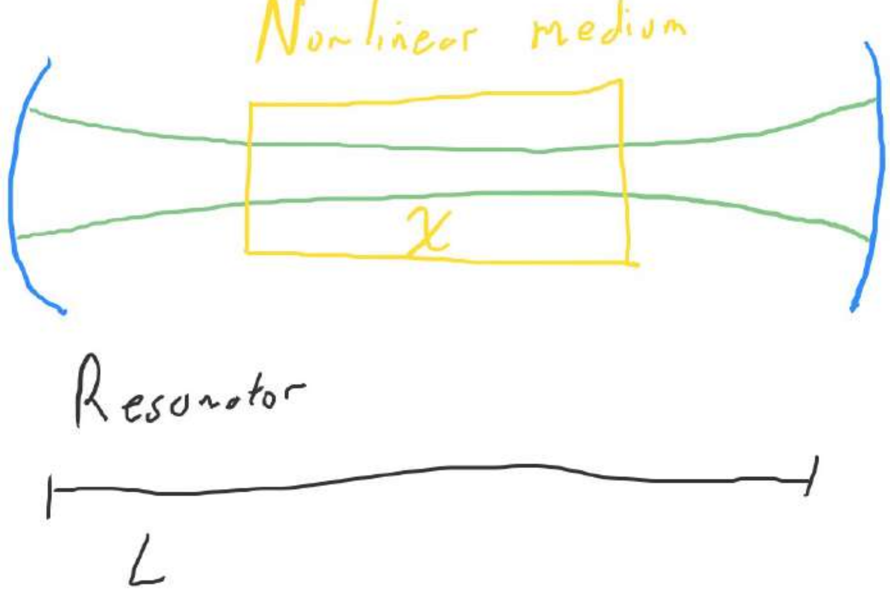
$$H_0^{(i)} = \hbar\omega_i\left(\hat{a}_i^\dagger\hat{a}_i + \frac{1}{2}\right)$$

(W&D, Ch. 5)

$$H = H_0 + V$$

$H_0 \downarrow$ $\hbar\omega\left(\hat{a}^\dagger\hat{a} + \frac{1}{2}\right)$ $\downarrow$ empty box

$V \hookrightarrow$? Interaction Hamiltonian



$$H = \sum_i H_0^{(i)}$$

$$H_0^{(i)} = \hbar\omega_i\left(\hat{a}_i^\dagger\hat{a}_i + \frac{1}{2}\right)$$

$$\vec{D} = \varepsilon_0 (1+\bar{\chi}) \vec{E} \rightarrow \text{Linear}$$

displacement vector

$$\bar{\chi} = \bar{\chi}(\vec{E}) = \bar{\chi}_0 + \vec{E}\,\bar{\chi}^{(2)} + \vec{E} \cdot \bar{\chi}^{(3)} \cdot \vec{E}$$

$$+ \dots$$

$\bar{\chi}_0 \rightarrow$ absorption
refraction

$$\bar{\chi}^{(2)} \Rightarrow E^2 \Rightarrow \left(\varepsilon e^{-i\omega t}\right)^2, \left(\varepsilon_1 e^{-i\omega_1 t} + \varepsilon_2 e^{-i\omega_2 t}\right)^2$$

$$\Downarrow$$

$$\varepsilon_1 e^{-2i\omega_1 t} + \varepsilon_2 e^{-2i\omega_2 t} + \varepsilon_1 \varepsilon_2^* e^{-i(\omega_1 - \omega_2)t}$$

$$+ |\varepsilon_1|^2 + |\varepsilon_2|^2 + \varepsilon_1 \varepsilon_2 e^{-i(\omega_1 + \omega_2)t}$$

For more on nonlinear optics, see A. Yariv, Quantum Electronics,

or R. Boyd, Nonlinear Opticsllowing

$$\vec{D} = \varepsilon_0 (1+\bar{\chi})\vec{E} \rightarrow \text{Linear}$$

$\downarrow$

Displacement vector

$$\bar{\chi} = \bar{\chi}(\vec{E}) = \bar{\chi}_0 + \vec{E}\,\bar{\chi}^{(2)} + \vec{E}\cdot\bar{\chi}^{(3)}\cdot\vec{E}$$

$+ \ldots$

$\bar{\chi}_0 \Rightarrow$ absorption
refraction

$$\bar{\chi}^{(2)} \Rightarrow E^2 \Rightarrow (\mathcal{E} e^{-i\omega t})^2, \; (\mathcal{E}_1 e^{-i\omega_1 t} + \mathcal{E}_2 e^{-i\omega_2 t})^2$$

Sub freq Gen

$\Downarrow$

SHG

$$\rightarrow \mathcal{E}_1 e^{-2i\omega_1 t} + \mathcal{E}_2 e^{-2i\omega_2 t} + \mathcal{E}_1 \mathcal{E}_2^* e^{-i(\omega_1-\omega_2)t}$$

optical retification

$$+ |\mathcal{E}_1|^2 + |\mathcal{E}_2|^2 + \mathcal{E}_1 \mathcal{E}_2 e^{-i(\omega_1+\omega_2)t}$$

Sum Freq gen

Quantum Electronics

Yariv

$H = H_0 + V$

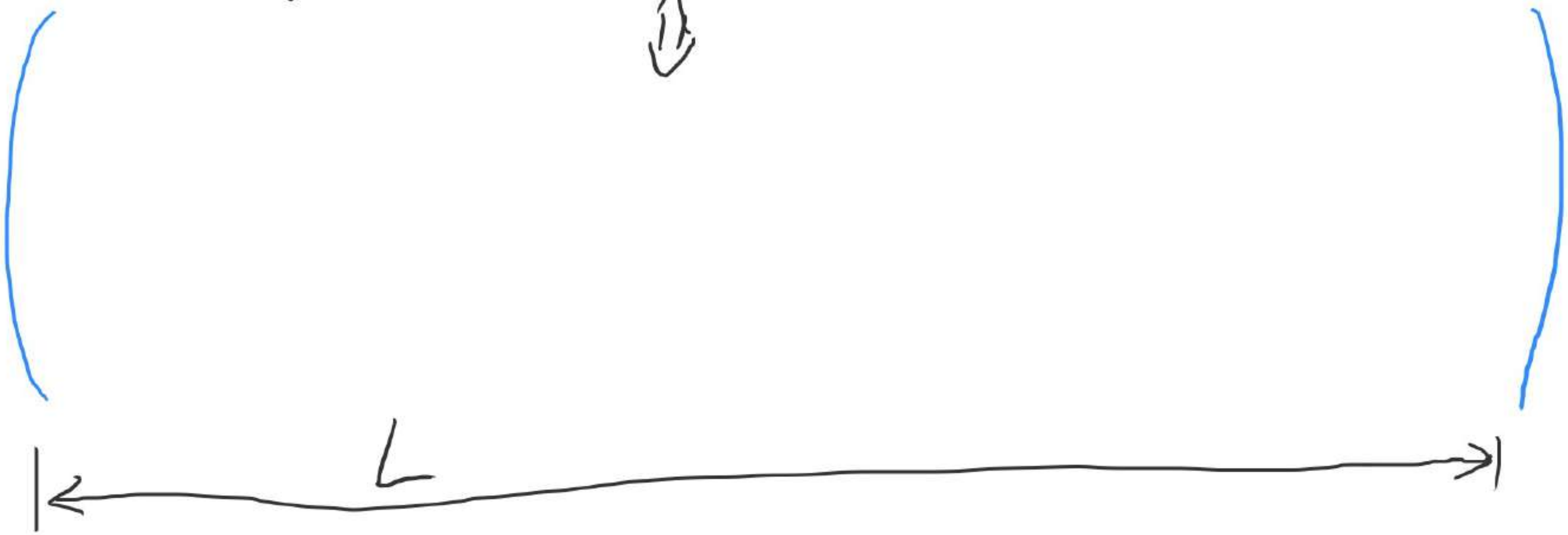
↳ Nonlinear response ($\bar{\chi}^{(n)}$) → Robert Boyd

Nonlinear Optics

Linear response ($\bar{\chi}$)

Driving (input energy)

time evolution of intracavity field



$$H = H_0 + V$$

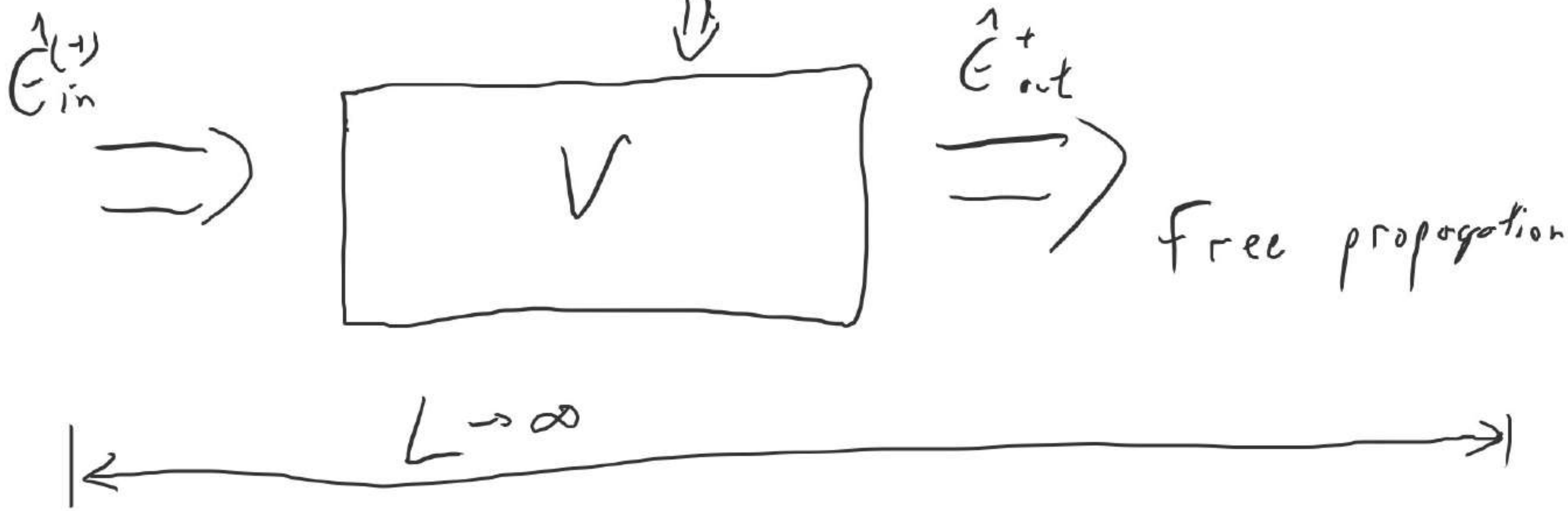
$\hookrightarrow$ Nonlinear response $(\chi^{(n)})$ $\rightarrow$ Yariv $\rightarrow$ Quantum Electronics

Robert Boyd $\rightarrow$ Nonlinear Optics

Linear response (χ)

Driving (input energy)

time evolution of intracavity field



Hamiltonian (single mode): $H = \hbar\omega\left(\hat{a}^{+}\hat{a} + \frac{1}{2}\right) + V$

Evolution of the system:

Schrödinger Representation: $|\psi(t)\rangle = U(t)|\psi(0)\rangle$

$$i\hbar \frac{d}{dt}|\psi(t)\rangle = H|\psi(t)\rangle$$

$$i\hbar \frac{d}{dt} U(t)|\psi(0)\rangle = H \cdot U(t)|\psi(0)\rangle$$

$$i\hbar \frac{d}{dt} U(t) = H \cdot U(t) \Rightarrow \boxed{U(t) = e^{-iHt/\hbar}}$$

For a review on Schrödinger and Heisenberg pictures, see

Quantum Mechanics, C. Cohen Tannoudji et. al V. 1Chap. G.III

Operator $\hat{A}_s \rightarrow \langle \hat{A} \rangle = \langle \Psi(t) | \hat{A} | \Psi(t) \rangle$

$$= \langle \Psi_0 | U^\dagger(t) \, \hat{A} \, U(t) | \Psi_0 \rangle$$

Heisenberg Picture $\rightarrow \hat{A}_H(t) = U^\dagger(t) \hat{A}_s U(t)$

↳ state remains constant

$$\frac{d}{dt} e^{iHt/\hbar} = \frac{iH}{\hbar} e^{iHt/\hbar}$$

$$\frac{d}{dt} \hat{A}_H(t) = \frac{d}{dt} \left[e^{iHt/\hbar} \hat{A}_s(t) e^{-iHt/\hbar} \right]$$

$$= \frac{i}{\hbar} \left[H \cdot \left(e^{iHt/\hbar} \hat{A}_s e^{-iHt/\hbar} \right) - \left(e^{iHt/\hbar} \hat{A}_s e^{iHt/\hbar} \right) H \right] + e^{iHt} \frac{d}{dt} \hat{A}_s(t) e^{-iHt/\hbar}$$

$$\frac{d}{dt}\hat{A}_H(t) = \frac{i}{\hbar}\left[\hat{H}, \hat{A}_H(t)\right] + \left(\frac{\partial}{\partial t} A_S(t)\right)_H$$

↳ Heiserg picture

$H = H_0 + V$

Intermidiate picture ⟷ "Interaction picture"

Walls ≠ Schiff

Interaction Picture (Walls) : $H = H_0 + V$

$$\langle \Psi_0 | U^{+}(t)\ \hat{A}_s\ \hat{U}^{-1}(t) | \Psi_0 \rangle$$

Schroedinger picture

$$\underbrace{\langle \Psi_0 | e^{iH_0t/\hbar}}_{\langle \Psi_I(t)|} \cdot \underbrace{e^{iVt/\hbar} \cdot \hat{A}_s(t)\, e^{-iVt/\hbar}}_{\hat{A}_I(t)} \cdot \underbrace{e^{-iH_0t/\hbar} | \Psi_0 \rangle}_{|\Psi_I(t)\rangle}$$

evolution of the operator under the Interaction Hamiltonian V

evolution of the state as a Free Field

Operator evolution ~ Heisenberg picture

$$\frac{d}{dt}\hat{A}_I = \frac{\frac{[A_I, H_I]}{i\hbar}}{} + \frac{\partial}{\partial t}\hat{A}_I(t)$$

$$\hat{\rho}_I = e^{-iH_0t/\hbar}|\psi_0\rangle\langle\psi_0|e^{iH_0t/\hbar}$$

→ evolution of the free cavity state

↓

Schrödinger picture

Driving Field:

$$H = \underbrace{\hbar\omega\left(\hat{a}^\dagger\hat{a} + \tfrac{1}{2}\right)}_{H_0} + \underbrace{i\hbar\left(\varepsilon^* \hat{a} - \varepsilon \hat{a}^\dagger\right)}_{V}$$

Interaction picture: $\hat{a}(t)$

$$\frac{d}{dt}\hat{a} = \frac{1}{i\hbar}[\hat{a}, V] = \frac{1}{i\hbar}[\hat{a}, \hat{a}^\dagger]\cdot(-i\hbar\varepsilon) = -\varepsilon$$

$$\hat{a}(t) = \hat{a}(0) - \varepsilon t$$

$$\hat{a}^\dagger(t) = \hat{a}^\dagger(0) - \varepsilon^* t$$

$$\frac{d}{dt}\hat{a}^\dagger = -\varepsilon^*$$

$$\Big(\qquad \qquad \Big)$$

$t > 0$

Driving Field:

$$H = \hbar\omega\left(\hat{a}^\dagger\hat{a} + \tfrac{1}{2}\right) + i\hbar\left(\varepsilon^*\hat{a} - \varepsilon\hat{a}^\dagger\right)$$

$$\underbrace{\hbar\omega\left(\hat{a}^\dagger\hat{a} + \tfrac{1}{2}\right)}_{H_0} \qquad \underbrace{i\hbar\left(\varepsilon^*\hat{a} - \varepsilon\hat{a}^\dagger\right)}_{V}$$

Interaction picture: $\hat{a}(t)$

$$\frac{d}{dt}\hat{a} = \frac{1}{i\hbar}[\hat{a}, V] = \frac{1}{i\hbar}[\hat{a}, \hat{a}^\dagger]\cdot(-i\hbar\varepsilon) = -\varepsilon$$

$$\frac{d}{dt}\hat{a}^\dagger = -\varepsilon^*$$

$$\begin{cases} \hat{a}(t) = \hat{a}(0) - \varepsilon t \\ \hat{a}^\dagger(t) = \hat{a}^\dagger(0) - \varepsilon^* t \end{cases}$$

$$\langle\hat{a}(t)\rangle = \langle\psi_0|\hat{a}(0)|\psi_0\rangle - \langle\psi_0|\psi_0\rangle\cdot\varepsilon t$$

$t \to t'$

$|\psi_0\rangle = 0 \rightsquigarrow \langle\hat{a}(t)\rangle = -\varepsilon t$

Quantum Optics – Coherent State

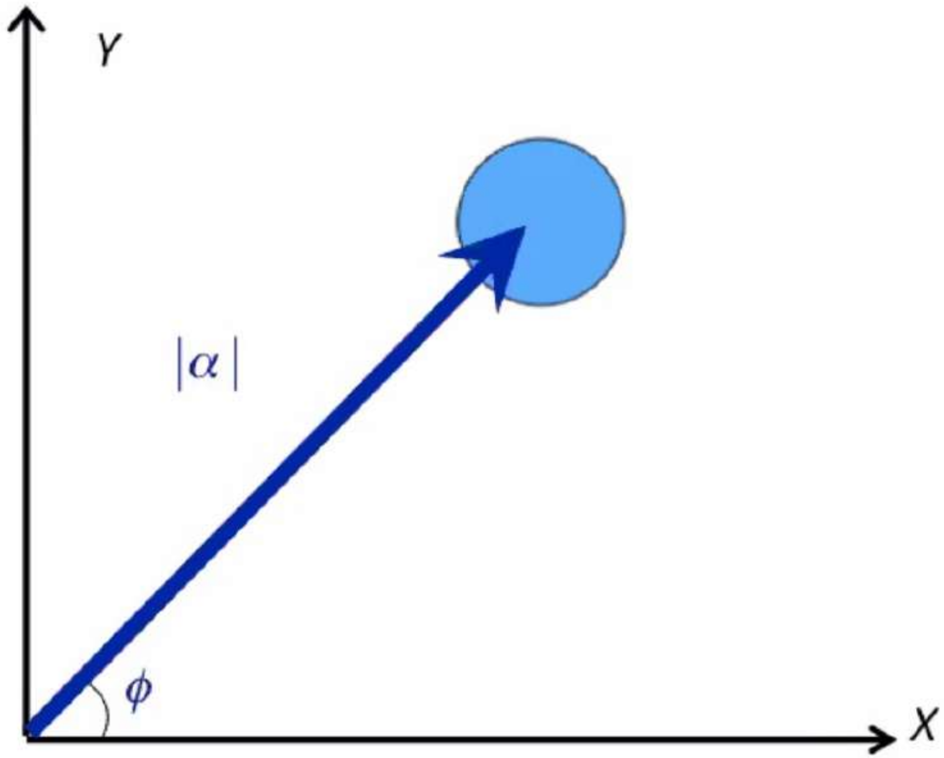
$$|\alpha\rangle = D(\alpha)|0\rangle$$

$$D(\alpha) = \exp(\alpha a^{\dagger} - \alpha^{*} a)$$

$\Longleftrightarrow$

$\hat{a}(t) = \hat{a}(0) + \alpha$

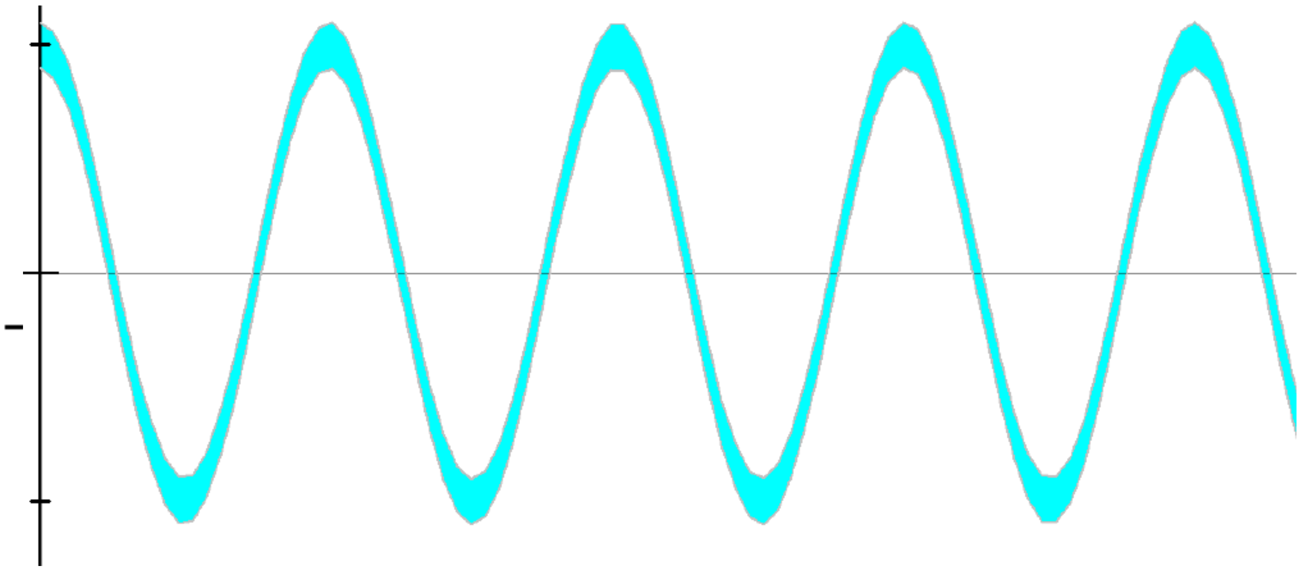
$\alpha = -\varepsilon t$

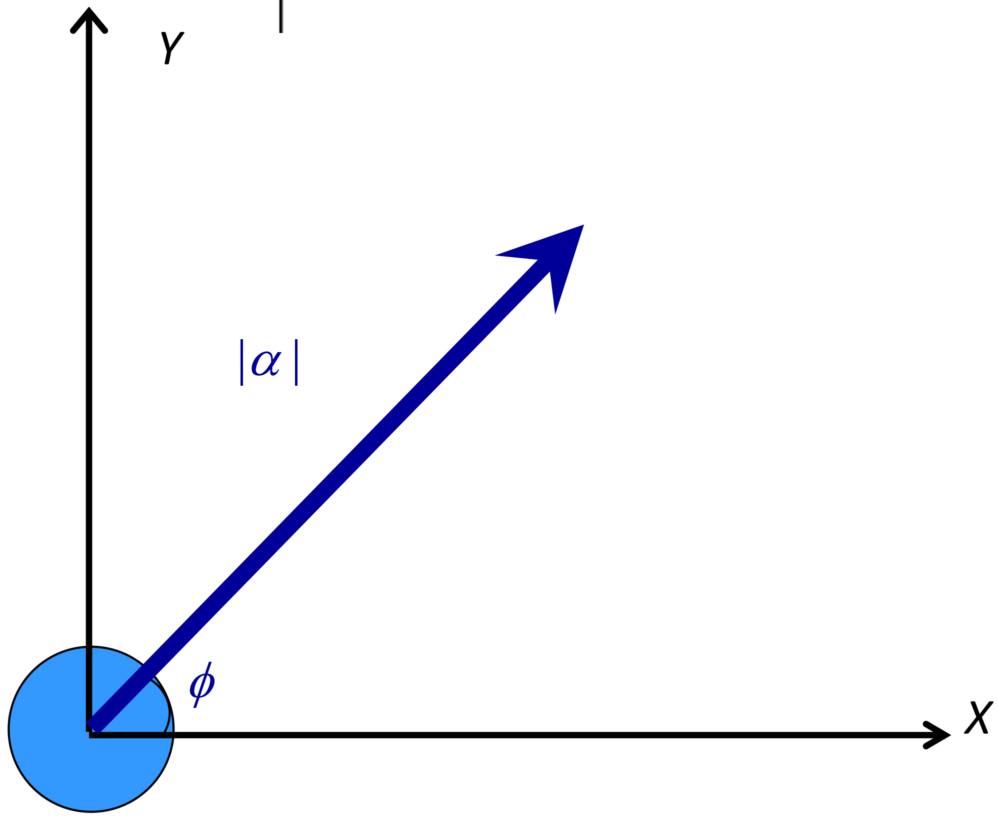


Quantum Optics – Coherent State

$$|\alpha\rangle = D(\alpha)|0\rangle$$

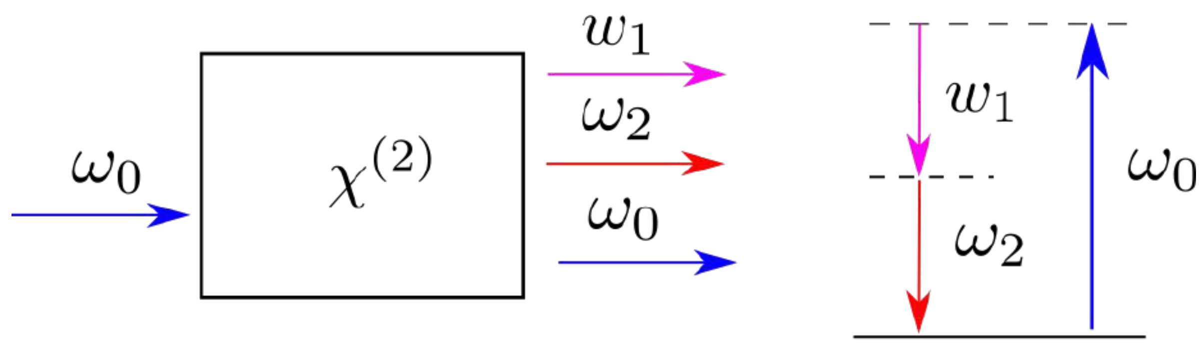
$$D(\alpha) = \exp(\alpha a^{\dagger} -$$



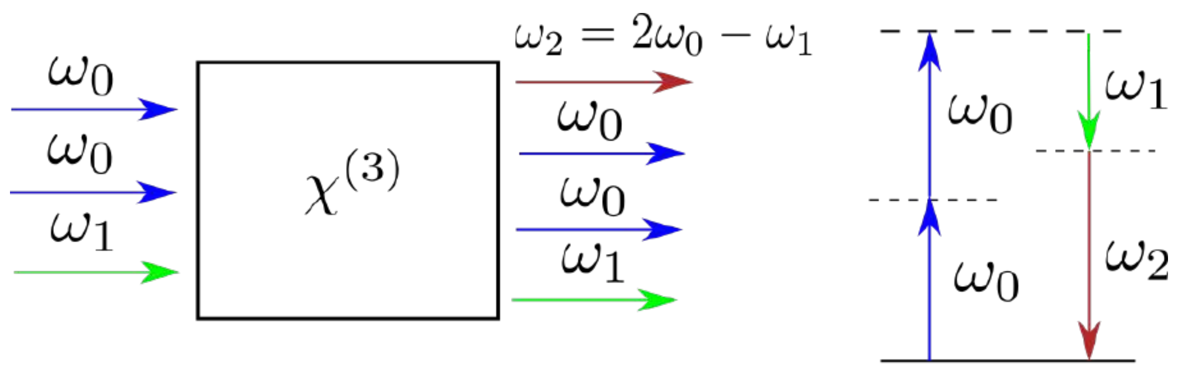


Comparing $\chi^{(2)}$ and $\chi^{(3)}$ nonlinearities

$$\hat{\mathscr{H}}_{\chi^2} = i\hbar\beta\hat{a}_0\hat{a}_1^\dagger\hat{a}_2^\dagger + h.c..$$



$$\hat{\mathscr{H}}_{\chi^3} = i\hbar\beta\hat{a}_0^2\hat{a}_1^\dagger\hat{a}_2^\dagger + h.c..$$



Degenerate Parametric Amplifier

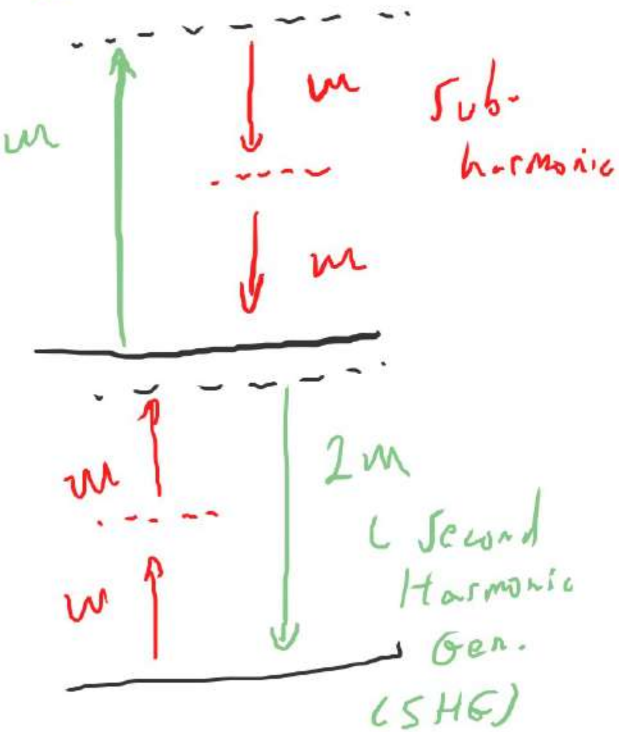
$$H = H_0 + i\hbar \frac{\chi}{2} \left(\hat{a}^{\dagger 2} - \hat{a}^2 \right)$$

$\chi \rightarrow$ nonlinear coupling

$\downarrow$

inharmonicity of the oscillator

NLO $\leftarrow$ $\underline{\chi^{(2)}} \cdot \mathcal{E} e^{-i2\omega t}$



$$V = \frac{i\hbar \chi}{2}\left(\hat{a}^{\dagger 2} - \hat{a}^{2}\right) \quad \Rightarrow \chi \in \mathbb{R}$$

if $\chi \in \mathbb{R} \to |\chi| e^{i\theta} \hat{a}^{\dagger 2}$

$$\frac{d}{dt}\hat{a}(t) = \frac{1}{i\hbar}[\hat{a}, \hat{V}] = \frac{i\hbar\chi}{i\hbar\, 2}[\hat{a}, \hat{a}^{\dagger 2}] = \chi\, \hat{a}^{\dagger}(t)$$

$$[\hat{a}, \hat{a}^{\dagger}] = \hat{a}\hat{a}^{\dagger} - \hat{a}^{\dagger}a = 1$$

$$[\hat{a}, \hat{a}^{\dagger 2}] = \underbrace{\hat{a}\hat{a}^{\dagger}}_{\hat{a}^{\dagger}\hat{a}+1}\hat{a}^{\dagger} - \hat{a}^{\dagger}\underbrace{\hat{a}^{\dagger}a}_{\hat{a}\hat{a}^{\dagger}-1} = \hat{a}^{\dagger}\hat{a}\hat{a}^{\dagger} + \hat{a}^{\dagger} - (\hat{a}^{\dagger}\hat{a}\hat{a}^{\dagger} + \hat{a}^{\dagger}) = 2\hat{a}^{\dagger}$$

$$\frac{d}{dt}\hat{a}^{+} = \frac{1}{i\hbar}[\hat{a}^{+},\hat{V}] = \chi\hat{a} \quad ; \quad \frac{d}{dt}\hat{a} = \chi\hat{a}^{+}$$

$$\frac{d^2}{dt^2}\hat{a}^{+} = \chi^2\hat{a}^{+} \quad ; \quad \frac{d^2}{dt^2}\hat{a} = \chi^2\hat{a}$$

$$\hat{a}^{+} = \hat{C}e^{\chi t} + \hat{D}e^{-\chi t} \quad ; \quad \hat{a} = \hat{A}e^{\chi t} + \hat{B}e^{-\chi t}$$

$$\Downarrow$$

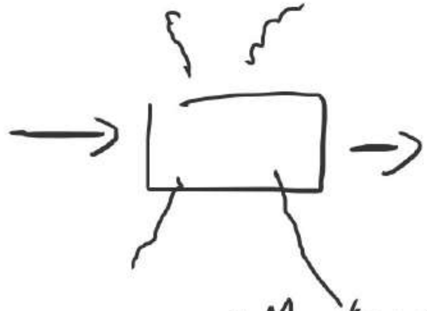
$$\hat{a}(t) = \hat{a}(0)\cosh(\chi t) + \hat{a}^{+}(0)\sinh(\chi t)$$

$$\hat{a}^{+}(t) = \hat{a}^{+}(0)\cosh(\chi t) + \hat{a}(0)\sinh(\chi t)$$

$$\cosh(\chi t) = \frac{e^{\chi t}+e^{-\chi t}}{2}$$

$$\sinh(\chi t) = \frac{e^{\chi t}-e^{-\chi t}}{2}$$

$$X = \hat{a} + \hat{a}^\dagger \; ; \; Y = \frac{a - a^\dagger}{i}$$



Markov equation

reservoirs

$$\frac{d}{dt}\hat{X} = \chi \hat{X} \; ; \; \frac{d}{dt}\hat{Y} = -\chi \hat{Y}$$

amplified

deamplified $\neq$ attenuated

$$\hat{X}(t) = e^{\chi t}\hat{X}(0) \; ; \; \hat{Y}(t) = e^{-\chi t}\hat{Y}(0)$$

amplification

compression

Quantum Optics – Coherent Squeezed States

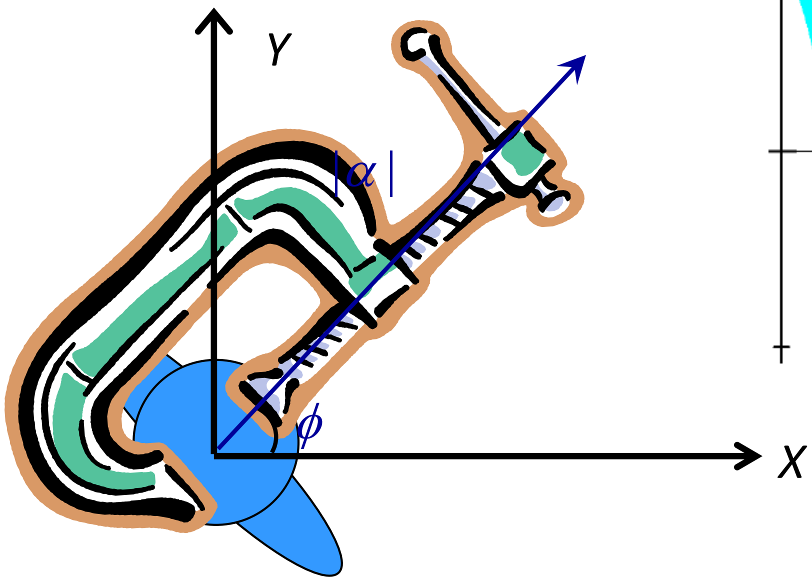
$$|\alpha\rangle = D(\alpha)|0\rangle$$

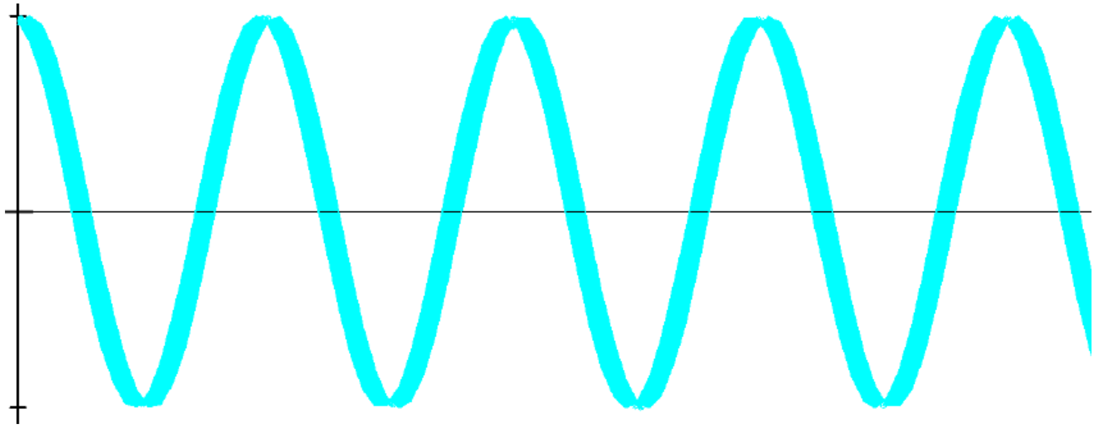
$$D(\alpha) = \exp(\alpha a^\dagger - \alpha^* a)$$

$$S(\varepsilon) = \exp(1/2\varepsilon^* a^2 - 1/2\varepsilon a^{\dagger 2})$$

$$\varepsilon = re^{2i\phi}$$

$$|\alpha, \varepsilon\rangle = D(\alpha)S(\varepsilon)|0\rangle$$





Two-mode squeezed state

$$\hat{H} = \hat{H}_0 + \hat{V} \; ; \quad \hat{H}_0 = \hbar\omega_1\left(\hat{a}_1^+\hat{a}_1 + \frac{1}{2}\right) + \hbar\omega_2\left(\hat{a}_2^+\hat{a}_2 + \frac{1}{2}\right)$$

$$\hat{V} = i\hbar\chi\left(\hat{a}_1^+\hat{a}_2^+ - \hat{a}_1\hat{a}_2\right)$$

$$\omega_0 = \omega_1 + \omega_2$$

Walls → Interaction picture

$$\frac{d}{dt}\hat{a}_1(t) = \frac{1}{i\hbar}[\hat{a}_1, \hat{V}] = \chi \hat{a}_2^\dagger$$

$$\boxed{[\hat{a}_i, \hat{a}_j^\dagger] = \delta_{ij}}$$

$$\frac{d}{dt}\hat{a}_2(t) = \frac{1}{i\hbar}[\hat{a}_2, \hat{V}] = \chi \hat{a}_1^\dagger$$

Solução:

$$\hat{a}_1(t) = \hat{a}_1(0)\cosh(\chi t) + \hat{a}_2^\dagger(0)\sinh(\chi t)$$

$$\hat{a}_2(t) = \hat{a}_2(0)\cosh(\chi t) + \hat{a}_1^\dagger(0)\sinh(\chi t)$$

Two coupled modes → Linear Combination

$$\hat{a}_+ = \frac{\hat{a}_1 + \hat{a}_2}{\sqrt{2}} \quad ; \quad \hat{a}_- = \frac{\hat{a}_1 - \hat{a}_2}{\sqrt{2}} \qquad [\hat{a}_+, \hat{a}_-] = 0$$

$$V = i\hbar\chi\left(\hat{a}_1^+ \hat{a}_2^+ - \hat{a}_1 \hat{a}_2\right) =$$

$$= i\hbar\chi\left[\left(\frac{\hat{a}_+ + \hat{a}_-}{\sqrt{2}}\right)^+ \left(\frac{\hat{a}_+ - \hat{a}_-}{\sqrt{2}}\right)^+ - h.c.\right]$$

$$= i\hbar\frac{\chi}{2}\left[\left(\hat{a}_+^{+2} - \hat{a}_+^2\right) - \left(\hat{a}_-^{+2} - \hat{a}_-^2\right)\right]$$

$$= V_+ + V_-$$ → a pair of squeezed states

Wigner function:

$$W = \frac{1}{4\pi^2} \exp\left\{ -\frac{1}{2}\left[\frac{(x_+ - x_{0+})^2}{e^{2\chi t}} + \frac{(y_+ - y_{0+})^2}{e^{-2\chi t}} \right] \right\} \times$$

$$\times \exp\left\{ -\frac{1}{2}\left[\frac{(x_- - x_{0-})^2}{e^{-2\chi t}} + \frac{(y_- - y_{0-})^2}{e^{2\chi t}} \right] \right\}$$

where $x_+ = \frac{x_1 + x_2}{\sqrt{2}}$; $x_- = \frac{x_1 - x_2}{\sqrt{2}}$; $y_+ = \frac{y_1 + y_2}{\sqrt{2}}$; $y_- = \frac{y_1 - y_2}{\sqrt{2}}$

Arbitrary displacement: $x_{0\pm} \to 0$, $y_{0\pm} \to 0$

$$\Rightarrow W(x_1, x_2, y_1, y_2) = \frac{1}{4\pi^2} \cdot \exp\left\{ -\frac{1}{2}\left[\frac{(x_1 + x_2)^2}{2r^{-2}} + \frac{(y_1 - y_2)^2}{2r^{-2}} + \overset{\text{Sqz}}{\overset{\downarrow}{\frac{(x_1 - x_2)^2}{2r^2}}} + \overset{\text{Sqz}}{\overset{\downarrow}{\frac{(y_1 + y_2)^2}{2r^2}}} \right] \right\}$$

$r = e^{-\chi t} < 1 \quad \chi \in \mathbb{R}, \chi > 0$

Arbitrary displacement: $x_{0\pm} \to 0$, $y_{0\pm} \to 0$

$$\Rightarrow W(x_1, x_2, y_1, y_2) = \frac{1}{4\pi^2} \cdot \exp\left\{-\frac{1}{2}\left[\frac{(x_1+x_2)^2}{2r^{-2}} + \frac{(y_1-y_2)^2}{2r^{-2}} + \frac{(x_1-x_2)^2}{2r} + \frac{(y_1+y_2)^2}{2r^2}\right]\right\}$$

$$r = e^{-\gamma t}$$

2 Squeezed states $\leftrightarrow$ Local $W(x_1, y_1) = ?$

Partial trace: $W(x_1, y_1) = \int W(x_1, x_2, y_1, y_2)\, dx_2\, dy_2$

$$W(x_1, y_1) = \frac{1}{2\pi\sigma^2} \exp\left[-\frac{x_1^2 + y_1^2}{2\cdot\sigma^2}\right]$$

$$\sigma^2 = \frac{r^4+1}{2r^2} \rightarrow \underline{\text{Thermal State}}\ !$$

$$\therefore \hat{X}^2 = (\hat{a}+\hat{a}^+)^2 = \hat{a}^2 + \hat{a}^{+2} + \hat{a}\hat{a}^+ + \hat{a}^+\hat{a} \; ; \; \hat{Y}^2 = \left[-i\,(\hat{a}-\hat{a}^+)\right]^2 = \hat{a}\hat{a}^+ + \hat{a}^+a - \hat{a}^2 - \hat{a}^{+2}$$

$$\hat{X}^2 + \hat{Y}^2 = 2\,(\hat{a}\hat{a}^+ + \hat{a}^+\hat{a}) = 2\,(2\hat{a}^+a + 1) = 4\cdot\left(\hat{n} + \frac{1}{2}\right)$$

$$\hat{n} = \frac{\hat{X}^2 + \hat{Y}^2 - 2}{4}$$

Thermal state: $\int x^2 W(x,y)\,dx\,dy = \sigma^2$

$$\langle \hat{n} \rangle = \frac{\sigma_x^2 + \sigma_y^2 - 2}{4} = \frac{\sigma^2 - 1}{2} \qquad \sigma^2 = 2\bar{n} + 1$$

$$\bar{n} = \frac{r^4 + 1}{4r^2} - \frac{1}{2} = \frac{r^4 + 1 - 2r^2}{4r^2} = \left(\frac{r^2 - 1}{2r}\right)^2$$

$$W(x_1, y_1) = \frac{1}{2\pi\sigma^2} \exp\left[-\frac{x_1^2 + y_1^2}{2\cdot\sigma^2}\right]$$

$$\therefore\ \hat{X}^2 = (\hat{a}+\hat{a}^\dagger)^2 = \hat{a}^2 + \hat{a}^{\dagger 2} + \hat{a}\hat{a}^\dagger + \hat{a}^\dagger\hat{a} \ ; \ \hat{Y}^2 = \left[-i\,(\hat{a}-\hat{a}^\dagger)\right]^2 = \hat{a}\hat{a}^\dagger + \hat{a}^\dagger a - \hat{a}^2 - \hat{a}^{\dagger 2}$$

$$\hat{X}^2 + \hat{Y}^2 = 2\,(\hat{a}\hat{a}^\dagger + \hat{a}^\dagger\hat{a}) = 2\,(2\hat{a}^\dagger a + 1) = 4\cdot\left(\hat{n}+\frac{1}{2}\right)$$

$$\hat{n} = \frac{\hat{X}^2 + \hat{Y}^2 - 2}{4} \qquad \langle\hat{X}^2\rangle = \Delta^2 X$$

<u>Thermal state</u>: $\int x^2\, W(x,y)\, dx\, dy = \sigma^2$

$$\langle\hat{n}\rangle = \frac{\sigma_x^2 + \sigma_y^2 - 2}{4} = \frac{\sigma^2 - 1}{2} \qquad \sigma^2 = 2\bar{n} + 1$$

$$\bar{n} = \frac{r^4+1}{4r^2} - \frac{1}{2} = \frac{r^4 + 1 - 2r^2}{4r^2} = \left(\frac{r^2-1}{2r}\right)^2$$

$$\Delta^2 X\, \Delta^2 Y \geq 1$$

$$W(x_1, y_1) = \frac{1}{2\pi\sigma^2} \exp\left[-\frac{x_1^2 + y_1^2}{2\cdot\sigma^2}\right]$$

Number state description

[Collett, PRA 38 2233 (1988)]

$$\hat{V} = i\hbar\chi\left(\hat{a}_1^\dagger \hat{a}_2^\dagger - \hat{a}_1 \hat{a}_2\right)$$

$$|\psi(t)\rangle = \exp\left[\chi t\left(\hat{a}_1^\dagger \hat{a}_2^\dagger - \hat{a}_1 \hat{a}_2\right)\right]|0\rangle$$

$$e^{\theta\left(\hat{a}_1^\dagger \hat{a}_2^\dagger - \hat{a}_1 \hat{a}_2\right)} = e^{\Gamma \hat{a}_1^\dagger \hat{a}_2^\dagger} \cdot e^{-g\left(\hat{a}_1^\dagger \hat{a}_2 + \hat{a}_1 \hat{a}_2^\dagger + 1\right)} e^{-\Gamma \hat{a}_1 \hat{a}_2}$$

$$\Gamma = \tanh\theta \quad ; \quad g = \ln(\cosh\theta)$$

$$\Rightarrow |\psi(t)\rangle = e^{-g} e^{\Gamma \hat{a}_1^\dagger \hat{a}_2^\dagger}|0\rangle = \sum_{n=0}^{\infty} \frac{\left[\tanh(\chi t)\right]^n}{\cosh(\chi t)} \cdot |n, n\rangle$$

1 2

$$|n, n\rangle = |n\rangle_1 \otimes |n\rangle_2$$

Trago parcial: $\rho_j(t) = \mathrm{Tr}_j\left\{|\psi(t)\rangle\langle\psi(t)|\right\} =$

Trago porcial : $\rho_i(t) = Tr_j\{|\psi(t)\rangle\langle\psi(t)|\} =$

$$= \sum_{n=0}^{\infty} \frac{\tanh(\chi t)^{2n}}{\cosh(\chi t)^2} |n\rangle\langle n|$$

$$\bar{n} = \sinh^2(\chi t) = \left(\frac{e^{\chi t} - e^{-\chi t}}{2}\right)^2 = \boxed{\left(\frac{r^2 - 1}{2r}\right)^2}$$

where: $r = e^{-\chi t}$

$$\cosh^2(\chi t) - \sinh^2(\chi t) = 1$$

$$\Rightarrow \rho_i(t) = \frac{1}{\cosh^2(\chi t)} \sum_{n=0}^{\infty} \left(\frac{\sinh^2 \chi t}{\cosh^2 \chi t}\right)^n |n\rangle\langle n|$$

$$= \frac{1}{\bar{n}+1} \sum_{n=0}^{\infty} \left(\frac{\bar{n}}{\bar{n}+1}\right)^n |n\rangle\langle n| \rightarrow \text{Thermal State}$$

Observing the quantum features Entanglement and Squeezing

Marcelo Martinelli
Laboratório de Manipulação
Coerente de Átomos e Luz

EPR and Entanglement

Anybody who is not shocked by quantum theory has not understood it.

Niels Bohr

MAY 15, 1935 PHYSICAL REVIEW VOLUME 47

Can Quantum-Mechanical Description of Physical Reality Be Considered Complete?

A. Einstein, B. Podolsky and N. Rosen, *Institute for Advanced Study, Princeton, New Jersey*

(Received March 25, 1935)

In a complete theory there is an element corresponding to each element of reality.

EPR's example

$W \simeq \delta(x_1 - x_2 - L)\delta(p_1 + p_2)$ (localized in $x_1 - x_2$ e $p_1 + p_2$)

We see therefore that, as a consequence of two different measurements performed upon the first system, the second system may be left in states with two different wave functions. On the other hand, since at the time of measurement the two systems no longer interact, no real change can take place in the second system in consequence of anything that may be done to the first system.

A measurement of x_1 yields x_2, as well as a measurement of p_1 gives p_2. But x_2 and p_2 *don't commute*! $\leftrightarrow [x, p] = i\hbar$

Bohr's reply

OCTOBER 15, 1935 PHYSICAL REVIEW VOLUME 48

Can Quantum-Mechanical Description of Physical Reality be Considered Complete?

N. Bohr, *Institute for Theoretical Physics, University, Copenhagen*

(Received July 13, 1935)

$$[q_1p_1]=[q_2p_2]=ih/2\pi,$$
$$[q_1q_2]=[p_1p_2]=[q_1p_2]=[q_2p_1]=0,$$

$$q_1=Q_1\cos\theta-Q_2\sin\theta \qquad p_1=P_1\cos\theta-P_2\sin\theta$$
$$q_2=Q_1\sin\theta+Q_2\cos\theta \qquad p_2=P_1\sin\theta+P_2\cos\theta.$$

$$[Q_1P_1]=ih/2\pi, \qquad [Q_1P_2]=0,$$

$$Q_1=q_1\cos\theta+q_2\sin\theta,$$

$$P_2=-p_1\sin\theta+p_2\cos\theta,$$

A tales of two systems

For strong entanglement, local information should vanish.

Meanwhile, global information is maximally kept

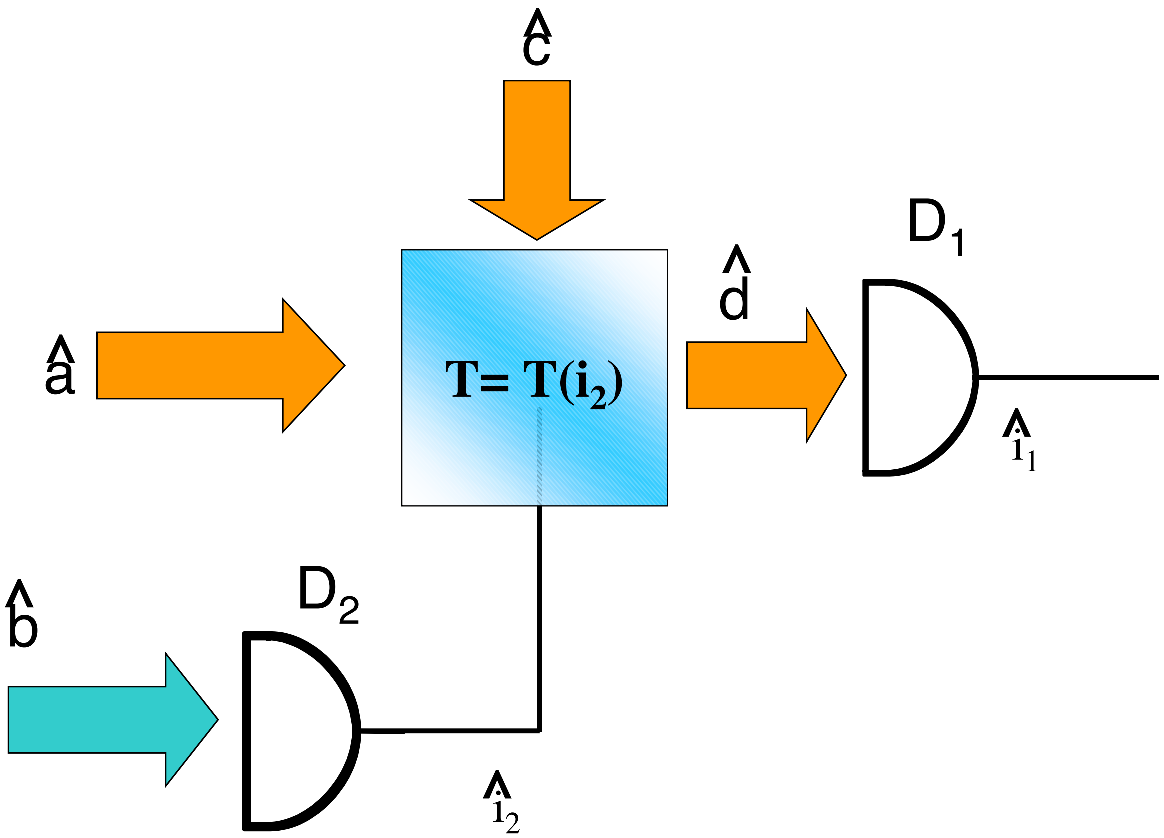
(bounded by the Uncerntainty Principle)!

Although there is a limitation for information in the quantum world, we

are allowed to have extreme nonlocal correlations.

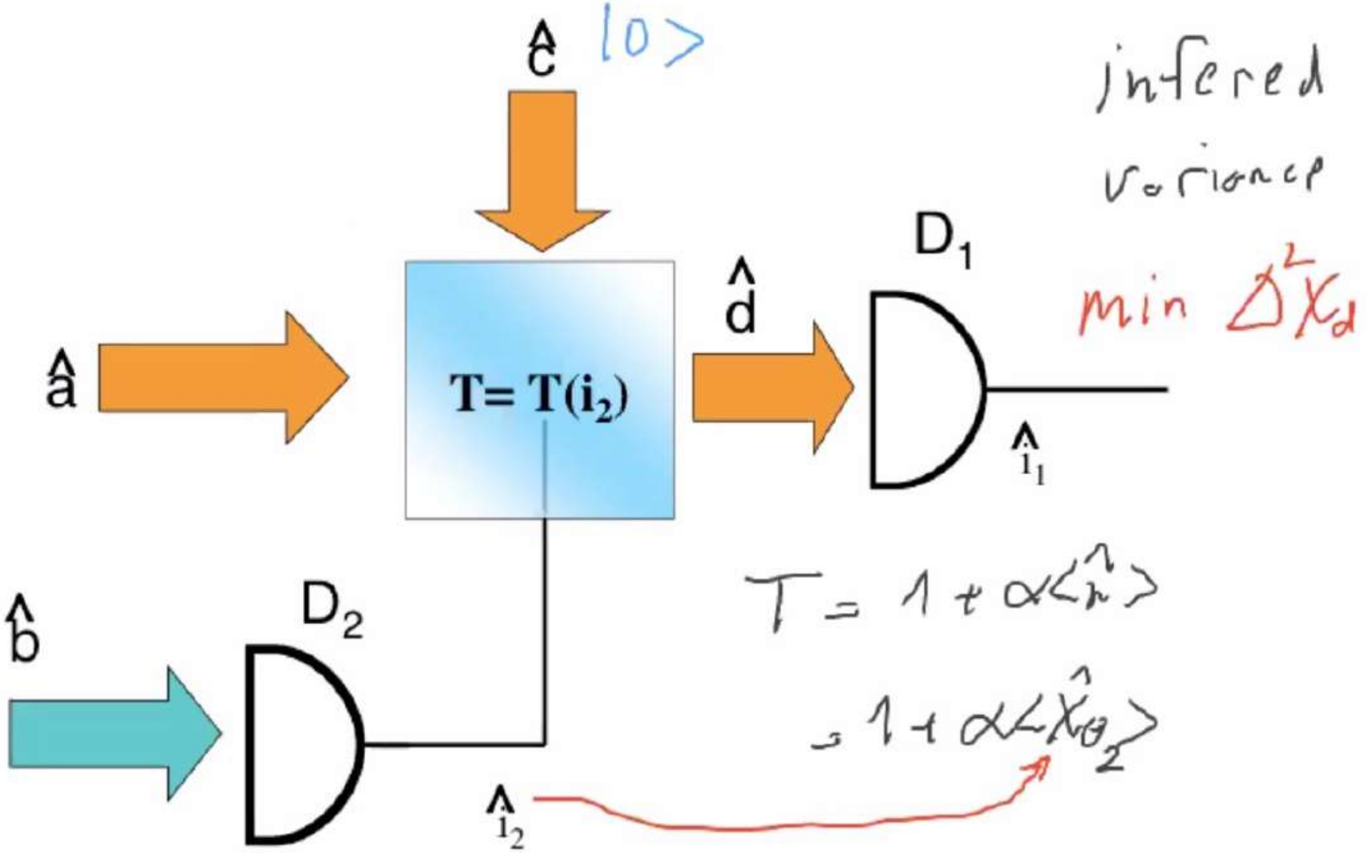
Few words about entanglement characterization

- "EPR" criterion [M. D. Reid, PRA **40**, 913 (1989), M. D. Reid and P. D. Drummond, PRL **60**, 2731 (1988) & PRA **40**, 4493 (1989)]

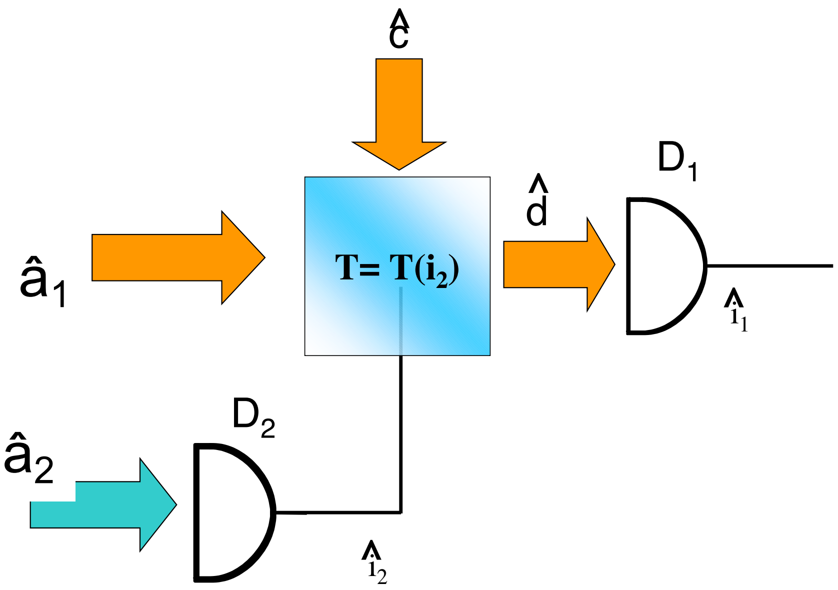


Few words about entanglement characterization

• "EPR" criterion [M. D. Reid, PRA **40**, 913 (1989), M. D. Reid and P. D. Drummond, PRL **60**, 2731 (1988) & PRA **40**, 4493 (1989)]



$$\delta\hat{p}_i = \hat{p}_i - \langle\hat{p}_i\rangle$$



$$\Delta^2\hat{p}_{\rm inf} = \Delta^2\hat{p}_1\left(1 - \frac{\langle\delta\hat{p}_1\delta\hat{p}_2\rangle^2}{\Delta^2\hat{p}_1\Delta^2\hat{p}_2}\right)$$

$$\Delta^2\hat{p}_{\rm inf}\,\Delta^2\hat{q}_{\rm inf} \geq 1$$

Entanglement Test - DGCZ

•DGCZ separability criterion:

$$\rho = \sum_i p_i \, \rho_i = \sum_i p_i \, \rho_i^1 \otimes \rho_i^2 \qquad [\hat{q}_i, \hat{p}_j] = 2i\delta_{ij}$$

$$\hat{u} = a\hat{q}_1 + \frac{1}{a}\hat{q}_2,$$
$$\hat{v} = a\hat{p}_1 - \frac{1}{a}\hat{p}_2,$$

Separability $\Rightarrow \langle(\Delta\hat{u})^2\rangle_\rho + \langle(\Delta\hat{v})^2\rangle_\rho \geq 2\,(a^2 + \frac{1}{a^2})$

Lu-Ming Duan, G. Giedke, J.I. Cirac, P. Zoller,
Inseparability criterion for continuous variable systems, Phys. Rev. Lett. **84**, 2722 (2000).

•After some (simple) algebra:

$$(\Delta^2 p_1 + \Delta^2 q_1 - 2)(\Delta^2 p_2 + \Delta^2 q_2 - 2) - (|c_p| + |c_q|)^2 \geq 0;$$

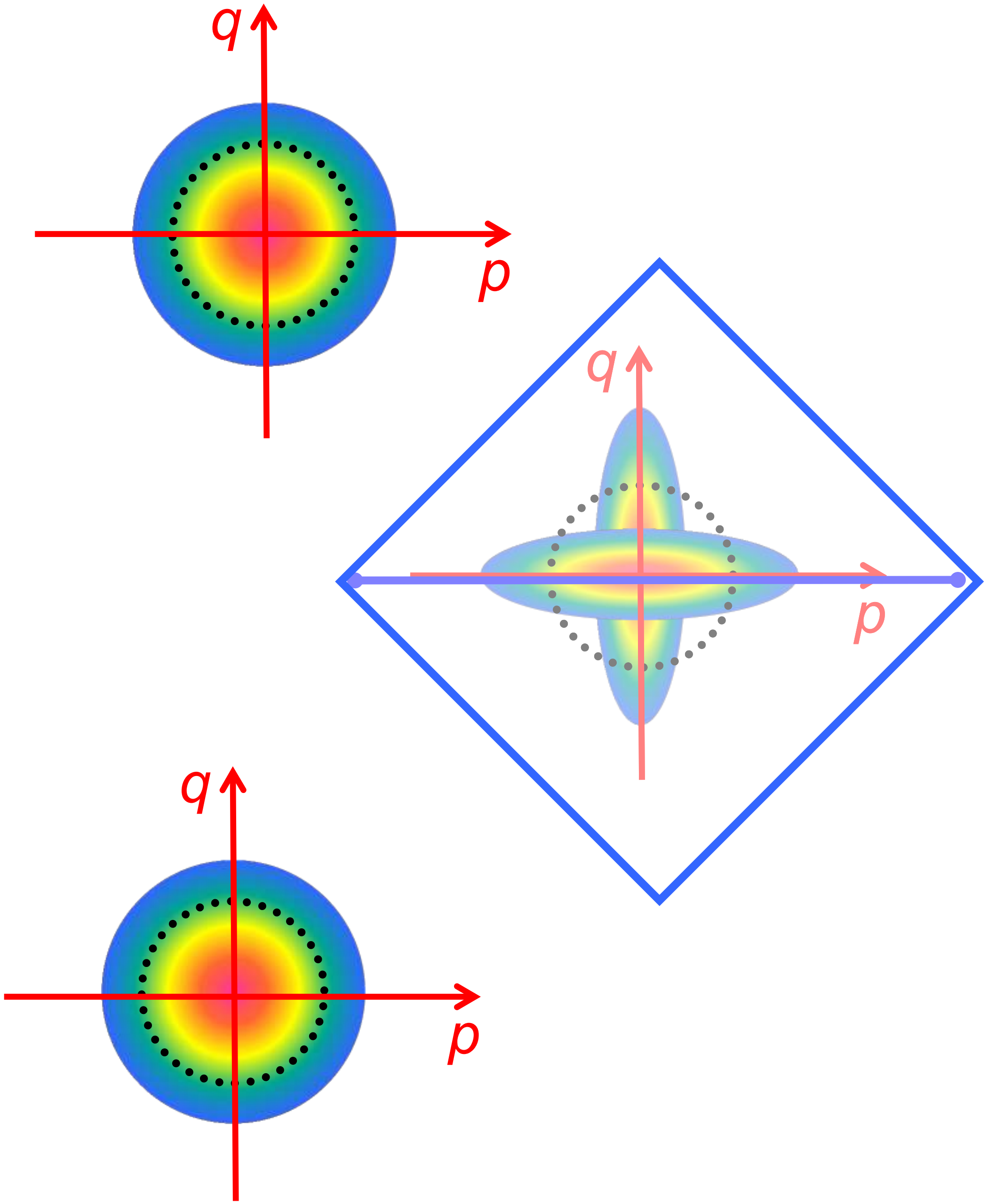
Entanglement Test - DGCZ

$$V = \begin{bmatrix} S_{p1} & C_{p1q1} & C_{p1p2} & C_{p1q2} \\ C_{p1q1} & S_{q1} & C_{q1p2} & C_{q1q2} \\ C_{p1p2} & C_{q1p2} & S_{p2} & C_{p2q2} \\ C_{p1q2} & C_{q1q2} & C_{p2q2} & S_{q2} \end{bmatrix}$$

$$C_{xixj} = \frac{1}{2}\langle\{x_i, x_j\}\rangle - \langle x_i\rangle\langle x_j\rangle \qquad S_{xj} = C_{xjxj}$$

$$\boxed{\begin{aligned}(\Delta^2 p_1 + \Delta^2 q_1 - 2)(\Delta^2 p_2 + \Delta^2 q_2 \; - \; 2) \\ -(|c_p| \; + \; |c_q|)^2 \geq 0;\end{aligned}}$$

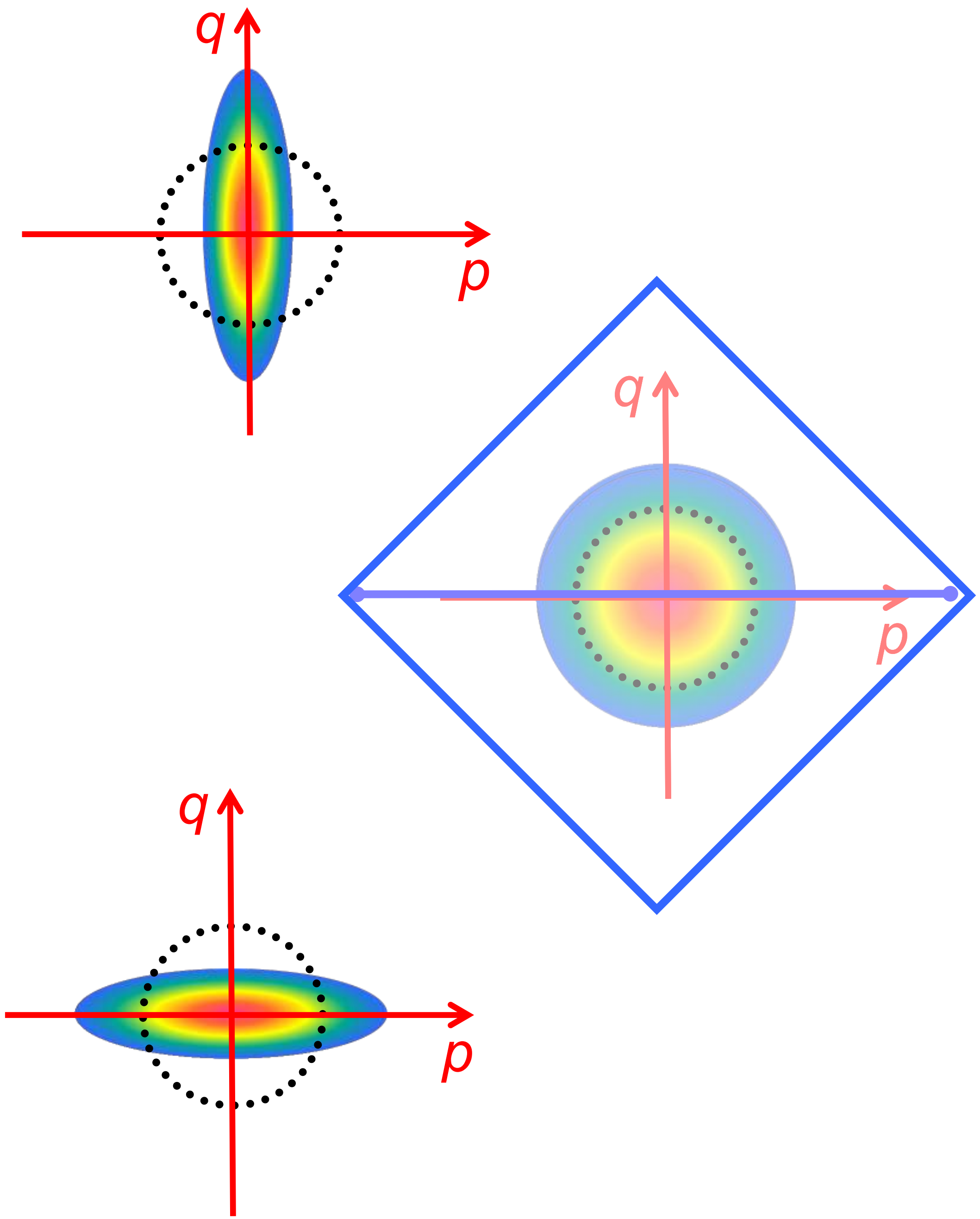
Entanglement Test - DGCZ



p_1+p_2 ,
q_1+q_2

p_1-p_2 ,
q_1-q_2

Entanglement Generation



p_1+p_2 ,
q_1+q_2

p_1-p_2 ,
q_1-q_2

Entanglement Test – Peres & Horodecki

- Positivity under Partial Transposition
  (discrete variables)

Separability Criterion for Density Matrices

Asher Peres*

PRL **77**, 1413 (1996)

$$\rho = \sum_A w_A \rho'_A \otimes \rho''_A \quad \Rightarrow \quad \sigma = \sum_A w_A (\rho'_A)^T \otimes \rho''_A$$

non-negative eigenvalues -> Separability

Entanglement Test - Simon

- Continuous variables:

Peres-Horodecki Separability Criterion for Continuous Variable Systems

R. Simon

PRL **84**, 2726 (2000)

$$PT: \qquad W(q_1, p_1, q_2, p_2) \longrightarrow W(q_1, p_1, q_2, -p_2)$$

$$V + \frac{i}{2}\Omega \geq 0 \quad \Rightarrow \quad \tilde{V} + \frac{i}{2}\Omega \geq 0$$

$$\tilde{V} = \Lambda V \Lambda$$

$$\Omega = \begin{pmatrix} J & 0 \\ 0 & J \end{pmatrix} \quad J = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \qquad \Lambda = \mathrm{diag}(1, 1, 1, -1)$$

Simplectic Eigenvalues >1

Diagonalize: $-(\Omega\tilde{V})^2$

Entanglement Test - Simon

$$V = \begin{bmatrix} S_{p1} & C_{p1q1} & C_{p1p2} & C_{p1q2} \\ C_{p1q1} & S_{q1} & C_{q1p2} & C_{q1q2} \\ C_{p1p2} & C_{q1p2} & S_{p2} & C_{p2q2} \\ C_{p1q2} & C_{q1q2} & C_{p2q2} & S_{q2} \end{bmatrix}$$

$$C_{xixj} = \frac{1}{2}\langle\{x_i, x_j\}\rangle - \langle x_i\rangle\langle x_j\rangle$$

$$S_{xj} = C_{xjxj}$$

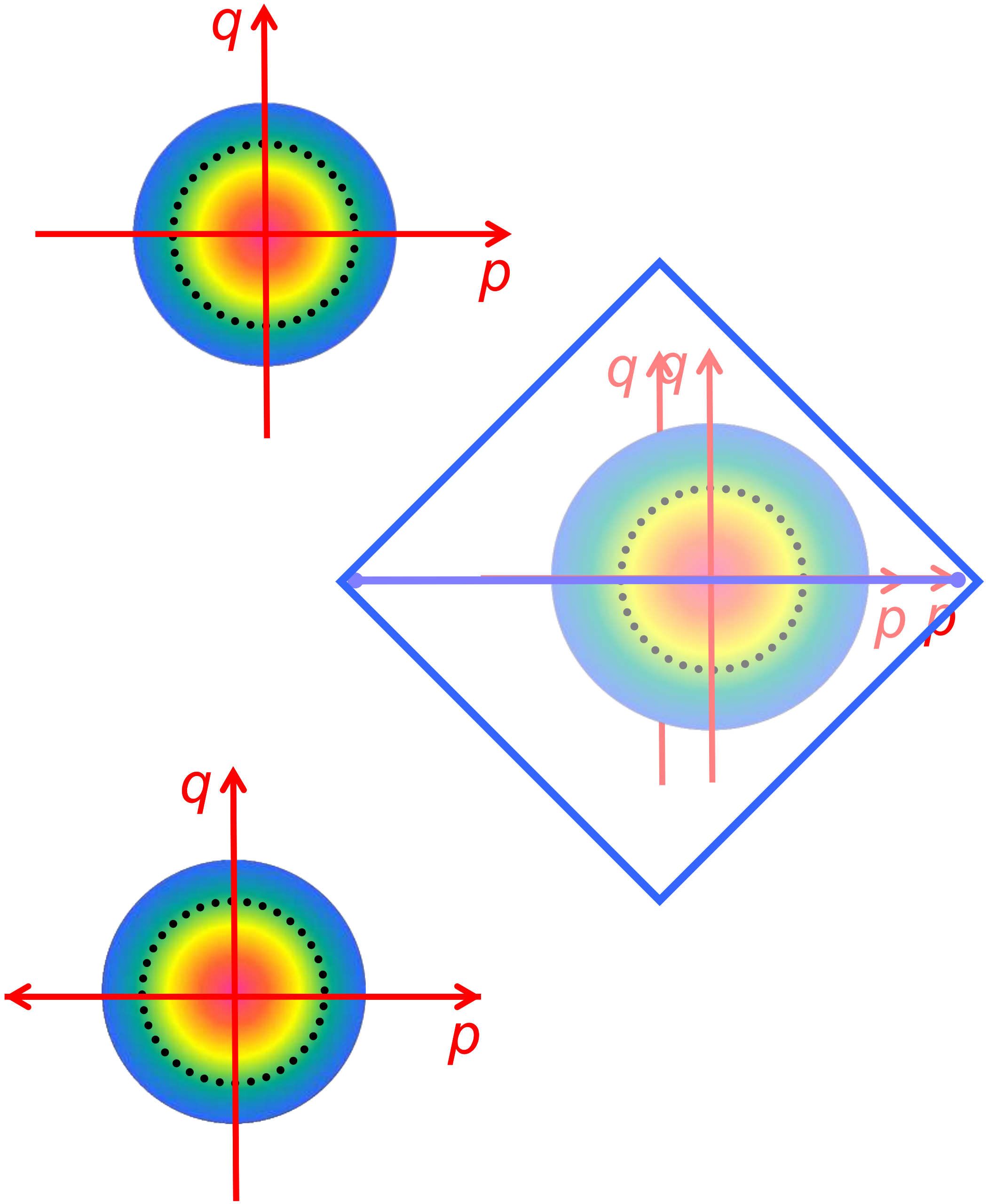
Entanglement Test - Simon

$$V = \begin{bmatrix} S_{p1} & C_{p1q1} & C_{p1p2} & -C_{p1q2} \\ C_{p1q1} & S_{q1} & C_{q1p2} & -C_{q1q2} \\ C_{p1p2} & C_{q1p2} & S_{p2} & -C_{p2q2} \\ -C_{p1q2} & -C_{q1q2} & -C_{p2q2} & S_{q2} \end{bmatrix}$$

$$C_{xixj} = \frac{1}{2}\langle\{x_i, x_j\}\rangle - \langle x_i\rangle\langle x_j\rangle$$

$$S_{xj} = C_{xjxj}$$

Entanglement Test - Simon



$p_1 - p_2$,
$q_1 + q_2$

$p_1 + p_2$,
$q_1 - q_2$

Tripartite Entanglement

- Extend DGCZ criterion to three variables

Detecting genuine multipartite continuous-variable entanglement

PHYSICAL REVIEW A **67**, 052315 (2003)
Peter van Loock[1] and Akira Furusawa[2]

$$\hat{u} \equiv h_1 \hat{x}_1 + h_2 \hat{x}_2 + h_3 \hat{x}_3, \quad \hat{v} \equiv g_1 \hat{p}_1 + g_2 \hat{p}_2 + g_3 \hat{p}_3,$$

$$\langle (\Delta \hat{u})^2 \rangle_\rho + \langle (\Delta \hat{v})^2 \rangle_\rho \geq f(h_1, h_2, h_3, g_1, g_2, g_3),$$

- Apply PPT to multiple partitions

Bound Entangled Gaussian States

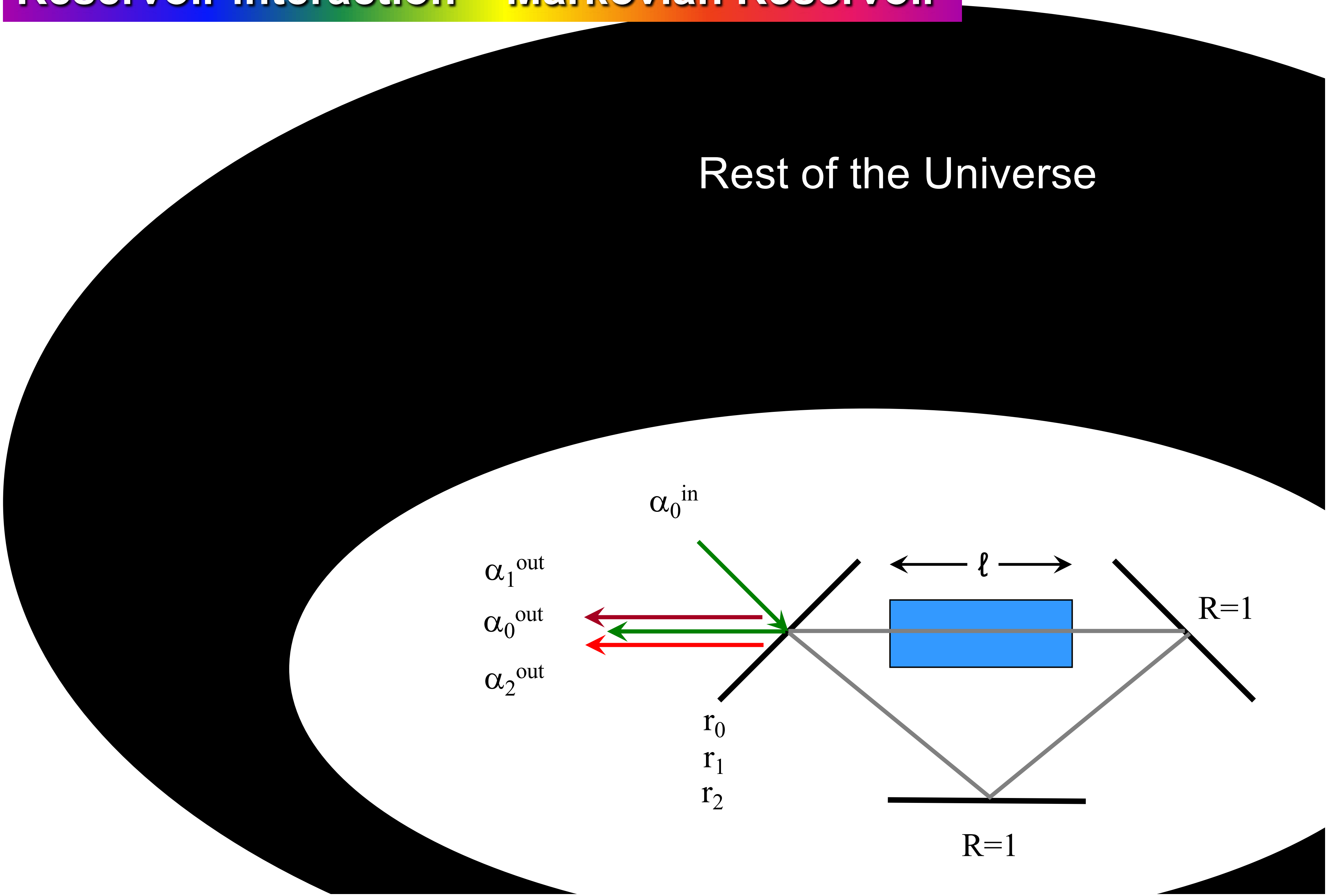
R. F. Werner* and M. M. Wolf†

PHYSICAL REVIEW LETTERS
VOLUME 86, NUMBER 16
DOI: 10.1103/PhysRevLett.86.3658

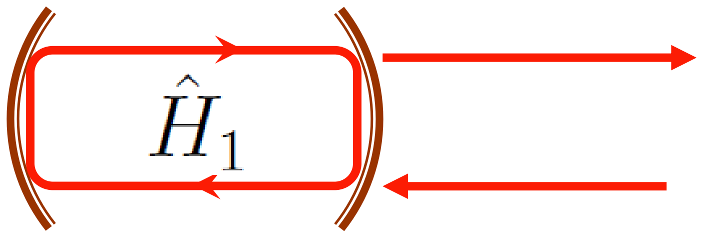
Gaussian states of $1 \times N$ systems

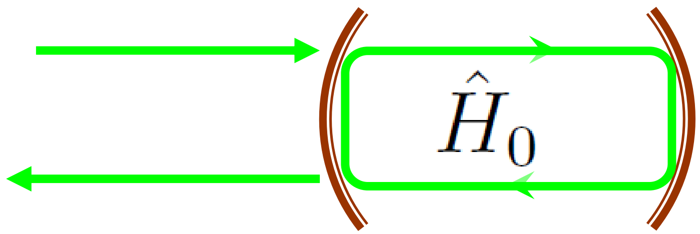
ppt implies separability.

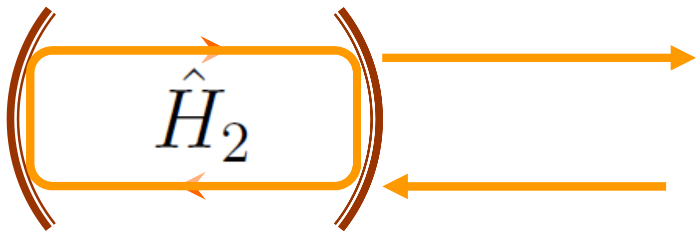
Playing with cavities: Reservoir interaction – Markovian Reservoir



Optical Parametric Oscillator (OPO) – Master Equation

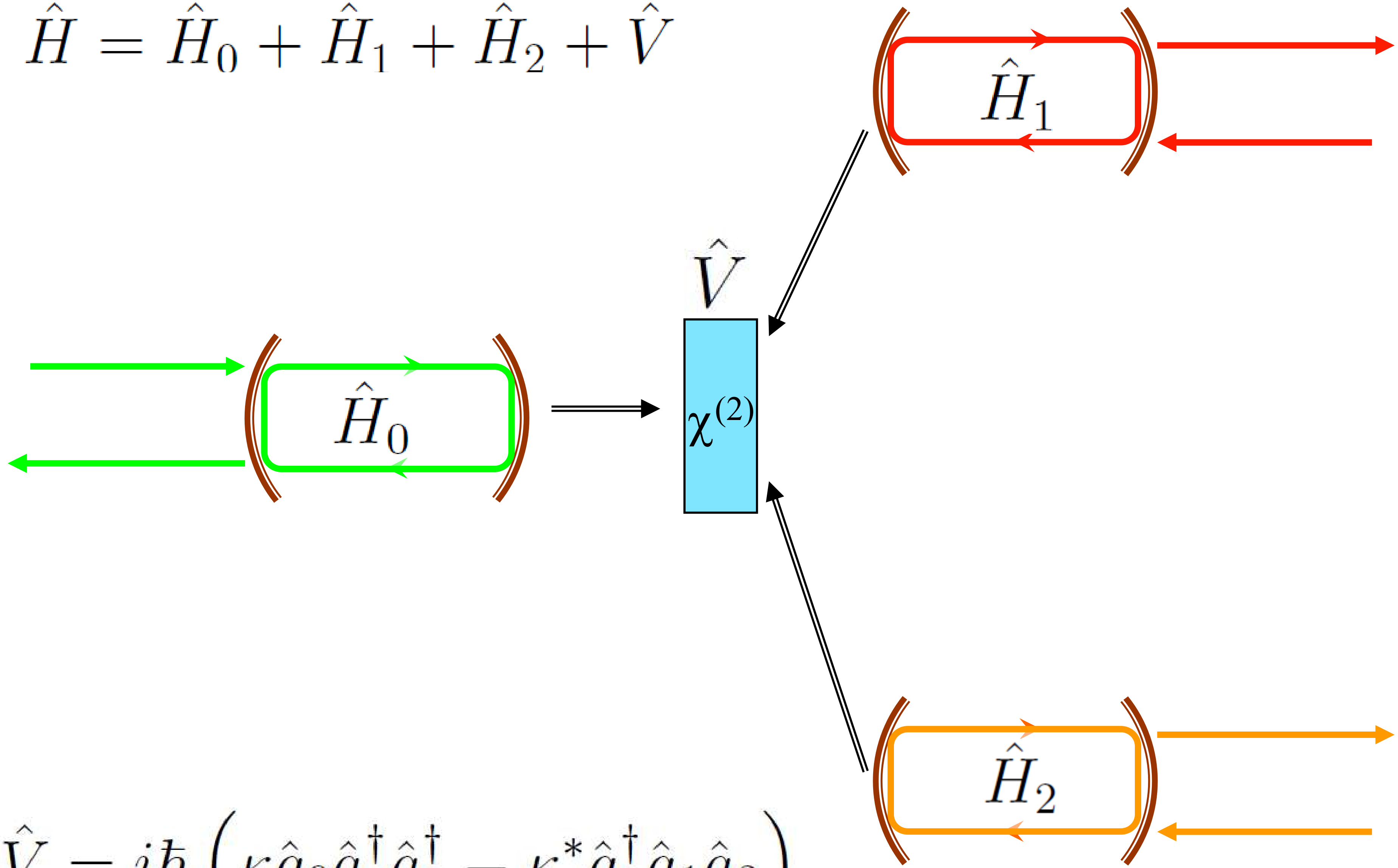






Optical Parametric Oscillator (OPO) – Master Equation

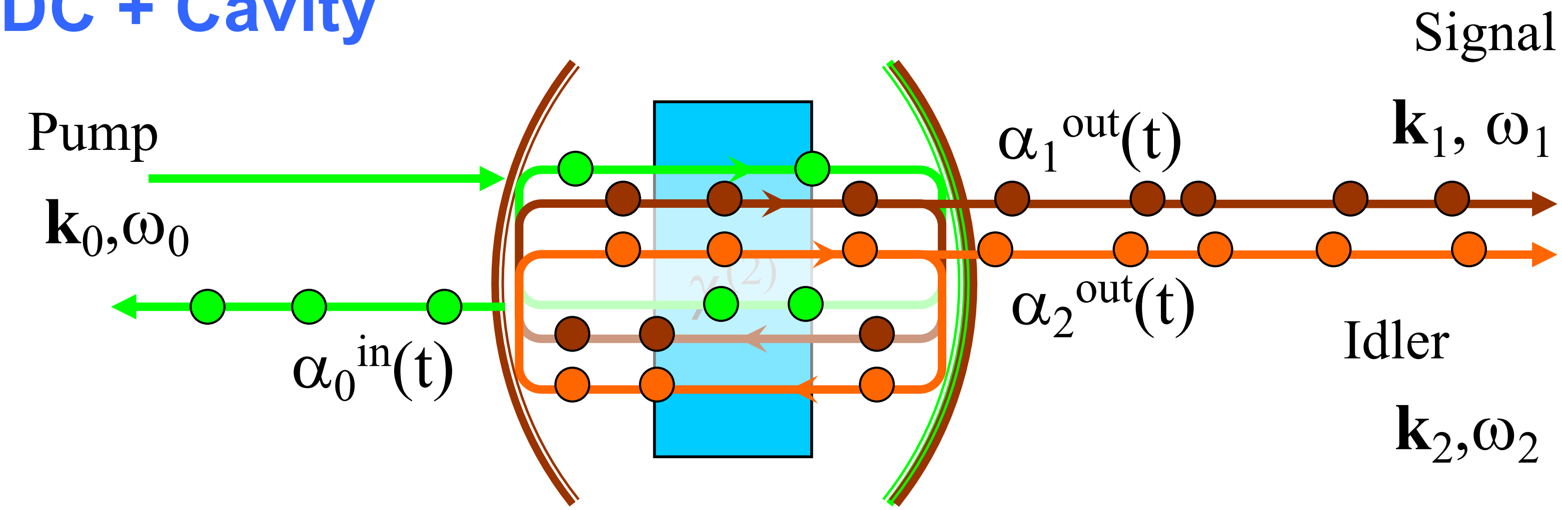
$$\hat{H} = \hat{H}_0 + \hat{H}_1 + \hat{H}_2 + \hat{V}$$



$$\hat{V} = i\hbar\left(\kappa \hat{a}_0 \hat{a}_1^\dagger \hat{a}_2^\dagger - \kappa^* \hat{a}_0^\dagger \hat{a}_1 \hat{a}_2\right)$$

OPO and Entanglement

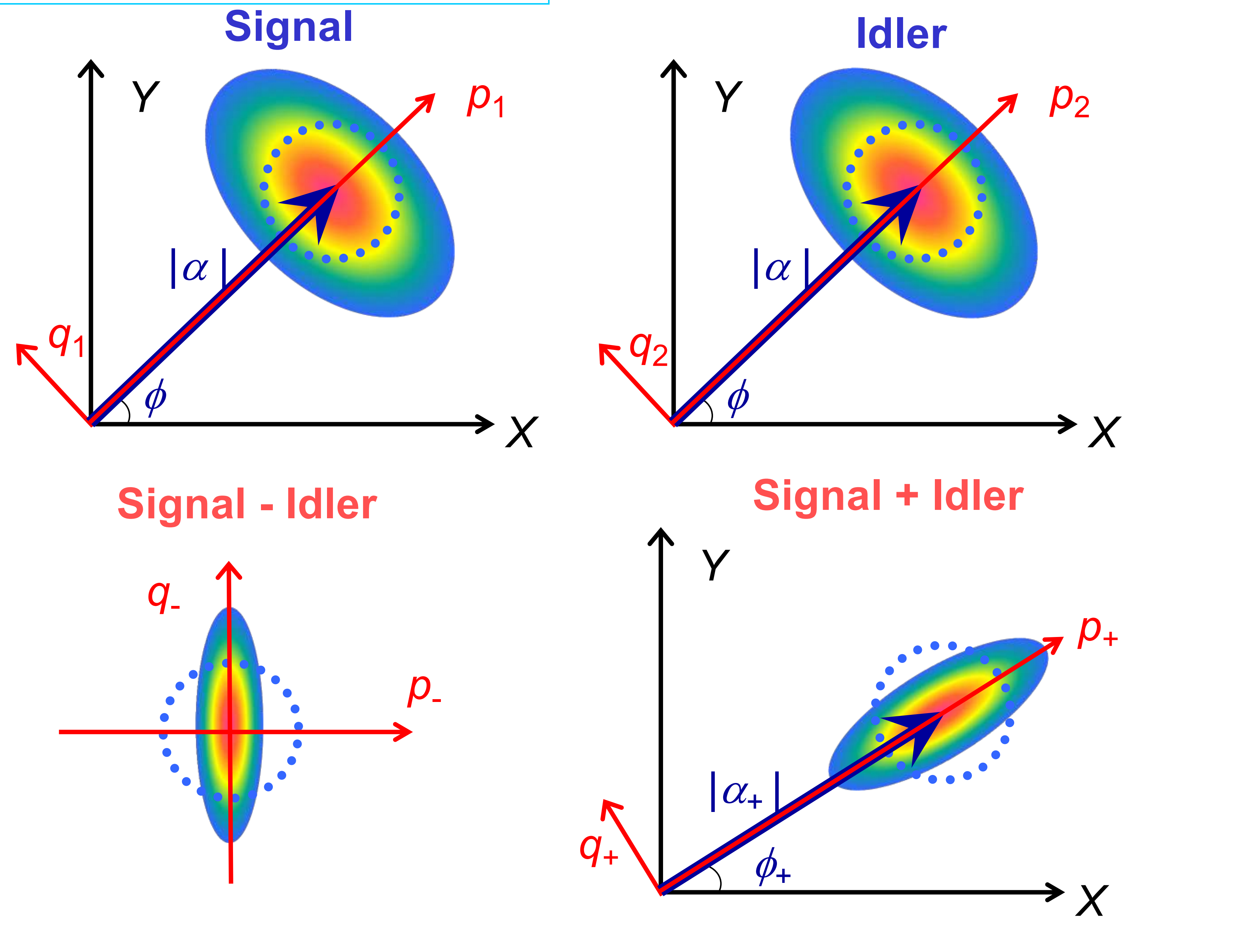
PDC + Cavity



Twin photons + phase correlation

- Sub-threshold
  - squeezed vacuum (degenerate case) - OPA
  - entangled fields (non-degenerate case)
- Above threshold: Intense entangled fields
  - Squeezing of the pump

Noise correlations



Energy Conservation

$$\omega_1 + \omega_2 = \omega_0$$

$$\delta I_1 - \delta I_2 = 0$$

Intensity Correlation

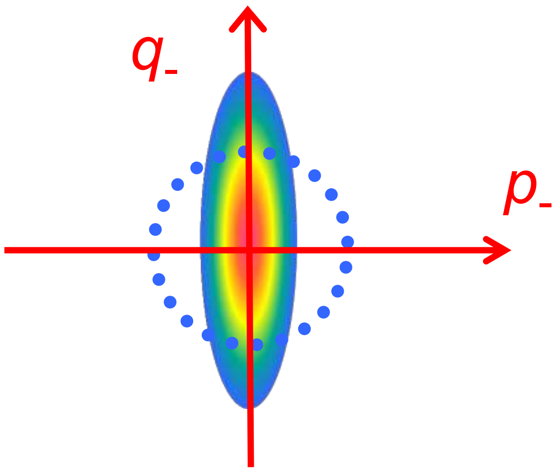
A. Heidmann *et al.*, PRL. **59**, 2555 (1987)

$$\delta\varphi_1 + \delta\varphi_2 = \delta\varphi_0$$

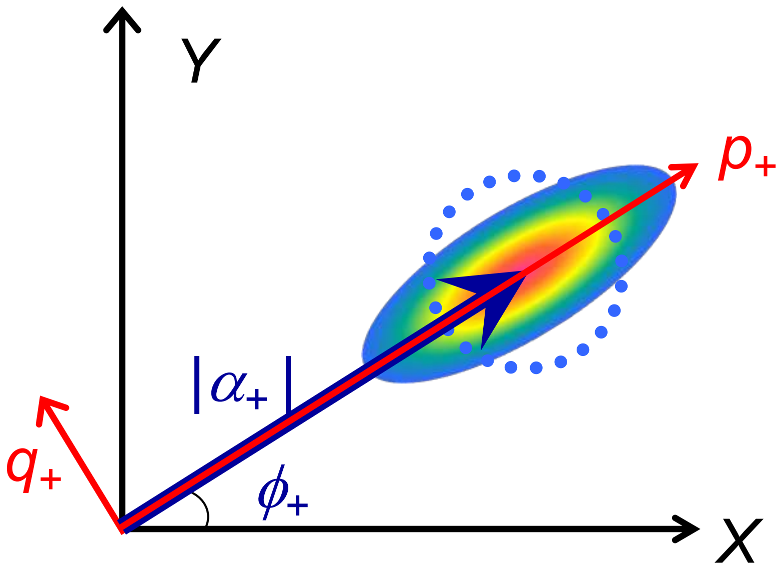
Phase Anti-correlation

A. S. Villar *et al.*, PRL **95**, 243603 (2005)

Signal - Idler



Signal + Idler



Usual treatment of the OPO: Master Equation

$$\frac{d\hat{\rho}}{dt} = \frac{i}{\hbar}\left[\hat{H}_0 + \hat{H}_1, \hat{\rho}\right] + \frac{\gamma}{2}\left[2\hat{a}\hat{\rho}\hat{a}^\dagger - \hat{a}\hat{a}^\dagger\hat{\rho} - \hat{\rho}\hat{a}\hat{a}^\dagger\right]$$

Quasi-probability representation

$$\frac{\partial P(\vec{X},t)}{\partial t} = \left[-\sum_i \frac{\partial}{\partial x_i} A_i(\vec{X},t) + \frac{1}{2}\sum_{ij} \frac{\partial}{\partial x_i}\frac{\partial}{\partial x_j} D_{ij}(\vec{X},t)\right] P(\vec{X},t)$$

$$\mathbb{D}(\vec{X},t) = \mathbb{B}(\vec{X},t)\mathbb{B}^T(\vec{X},t)$$

Langevin Equation

$$\frac{d\vec{X}}{dt} = \mathbb{A}(\vec{X},t) + \mathbb{B}(\vec{X},t)\vec{X}^{in}(t)$$

Usual treatment of the OPO: Langevin Equation

Linearization

$$\frac{d\delta\vec{X}(t)}{dt} = \mathbb{A}\delta\vec{X}(t) + \mathbb{B}\vec{X}^{in}(t)$$

Input – Output Formalism

$$\delta\vec{X}^{out}(t) = \mathbb{B}\delta\vec{X}(t) - \mathbb{I}\vec{X}^{in}(t)$$

Frequency Domain

$$\vec{X}(\Omega) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} \delta\vec{X}(t)exp(-i\Omega t)dt$$

$$\vec{X}(\Omega) = \left[-(\mathbb{A} + i\Omega\mathbb{I})^{-1}\mathbb{B}\right]\vec{X}^{in}(\Omega)$$

$$\vec{X}^{out}(\Omega) = -\left[\mathbb{B}(\mathbb{A} + i\Omega\mathbb{I})^{-1}\mathbb{B} + \mathbb{I}\right]\vec{X}^{in}(\Omega)$$

$$\vec{X}(\Omega) = -\mathbb{M}_I(\Omega)\vec{X}^{in}(\Omega)$$

$$\vec{X}^{out}(\Omega) = -\mathbb{M}_O(\Omega)\vec{X}^{in}(\Omega).$$

$$\mathbb{M}_I(\Omega) = (\mathbb{A} + i\Omega\mathbb{I})^{-1}\mathbb{B}$$

$$\mathbb{M}_O(\Omega) = \mathbb{I} + \left[\mathbb{B}(\mathbb{A} + i\Omega\mathbb{I})^{-1}\mathbb{B}\right]$$

Covariance Matrix X Spectral Matrix

$$\mathbb{V}(t,t+\tau) = \mathbb{V}(\tau) = \langle \delta\vec{X}^{out}(t)[\delta\vec{X}^{out}(t+\tau)]^T\rangle \qquad \mathbb{S}(\Omega) = \langle \vec{X}^{out}(\Omega)[\vec{X}^{out}(-\Omega)]^T\rangle$$

$$\mathbb{V}(\tau) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} S(\Omega)\exp(i\Omega\tau)d\Omega$$

Complete description of the state: Wigner function (for a Gaussian State)

$$W(\vec{X}) = \frac{1}{4\pi^2\sqrt{\det \mathbb{V}_i}}\exp\left(-\frac{1}{2}\vec{X}^T\mathbb{V}_i^{-1}\vec{X}\right)$$

Where is the complete information about the OPO state?

S will present the Fourier transform of a two-time correlation matrix V(τ).

Therefore, it will correspond to a Covariance Matrix for a pair of sidebands of the carrier modes.

The covariance matrix for the carrier is given by V(τ=0) that is generally of limited access due to excess noise of the driving fields.

Spectral Covariance Matrix

$$V = \begin{bmatrix} S_{p1} & C_{p1q1} & C_{p1p2} & C_{p1q2} & C_{p1p0} & C_{p1q0} \\ C_{p1q1} & S_{q1} & C_{q1p2} & C_{q1q2} & C_{q1p0} & C_{q1q0} \\ C_{p1p2} & C_{q1p2} & S_{p2} & C_{p2q2} & C_{p2p0} & C_{p2q0} \\ C_{p1q2} & C_{q1q0} & C_{p2q2} & S_{q2} & C_{q2p0} & C_{q2q0} \\ C_{p1p0} & C_{q1p0} & C_{p2p0} & C_{q2p0} & S_{p0} & C_{p0q0} \\ C_{p1q0} & C_{q1q0} & C_{p2q0} & C_{q2q0} & C_{p0q0} & S_{q0} \end{bmatrix}$$

$$C_{xixj} = \frac{1}{2}\langle\{x_i, x_j\}\rangle - \langle x_i\rangle\langle x_j\rangle \qquad S_{xj} = C_{xjxj}$$

36 independent terms !

Spectral Covariance Matrix

$$V = \begin{bmatrix} S_{p1} & 0 & C_{p1p2} & 0 & C_{p1p0} & 0 \\ 0 & S_{q1} & 0 & C_{q1q2} & 0 & C_{q1q0} \\ C_{p1p2} & 0 & S_{p2} & 0 & C_{p2p0} & 0 \\ 0 & C_{q1q0} & 0 & S_{q2} & 0 & C_{q2q0} \\ C_{p1p0} & 0 & C_{p2p0} & 0 & S_{p0} & 0 \\ 0 & C_{q1q0} & 0 & C_{q2q0} & 0 & S_{q0} \end{bmatrix}$$

$$C_{xixj} = \frac{1}{2}\langle\{x_i, x_j\}\rangle - \langle x_i\rangle\langle x_j\rangle \qquad S_{xj} = C_{xjxj}$$

18 independent terms !

Twin beams ($P_0 > P_{th}$)

Squeezed vacuum ($P_0 < P_{th}$, degenerate)

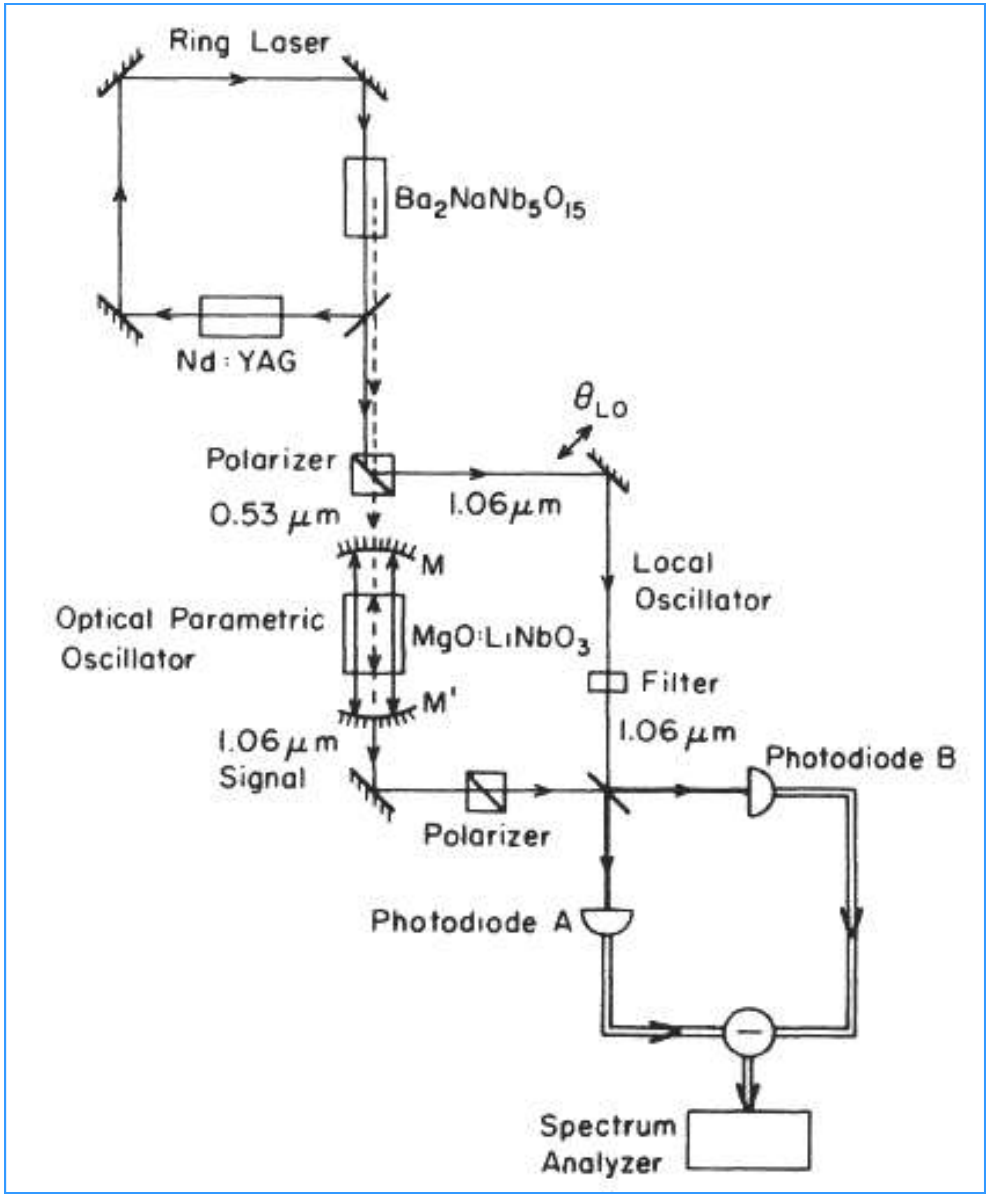
$$V = \begin{bmatrix} S_{p-} & 0 & 0 & 0 & 0 & 0 \\ 0 & S_{q-} & 0 & 0 & 0 & 0 \\ 0 & 0 & S_{p+} & 0 & C_{p+p0} & 0 \\ 0 & 0 & 0 & S_{q+} & 0 & C_{q+q0} \\ 0 & 0 & C_{p+p0} & 0 & S_{p0} & 0 \\ 0 & 0 & 0 & C_{q+q0} & 0 & S_{q0} \end{bmatrix}$$

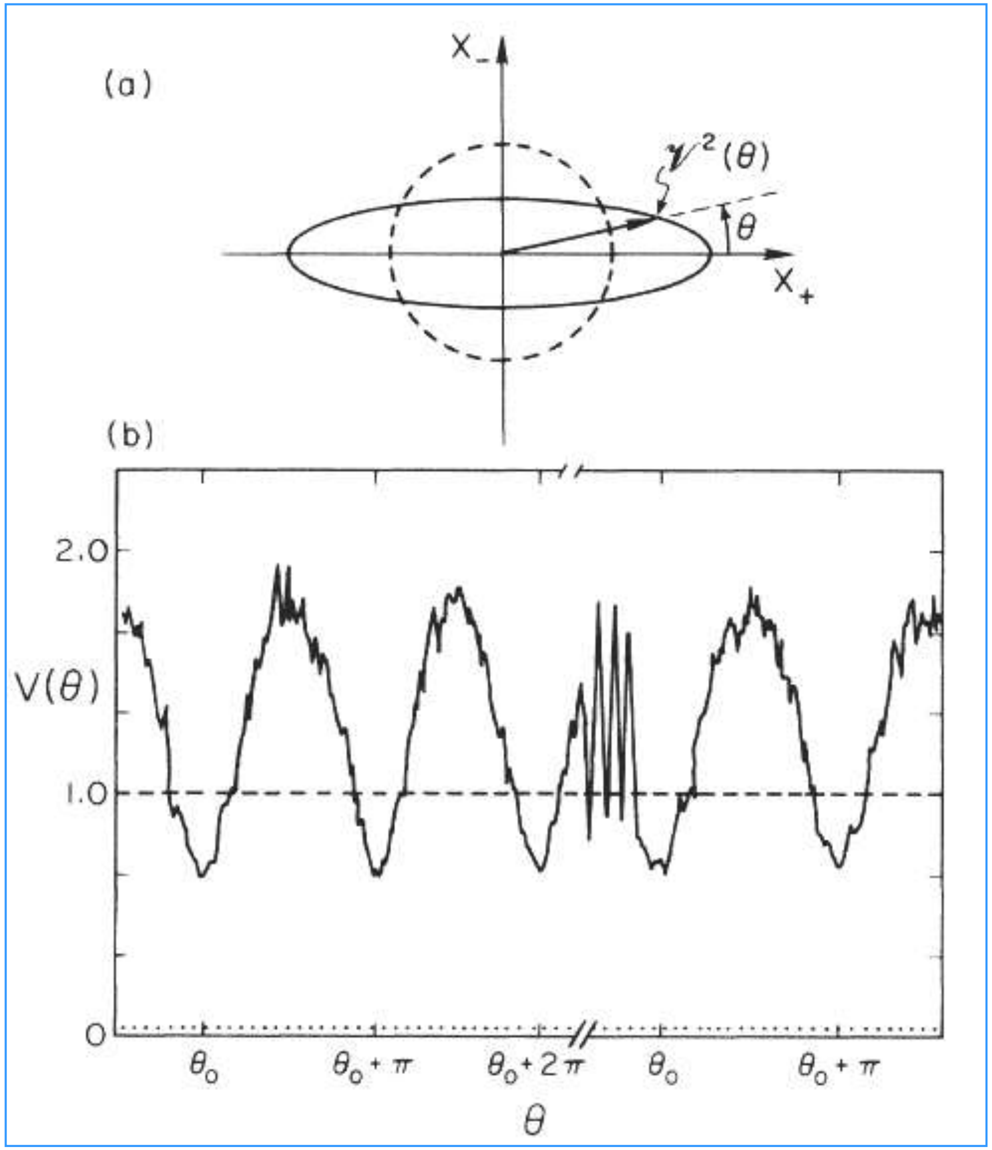
Entangled fields
- Vacuum ($P_0 < P_{th}$, maximum entanglement)
- Intense beams ($P_0 > P_{th}$)

Pump Squeezing ($P_0 > P_{th}$)

Generation of Squeezed States by Parametric Down Conversion

Ling-An Wu, H. J. Kimble, J. L. Hall,[a] and Huifa Wu
Department of Physics, University of Texas at Austin, Austin, Texas 78712
(Received 11 September 1986)



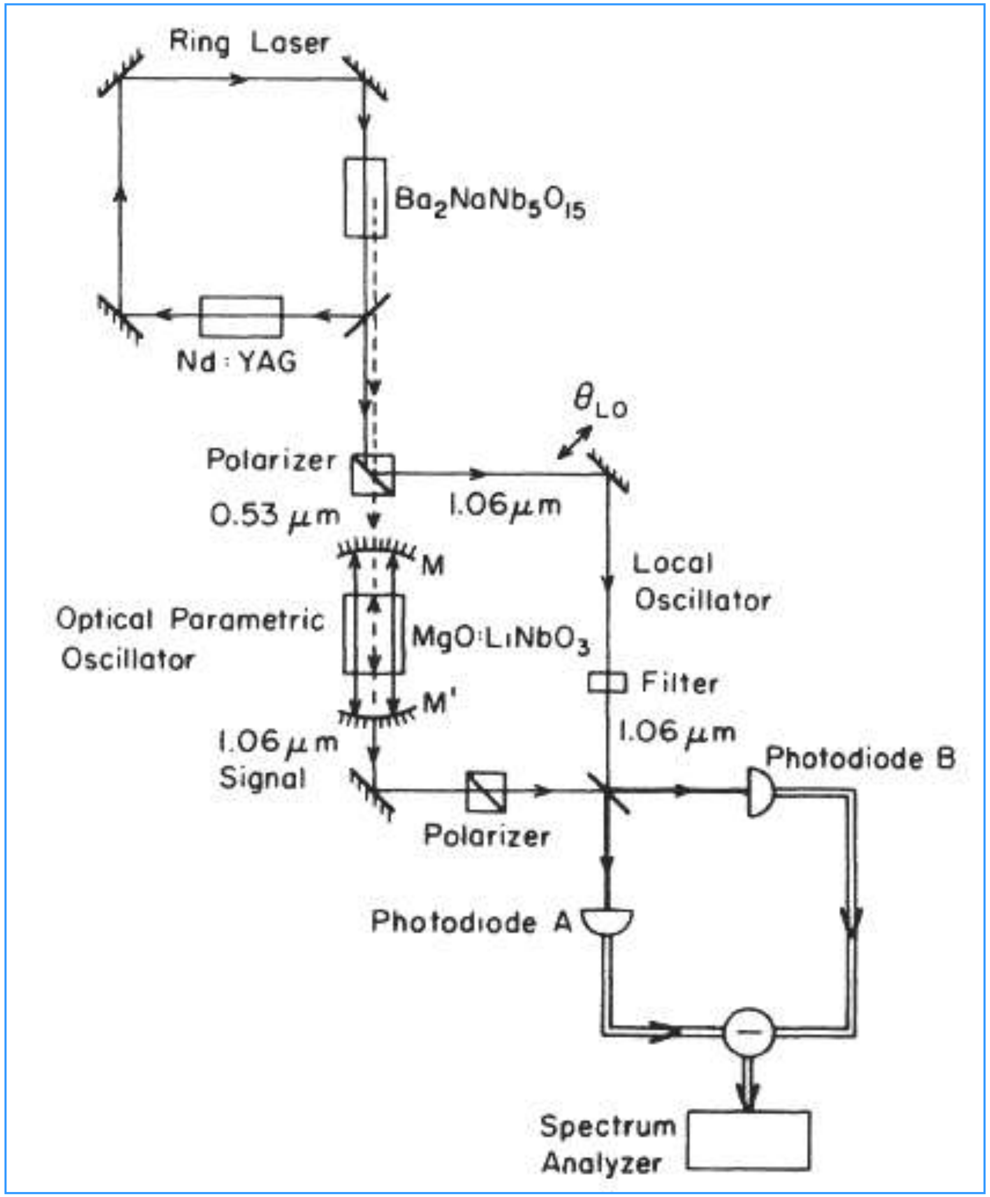


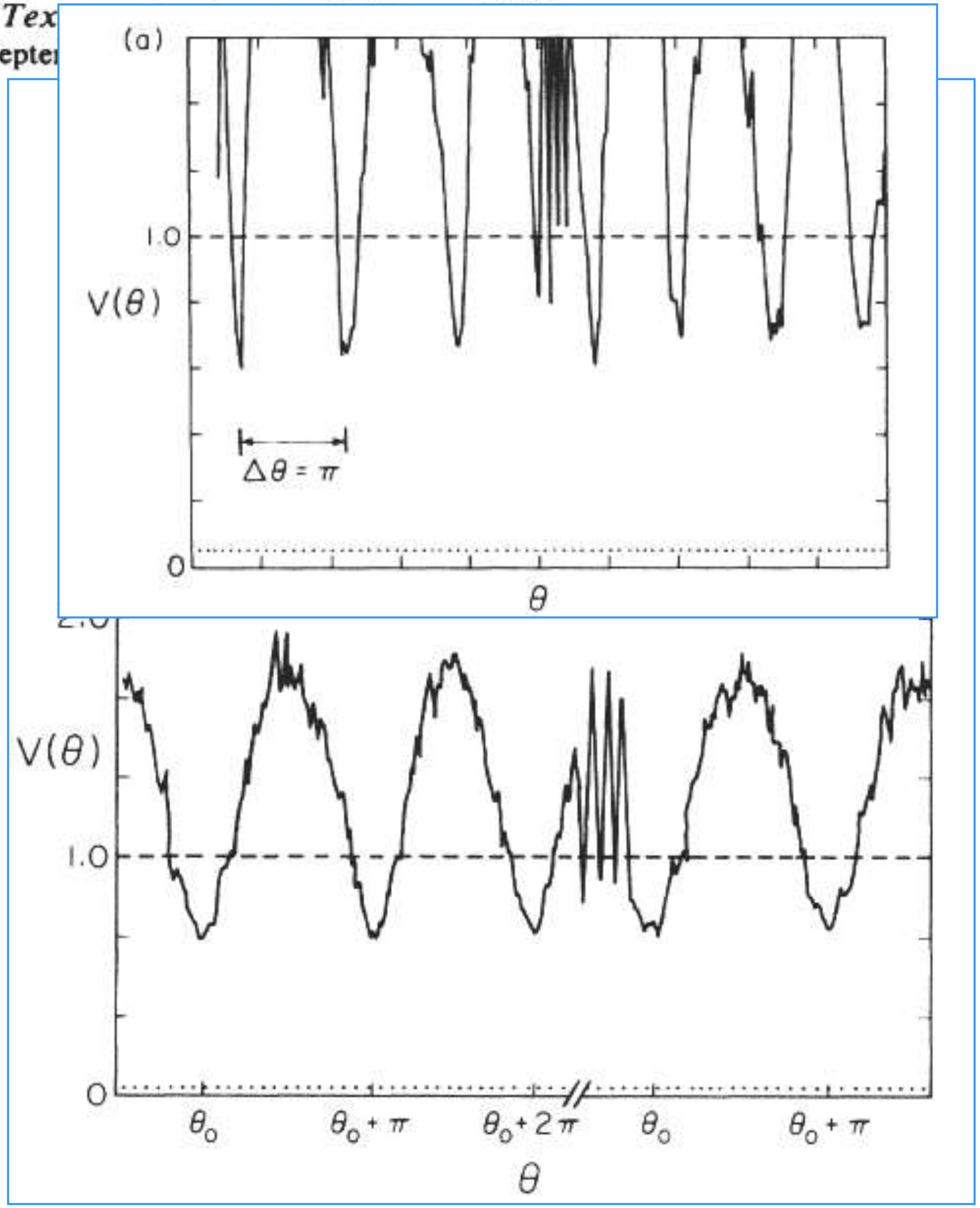
Generation of Squeezed States by Parametric Down Conversion

Ling-An Wu, H. J. Kimble, J. L. Hall,[a] and Huifa Wu

Department of Physics, University of Tex

(Received 11 Septe



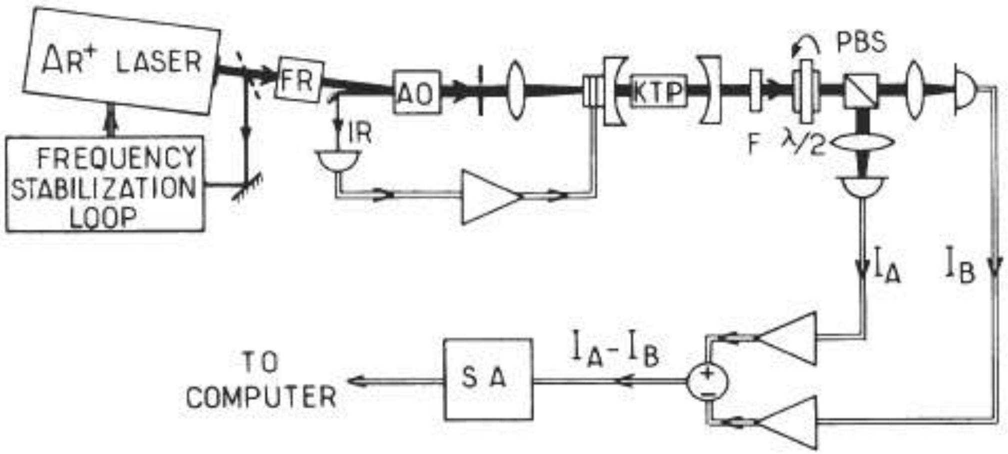


Observation of Quantum Noise Reduction on Twin Laser Beams

A. Heidmann, R. J. Horowicz, S. Reynaud, E. Giacobino, and C. Fabre
Laboratoire de Spectroscopie Hertzienne de l'Ecole Normale Supérieure, Université Pierre et Marie Curie, 75252 Paris Cedex 05, France

and

G. Camy
Laboratoire de Physique des Lasers, Université de Paris Nord, 93430 Villetaneuse, France
(Received 3 August 1987)



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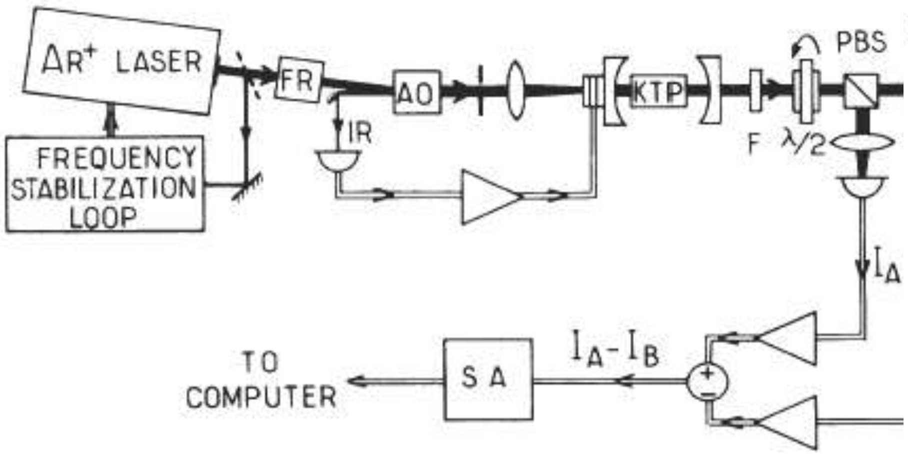
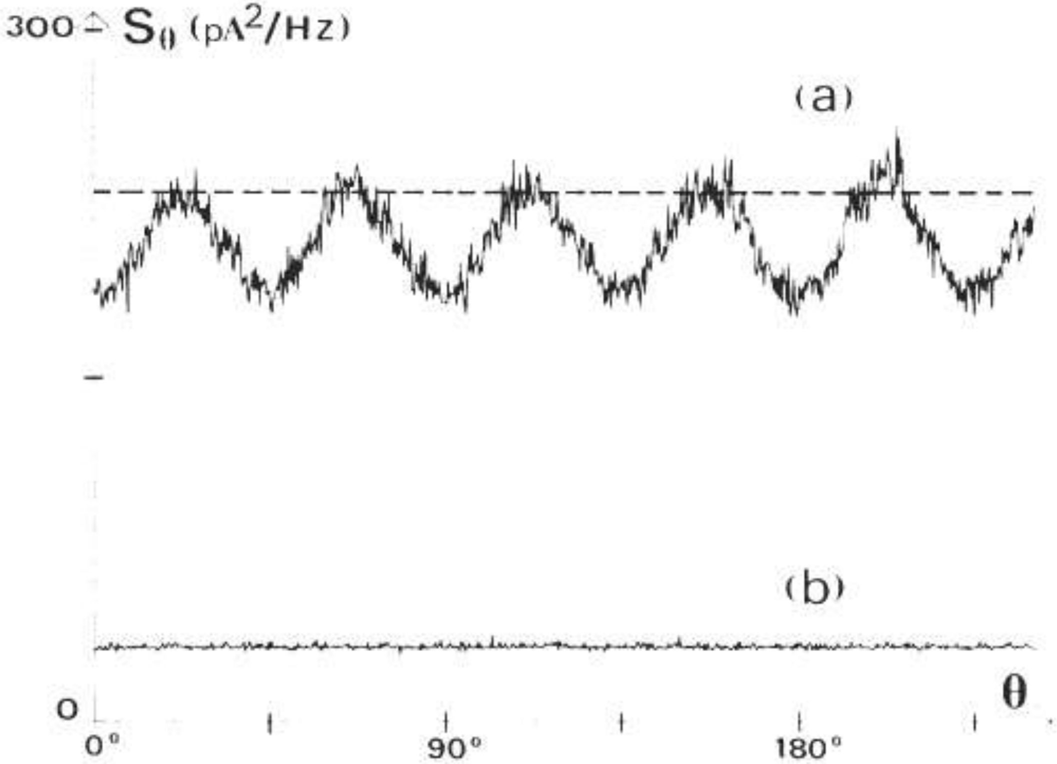




Experimental investigation of amplitude and phase quantum correlations in a type II optical parametric oscillator above threshold: from nondegenerate to degenerate operation

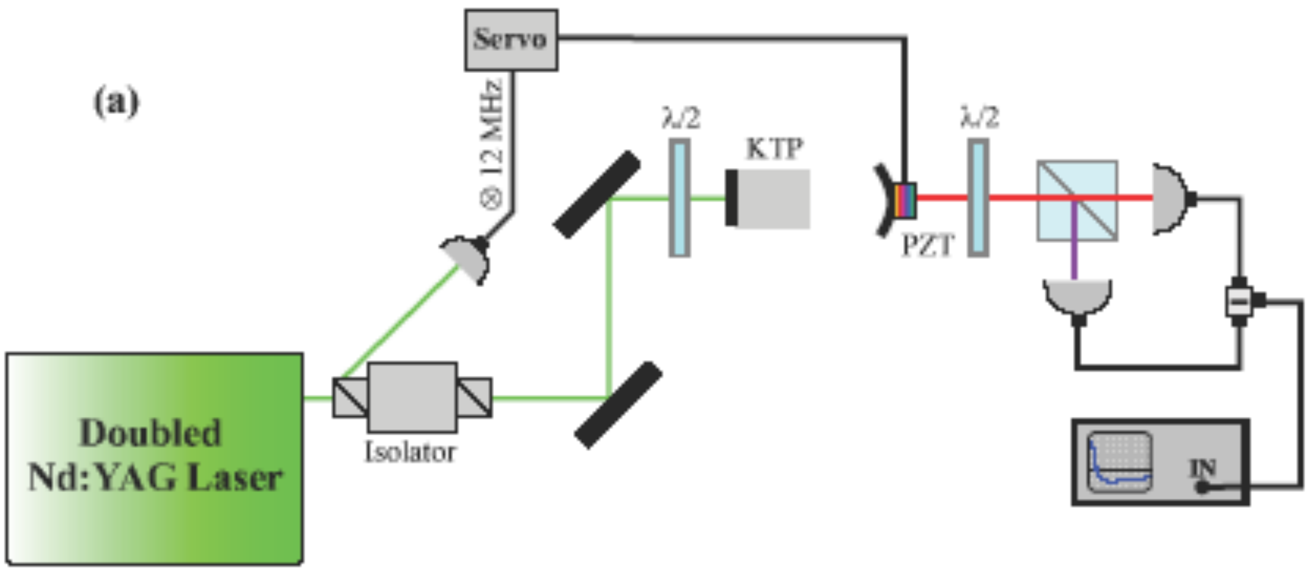
Julien Laurat, Laurent Longchambon, and Claude Fabre

Laboratoire Kastler Brossel, Case 74, Université Pierre et Marie Curie, 4 Place Jussieu, 75252 Paris Cedex 05, France

Thomas Coudreau

Laboratoire Kastler Brossel, Case 74, Université Pierre et Marie Curie, 4 Place Jussieu, 75252 Paris Cedex 05, France, and Laboratoire Matériaux et Phénomènes Quantiques, Case 7021, Université D. Diderot, 2 Place Jussieu, 75251 Paris Cedex 05, France

(a)



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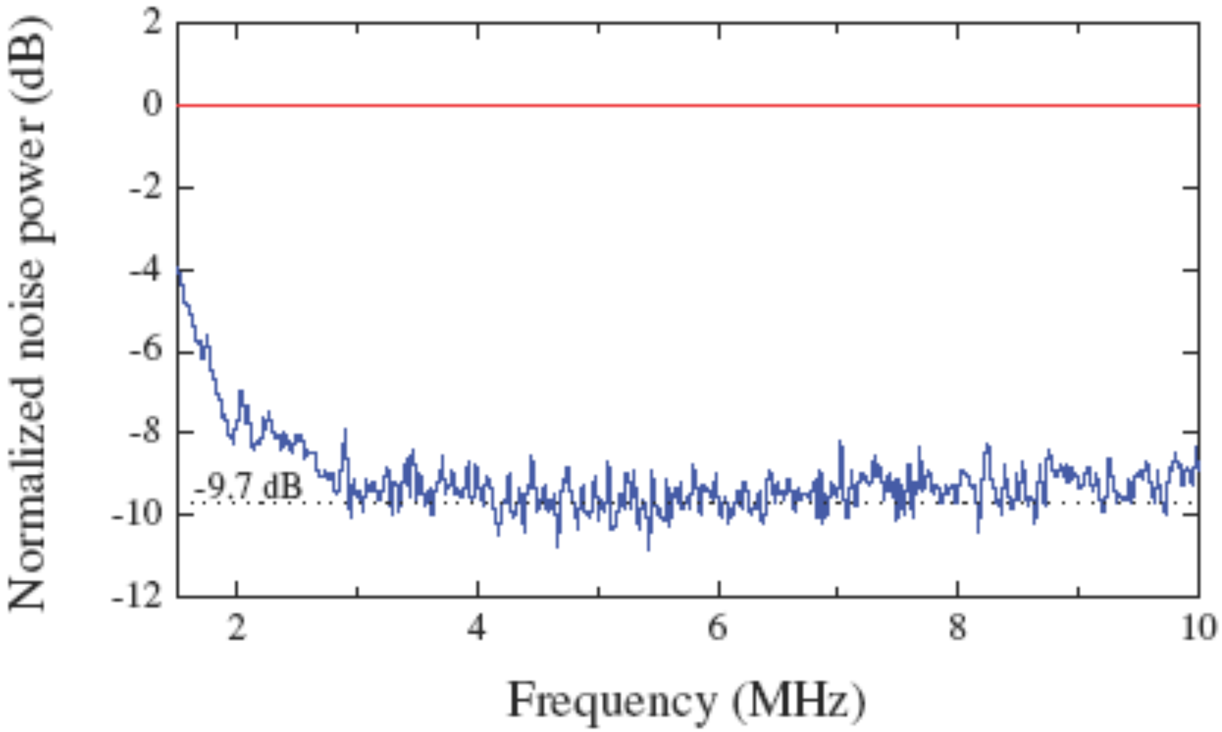
and Laboratoir u, 75251



Quantum Correlations of Phase in Nondegenerate Parametric Oscillation

M. D. Reid

Physics Department, University of Waikato, Hamilton, New Zealand

and

P. D. Drummond

Physics Department, University of Auckland, Auckland, New Zealand

(Received 17 November 1987)

The squeezing spectrum for nondegenerate parametric oscillation above threshold is calculated, including phase diffusion. A *nonclassical* correlation in phase *and* intensity occurs which is an example of the Einstein-Podolsky-Rosen paradox, even in fields of large photon number.

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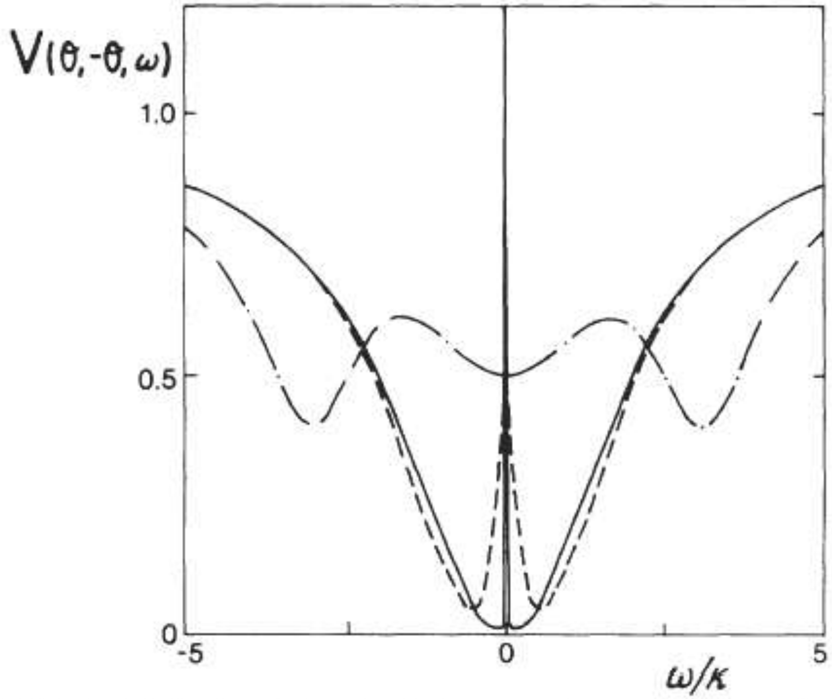


FIG. 1. Plot of $V(\theta,-\theta,\omega)$, the spectrum of fluctuation in the signal and idler quadrature amplitude difference $X_1^\theta - X_2^{-\theta}$: Solid line, near threshold. $E/E_T=1.01$, $\kappa_3/\kappa=0.01$, $I=10$. Dash-dotted line, well above threshold with a good pump. $E/E_T=50$, $\kappa_3/\kappa=0.1$ $(I^0>10)$. Dashed line, well above threshold with excellent pump. $E/E_T=20$, $\kappa_3/\kappa=0.01$ $(I^0>10)$.

Europhys. Lett., **40** (1), pp. 25-30 (1997)

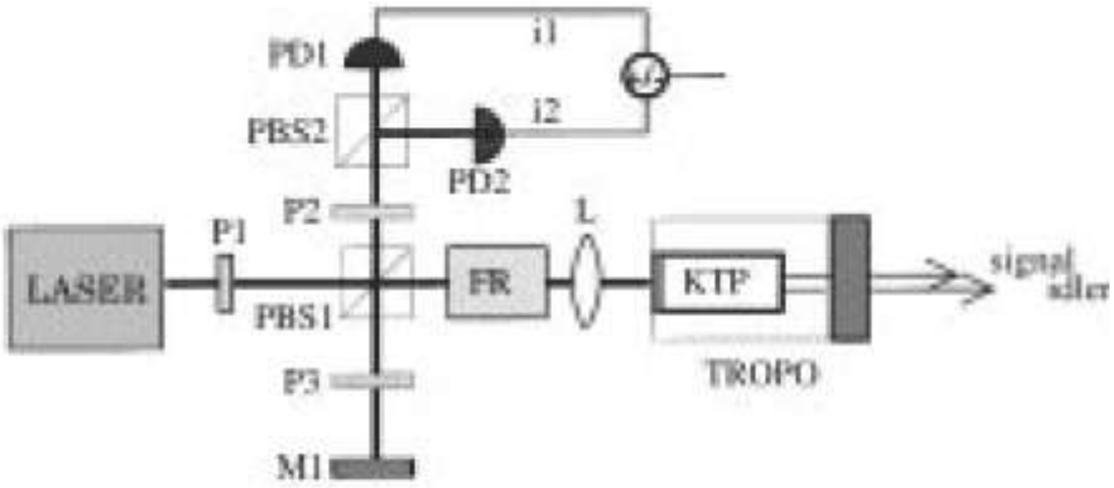
Observation of squeezing using cascaded nonlinearity

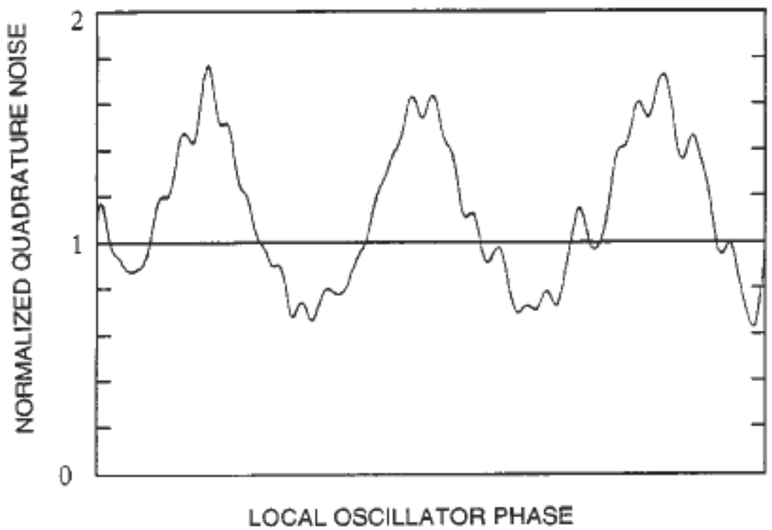
K. Kasai(*), Gao Jiangrui(**) and C. Fabre

Laboratoire Kastler Brossel (***) UPMC - Case 74 75252 Paris Cedex 05, France

(received 20 January 1997; accepted in final form 2 September 1997)

Abstract. – We have observed that the pump beam reflected by a triply resonant optical parametric oscillator, after a cascaded second-order nonlinear interaction in the crystal, is significantly squeezed. The maximum measured squeezing in our device is 30% (output beam squeezing inferred: 48%). The direction of the noise ellipse depends on the cavity detuning and can be adjusted from intensity squeezing to phase squeezing.

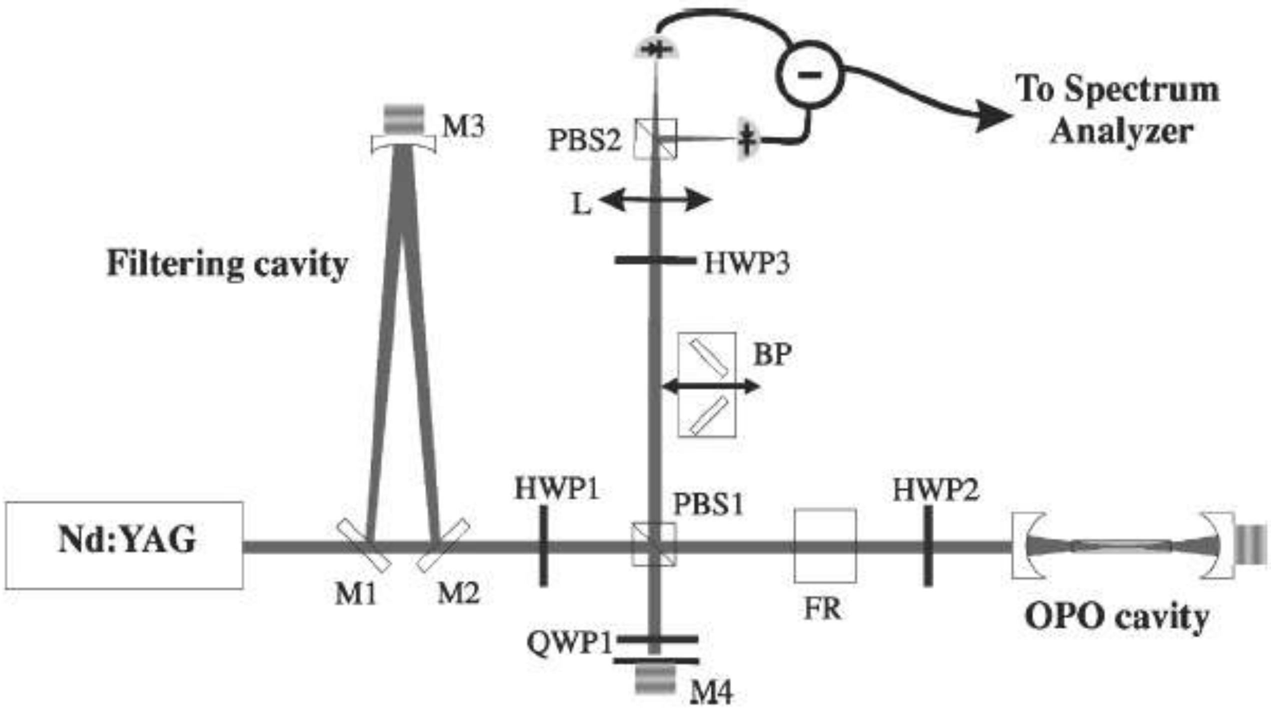




Generation of bright squeezed light at 1.06 μm using cascaded nonlinearities in a triply resonant cw periodically-poled lithium niobate optical parametric oscillator

K. S. Zhang, T. Coudreau,* M. Martinelli, A. Maître, and C. Fabre
Laboratoire Kastler Brossel, Université Pierre et Marie Curie, case 74, 75252 Paris cedex 05, France

We have used an ultralow threshold continuous-wave optical parametric oscillator (OPO) to reduce the quantum fluctuations of the reflected pump beam below the shot noise limit. The OPO consisted of a triply resonant cavity containing a periodically poled lithium niobate crystal pumped by a Nd:YAG (yttrium aluminum garnets) laser and giving signal and idler wavelengths close to 2.12 μm and a threshold as low as 300 μW. We detected the quantum fluctuations of the pump beam reflected by the OPO using a slightly modified homodyne detection technique. The measured noise reduction was 30% (inferred noise reduction at the output of the OPO 38%).



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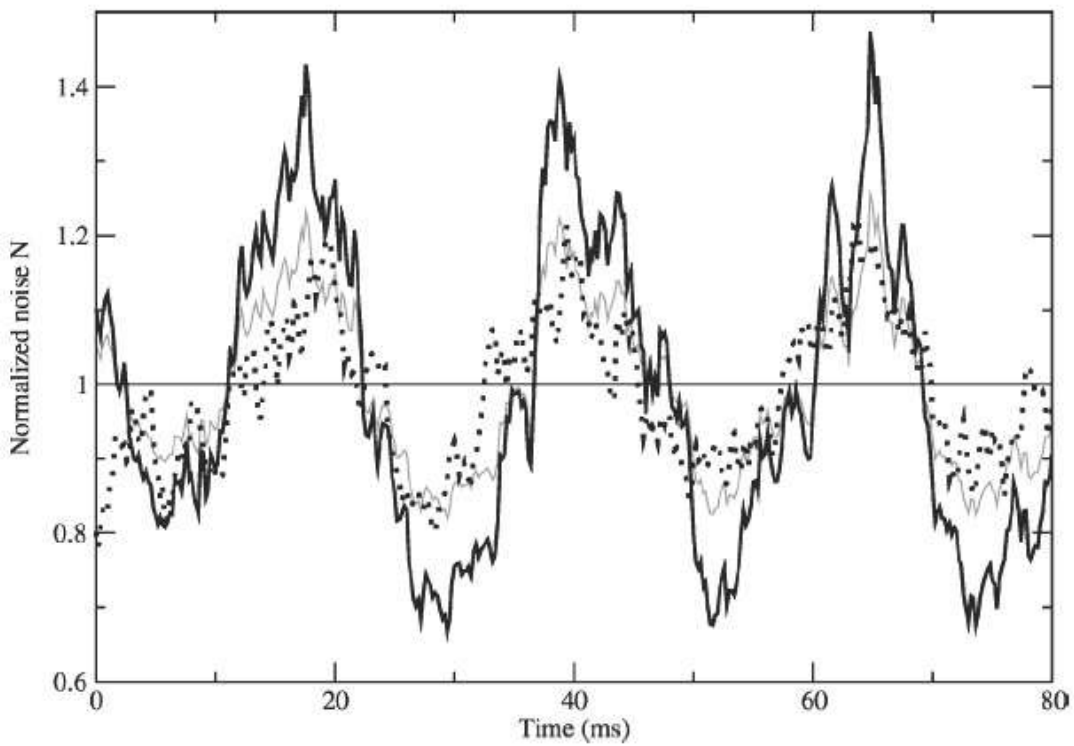

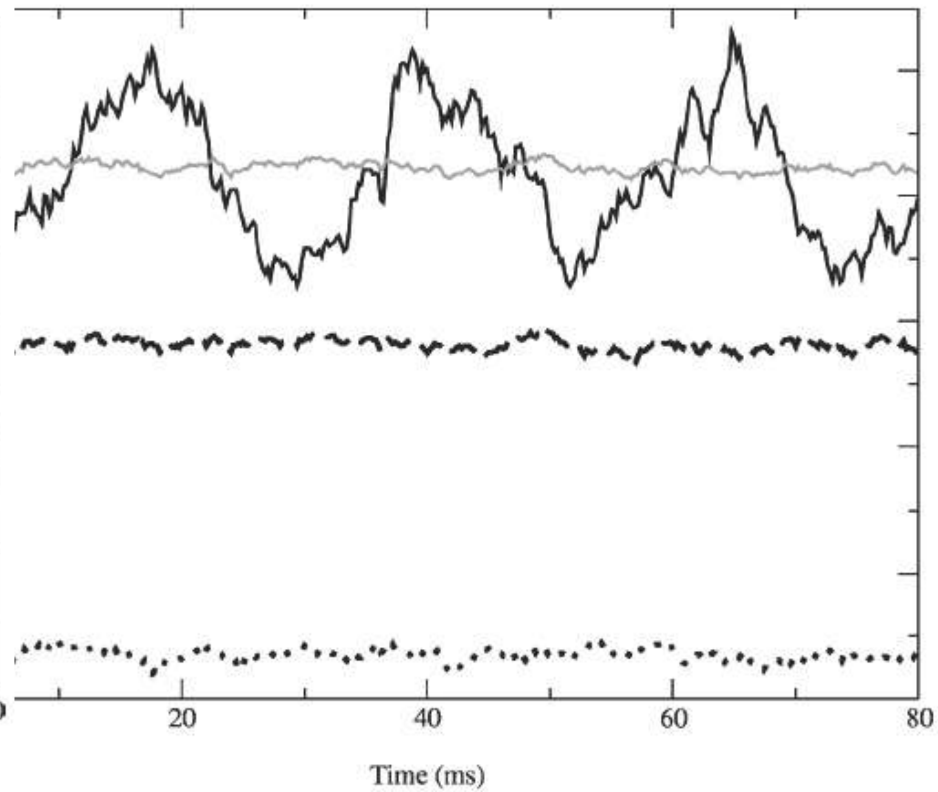

Realization of the Einstein-Podolsky-Rosen Paradox for Continuous Variables

Z. Y. Ou, S. F. Pereira, H. J. Kimble, and K. C. Peng[a]

Norman Bridge Laboratory of Physics 12-33, California Institute of Technology, Pasadena, California 91125

(Received 20 February 1992)

The Einstein-Podolsky-Rosen paradox is demonstrated experimentally for dynamical variables having a continuous spectrum. As opposed to previous work with discrete spin or polarization variables, the continuous optical amplitudes of a signal beam are inferred in turn from those of a spatially separated but strongly correlated idler beam generated by nondegenerate parametric amplification. The uncertainty product for the variances of these inferences is observed to be 0.70 ± 0.01, which is below the limit of unity required for the demonstration of the paradox.

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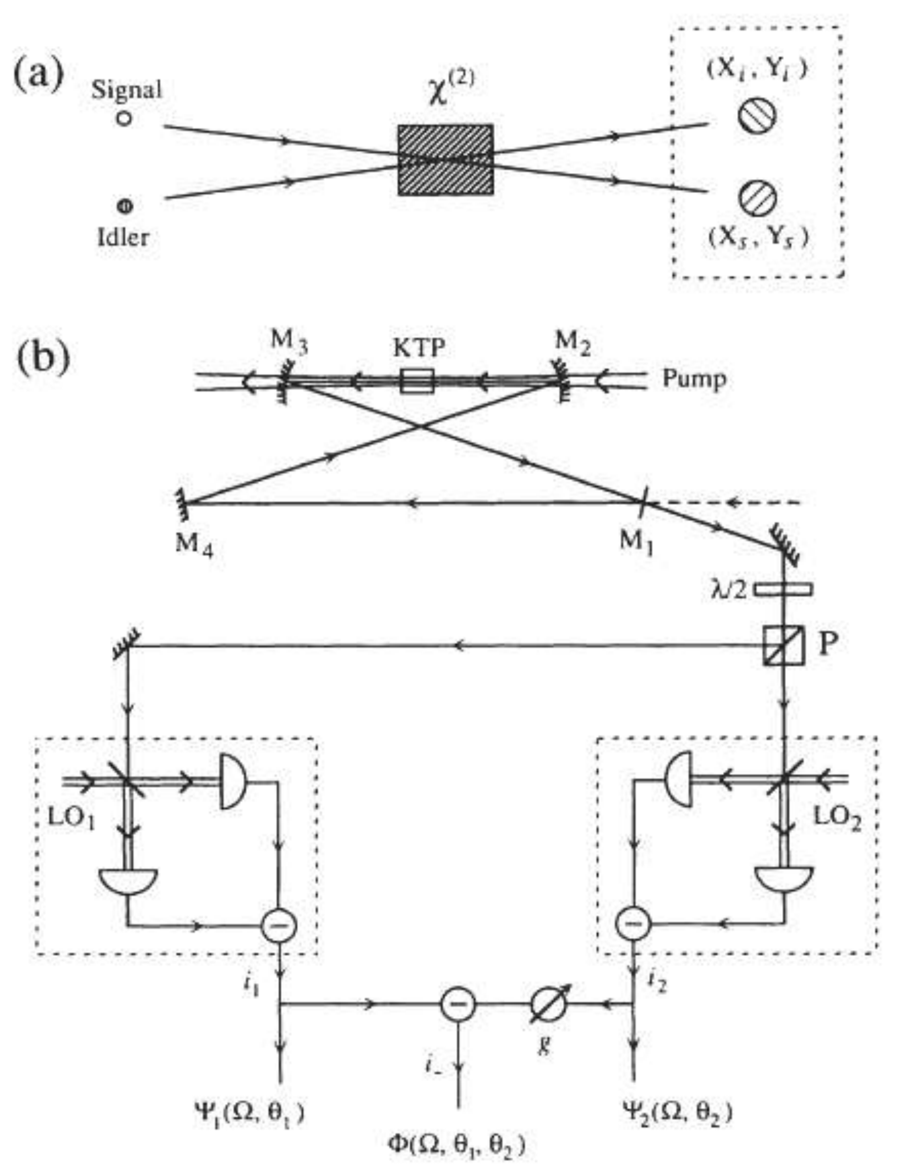
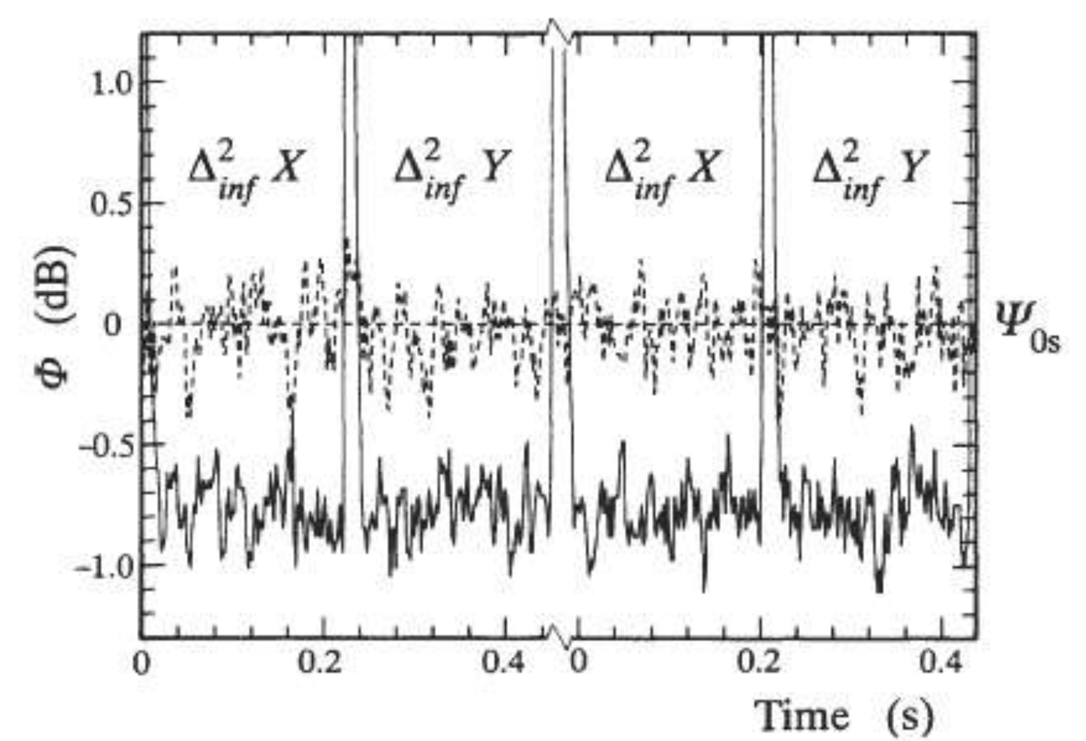


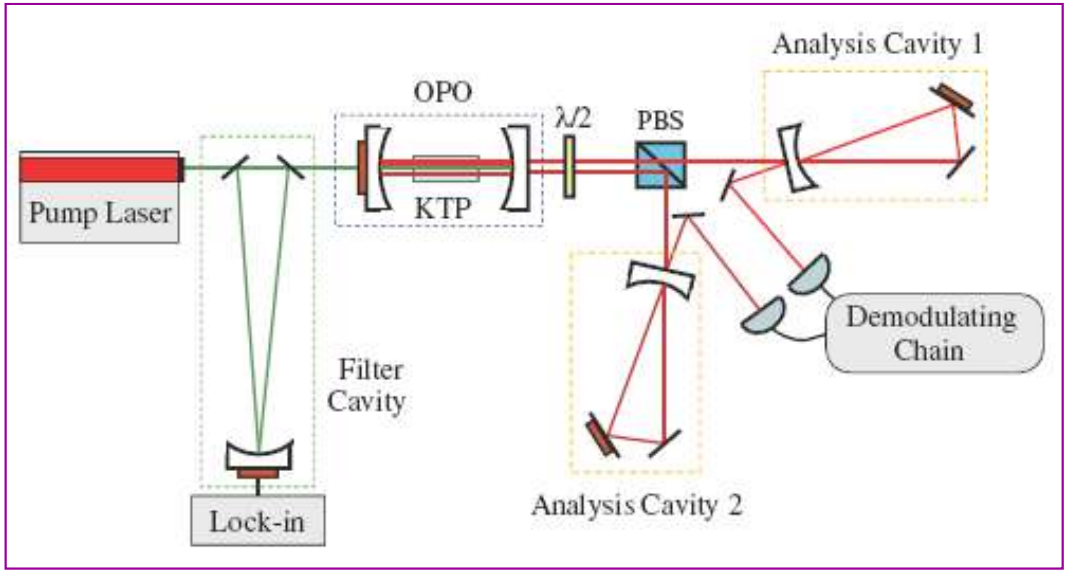


$$\Delta^2_{\rm inf} X \, \Delta^2_{\rm inf} Y = 0.70 \pm 0.01$$

Generation of Bright Two-Color Continuous Variable Entanglement

A. S. Villar, L. S. Cruz, K. N. Cassemiro, M. Martinelli, and P. Nussenzveig*
Instituto de Física, Universidade de São Paulo, Caixa Postal 66318, 05315-970 São Paulo, São Paulo, Brazil

We present the first measurement of squeezed-state entanglement between the twin beams produced in an optical parametric oscillator operating above threshold. In addition to the usual squeezing in the intensity difference between the twin beams, we have measured squeezing in the sum of phase quadratures. Our scheme enables us to measure such phase anticorrelations between fields of different frequencies. In the present measurements, wavelengths differ by ≈ 1 nm. Entanglement is demonstrated according to the Duan *et al.* criterion [Phys. Rev. Lett. **84**, 2722 (2000)] $\Delta^2 \hat{p}_- + \Delta^2 \hat{q}_+ = 1.41(2) < 2$. This experiment opens the way for new potential applications such as the transfer of quantum information between different parts of the electromagnetic spectrum.



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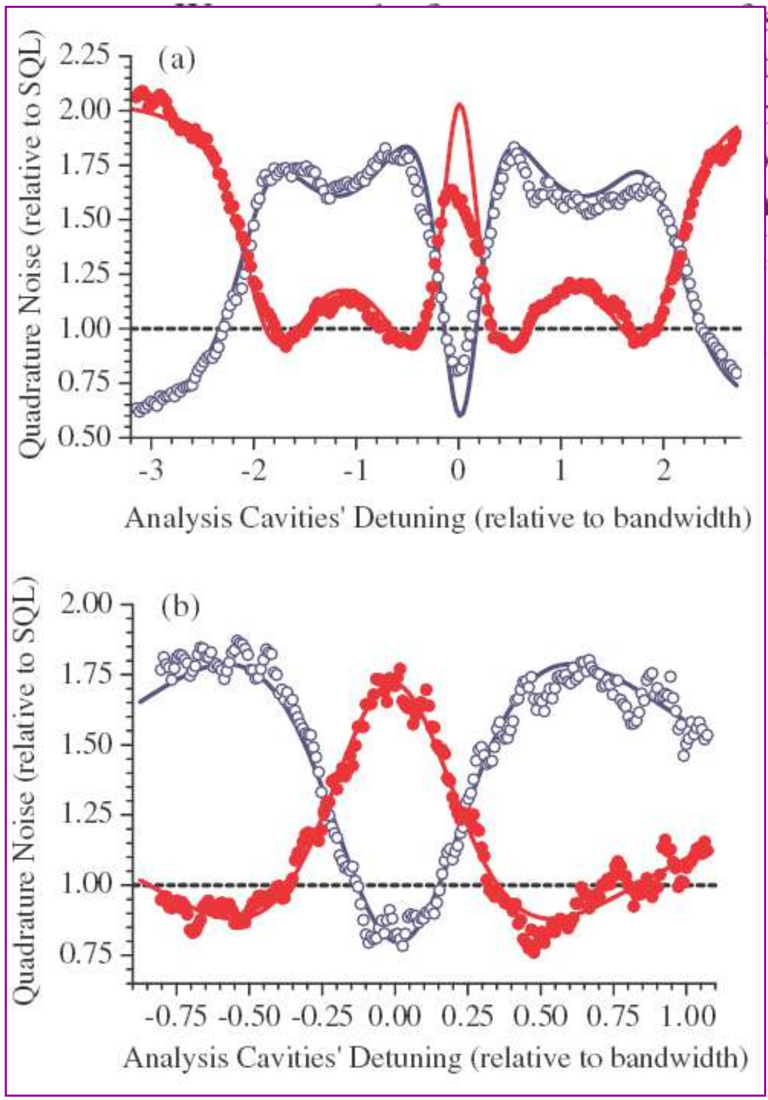

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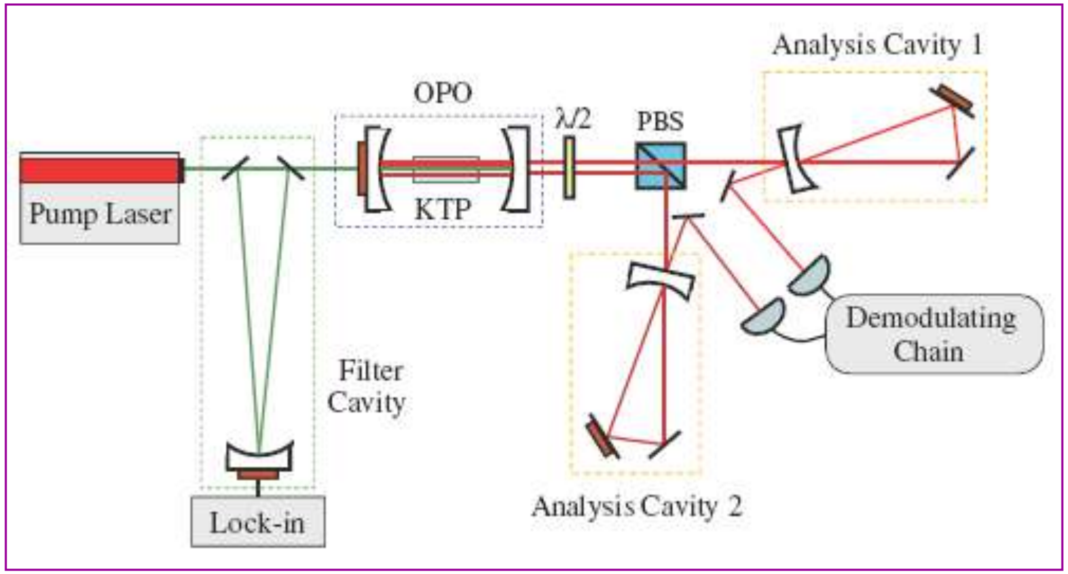
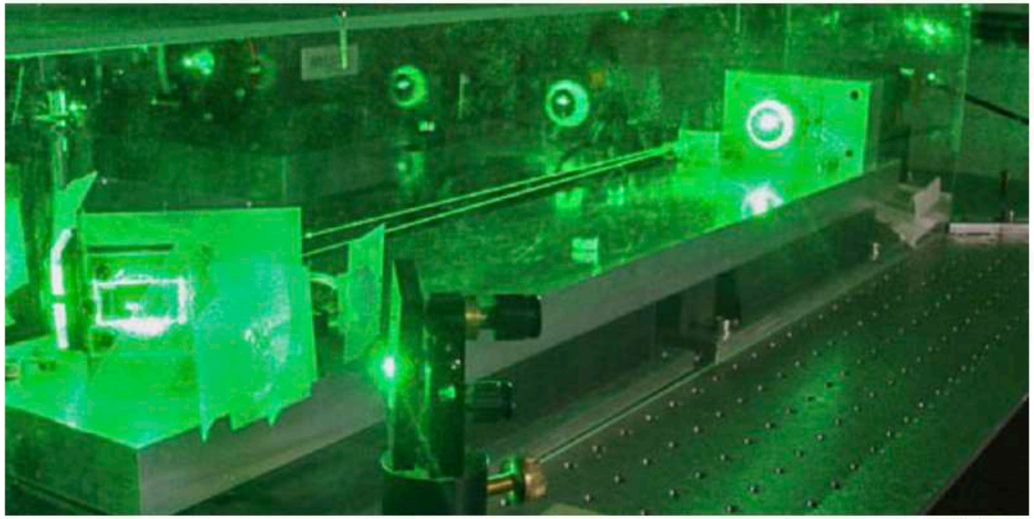


Synopsis: A Sextet of Entangled Laser Modes

August 13, 2018

Researchers have entangled six modes of a laser cavity—a record number for such a device.



M. Martinelli/University of São Paulo

An optical cavity is like a quantum guitar string. Both a plucked string and a "plucked" cavity can sustain oscillations at one or more of their resonant frequencies, or modes, for example.

Exploring six modes of an optical parametric oscillator

Luis F. Muñoz-Martínez, Felippe Alexandre Silva Barbosa, Antônio Sales Coelho, Luis Ortiz-Gutiérrez, Marcelo Martinelli, Paulo Nussenzveig, and Alessandro S. Villar

Phys. Rev. A 98, 023823 (2018)

Published August 13, 2018

Hexapartite Entanglement in an above-Threshold Optical Parametric Oscillator

F.A.S. Barbosa, A.S. Coelho, L.F. Muñoz-Martínez, L. Ortiz-Gutiérrez, A. S. Villar, P. Nussenzveig, and M. Martinelli

Phys. Rev. Lett. 121, 073601 (2018)

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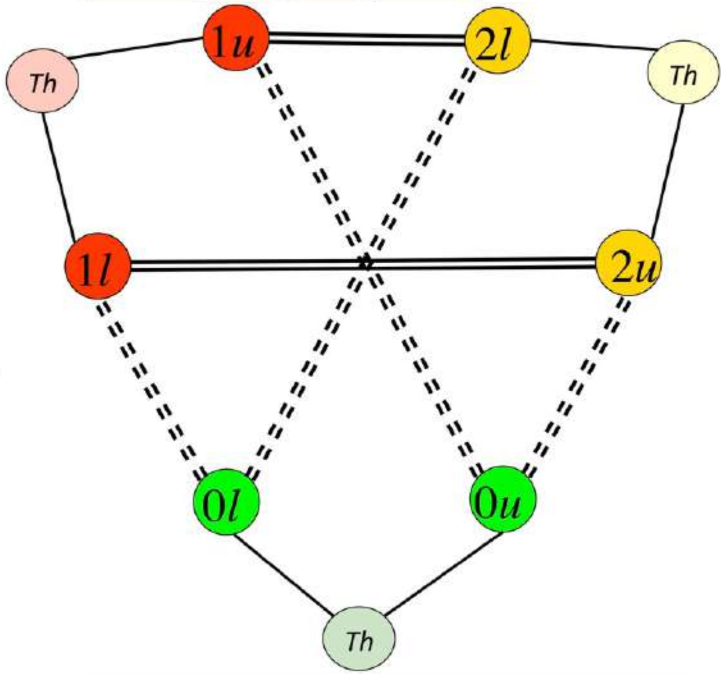

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$$\hat{H}_\chi(\Omega) = -i\hbar\frac{\chi}{\tau}\Big[\alpha^*_{\omega_0}\left(\hat{a}^{(1)}_{\omega_1+\Omega}\hat{a}^{(2)}_{\omega_2-\Omega} + \hat{a}^{(1)}_{\omega_1-\Omega}\hat{a}^{(2)}_{\omega_2+\Omega}\right) + \alpha_{\omega_1}\left(\hat{a}^{(0)\dagger}_{\omega_0+\Omega}\hat{a}^{(2)}_{\omega_2+\Omega} + \hat{a}^{(0)\dagger}_{\omega_0-\Omega}\hat{a}^{(2)}_{\omega_2-\Omega}\right) + \alpha_{\omega_2}\left(\hat{a}^{(0)\dagger}_{\omega_0+\Omega}\hat{a}^{(1)}_{\omega_1+\Omega} + \hat{a}^{(0)\dagger}_{\omega_0-\Omega}\hat{a}^{(1)}_{\omega_1-\Omega}\right) - \mathrm{h.c.}\Big]$$

$$\hat{H}_g(\Omega) = \sum_{n=0}^{2}\sum_{j=1}^{3} -\hbar g_{nj}\left[\alpha_{\omega_n}\left(\hat{a}^{(n)\dagger}_{\omega_n-\Omega}\hat{d}^{(j)\dagger}_{\Omega} + \hat{a}^{(n)\dagger}_{\omega_n+\Omega}\hat{d}^{(j)}_{\Omega}\right) + \mathrm{h.c.}\right],$$

São Paulo

Hexapartite Entanglement in an above-Threshold Optical Parametric Oscillator

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Phys. Rev. Lett. 121, 073601 (2018)

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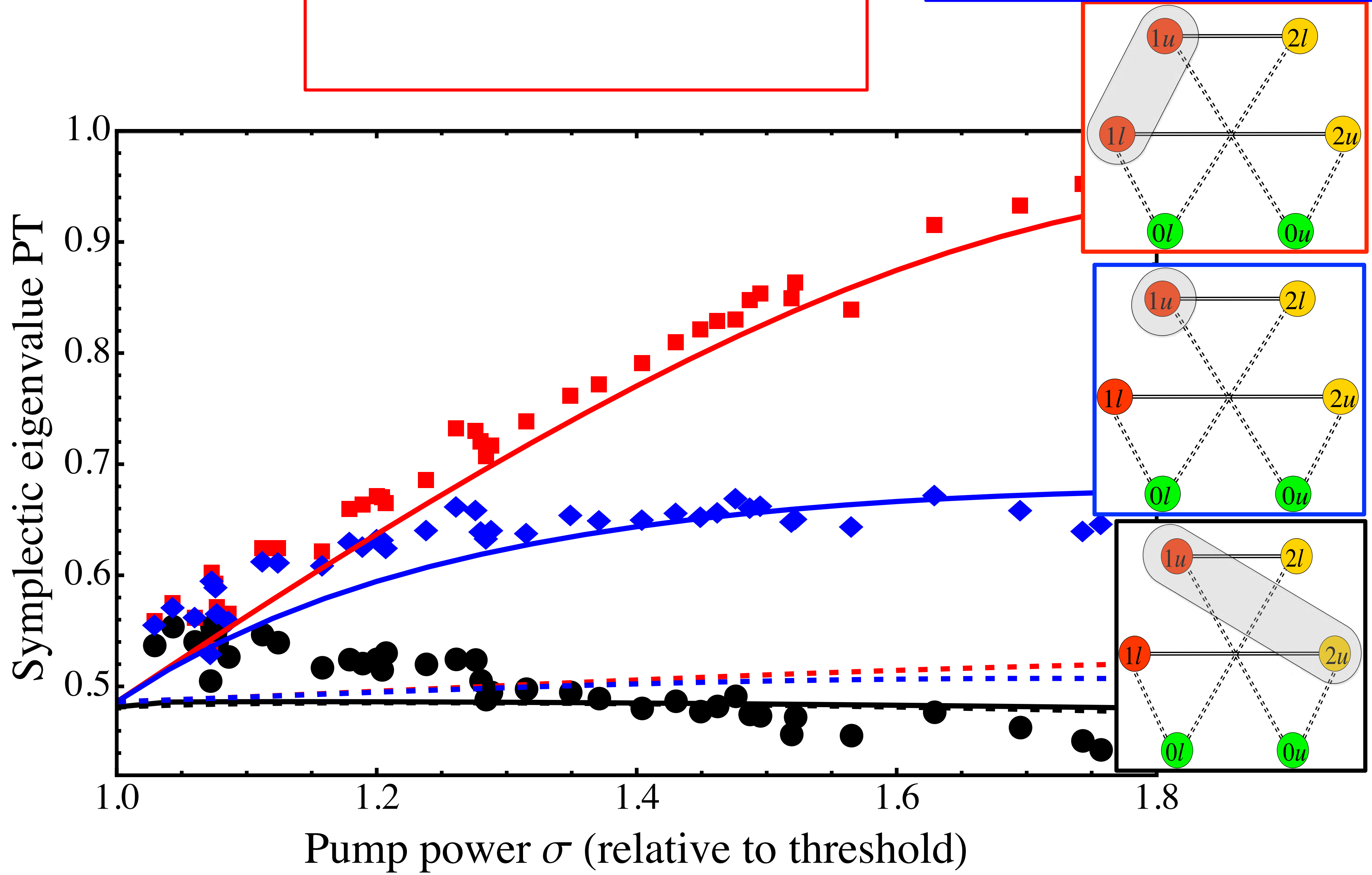
Strong entanglement involving modes connected by squeezing operators.

Sideband modes are affected by noncorrelated phase noise: phonons!

If a single mode (from the squeezed pair) is taken aside, entanglement persists. Phase noise, once again, degrades correlations.



If the modes from a two mode squeezed operation are kept aside, entanglement grows for increasing pump (i. e., growing downconverted power).

Pump coupling grows, but limited by phonon noise.