Elucidating Pion and Kaon in Continuum QCD Approach

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Physics Opportunities at an Electron-Ion Collider POETIC 10

May 5, 2023, Sao Paulo, Brazil

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QCD: Facts and Challenges

QCD is characterized by two **emergent** phenomena: **confinement** and dynamical generation of mass (**DGM**).

- > Quarks and gluons do not reach detectors.
- Formation of color-singlet bound states: "Hadrons" mesons, baryons, tetraquarks, molecules

 $\begin{aligned} \mathcal{L}_{\text{QCD}} &= \sum_{j=u,d,s,\dots} \bar{q}_j [\gamma_\mu D_\mu + m_j] q_j + \frac{1}{4} G^a_{\mu\nu} G^a_{\mu\nu}, \\ D_\mu &= \partial_\mu + ig \frac{1}{2} \lambda^a A^a_\mu, \\ G^a_{\mu\nu} &= \partial_\mu A^a_\nu + \partial_\nu A^a_\mu - g f^{abc} A^b_\mu A^c_\nu, \end{aligned}$

Emergence of hadron masses (EHM)
 " from QCD dynamics



QCD: Current Understanding and Challenges

Origins of **confinement** and **dynamical mass generation** can be traced back to the Green functions of **quarks** and **gluons**.

These emergent phenomena of **QCD**, non-existent in perturbation theory are naturally linked to the infrared enhancement of the **strong running coupling**.

The effects of of the pattern of dynamical mass generation are traceable in the Q^2 evolution of the π and K and form factors explored and planned in the JLab and the EIC.



QCD – Schwinger-Dyson Equations

Gauge Technique – Non Perturbative Solutios

- A. Salam, R. Delbourgo, Phys. Rev. 135 (1964) 6, B1398-B1427.
 DCSB Non-perturbative QED
- P.I. Fomin, V.A. Miransky, Phys. Lett. B64 (1976) 166-168.
 DCSB Non-abelian Gauge Theories
- V. Miransky, V. Gusynin, Y. Sitenko, Phys. Lett. B100 (1981) 157-162
 DCSB MT Model Vector Mesons
- P. Maris, P. Tandy, Phys. Rev. C60 (1999)



$$S(p^{2}, \mu^{2}) = \frac{Z(p^{2}, \mu^{2})}{i \gamma \cdot p + M(p^{2})}$$



π and K: Bound States and Goldstone Bosons

The pattern of **dynamical chiral symmetry breaking** and the **Bethe-Salpeter amplitude** to be computed by solving the **Bethe-Salpeter equation**.



M(k) [GeV]

π and K: Probing Quarks with Photons

In studying the **elastic form factors**, it is the **photon** which probes the **dressed guarks** inside the **bound states**, highlighting the importance of the **quark-photon vertex**.

Gauge covariance (WTI,TTI, LKFT), kinematic singularities, perturbation theory, multiplicative renormalizability

$$\begin{split} \Gamma^{L}_{\mu}(p,k,q) &= \sum_{i=1}^{4} \lambda_{i} L^{i}_{\mu}(p,k) \\ L^{1}_{\mu} &= \gamma_{\mu} \\ L^{2}_{\mu}(p,k) &= (\not\!\!\!\!p + \not\!\!\!\!k)(p+k)_{\mu} \\ L^{3}_{\mu}(p,k) &= -(p+k)_{\mu} \\ L^{4}_{\mu}(p,k) &= -\sigma_{\mu\nu} (p+k)^{\nu} \end{split}$$

1

1

1

1

$$\begin{split} r^{T}_{\mu}(p,k,q) &= \sum_{i=1}^{8} \tau_{i}(p^{2},k^{2},q^{2})T^{i}_{\mu}(p,k) \\ r^{1}_{\mu} &= p_{\mu}(k \cdot q) - k_{\mu}(p \cdot q) , \\ r^{2}_{\mu} &= \left[p_{\mu}(k \cdot q) - k_{\mu}(p \cdot q) \right] (\not \!\!\!\!/ p \!\!\!\!\!\!\!\!\!/ k) , \\ r^{3}_{\mu} &= q^{2} \gamma_{\mu} - q_{\mu} \not \!\!\!\!\!/ g , \\ r^{4}_{\mu} &= q^{2} \left[\gamma^{\mu} \left(\not \!\!\!\!\!\!\!\!\!\!/ k \!\!\!\!\!\!\!\!\!\!/ p \right) - \left(k + p \right)^{\mu} \right] \\ &\quad + 2(k - p)^{\mu} \sigma_{\nu\lambda} p^{\nu} k^{\lambda} , \\ r^{5}_{\mu} &= -\sigma_{\mu\nu} q^{\nu} , \\ r^{6}_{\mu} &= \gamma_{\mu}(p^{2} - k^{2}) + (p + k)_{\mu} \not \!\!\!\!/ g , \\ r^{7}_{\mu} &= \frac{1}{2}(p^{2} - k^{2}) \left[\gamma_{\mu}(\not \!\!\!\!\!\!\!\!/ p \!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!/ k) - (p + k)_{\mu} \right] \\ &\quad - (p + k)_{\mu} \sigma_{\nu\lambda} p^{\nu} k^{\lambda} , \end{split}$$

$$e^{-}$$

 g^{-}
 p^{-}
 p



q, C.D. Roberts 205 (2012)

El Bennich, E. Rojas 028 (2019) 07 (2023) 7, 073008

J. Roberto Lessa, F. Serna, B. El-Bennich, AB, O. Oliveira, Phys. Rev. D 107 (2023) 7, 074017

π Electromagnetic Form Factor



D 105 (2022) 7, 074013

Phys. Rev. Lett. 111 (2013) 092001

Phys. Rev. D 102 (2020) 9, 094519

The electromagnetic form factor of π is central to the Jlab and planned EIC research.

Jefferson Lab: Lattice Hadron Physics Collaboration, Frederic D.R. Bonnet, et. al. Phys. Rev. D 72 (2005) 054506

Our computation is consistent with the valence quark distribution amplitude (PDA) which is broad and concave at the hadronic scale.

The results agree with earlier computation of the electromagnetic form factors of π .

L. Chang, I.C. Cloet, C.D. Roberts, S.M. Schmidt, P. Tandy, Phys. Rev. Lett. 111 (2013) 14, 141802

K Electromagnetic Form Factor



A. Miramontes et. al., Phys. Rev. D 105 (2022) 7, 074013

J. Segovia et. Al. , Phys. Lett. B 731 (2014) 13-18

The electromagnetic form factors of π and K can be measured till Q² ~ 8 GeV² (10-40 GeV²) and 5 GeV² (10-20 GeV²) respectively in the **12 GeV** upgrade of **JLab (EIC)**.

A. Aguilar, Eur. Phys. J. A55 (2019) 10, 190

The results obtained from the **Schwinger-Dyson equations** appear robust under the calculational scheme employed and ingredients used as in the computation of the **transition form factors** and the **distribution amplitudes** of mesons.

π and K Form Factors: Probing the Standard Model

A muon with spin s has a **magnetic moment**: $\mu = g \frac{e}{2m} s$

The factor **g** is called the gyro-magnetic factor. The **Dirac equation** for a charged elementary fermion with spin 1/2 implies g = 2.

The anomalous magnetic moment is the deviation from g = 2, parameterized by $a_{\mu} = (g-2)/2$. It appears due to radiative corrections. **Renormalization** of **QED** was established in 1943 and 1947-1948 by Tomonaga, Schwinger and Feynman.

The leading contribution to a_{μ} , calculated by Schwinger in 1949, is:

$$a_{\mu} = \frac{\alpha}{2\pi}$$

The **amplitude** of a muon scattering off an external electromagnetic field A is: $(q=p_2-p_1)$:

$$\mathcal{M} = -ie\langle \mu_{p_2} | J^{\mu}(0) | \mu_{p_1} \rangle A_{\mu}(q)$$

$$\langle \mu_{p_2} | J^{\mu}(0) | \mu_{p_1} \rangle = \bar{u}_{p_2} \Gamma^{\mu}(p_2, p_1) u_{p_1}$$
$$\Gamma^{\mu}(p_2, p_1) = \left[F_D(q^2) \gamma^{\mu} + F_P(q^2) \frac{i\sigma^{\mu\nu}q_{\nu}}{2m} \right]$$

π and K Form Factors : Probing the Standard Model



π Form Factor at Large Q²



The 2015 LONG RANGE PLAN for NUCLEAR SCIENCE



The study of the **pion form factor** is one of the **flagship goals** of the **JLab 12-GeV upgrade**... regime in which the phenomenology of **QCD** begins a **transition** from **large-** to **short-distance** scale behavior.

The **pion form factor** can potentially be measured till **Q² ~ 6-8** in the **12 GeV** upgrade of the **JLab**.

Courtesy Garth Huber



π Form Factor at Large Q²

With a potential and desirable next 22-GeV upgrade of the JLab, the pion electromagnetic form factor could be measured till Q² ~ 15 GeV².

Jan. 23-25, 2023



Courtesy Garth Huber

π and K Form Factor at Large Q²

Science Question: Can we get quantitative guidance on the emergent pion mass mechanism? Key measurement: Pion form factor data for Q²: 10-40 GeV².

Science Question: How much interference is between emergent and Higgs mass mechanism? Key measurement: Kaon form factor data for Q²: 10-20 GeV².



π and K: Towards a 3-Dimensional Picture



Towards Algebraic Models



The Algebraic Model (AM)

- It retains the constant term from original models, setting it to M_q.
- There is a term linear in w with the coefficient (M_h² - M_q²). For same flavored quarks, it ceases to contribute by construction.
- ▷ There is a quadratic term w² with coefficient m_M². The condition $|M_{\bar{h}} M_{q}| \le m_{M} \le M_{\bar{h}} + M_{q}$

 $|mn|_{h} = mn|_{q} = mm|_{h} = mn|_{h}$

It guarantees the **positivity** of

$$\Lambda^2(w)$$

The quark propagator:

$$egin{aligned} S_{q(ar{h})}(k) &= [-i\gamma\cdot k + M_{q(ar{h})}]\Delta(k^2,M^2_{q(ar{h})})\ \Delta(s,t) &= (s+t)^{-1} \end{aligned}$$

Bethe-Salpeter Amplitude:

$$n_{\rm M}\Gamma_{\rm M}(k,P) = i\gamma_5 \int_{-1}^1 dw \,\rho_{\rm M}(w) [\hat{\Delta}(k_w^2,\Lambda_w^2)]^{\nu}$$
$$\hat{\Delta}(s,t) = t\Delta(s,t), \ k_w = k + (w/2)P$$

 $M_{q(\bar{h})}$ is constituent quark mass for a given flavor $n_{\rm M}$ is a normalization constant $\rho_{\rm M}(w)$ is a spectral density $\Lambda^2(w) = M_q^2 - \frac{1}{4}(1-w^2)m_{\rm M}^2 + \frac{1}{2}(1-w)(M_{\bar{h}}^2 - M_q^2)$

The Light Front Wavefunction

For a quark in pseudo-scalar meson M, the leading twist (2-particle) light front wave function, ψ_M , can be obtained via the light front projection of the meson's **BSWF**.



Bethe-Salpeter Wavefunction:

$$\chi_{\rm M}(k,P) = S_q(k+P/2)\Gamma_{\rm M}(k,P)S_{\bar{h}}(k-P/2)$$
Light Front Wavefunction
$$\psi_{\rm M}^q(x,k_{\perp}^2) = \operatorname{tr} \int_{dk_{\parallel}}^{\delta} (n \cdot k - xn \cdot P)\gamma_5 \gamma \cdot n\chi_{\rm M}(k-P/2,P)$$
n lightlike, $n^2 = 0$ and $n \cdot P = -m_{\rm M}$
BSA: $f_{\rm M}\phi_{\rm M}^q(x) = \frac{1}{16\pi^3} \int d^2k_{\perp}\psi_{\rm M}^q(x,k_{\perp}^2)$
The Algebraic Model
$$\psi_{\rm M}^q(x,k_{\perp}^2) = 16\pi^2 f_{\rm M} \frac{\nu \Lambda_{1-2x}^{2\nu}}{(k_{\perp}^2 + \Lambda_{1-2x}^2)^{\nu+1}} \phi_{\rm M}^q(x)$$

The GPDs from the overlap representation



From the PDAs to the LFWFs



 $[1 - 2.509\sqrt{x\bar{x}} + 2.025x\bar{x}]$ $\phi_K^u(x) = 18.04 \ x\bar{x}$ $[1 + 5x^{0.032}\bar{x}^{0.024} - 5.97x^{0.064}\bar{x}^{0.048}]$ Drawing upon accumulated information on the **PDAs** of π and **K**, we parameterize them as corrections to the asymptotic PDA form on the hadronic scale.

LFWF $\psi^q_{\rm M}(x,k_{\perp}^2)$

PDA

$$\frac{16\pi^2 f_{\rm M} \nu \Lambda_{1-2x}^{2\nu}}{(k_{\perp}^2 + \Lambda_{1-2x}^2)^{\nu+1}} \phi_{\rm M}^q(x)$$



From the LFWFs to the GPDs





From the GPDs to the PDFs



Completing the Cycle – Back to Form Factors

The **electromagnetic form factors** using our **algebraic model** can be obtained either through the knowledge of the **GPDs** or the direct evaluation of the **triangle diagram**.

Such an exercise provides stringent constraints on the efficacy of the **algebraic model** we have constructed by direct comparison with the refined calculation of these **form factors**.



Completing the Cycle – Back to Form Factors

We can extend this analysis of the Algebraic Model to compute the pion electromagnetic form factors to larger Q² range: 0-40 GeV² which would likely cover the photon virtualities accessible to the JLab12, JLab22 and EIC programs:



Completing the Cycle – Back to Form Factors

There is an analysis underway of the **kaon electromagnetic form factor** till **5.5 GeV**² of the data obtained in **JLab E12-09-011** experiment.

Courtesy Garth Huber

Algebraic Model results



Summary and Outlook

- The interplay of QCD akin truncations of Schwinger-Dyson equations and algebraic model based upon these studies shed important light on the internal structure of pion and kaon.
- QCD akin refined computation of pion and kaon electromagnetic form factors at low and intermediate virtualities of the probing photon in electroproduction processes:

A. Miramontes AB, K. Raya, P. Roig, Phys. Rev. D 105 (2022) 7, 074013 L. Chang, I.C. Cloët, C.D. Roberts, S.M. Schmidt, P.C. Tandy, Phys. Rev. Lett. 111 (2013) 14, 141802

Results for the pion electromagnetic form factor at large photon virtualities accessible to the potential 22GeV upgrade of the JLab and EIC are also available:

L. Chang, I.C. Cloët, C.D. Roberts, S.M. Schmidt, P.C. Tandy, Phys. Rev. Lett. 111 (2013) 14, 141802 J. Arrington, et al. (Feb 23, 2021, J.Phys. G 48 (2021) 7, 075106

More recently, pion and kaon form factors have been computed in the the time-like region

A.S. Miramontes, H. Sanchis Alepuz, R. Alkofer, Phys. Rev. D 103 (2021) 11, 116006 A.S. Miramontes, AB, Phys. Rev. D 107 (2023) 1, 014016

Summary and Outlook

Carefully constructed Algebraic Models can enable computation of the GPDs, PDFs and EFF with relative ease which is reminiscent of a contact interaction while mimicking the reliability of QCD akin refined truncations of Schwinger-Dyson equations.

L. Albino, M. Higuera, K. Raya, AB Phys. Rev. D 106 (2022) 3, 034003

- Despite these encouraging results and synergy with experimental endeavors at JLab and EIC, further improvements and extensions in the continuum QCD approach are desirable.
- Deeper research into the theoretical foundations of the truncations involved at the level of the Green functions of the fundamental degrees of freedom, i.e., quarks, gluons, as well as quark-gluon and gluon-gluon interactions continues vigorously.
- Schwinger-Dyson equations have also been of substantial success in the studies of baryons such as the transition form factors of nucleon to its excited states which is a hallmark of CLAS, CLAS12 and CLAS22 programs at JLab and hold the promise to offer a reliable tool for the future JLab and EIC era research into the heart of hadronic matter.

 $N \rightarrow N^*(1520)$ Transition Form Factors

Thank you for your attention