Evidence of the Schwinger Mechanism in QCD

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Based on:

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• Motivation - Emergence of a dynamical gluon mass

• Difficulties to generate a gluon mass → seagull cancellation

• Schwinger Mechanism in $\text{QCD}$ and the presence of massless in the fundamental vertices

• Dynamical generation of the massless poles – Bethe Salpeter equation

• Displacement of the Ward identity – the smoking gun signal of the Schwinger Mechanism in $\text{QCD}$

• Conclusions
Dynamical generation of a gluon mass

- The gauge fields (gluons) are massless at the level \( \mathcal{QCD} \) lagrangian

\[
\mathcal{L}_{YM} = -\frac{1}{4} G^a_{\mu \nu} G^a_{\mu \nu} + \frac{1}{2\xi} (\partial^\mu A^a_\mu)^2 + \bar{c}^a (-\partial^\mu D^a_{\mu \nu}) c^c
\]

where the gluonic field strength tensor

\[
G^a_{\mu \nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{abc} A^b_\mu A^c_\nu
\]

- A mass term \((m^2 A^2_\mu)\) is forbidden by gauge invariance.

- The mechanism should not generate quadratic divergences \(\Rightarrow\) to renormalize them away you must add a mass term.
QCD coupling constant

The strongest force in the nature turns off at large momentum values.

Asymptotic freedom
Objects of interest: Green's functions

- Full propagators defined as vacuum expectation value of the fields

\[
\langle \Omega | T \{ A^a_\mu(x) A^b_\nu(y) \} | \Omega \rangle := -i \Delta^{ab}_{\mu\nu}(x - y)
\]

\[
\langle \Omega | T \{ c^a(x) \bar{c}^b(y) \} | \Omega \rangle := i D^{ab}(x - y)
\]

Gluon propagator

 Ghost propagator

\[
\langle \Omega | T \{ \psi(x) \bar{\psi}(y) \} | \Omega \rangle := i S(x - y)
\]

Quark propagator
Off-shell QCD Green’s functions

Green’s functions:

Propagators and vertices

Although they are:

• Gauge-dependent

• Renormalization point ($\mu$) and scheme-dependent

However

• They capture characteristic features of the underlying dynamics, both perturbative and non-perturbative.

• When appropriately combined they give rise to physical observables.

Crucial pieces for completing the QCD puzzle
The nonperturbative QCD problems

The Green’s functions are crucial for exploring the outstanding nonperturbative problems of QCD:

Emergence of mass scale (quark and gluon masses)

Bound states formation

Confinement
Non-pertubative tools

- Non-perturbative physics requires special tools.
- For QCD we have (first principles):
  - Lattice simulations

- Space-time is discretized;
- The precision depends on the lattice spacing parameter and volume.
Insightful computational framework

Equations of motion for off-shell Green's functions.

Derived formally from the generating functional.

\[
\left( \frac{1}{\mu q \rightarrow \nu} \right)^{-1} = \left( \frac{1}{\mu q \rightarrow \nu} \right)^{-1} - \frac{1}{\mu q \rightarrow \nu}
\]

Infinite system of coupled non-linear integral equations

Inherently non-perturbative, but at the same time captures the perturbative behavior \( \rightarrow \) It accommodates the full range of physical momenta.
Difficulties with SDEs

- The need for truncations is evident

✓ No obvious expansion parameter, so, no formal way of estimating the size of the omitted terms. However, it seems that the “projection” of higher Green’s functions on the lower ones is “small”.

✓ Casual truncation interferes with the symmetries encoded in the form of the SDEs

\[ q^\mu \Pi_{\mu\nu}(q) = 0 \quad \Rightarrow \quad \Pi_{\mu\nu}(q) = \left[ g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2} \right] \Pi(q^2) \]

- Self-consistent truncation scheme must be used.

\[ \Pi_{\mu\nu}(q) \text{ is the gluon self-energy} \]

\[ \text{It is transverse} \]
Retaining only (a) and (b) is not correct even at one loop.

\[ q^\mu \Pi_{\mu\nu}(q)|_{(a)+(b)} \neq 0 \]

Adding (c) is not sufficient for a full analysis → beyond one loop.

\[ q^\mu \Pi_{\mu\nu}(q)|_{(a)+(b)+(c)} \neq 0 \]
The main problem is that fully dressed vertices satisfy STIs instead of WI.

\[ q^\alpha \Gamma_{\alpha \mu \nu}(q, r, p) = F(q) \left[ \Delta^{-1}(p)P_\nu^\alpha(p)H_{\mu \alpha}(r, p) - \Delta^{-1}(r)P_\mu^\alpha(r)H_{\nu \alpha}(p, r) \right] \]

All diagrams must conspire to maintain intact crucial properties of the theory.

If one truncates “naively”, i.e., just by dropping diagrams without a guiding principle \( \rightarrow \) one will violate the fundamental transversality property.

To avoid that \( \rightarrow \) use SDE in the Pinch Technique - Background field method (PT-BFM) formalism

Split the gauge field

\[ A^a_\mu \rightarrow B^a_\mu + Q^a_\mu \]
Pinch Technique - Background Field Method

\[ \Delta(q) = \tilde{\Delta}(q)[1 + G(q)] \]

\[ q^{\mu}[(a_1) + (a_2)]_{\mu\nu} = 0 \]

\[ q^{\mu}[(a_3) + (a_4)]_{\mu\nu} = 0 \]

\[ q^{\mu}[(a_5) + (a_6)]_{\mu\nu} = 0 \]

**Transversality** is enforced separately for gluon and ghost loops, and order by order in the “dressed-loop” expansion!

\[ q^{\mu}\tilde{\Gamma}_{\mu\alpha\beta}(q, r, p) = i\Delta_{\alpha\beta}^{-1}(r) - i\Delta_{\alpha\beta}^{-1}(p) \]

\[ q^{\mu}\tilde{\Gamma}(q, r, -p) = D^{-1}(p) - D^{-1}(r) \]

\[ q^{\mu}\tilde{\Gamma}_{\mu\alpha\beta\gamma} = f^{mne} f^{fer} \Gamma_{\alpha\beta\gamma} + f^{mne} f^{fer} \Gamma_{\beta\gamma\alpha} + f^{mne} f^{fer} \Gamma_{\gamma\alpha\beta} \]

Emergence of a dynamical gluon mass
• Gluon self-interactions generate a dynamical mass.

• Lattice QCD: The gluon propagator saturates in the deep infrared.

The gluon propagator (in Landau gauge)

\[ \Delta^{\mu\nu}(q) = \left[ g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2} \right] \Delta(q^2) \]

• IR saturation is explained through gluon mass generation

\[ \Delta(q^2) \sim \frac{1}{q^2 + m^2 + q^2 c \ln \left( \frac{q^2 + m^2}{\Lambda^2} \right) + \cdots} \]

\[ \Delta(0)^{-1} = m^2 \]

A deep mechanism is at work: **dynamics and symmetry are tightly interlocked**
• Properly regularized perturbation theory cannot generate mass at any finite order.

• Perturbative results are plagued with unphysical divergences (Landau pole).

• For example: Gluon propagator

\[ \Delta(q^2) \sim \frac{1}{q^2 \left[ 1 + c \ln \left( \frac{q^2}{\Lambda^2} \right) + \cdots \right]} \]

However, the theory cures its divergences nonperturbatively.
Question 1: How can one generate a gluon mass (saturation of the gluon propagator a zero momentum) without breaking the gauge symmetry?

It is not so simple because the QCD displays a similar cancellation that protects the photon to be massive in QED.
Seagull cancellation

\[ \tilde{\Delta}^{-1}(q^2) = \tilde{\Delta}^{-1}_0(q^2) + \tilde{\Pi}^{(1)}(q^2) + \tilde{\Pi}^{(2)}(q^2) + \tilde{\Pi}^{(3)}(q^2) \]

\[ \Delta(q) = \tilde{\Delta}(q)[1 + G(q)] \]

\[ \tilde{\Delta}^{-1}(q^2) = q^2 + i \left[ \tilde{\Pi}^{(1)}(q^2) + \tilde{\Pi}^{(2)}(q^2) + \tilde{\Pi}^{(3)}(q^2) \right] \]

PT- BFM Ward-Takahashi identities + Seagull cancellation = \[ \tilde{\Delta}^{-1}(0) = 0 \]

Seagull cancellation (valid in dimensional regularization)

\[ \int_k k^2 \frac{\partial f(k^2)}{\partial k^2} + \frac{d}{2} \int_k f(k^2) = 0 \]
Seagull identity in the Scalar QED

\[ \Pi_{\mu\nu}^{(1)}(q) = (d_1)_{\mu\nu} + (d_2)_{\mu\nu}, \]

where

\[ (d_1)_{\mu\nu} = e^2 \int_k (2k + q)_{\mu} D(k) D(k + q) \Gamma_{\nu}(-q, k + q, -k), \]

\[ (d_2)_{\mu\nu} = -2e^2 g_{\mu\nu} \int_k D(k^2), \]

Taking the limit \( q \to 0 \), we have that \( g^{\mu\nu} \) component

\[ d_1 = \frac{2e^2}{d} \int_k k_{\mu} D^2(k^2) \Gamma_{\mu}(0, k, -k), \]

\[ d_2 = -2e^2 \int_k D(k^2). \]

Thus, we find

\[ \Pi_{\mu\nu}^{(1)}(0) = -\frac{4e^2}{d} \left[ \int_k k^2 \frac{\partial D(k^2)}{\partial k^2} + \frac{d}{2} \int_k D(k^2) \right]. \]

\[ \Pi_{\mu\nu}^{(1)}(0) = 0 \]

The full vertex satisfies the WTI

\[ q^\mu \Gamma_{\mu}(q, r, -p) = D^{-1}(p) - D^{-1}(r) \]

and the WIs

\[ \Gamma_{\mu}(0, r, -r) = \frac{\partial}{\partial r^\mu} D^{-1}(r) \]

**Question 2:** How can one evade the seagull cancellation and get a gluon mass?

**Answer:** Introduce massless poles to trigger the Schwinger Mechanism


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**Gauge invariance and mass**

A gauge boson may acquire a mass, even if the gauge symmetry forbids a mass term at the level of the fundamental Lagrangian, provided that its vacuum polarization function develops a pole at zero momentum transfer.
Schwinger Mechanism

• Schwinger-Dyson Equation for the gauge boson

\[
\left( \begin{array}{l}
\mu \\
\nu
\end{array} \right) = \left( \begin{array}{l}
\mu \\
\nu
\end{array} \right) - \left( \begin{array}{l}
\mu \\
\nu
\end{array} \right)
\]

\[
\Delta^{-1}(q^2) = q^2 [1 + \Pi(q^2)]
\]

• If the vacuum polarization has a pole in \(q^2 = 0\) with positive residue \(m^2\), i.e.

\[
\Pi(q^2) = \frac{m^2}{q^2}
\]

• Then

\[
\Delta^{-1}(q^2) = q^2 + m^2
\]

\[
\Delta^{-1}(0) = m^2
\]

Dynamical mass generation requires the emergence of massless poles in the vacuum polarization \(\rightarrow\) coming from the vertices (nonpertubative origin)
Vertices with massless poles in QCD

- To evade the seagull cancellation, let us introduce poles of the type $1/q^2$ in the full three gluon vertex.

$$\Gamma_{\alpha\mu\nu}(q, r, p) = \frac{q_\alpha}{q^2} \tilde{C}_{\mu\nu}(q, r, p)$$

$$= \frac{q_\alpha}{q^2} \left[ \tilde{C}_1 g_{\mu\nu} + \tilde{C}_2 i_{\mu\nu} + \tilde{C}_3 p_{\mu} p_{\nu} + \tilde{C}_4 r_{\mu} p_{\nu} + \tilde{C}_5 p_{\mu} r_{\nu} \right]$$

$P^\alpha_\alpha'(q) P^\mu_{\mu'}(r) P^\nu_{\nu'}(p) \tilde{V}_{\alpha\mu\nu}(q, r, p) = 0$

But what happens with the Ward Takahashi identity that this vertex satisfies?
Ward identity in the presence of poles

\[ \tilde{\Pi}_{\alpha \mu \nu}(q, r, p) = \tilde{\Gamma}_{\alpha \mu \nu}(q, r, p) + \frac{q^\alpha}{q^2} \tilde{\mathcal{C}}_{\mu \nu}(q, r, p) \]

\[ q^\alpha \tilde{\Pi}_{\alpha \mu \nu}(q, r, p) = i \Delta_{\mu \nu}^{-1}(r) - i \Delta_{\mu \nu}^{-1}(p) \]

\[ \tilde{V}_{\alpha \mu \nu}(q, r, p) = \frac{q^\alpha}{q^2} \tilde{\mathcal{C}}_{\mu \nu}(q, r, p) \]

\[ \tilde{\mathcal{C}}_{\mu \nu}(q, r, p) = \tilde{\mathcal{C}}_1 g_{\mu \nu} + \cdots \]

Same WTI identity!

Using that \( \tilde{\mathcal{C}}_{\mu \nu}(0, r, -r) = 0 \), we obtain (keep only terms linear in \( q \))

\[ q^\alpha \tilde{\Gamma}_{\alpha \mu \nu}(0, r, -r) + \tilde{\mathcal{C}}_{\mu \nu}(0, r, -r) + q^\alpha \left\{ \frac{\partial}{\partial q^\alpha} \tilde{\mathcal{C}}_{\mu \nu}(q, r, p) \right\}_{q=0} = -iq^\alpha \frac{\partial \Delta_{\mu \nu}^{-1}(r)}{\partial r^\alpha} \]

Using that \( \tilde{\mathcal{C}}_{\mu \nu}(0, r, -r) = 0 \), we obtain (keep only terms linear in \( q \))

\[ \tilde{\Gamma}_{\alpha \mu \nu}(0, r, -r) = -i \frac{\partial \Delta_{\mu \nu}^{-1}(r)}{\partial r^\alpha} - \left\{ \frac{\partial}{\partial q^\alpha} \tilde{\mathcal{C}}_{\mu \nu}(q, r, p) \right\}_{q=0} \]

Ward identity suffers a displacement

\[ \mathcal{C}(r^2) := \left[ \frac{\partial \mathcal{C}_1(q, r, p)}{\partial p^2} \right]_{q=0} \]
Evading the seagull identity

\[ \tilde{\Delta}^{-1}(q^2) = \lim_{q \to 0} \tilde{\Delta}^{-1}(q^2) \]

\[ \tilde{\Delta}^{-1}(0) = \lim_{q \to 0} \tilde{\Delta}^{-1}(0) \]

\[ \Delta^{-1}(0) \sim \int k^2 \Delta^2(k) C(r^2) \]

Saturation!
Gluon mass Generated related to the massless poles

Question 3: How the QCD dynamics generates the massless poles which appear in the fundamental vertices?
**Dynamical equation for the massless pole**

Equation for the full vertex

\[ \Pi_\mu(q, r, p) \]

Substitute:

\[ \tilde{\Gamma}_\alpha^{\mu
u}(q, r, p) = \tilde{\Gamma}_\alpha^{\mu
u}(q, r, p) + \tilde{\Gamma}_\alpha^{\mu
u}(q, r, p) + \cdots \]

Take the limit

\[ q \to 0 \]

It is the Bethe-Salpeter Equation for the massless pole!

\[ C(r^2) = \alpha_s \int_k C(k^2) \Delta^2(k) \mathcal{K}_{11}(k, r) \]

Describes the formation of the dynamical formation of massless pole
Dynamical equation for the massless pole

\[ C(r^2) = \alpha_s \int_k C(k^2) \Delta^2(k) K_{11}(k, r) \]

- Eigenvalue problem
- Solution when \( \alpha_s \approx 0.3 \) @ \( \mu = 4.3 \text{ GeV} \)
- Directly connected with the gluon mass

\[ m^2 = \Delta^{-1}(0) \sim \int_k k^2 \Delta^2(k) C(r^2) \]

- Gluon propagator acquires a mass (self-stabilizing effect).

The theory solves its infrared problems

Question 4: Is there a way to confirm that the action of the Schwinger Mechanism in QCD using the lattice results?
**The signal: Displacement of the Ward identity**

**Schwinger mechanism off**

Ward Takahashi identity

\[ q^\mu \Gamma_\mu(q, r, -p) = D^{-1}(p) - D^{-1}(r) \]

pole-free

\[ q \to 0 \]

\[ p \to r \]

Taylor expansion

**Ward identity**

\[ \Gamma_\mu(0, r, -r) = \frac{\partial D^{-1}(r)}{\partial r^\mu} \]

Tensorial decomposition (Soft photon limit)

\[ \Gamma_\mu(0, r, -r) = L_{sg}(r^2) r_\mu \]

\[ L_{sg}(r^2) = 2 \frac{\partial D^{-1}(r)}{\partial r^2} \]

**Schwinger mechanism on**

\[ \Pi_\mu(q, r, -p) = \Gamma_\mu(q, r, -p) + \frac{q_\mu}{q^2} \mathcal{C}(q, r, -p) \]

pole-free

The Ward Takahashi identity **does not change**

\[ q^\mu \Gamma_\mu(q, r, -p) = q^\mu \Gamma_\mu(q, r, -p) + \mathcal{C}(q, r, -p) \]

\[ = D^{-1}(p) - D^{-1}(r) \]

\[ q \to 0 \]

\[ p \to r \]

Taylor expansion

**Ward identity**

\[ \Gamma_\mu(0, r, -r) = \frac{\partial D^{-1}(r)}{\partial r^\mu} - 2r_\mu \left[ \frac{\partial \mathcal{C}(q, r - p)}{\partial p^2} \right]_{q=0} \]

\[ L_{sg}(r^2) = 2 \frac{\partial D^{-1}(r)}{\partial r^2} - 2\mathcal{C}(r^2) \]

displacement
Displacement of the WI of the three-gluon vertex

- In the case of the full vertex three gluon vertex, we obtain

\[
C(r^2) = L_{sg}(r^2) - F(0) \left\{ \frac{\mathcal{W}(r^2)}{r^2} \Delta^{-1}(r^2) + \left[ \frac{d\Delta^{-1}(r^2)}{dr^2} \right] \right\},
\]

**displacement**

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A.C.A., M.N. Ferreira, and J.Papavassiliou, Phys. Rev. D 105, no.1, 014030 (2022);
Displacement of the WI of the three-gluon vertex

\[ C(r^2) = L_{sg}(r^2) - F(0) \left\{ \frac{W(r^2)}{r^2} \Delta^{-1}(r^2) + \left[ \frac{d \Delta^{-1}(r^2)}{dr^2} \right] \right\} , \]

Form factor of the three-gluon vertex

Displacement of the WI of the three-gluon vertex

\[ \mathbb{C}(r^2) = L_{sg}(r^2) - F(0) \left\{ \frac{\mathcal{W}(r^2)}{r^2} \Delta^{-1}(r^2) + \left[ \frac{d\Delta^{-1}(r^2)}{dr^2} \right] \right\}, \]

**Displacement**

**Ghost dressing function**

\[ D(q^2) = \frac{F(q^2)}{q^2} \]

**Ghost propagator**

O. Oliveira and P. J. Silva, PoS QCD-TNT09, 033 (2009)

Displacement of the WI of the three-gluon vertex

\[
\mathcal{C}(r^2) = L_{sg}(r^2) - F(0) \left\{ \frac{\mathcal{W}(r^2)}{r^2} \Delta^{-1}(r^2) + \left[ \frac{d\Delta^{-1}(r^2)}{dr^2} \right] \right\},
\]

O. Oliveira and P. J. Silva, PoS QCD-TNT09, 033 (2009)

Displacement of the WI of the three-gluon vertex

\[
C(r^2) = \mathcal{L}_{sg}(r^2) - F(0) \left\{ \mathcal{W}(r^2) \frac{\Delta^{-1}(r^2)}{r^2} + \left[ \frac{d\Delta^{-1}(r^2)}{dr^2} \right] \right\},
\]

- No lattice results for \( \mathcal{W}(r^2) \)
- Computed from its own SDE using lattice inputs

The result is dominated by a particular projection of the three-gluon vertex, evaluated on the lattice

\[
\overline{\Pi}_{\alpha\mu\nu}(q, r, p) = P_{\alpha'\alpha}(q)P_{\mu'\mu}(r)P_{\nu'\nu}(p)L_{sg}(s^2)
\times \left[ (q - r)^{\nu'} g^{\mu'\alpha'} + (r - p)^{\alpha'} g^{\mu'\nu'} + (p - q)^{\mu'} g^{\nu'\alpha'} \right]
\]

Model-independent determination of the displacement function

- The lattice is “blind” to specific dynamical mechanisms

Conclusions

• The apparent simplicity of the QCD Lagrangian conceals an enormous wealth of dynamical patterns, giving rise to a vast array of complex emergent phenomena.

• Gluon self-interactions generate a dynamical mass scale in the gauge sector of QCD.

• Dynamics and symmetry are tightly intertwined:

  Displacement of the Ward identity  \[\rightarrow\]  BSE amplitude for pole formation

• Smoking gun signal corroborates the action of the Schwinger mechanism in QCD and the emergence of a dynamical gluon mass.