Anomalies in Deep Virtual Compton Scattering

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Outline

• Chiral & trace anomalies in QCD
• Anomaly in polarized DIS & proton’s spin puzzle: History
• QCD Compton Scattering: Calculation of box diagrams
  ▪ Polarized case
  ▪ Unpolarized case
Recap on chiral anomaly in QCD:

- Lagrangian invariant under global chiral rotation: \( \psi \rightarrow e^{i \alpha \gamma_5} \psi \)

- Axial-vector current: \( J_5^\mu = \sum_f \bar{\psi}_f \gamma^\mu \gamma_5 \psi_f \)
Recap on chiral anomaly in QCD:

- Lagrangian invariant under global chiral rotation: $\psi \rightarrow e^{i\alpha \gamma_5} \psi$

- Axial-vector current: $J_5^\mu = \sum_f \bar{\psi}_f \gamma^\mu \gamma_5 \psi_f$

- But measure of the path integral is not invariant, which breaks the conservation of the axial current

K. Fujikawa, PRL 1979
Chiral anomaly

Anomaly equation:

$$\partial_\mu J_5^\mu = -\frac{n_f\alpha_s}{4\pi} F^{\mu\nu} \tilde{F}_{\mu\nu}$$

$$\tilde{F}^{\mu\nu} = \frac{1}{2} \epsilon^{\mu\nu\rho\sigma} F_{\rho\sigma}$$

A fundamental property of axial-vector current is the anomaly equation
Chiral anomaly

Anomaly equation:

$$\partial_\mu J^\mu_5 = -\frac{n_f\alpha_s}{4\pi} F_{\mu\nu} \tilde{F}_{\mu\nu}$$

$$\tilde{F}_{\mu\nu} = \frac{1}{2} \varepsilon^{\mu\nu\rho\sigma} F_{\rho\sigma}$$

A fundamental property of axial-vector current is the anomaly equation

Adler – Bell - Jackiw chiral anomaly

Famous example: ABJ anomaly contribution to \(\pi^0 \rightarrow 2\gamma\)

In the chiral limit, without the anomaly, \(\pi^0\) does not decay!

\(\tau = (0.84 \pm 0.04) \times 10^{-16}\) s
Anomaly equation:

\[ \partial_\mu J_5^\mu = -\frac{n_f\alpha_s}{4\pi} F^{\mu\nu} \tilde{F}_{\mu\nu} \]

\[ \tilde{F}^{\mu\nu} = \frac{1}{2} \epsilon^{\mu\nu\rho\sigma} F_{\rho\sigma} \]

A fundamental property of axial-vector current is the anomaly equation.

A perturbative solution to anomaly equation:

Calculation in off-forward kinematics \((l = p_2 - p_1)\):

\[ \langle p_2 | J_5^\mu | p_1 \rangle = \frac{n_f\alpha_s}{4\pi} \frac{i l^\mu}{l^2} p_2 | F_{a}^{\alpha\beta} \tilde{F}_{a}^{\alpha\beta} | p_1 \rangle \]

Triangle diagram is dominated by infra-red pole.
**Chiral anomaly**

**Axial Form Factors:**

\[
\langle P_2 | J_5^\mu | P_1 \rangle = \bar{u}(P_2) \left[ \gamma^\mu \gamma_5 g_A(l^2) + \frac{l^\mu \gamma_5}{2M} g_P(l^2) \right] u(P_1)
\]

A fundamental property of axial vector current is the anomaly equation:

\[g_A(0) = \Delta \Sigma \]  
Fraction of proton spin carried by quarks

**Massless pole in pseudo scalar Form Factor?**  
\[g_P(l^2) \sim \frac{1}{l^2}\]

**A perturbative solution**

Calculation in off-forward kinematics \((l = p_2 - p_1)\):

\[
\langle p_2 | J_5^\mu | p_1 \rangle = \frac{n_f \alpha_s}{4\pi} \left( \frac{i l^\mu}{l^2} \right) p_2 | F_{a}^{\alpha\beta} \tilde{F}_{a}^{\alpha\beta} | p_1 \rangle
\]

Triangle diagram is dominated by infra-red pole
Chiral anomaly

**Axial Form Factors:**

\[
\langle p_2 | J_5^\mu | p_1 \rangle = \bar{u}(p_2) \left[ \gamma^\mu \gamma_5 g_A(l^2) + \frac{l^\mu \gamma_5}{2M} g_P(l^2) \right] u(p_1)
\]

A fundamental property of axial-vector current is the anomaly equation

A perturbative solution: \( g_A(0) = \Delta \Sigma \) - Fraction of proton spin carried by quarks

**Massless pole in pseudo scalar Form Factor?** \( g_P(l^2) \approx \frac{1}{l^2} \) - \xmark

In QCD, we expect: \( g_P(l^2) \sim \frac{1}{l^2 - m_{\eta'}^2} \)

**Calculation in off-forward kinematics** \((l = p_2 - p_1)\):

\[
\langle p_2 | J_5^\mu | p_1 \rangle = \frac{n_f \alpha_s}{4\pi} \frac{i l^\mu}{l^2} p_2 \left| F_{a \alpha \beta} \tilde{F}_{a \alpha \beta} \right| p_1
\]

Triangle diagram is dominated by infra-red pole

eta meson mass generation
Chiral anomaly

Taking divergence of axial-vector matrix element:

\[
2Mg_A(l^2) + \frac{l^2}{2M} g_P(l^2) = \frac{i\langle P_2| \frac{n_f \alpha_s}{4\pi} F \tilde{F}(l^2)|P_1 \rangle}{\bar{u}(P_2)\gamma_5 u(P_1)}
\]

Pole cancellation at Form Factor level:

\[
\frac{g_P(l^2)}{2M} = -\frac{2Mg_A(l^2)}{l^2} + \frac{i\langle P_2| \frac{n_f \alpha_s}{4\pi} F \tilde{F}(l^2)|P_1 \rangle}{l^2\bar{u}(P_2)\gamma_5 u(P_1)}
\]

\[
\Rightarrow -\frac{i}{l^2} \left( \frac{\langle P_2| \frac{n_f \alpha_s}{4\pi} F \tilde{F}|P_1 \rangle}{\bar{u}(P_2)\gamma_5 u(P_1)} \big|_{l^2=0} - \frac{\langle P_2| \frac{n_f \alpha_s}{4\pi} F \tilde{F}|P_1 \rangle}{\bar{u}(P_2)\gamma_5 u(P_1)} \right)
\]

Same pole as what one naively gets from perturbation theory

In QCD, we expect:

\[
g_P(l^2) \sim \frac{1}{l^2 - m_{\eta}^2}
\]

Massless pole in pseudo scalar Form Factor? No.
Recap on trace anomaly in QCD:

- Lagrangian invariant under scale transformation: $x^\mu \rightarrow e^\sigma x^\mu \quad \phi \rightarrow e^{-D\sigma} \phi$

- Dilatation current: $D^\mu = \Theta^{\mu\nu} x_\nu$

**Energy Momentum Tensor (EMT)**

$$\Theta^{\mu\nu} = -F^{\mu\lambda} F_{\lambda}^\nu + \frac{\eta^{\mu\nu}}{4} F^2 + i\bar{\psi} \gamma^\mu (\vec{D}^\nu) \psi$$
Recap on trace anomaly in QCD:

- Lagrangian invariant under scale transformation
  \[ x^\mu \rightarrow e^\sigma x^\mu \quad \phi \rightarrow e^{-D\sigma} \phi \]

- Dilatation current:
  \[ D^\mu = \Theta^{\mu\nu} x_\nu \]
  \( \Theta^{\mu\nu} \): Energy Momentum Tensor (EMT)

- Conformal symmetry explicitly broken by quantum effects
  \[ \partial_\mu D^\mu = \Theta_\mu^\mu \neq 0 \]
Recap on trace anomaly in QCD:

- A quantum anomaly in the trace of its energy momentum tensor (conformal anomaly) breaks conformal invariance

**Trace anomaly:**

\[
\Theta^\mu_\mu = \frac{\beta(g)}{2g} F^{\mu\nu} F_{\mu\nu}
\]

\(\Theta^{\mu\nu}:\) Energy Momentum Tensor (EMT)
Recap on trace anomaly in QCD:

**Trace anomaly**: A quantum anomaly in the trace of its energy momentum tensor (conformal anomaly) breaks conformal invariance.

**Trace anomaly** in QCD:

**Fundamentally important in QCD**: Trace anomaly is the origin of hadron masses.

\[
\Theta^\mu_\mu = \frac{\beta(g)}{2g} F^{\mu\nu} F_{\mu\nu}
\]

Efforts detailed in a decade worth of reports:
Recap on trace anomaly in QCD:

- A quantum anomaly in the trace of its energy momentum tensor (conformal anomaly) breaks conformal invariance

Trace anomaly:

\[ \Theta_\mu^\nu = \frac{\beta(g)}{2g} F^{\mu\nu} F_{\mu\nu} \]

\( \Theta^{\mu\nu} \): Energy Momentum Tensor (EMT)

A perturbative solution to anomaly equation:

Calculation in off-forward kinematics \((l = p_2 - p_1)\):

\[
\langle p_2 | \Theta_{\text{QED}}^{\mu\nu} | p_1 \rangle = -\frac{e^2}{24\pi^2 l^2} \left( p^\mu p^\nu + \frac{l_\mu l_\nu - l^2 g^{\mu\nu}}{4} \right) \langle p_2 | F^{\alpha\beta} F_{\alpha\beta} | p_1 \rangle
\]

Triangle diagram is dominated by infra-red pole
Recap on trace anomaly in QCD:

- A quantum anomaly in the trace of its energy momentum tensor (conformal anomaly) breaks conformal invariance.

Trace anomaly:

- Energy Momentum Tensor (EMT)

A perturbative solution to anomaly equation:

\[ \langle p_2 | \Theta_{\mu\nu}^{\mu\nu} | p_1 \rangle = \frac{1}{M} \bar{u}(p_2) \left[ P^{\mu} P^{\nu} A_f + (A_f + B_f) \frac{P^{(\mu \sigma \nu \rho)} p_{\rho}}{2} + \frac{D_f}{4} (l^{\mu} l^{\nu} - g^{\mu\nu} l^2) + M^2 \tilde{C}_f g^{\mu\nu} \right] u(p_1) \]

Massless poles in Gravitational Form Factors:

\[ A_f(l^2), B_f(l^2), D_f(l^2) \sim \frac{1}{l^2} \]

Calculation in off-forward kinematics \((l = p_2 - p_1)\):

\[ \langle p_2 | \Theta_{\mu\nu}^{\mu\nu} | p_1 \rangle = -\frac{e^2}{24\pi^2 l^2} \left( p^{\mu} p^{\nu} + \frac{l^{\mu} l^{\nu} - l^2 g^{\mu\nu}}{4} \right) \langle p_2 | F^{\alpha\beta} F_{\alpha\beta} | p_1 \rangle \]

Triangle diagram is dominated by infra-red pole.
Recap on trace anomaly in QCD:

- Trace anomaly: A quantum anomaly in the trace of its energy momentum tensor (conformal anomaly) breaks conformal invariance.

- Trace anomaly: Energy Momentum Tensor (EMT)

A perturbative solution to anomaly equation:

- Giannotti, Mottola (2009)

\[
\langle p_2 | \Theta^{\mu\nu}_{f} | p_1 \rangle = \frac{1}{M} \bar{u}(p_2) \left[ P^\mu P^\nu A_f + \left( A_f + B_f \right) \frac{P^{(\mu}i_{\sigma})^\nu\rho l_\rho}{2} + \frac{D_f}{4} \left( l^\mu l^\nu - g^\mu\nu l^2 \right) + M^2 \bar{C}_f g^{\mu\nu} \right] u(p_1)
\]

- Calculation in off-forward kinematics

- Triangle diagram is dominated by infra-red pole

In QCD, we expect:

\[
\frac{1}{l^2} \to \frac{1}{l^2 - m_G^2}
\]

- Infrared mass generations

- Fujita, Hatta, Sugimoto, Ueda (2022)

- Mamo, Zahed (2019)
Anomaly in polarized DIS: History

The role of chiral anomaly in polarized DIS is a well-known old story
Anomaly in polarized DIS: History

Polarized DIS & proton’s spin puzzle:

\[ g_1 \text{ can be extracted from longitudinal double spin asymmetry:} \]

\[ A_{LL} = \frac{\mu^\uparrow p^\uparrow - \mu^\uparrow p^\uparrow}{\mu^\uparrow p^\uparrow + \mu^\uparrow p^\uparrow} \approx \left(1 + \frac{\sigma_L}{\sigma_T}\right) \frac{2x g_1}{F_2} \]

First moment of \( g_1 \):

\[
\int_0^1 dx g_1(x) = \frac{1}{9} (\Delta u + \Delta d + \Delta s) + \frac{1}{12} (\Delta u - \Delta d) + \frac{1}{36} (\Delta u + \Delta d - 2\Delta s) + \mathcal{O}(\alpha_s)
\]

\[ g_A(0) = \Delta \Sigma \text{ : Fraction of proton spin carried by quarks} \]

Deep inelastic scattering (DIS) experiments showed that quarks carry only about 30% of the proton’s spin: \( \Delta \Sigma \approx 0.32 \), which is much smaller than predicted by the quark model \( \Delta \Sigma \sim 1 \) - spin puzzle
Anomaly in polarized DIS: History

Polarized DIS & proton's spin puzzle:

\[ g_1 \text{ can be extracted from longitudinal double spin asymmetry:} \]

\[ A_{LL} = \frac{\mu^\uparrow p^\uparrow - \mu^\uparrow p^\downarrow}{\mu^\uparrow p^\uparrow + \mu^\uparrow p^\downarrow} \approx \left(1 + \frac{\sigma_L}{\sigma_T}\right) \frac{2xg_1}{F_2} \]

First moment of \( g_1 \):

\[
\int_0^1 dx g_1(x) = \frac{1}{9}(\Delta u + \Delta d + \Delta s) + \frac{1}{12}(\Delta u - \Delta d) + \mathcal{O}(\alpha_s)
\]

Calculate "box diagram", which is a controversial diagram
Anomaly in polarized DIS: History

One-loop correction to $g_1$ (gluon channel):

$$g_1(x) \sim \frac{\alpha_s}{2\pi} \left( \ln \frac{Q^2}{m_q^2} \Delta P_{qq}(x) + \delta C_{qq}(x) \right) \otimes \Delta G(x)$$

Polarized DGLAP splitting function: $\Delta P_{qq}(x) = 2x - 1$

Hard coefficient function (mass regularization):

$$\delta C_{qq}(x) = (2x - 1) \left( \ln \frac{1-x}{x} - 1 \right) + 2(1-x)$$

A lot of controversy over this term in the past

(same result in DR)
One-loop correction to $a_2$ (gluon channel):

The “1 − $x$” comes from the infrared region of the box diagram:

\[
(1 - x) \int_0^{Q^2} dk^2 \frac{m_q^2}{(k^2 + m_q^2)^2} = \text{finite!}
\]

But the coefficient function is supposed to be dominated by UV physics ...

Polarized DGLAP splitting function: $\Delta P_{qq}(x) = 2x - 1$

Hard coefficient function (mass regularization):

\[
\delta C_{qq}(x) = (2x - 1) \left( \ln \frac{1 - x}{x} - 1 \right) + 2(1 - x)
\]

A lot of controversy over this term in the past
Anomaly in polarized DIS: History

A solution of spin problem?

The "1 - x" comes from the infrared region of the box diagram.

One-loop correction to α_s (gluon channel):

\[ p = xP \]

But the coefficient function is supposed to be dominated by large x.

Consider this "anomalous" contribution as a part of "intrinsic spin":

\[ \Delta \tilde{\Sigma} = \Delta \Sigma + \frac{n_f \alpha_s}{2\pi} \Delta G \]

Expect \( \Delta \tilde{\Sigma} \sim 1 \)

If \( \Delta G \) is large & positive, this can explain the smallness of \( \Delta \Sigma \)!

THE ROLE OF THE AXIAL ANOMALY IN MEASURING SPIN-DEPENDENT PARTON DISTRIBUTIONS

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THE ANOMALOUS GLUON CONTRIBUTION TO POLARIZED LEPTOPRODUCTION

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Anomaly in polarized DIS: History

**Critique 1**

Gluonic contribution to $g_1$ and its relationship to the spin-dependent parton distributions

Geoffrey T. Bodwin and Jianwei Qiu*

usual forms of the quark sum rules. We conclude that the size of the gluonic contribution to the first moment of $g_1$ is entirely a matter of the convention used in defining the quark distributions.

Polarized DGLAP splitting function: $\Delta P_{qq}(x) = 2x - 1$

Hard coefficient function (mass regularization):

$$\delta C_{qq}(x) = (2x - 1) \left( \ln \frac{1 - x}{x} - 1 \right) + 2(1 - x)$$

A lot of controversy over this term in the past
Anomaly in polarized DIS: History

Critique 2

Calculation in forward kinematics: No infrared pole!

Calculation in off-forward kinematics \((l = p_2 - p_1)\):

\[
(p_2|J^\mu_5|p_1) = \frac{n_f \alpha_s}{4\pi} \frac{il^\mu}{l^2} p_2|F_{\alpha\beta}^a \tilde{F}_{\alpha\beta}^a|p_1
\]

Triangle diagram is dominated by infra-red pole

Polarized DGLAP splitting function:

\[
(1 - x) \int_0^{Q^2} dQ^2 \rho_{1q^2} \rho_{2q^2} = \frac{1}{2p^+} \langle p, \pm | j_{5+}^+ | p, \pm \rangle = \mp \frac{\alpha_s N_f}{2\pi}
\]

(Carlitz, Collins, Mueller)

But the coefficient function is supposed to be infrared safe?

THE \(g_1\) PROBLEM: DEEP INELASTIC ELECTRON SCATTERING AND THE SPIN OF THE PROTON*

R.L. JAFFE and Aneesh MANOHAR**

not local in the sense required. It is therefore not necessarily protected from infrared effects, in particular from the non-perturbative effects which give the \(\eta'\) a mass*.

regularization-independent. In QCD, the pole at \(l^2 = 0\) is unphysical and is cancelled by non-triangle contributions to the matrix element of \(A_\mu^0\). With the aid

\[\text{Jaffe, Manohar (1990)}\]
Anomaly in polarized DIS: History

Critique 2

Calculation in forward kinematics: (\(l = p_2 - p_1\)):

\[
\langle p_2 | J_5^\mu | p_1 \rangle = \frac{n_f \alpha_s}{4\pi} \frac{il^\mu}{l^2} p_2 | F_{a\beta} \tilde{F}_{a\beta} | p_1 \rangle
\]

Triangle diagram is dominated by infra-red pole

Box diagram can be viewed as a non-local generalization of triangle diagram

If triangle is dominated by anomaly pole, trace of that should be visible in box diagram

Calculation in off-forward kinematics (\(l = p_2 - p_1\)):

\[
\langle p_2 | J_5^\mu | p_1 \rangle = \frac{n_f \alpha_s}{4\pi} \frac{il^\mu}{l^2} p_2 | F_{a\beta} \tilde{F}_{a\beta} | p_1 \rangle
\]

Polarized DGLAP splitting function:

Hard coefficient function (mass regularization):

A lot of controversy over this term in the past (Carlitz, Collins, Mueller)

But the coefficient function is supposed to be

Not local in the sense required. It is therefore not necessarily protected from infrared effects, in particular from the non-perturbative effects which give the \(q^\prime\) a mass*.

Regularization-independent. In QCD, the pole at \(l^2 = 0\) is unphysical and is cancelled by non-triangle contributions to the matrix element of \(A^\mu\). With the aid

R.L. JAFFE and Aneesh MANOHAR**
Imprint of Anomalies in DIS

First calculation of box diagram with $l^2 \neq 0$:

Anomaly equation:

The role of the chiral anomaly in polarized deeply inelastic scattering I: Finding the triangle graph inside the box diagram in Bjorken and Regge asymptotics

Andrey Tarasov\textsuperscript{1,2} and Raju Venugopalan\textsuperscript{3}

The role of the chiral anomaly in polarized deeply inelastic scattering II: Topological screening and transitions from emergent axion-like dynamics

Andrey Tarasov\textsuperscript{1,2} and Raju Venugopalan\textsuperscript{3}

Andrey & Raju demonstrated within world-line formalism that to capture the physics of anomaly we need to calculate everything in off-forward kinematics for polarized DIS

Calculation in off-forward kinematics $(l = p_2 - p_1)$:

$$\langle p_2 | J_5^{\mu} | p_1 \rangle = \frac{n_f \alpha_s}{4\pi} \frac{il^\mu}{l^2} \langle p_2 | F_{a\alpha\beta} \tilde{F}_{a\alpha\beta} | p_1 \rangle$$

Triangle diagram is dominated by infra-red pole
Imprint of Anomalies in QCD Compton scattering

First calculation of box diagram with $l^2 \neq 0$

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arXiv: 2210.13419 (2022)

Chiral and trace anomalies in Deeply Virtual Compton Scattering

Shohini Bhattacharya,$^1,\ast$ Yoshitaka Hatta,$^{1,2,\dagger}$ and Werner Vogelsang$^{3,\ddagger}$

We explored the physics of anomaly in DVCS using Feynman-diagram approach
Imprint of Anomalies in QCD Compton scattering

Kinematics:

\[ q_1 = q + \frac{l}{2} \quad q_2 = q - \frac{l}{2} \]

\[ p_1 = p - \frac{l}{2} \quad p_2 = p + \frac{l}{2} \]

\[ t = t^2 \neq 0 \]

Calculation of imaginary part of anti-symmetric/symmetric \((\mu, \nu)\) of Compton amplitude with non-zero \(t\)
Imprint of Anomalies in QCD Compton scattering

**Kinematics:**

\[ q_1 = q + \frac{l}{2} \quad \text{and} \quad q_2 = q - \frac{l}{2} \]

Usual rationale is that keeping \( t = l^2 \neq 0 \) produces higher twist corrections \( \sim \frac{t}{Q^2} \)

But there are surprises ...

\[ p_1 = p - \frac{l}{2} \quad \text{and} \quad p_2 = p + \frac{l}{2} \]

Calculation of imaginary part of anti-symmetric/symmetric \((\mu, \nu)\) of Compton amplitude with non-zero \( t \)
Antisymmetric part of Compton amplitude

\[-\epsilon^{\alpha\beta\mu\nu} P_\beta \text{Im} T^{\text{asym}}_{\mu\nu}\]
Antisymmetric part of Compton amplitude

\[-e^{\alpha \beta \mu \nu} P_{\beta} \text{Im} T_{\mu \nu}^{\text{asy}} \approx \frac{1}{2 \pi} \frac{\alpha_s}{2} \left( \sum_j e_j^2 \right) \bar{u}(P_2) \left[ \left( \Delta P_{g g} \ln \frac{Q^2}{l^2} + \delta C_g^{\text{off}} \right) \otimes \Delta G(x_B) \gamma^\alpha \gamma_5 + \frac{l^\alpha}{l^2} \delta C_g^{\text{anom}} \otimes \tilde{F}(x_B) \gamma_5 \right] u(P_1)\]

Expected terms:

Splitting function
\[\Delta P_{g g}(\hat{x}) = 2T_R(2\hat{x} - 1)\]

Coefficient function
\[\delta C_g^{\text{off}}(\hat{x}) = 2T_R(2\hat{x} - 1) \left( \ln \frac{1}{\hat{x}(1 - \hat{x})} - 1 \right)\]

Recall: In DR, one obtains
\[\Delta P_{g g} \frac{1}{\epsilon} + \delta C_g^{\text{MS}}\]
\[\delta C_g^{\text{MS}}(\hat{x}) = 2T_R(2\hat{x} - 1) \left( \ln \frac{1 - \hat{x}}{\hat{x}} - 1 \right) + 4T_R(1 - \hat{x})\]
Imprint of Anomalies in QCD Compton scattering

Antisymmetric part of Compton amplitude

\[-\epsilon^{\mu\nu\lambda} P_\lambda \text{Im} T_{\mu\nu}^{\text{asym}} \approx \frac{1}{2} \frac{\alpha_s}{2\pi} \sum_f e_f^2 \bar{u}(P_2) \left[ \left( \Delta P_{qg} \ln \frac{Q^2}{l^2} + \delta C_g^{\text{off}} \right) \otimes \Delta G(x_B) \gamma^\sigma \gamma_5 \right] \frac{l^\alpha}{l^2} \delta C_g^{\text{anom}} \otimes \bar{F}(x_B) \gamma_5 u(P_1)\]

Pole term
In agreement with Tarasov, Venugopalan

Coefficient function \( \delta C_g^{\text{anom}}(\hat{x}) = 4T_R(1 - \hat{x}) \)

Twist-4 GPD:

\[\bar{F}(x, l^2) = \frac{iP^+}{\bar{u}(P_2)\gamma_5 u(P_1)} \int \frac{dz^-}{2\pi} e^{i x P^+ z^-} \langle P_2 | F_a^{\mu\nu}(-z^-/2) \bar{F}_a^{\mu\nu}(z^-/2) | P_1 \rangle\]

(Non-local) chiral anomaly manifests itself in high energy scattering amplitude

Same controversial “1-x” term as in previous calculations! (see earlier slide)
**Imprint of Anomalies in QCD Compton scattering**

Antisymmetric part of Compton amplitude

\[
-e^{\alpha \beta \mu \nu} P_\beta \text{Im} T^\text{asy} \approx \frac{1}{2 \pi} \sum_f e_f^2 \bar{u}(P_2) \left[ \Delta \frac{Q^2}{l^2} + \delta C_g^\text{off} \right] \otimes \Delta G(x_B) \gamma^\nu \gamma_5 + \frac{l^\alpha}{l^2} \delta C_g^\text{anom} \otimes \tilde{F}(x_B) \gamma_5 \right] u(P_1)
\]

**The QCD factorization theorem:** Collins, Freund; Ji, Osborne (1998)

\[
-e^{\alpha \beta \mu \nu} P_\beta \text{Im} T^\text{asy} = \frac{1}{2} \sum_f e_f^2 \bar{u}(P_2) \left[ \gamma^\alpha \gamma_5 (\tilde{H}_f(x_B, \xi, l^2) + \tilde{H}_f(-x_B, \xi, l^2)) + \frac{l^\alpha \gamma_5}{2M} \left( \tilde{E}^\text{bare}_{x_B, \xi, l^2} + \tilde{E}^\text{bare}_{-x_B, \xi, l^2} \right) \right] u(P_1)
\]

\[+ O(\alpha_s) \rightarrow O(1/Q^2),\]

Twist-2 GPDs to all orders

But no suppression in \(1/Q^2\)!

(Non-local) chiral anomaly manifests itself in high energy scattering amplitude & apparently breaks QCD factorization
Imprint of Anomalies in QCD Compton scattering

The QCD factorization theorem: Collins, Freund; Ji, Osborne (1998)

Twist-2 GPDs to all orders

(Non-local) chiral anomaly manifests itself in high energy scattering amplitude & apparently breaks QCD factorization
Imprint of Anomalies in QCD Compton scattering

Antisymmetric part of Compton amplitude

Elusive pole

ONE-LOOP QCD CORRECTIONS TO DEEPLY-VIRTUAL COMPTON SCATTERING: THE PARTON HELICITY-INDEPENDENT CASE

Xiangdong Ji and Jonathan Osborne

Predictions from conformal algebra for the deeply virtual Compton scattering.

A.V. Belitsky, D. Müller

NLO Corrections to Deeply-Virtual Compton Scattering

L. Mankiewicz, G. Piller, E. Stein, M. Vänttinen and T. Weigl

NLO corrections to timelike, spacelike and double deeply virtual Compton scattering.

B. Pire, L. Szymanowski and J. Wagner

Anomalous contribution to GPD $\tilde{E}$ at one loop

Collins, Freund; Ji, Osborne (1998)

Twist-2 GPDs

(Non-local chiral anomaly manifests itself in high energy scattering amplitude &)

To all orders

Twist-2 GPDs
Elusive pole

Pole was unnoticed in the GPD literature because one typically assumes

\[ l^\mu = -2\xi p^\mu \rightarrow t = l^2 = 0 \]

before loop integration

Usual rationale: Corrections supposedly higher twist

\[ \frac{t}{Q^2} \]
Pole was unnoticed in the GPD literature because one typically assumes

\[ l^\mu = -2\xi p^\mu \rightarrow t = l^2 = 0 \]

before loop integration

Usual rationale: Corrections supposedly higher twist

\[ \frac{t}{Q^2} \]

However, box diagram is power-divergent in the IR!

Still, pole was never seen before because:

\[ \langle p_2 | F_{\mu \nu} \mathcal{F}_{\mu \nu} | p_1 \rangle \propto \epsilon^{\mu \nu \alpha \beta} l_\mu p_\nu \epsilon_1^\alpha \epsilon_2^\beta \]

\[ \rightarrow 0 \quad \text{when} \quad l^\mu \propto p^\mu \]
Imprint of Anomalies in QCD Compton scattering

Perturbative calculations suggest that massless poles are induced in GPD $\tilde{E}$

However, we know there are no massless poles in axial form factor (moment of GPD $\tilde{E}$)

$$g_P(l^2) = \int dx \tilde{E}(x) \sim \frac{1}{l^2}$$

Pole contribution to GPD $\tilde{E}$ at one loop

The QCD factorization theorem: Collins, Freund; Ji, Osborne (1998)

Deeply tied to the UA(1) problem: Why is the $\eta'$ so massive (957 MeV)?

Twist-2 GPDs to all orders

(Non-local) chiral anomaly manifests itself in high energy scattering amplitude & apparently breaks QCD factorization
Imprint of Anomalies in QCD Compton scattering

The QCD factorization theorem: Collins, Freund, Ji, Osborne (1998)

\[-\epsilon^{\alpha\beta\mu\nu} P_3 \text{Im} T_{\mu\nu}^{\text{asy}} = \frac{1}{2} \sum_f e_f^2 \bar{u}(P_2) \left[ \gamma^\alpha \gamma_5 (\tilde{H}_f(x_B, \xi, l^2) + \tilde{H}_f(-x_B, \xi, l^2)) + \frac{l^\alpha \gamma_5}{2M} (\tilde{E}_f^{\text{bare}}(x_B, \xi, l^2) + \tilde{E}_f^{\text{bare}}(-x_B, \xi, l^2)) \right] u(P_1) + \mathcal{O}(\alpha_s) + \mathcal{O}(1/Q^2),\]

Twist-2 GPDs to all orders

(Non-local) chiral anomaly manifests itself in high energy scattering amplitude & apparently breaks QCD factorization
Imprint of Anomalies in QCD Compton scattering

Redefine

Perturbative pole in GPD

\[
\int \frac{dz^-}{4\pi} e^{ixp^+z^-} \langle p_2 | \bar{\psi}(-z^-/2)\gamma^+\gamma_5\psi(z^-/2) | p_1 \rangle_{\text{pole}} \sim \frac{\alpha_s}{2\pi} \frac{2il^+}{l^2} (1 - \hat{x}) \otimes \delta(1 - \hat{x}) \epsilon_1 \epsilon_2 \not{l} \not{p}
\]

Same pole in one-loop calculation!

Perhaps not an ad hoc argument!

The pole “belongs” to GPD

Chiral anomaly manifests itself in high energy scattering amplitude & apparently breaks QCD factorization

Twist-2 GPDs to all orders

The QCD factorization theorem:

Collins, Freund; Ji, Osborne (1998)
Imprint of Anomalies in QCD Compton scattering

Redefine

\[ \tilde{E}_f(x_B, l^2) + \tilde{E}_f(-x_B, l^2) = \tilde{E}_f^{\text{bare}}(x_B, l^2) + \tilde{E}_f^{\text{bare}}(-x_B, l^2) + \frac{\alpha_s}{2\pi} \frac{2M}{l^2} \delta C_g^{\text{anom}} \otimes \tilde{F}(x_B, l^2) \]

"Bare GPD" (tree level) \hspace{1cm} Perturbative pole (one loop)

Postulate that the perturbative pole cancels the pre-existing pole in "bare" GPD:

\[ \tilde{E}_f^{\text{bare}}(x_B, l^2) + \tilde{E}_f^{\text{bare}}(-x_B, l^2) \approx -\frac{\alpha_s}{2\pi} \frac{2M}{l^2} \delta C_g^{\text{anom}} \otimes \tilde{F}(x_B, l^2 = 0) \]

Postulate that the "renormalized" GPD integrates to \( g_P(l^2) \):

\[ g_P(l^2) = \sum_f \int_{-1}^{1} dx \tilde{E}_f(x, \xi, l^2) = \sum_f \int_{0}^{1} dx (\tilde{E}_f(x, \xi, l^2) + \tilde{E}_f(-x, \xi, l^2)) \]
Imprint of Anomalies in QCD Compton scattering

Redefine

\[ \tilde{E}_f(x_B, l^2) + \tilde{E}_f(-x_B, l^2) = \tilde{E}^\text{bare}_f(x_B, l^2) + \tilde{E}^\text{bare}_f(-x_B, l^2) + \frac{\alpha_s}{2\pi} \frac{2M}{l^2} \delta C^\text{anom}_g \otimes \tilde{F}(x_B, l^2) \]

"Bare GPD" (tree level) \quad Perturbative pole (one loop)

Pole cancellation at \( \int dx \)

We find:

\[ g_P(l^2) = \frac{i}{l^2} \left( \left. \frac{P_2}{4\pi} F \hat{F} \right| P_1 \right) - \left. \left. \frac{P_2}{4\pi} F \hat{F} \right| P_1 \right|_{l^2=0} \]

See earlier slide

The QCD factorization theorem: Collins, Freund, Ji, Osborne (1998)

Twist-2 GPDs to all orders
Imprint of Anomalies in QCD Compton scattering

The QCD factorization theorem: Collins, Freund, Ji, Osborne (1998)

Twist-2 GPDs to all orders

"We demonstrate that the dynamical interplay between the physics of the anomaly, and that of the isosinglet pseudoscalar \( U_A(1) \) sector of QCD resolves both problems simultaneously: the lifting of the \( \eta' \) pole by topological mass generation of the \( \eta' \) and the cancellation of the anomaly pole"

- Tarasov, Venugopalan

See also Jaffe Manohar, 1990
Imprint Equivalence with the iso vector caseettinging

**Example: Iso vector axial current**

\[ J_{5a}^\alpha = \sum_q \bar{q} \gamma^\alpha \gamma_5 \frac{\tau^a}{2} q \]

**Axial Form Factors:**

\[ \langle P_2 | J_{5a}^\alpha | P_1 \rangle = \bar{u}(P_2) \left[ \gamma^\alpha \gamma_5 F_A(t) + \frac{\langle l^\alpha \rangle}{2M} F_P(t) \right] \frac{\tau^a}{2} u(P_1) \]

**Current conservation (chiral symmetry) leads to:**

\[ F_P(t) \approx \frac{-2M^2 g_A^{(3)}}{t}, \quad (t \to 0) \]

where \( g_A^{(3)} = F_A(0) \approx 1.3 \) is the isovector axial coupling constant. The pole is generated by the exchange of the massless pion which is the Nambu-Goldstone boson of spontaneously broken chiral symmetry.

“We demonstrate that the dynamical interplay between the physics of the anomaly, and that of the isosinglet pseudoscalar \( U_A(1) \) sector of QCD resolves both problems simultaneously: the lifting of the \( \bar{\eta} \) pole by topological mass generation of the \( \eta' \) and the cancellation of the anomaly pole”

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Iso vector GPD:

$$F_P(t) = \int_{-1}^{1} dx \left( \tilde{E}_u(x, \xi, t) - \tilde{E}_d(x, \xi, t) \right)$$

In QCD, we expect:

$$\tilde{E}_u(x, \xi, t) - \tilde{E}_d(x, \xi, t) \sim \theta(\xi - |x|) \frac{g_A^{(3)}}{t}$$

$$\tilde{E}_u - \tilde{E}_d \sim \frac{1}{l^2 - m^2_{\eta'}}$$

(Penttinen, Polyakov, Goeke)
Imprint of Anomalies in QCD Compton scattering

Symmetric part of Compton amplitude \((\xi \neq 0)\)
Imprint of Anomalies in QCD Compton scattering

Symmetric part of Compton amplitude \((\xi \neq 0)\)

Pole! (New result)

\[
(H_f(x_B, \xi, l^2) - H_f(-x_B, \xi, l^2)) = \left( H_f^{\text{bare}}(x_B, \xi, l^2) - H_f^{\text{bare}}(-x_B, \xi, l^2) \right) \\
+ \frac{\alpha_s}{2\pi} \frac{1}{l^2} C_{\text{anom}} \otimes' \mathcal{F}(x_B, \xi, l^2)
\]

\[
(E_f(x_B, \xi, l^2) - E_f(-x_B, \xi, l^2)) = \left( E_f^{\text{bare}}(x_B, \xi, l^2) - E_f^{\text{bare}}(-x_B, \xi, l^2) \right) \\
- \frac{\alpha_s}{2\pi} \frac{1}{l^2} C_{\text{anom}} \otimes' \mathcal{F}(x_B, \xi, l^2)
\]

“Bare GPD” (tree level)

Perturbative pole (one loop)
Imprint of Anomalies in QCD Compton scattering

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\]

\[
(E_f(x_B, \xi, l^2) - E_f(-x_B, \xi, l^2)) = \left( E_f^{\text{bare}}(x_B, \xi, l^2) - E_f^{\text{bare}}(-x_B, \xi, l^2) \right) - \frac{\alpha_s}{2\pi} \frac{1}{l^2} C^{\text{anom}} \otimes' \mathcal{F}(x_B, \xi, l^2)
\]

Hatta, Zhao (2020); Radyushkin, Zhao (2021)

Twist-4 GPD:

\[
\mathcal{F}(x, \xi, l^2) = -4xP^+ M \int \frac{dz^-}{2\pi} e^{ixP^+ z^-} \langle P_2 | F^{\mu\nu}(-z^-/2) F_{\mu\nu}(z^-/2) | P_1 \rangle \frac{\bar{u}(P_2) u(P_1)}{\bar{u}(P_2) u(P_1)}
\]

“Bare GPD” (tree level)

Perturbative pole (one loop)

(Non-local) trace anomaly manifests itself in high energy scattering amplitude & apparently breaks QCD factorization
Imprint of Anomalies in QCD Compton scattering

Symmetric part of Compton amplitude \( (\xi \neq 0) \)

Pole! (New result)

\[
(H_f(x_B, \xi, l^2) - H_f(-x_B, \xi, l^2)) = \left( H_f^{\text{bare}}(x_B, \xi, l^2) - H_f^{\text{bare}}(-x_B, \xi, l^2) \right) + \frac{\alpha_s}{2\pi} \frac{1}{l^2} \mathcal{C}^{\text{anom}} \otimes' \mathcal{F}(x_B, \xi, l^2)
\]

The pole “belongs” to GPD

Perturbative pole in GPD

\[
\int \frac{dk^- d^2k_\perp}{(2\pi)^{3-2\epsilon}} \left[ \left( \gamma_\alpha(k + \frac{l}{2}) \gamma^\beta(k - \frac{p}{2}) \right) \epsilon_\alpha(p + l/2) \epsilon_\beta(p - l/2) \right] \frac{1}{(p-k)^2(1-\xi^2)} \frac{1}{1-x} \left( F_1^F \right) \]

Same pole in one-loop calculation!
Symmetric part of Compton amplitude \((\xi \neq 0)\)

Pole! (New result)

\[
\langle P_2| (\Theta)_{\alpha}^\mu |P_1 \rangle = M \left( A(t) + \frac{B(t)}{4M^2} t - \frac{3D(t)}{4M^2} t \right) \bar{u}(P_2) u(P_1) = \langle P_2 | \frac{\beta(g)}{2g} F_{\mu\nu} F_{\nu\mu} | P_1 \rangle
\]

In the absence of trace anomaly:

\[
\frac{3D(t)}{4M^2} \approx \frac{1}{t}, \quad (t \to 0)
\]

\[
(E_f(x_B, \xi, l^2) - E_f(-x_B, \xi, l^2)) = (E_f^\text{bare}(x_B, \xi, l^2) - E_f^\text{bare}(-x_B, \xi, l^2)) + \frac{\alpha_s}{2\pi} \frac{1}{l^2} C_{\text{anom}} \otimes' F(x_B, \xi, l^2)
\]

Hatta, Zhao (2020); Radyushkin, Zhao (2021)

Twist-4 GPD:

\[
\mathcal{F}(x, \xi, l^2) = -4xP^+ M \int \frac{dz}{2\pi} e^{izP^+z} \int dz' e^{iz'P^+z'} \frac{\langle P_2 | F_{\mu\nu}^a (-z'/2) F_{\mu\nu}^a (z'/2) | P_1 \rangle}{\bar{u}(P_2) u(P_1)}
\]

“Bare GPD” (tree level)

Perturbative pole (one loop)

(Non-local) trace anomaly manifests itself in high energy scattering amplitude & apparently breaks QCD factorization
Imprint of Anomalies in QCD Compton scattering

Trace of EMT: Symmetric part of Compton amplitude \((\xi \neq 0)\)

\[
\langle P_2 | (\Theta)_a^\mu | P_1 \rangle = M \left( A(t) + \frac{B(t)}{4M^2} t - \frac{3D(t)}{4M^2} t \right) \bar{u}(P_2) u(P_1) = \langle P_2 | \frac{\beta(g)}{2g} F^{\mu\nu} F_{\mu\nu} | P_1 \rangle
\]

Pole! (New result)

In the absence of trace anomaly:

\[
\frac{3D(t)}{4M^2} \approx \frac{1}{t}, \quad (t \to 0)
\]

D term

\[
\frac{3D(t)}{4M^2} \approx \frac{1}{t} \left( A(t) - \frac{\langle P_2 | \frac{\beta(g)}{2g} F^{\mu\nu} F_{\mu\nu} | P_1 \rangle}{M \bar{u}(P_2) u(P_1)} \right)
\]

Perturbative pole (one loop)

(Non-local) trace anomaly manifests itself in high energy scattering amplitude & apparently breaks QCD factorization.

Glueball mass generations
Imprint of Anomalies in QCD Compton scattering

Real part of Compton amplitude: (SB, Hatta, Vogelsang, In Preparation)
Imprint of Anomalies in QCD Compton scattering

Real part of Compton amplitude: (SB, Hatta, Vogelsang, In Preparation)

Example: Antisymmetric case

\[ \sim \frac{16\langle F \tilde{F} \rangle}{l^2(-1 + \xi^2)} \left( -(-1 + x) \ln \frac{x - 1}{x} + (x - \xi) \ln \frac{x - \xi}{x} \right) - (x \to -x) \]

Pole in real part!

\[ + \bar{\kappa}_{qg} (\hat{x}, \hat{\xi}) \ln \frac{Q^2}{l^2} \]

\[ + \bar{C}_1^{qg} (\hat{x}, \hat{\xi}) \]

Same structure for convolution!

Even in real part, same mechanism should cancel pole as in imaginary part
Imprint of Anomalies in QCD Compton scattering

Real part of Compton amplitude: (SB, Hatta, Vogelsang, In Preparation)

Example: Antisymmetric case

\[ \sim \frac{16 \langle F \tilde{F} \rangle}{l^2 (-1 + \xi^2)} \left( -(-1 + x) \ln \frac{x - 1}{x} + (x - \xi) \ln \frac{x - \xi}{x} \right) - (x \to -x) \]

\[ + \tilde{K}_{qg}(\hat{x}, \hat{\xi}) \ln \frac{Q^2}{l^2} \]

\[ + \tilde{C}_1^q(\hat{x}, \hat{\xi}) \]

Reproduced the known logarithms from literature

\[ \tilde{K}_{qg}(\hat{x}, \hat{\xi}) = \frac{2\hat{x} - 1 - \hat{\xi}^2}{2(1 - \hat{\xi}^2)^2} \ln \frac{\hat{x} - 1}{\hat{x}} - \frac{\hat{x} - \hat{\xi}}{(1 - \hat{\xi}^2)^2} \ln \frac{\hat{x} - \hat{\xi}}{\hat{x}} - (\hat{x} \to -\hat{x}) \]

Ji, Osborne; Belitsky, Mueller
Imprint of Anomalies in QCD Compton scattering

Real part of Compton amplitude: (SB, Hatta, Vogelsang, In Preparation)

Example: Antisymmetric case

\[ \sim \frac{16 \langle F \tilde{F} \rangle}{l^2 (-1 + \xi^2)} \left( -(-1 + x) \ln \frac{x - 1}{x} + (x - \xi) \ln \frac{x - \xi}{x} \right) - (x \to -x) \]

\[ + \tilde{\kappa}_{qg}(\hat{x}, \hat{\xi}) \ln \frac{Q^2}{l^2} \]

\[ + \tilde{C}^g_1(\hat{x}, \hat{\xi}) \]

Coefficient function

\[ \delta \tilde{C}^g_1(\hat{x}, \hat{\xi}) = \frac{2\hat{x} - 1 - \hat{\xi}^2}{(1 - \hat{\xi}^2)^2} \ln \frac{\hat{x} - 1}{\hat{x}} + 2 \frac{\hat{x} - \hat{\xi}}{(1 - \hat{\xi}^2)^2} \ln \frac{\hat{x} - \hat{\xi}}{\hat{x}} 
\]

\[ + \frac{2\hat{x} - 1 - \hat{\xi}^2}{2(1 - \hat{\xi}^2)^2} \ln^2 \frac{\hat{x} - 1}{\hat{x}} + \frac{\hat{\xi}}{1 - \hat{\xi}^2} \ln^2 \frac{\hat{x} - \hat{\xi}}{\hat{x}} - \frac{\hat{x}}{(1 - \hat{\xi}^2)^2} \ln \frac{\hat{x} - \hat{\xi}}{\hat{x}} \ln \frac{\hat{x} + \hat{\xi}}{\hat{x}} 
\]

\[ + \frac{2\hat{\xi}}{(1 - \hat{\xi}^2)^2} \text{Li}_2 \frac{2\hat{\xi}}{\hat{x} - \hat{\xi}^2} + \frac{2\hat{x} - 1 - \hat{\xi}^2}{(1 - \hat{\xi}^2)^2} \left( \text{Li}_2 \frac{1 - \hat{\xi}}{1 - \hat{x}} + \text{Li}_2 \frac{1 + \hat{\xi}}{1 - \hat{x}} \right) - (\hat{x} \to -\hat{x}) \]

Relation to the $\overline{MS}$ scheme

May be possible
Imprint of Anomalies in QCD Compton scattering

Quark-channel diagrams in DVCS:  (SB, Hatta, Vogelsang, In Preparation)
Imprint of Anomalies in QCD Compton scattering

Quark-channel diagrams in DVCS: (SB, Hatta, Vogelsang, In Preparation)

Example: Antisymmetric case

\[ \sim \frac{1}{l^2} + \hat{k}_{qq}(\hat{x}, \hat{\xi}) \ln \frac{Q^2}{l^2} + \delta \hat{C}_{1}^{q}(\hat{x}, \hat{\xi}) - (\hat{x} \to -\hat{x}) \]

No pole!
Imprint of Anomalies in QCD Compton scattering

Quark-channel diagrams in DVCS: (SB, Hatta, Vogelsang, In Preparation)

Example: Antisymmetric case

\[
\tilde{\kappa}_{qq}(\hat{x}, \hat{\xi}) \sim \frac{1}{l^2} + \hat{x}^2 + 2\hat{\xi}^2 \ln \frac{Q^2}{l^2} + \delta \tilde{C}_1^q(\hat{x}, \hat{\xi}) - (\hat{x} \to -\hat{x})
\]

Reproduced the known logarithms from literature

\[
\tilde{\kappa}_{qq}(\hat{x}, \hat{\xi}) = \frac{3}{2(1-x)} + \frac{\hat{x}^2 + 1 - 2\hat{\xi}^2}{(1-\hat{\xi}^2)(1-x)} \ln \frac{\hat{x} - 1}{\hat{x}} - \frac{(\hat{x} - \hat{\xi})(1 + \hat{x}^2 + 2\hat{x}\hat{\xi})}{(1-\hat{\xi}^2)(1-x)} \ln \frac{\hat{x} - \hat{\xi}}{\hat{x}} - (\hat{x} \to -\hat{x})
\]

Ji, Osborne; Belitsky, Mueller
Imprint of Anomalies in QCD Compton scattering

Quark-channel diagrams in DVCS:  (SB, Hatta, Vogelsang, In Preparation)

Example: Antisymmetric case

\[ \sim \frac{1}{l^2} + \bar{\kappa}_{qq}(\hat{x}, \hat{\xi}) \ln \frac{Q^2}{l^2} + \delta \tilde{C}_1^q(\hat{x}, \hat{\xi}) \rightarrow (\hat{x} \rightarrow -\hat{x}) \]

Coefficient function

(not shown here in detail)

\[ \delta \tilde{C}_1^q(\hat{x}, \hat{\xi}) = -\left( \frac{Q^2}{-l^2} \right)^\epsilon \frac{3}{\epsilon^2(1 - \hat{x})} \frac{3}{2\epsilon(1 - \hat{x})} + \cdots - (\hat{x} \rightarrow -\hat{x}) \]

Unexpected double IR pole
Imprint of Anomalies in QCD Compton scattering

Quark-channel diagrams in DVCS: (SB, Hatta, Vogelsang, In Preparation)

Example: Antisymmetric case

\[ \sim \frac{1}{l^2} \left[ \tilde{\kappa}_{qq}(\hat{x}, \hat{\xi}) \ln \frac{Q^2}{l^2} + \delta \tilde{C}_1^q(\hat{x}, \hat{\xi}) \right] \quad (\hat{x} \to -\hat{x}) \]

Coefficient function (not shown here in detail)

Unexpected single IR pole

\[ \delta \tilde{C}_1^q(\hat{x}, \hat{\xi}) = -\left( \frac{Q^2}{-l^2} \right)^\epsilon \frac{3}{\epsilon^2(1 - \hat{x})} - \frac{3}{2\epsilon(1 - \hat{x})} + \ldots \quad (\hat{x} \to -\hat{x}) \]
Imprint of Anomalies in QCD Compton scattering

Quark-channel diagrams in DVCS: (SB, Hatta, Vogelsang, In Preparation)

It looks like factorization is broken due to the unexpected double, single IR poles.

But, when you compute GPD itself, you find the same double, single IR poles! These poles can be systematically absorbed into GPD.

unexpected single IR pole

\[ \delta \hat{C}_1^q(\hat{x}, \hat{\xi}) = -\frac{Q^2}{\epsilon^2(1 - \hat{x})} - \frac{3}{2\epsilon(1 - \hat{x})} + \cdots - (\hat{x} \to -\hat{x}) \]

unexpected double IR pole
Imprint of Anomalies in QCD Compton scattering

Quark-channel diagrams in DVCS:  (SB, Hatta, Vogelsang, In Preparation)

Remarks:

- Off-forwardness is an alternative factorization scheme that clarifies the physics of anomaly
- But, when you compute GPD itself, you find the same double, single IR poles! These poles can be systematically absorbed into GPD
- Novel connection between twist 2 & twist 4 sectors at the density level due to anomaly

\[ \delta \hat{C}_1^q(\hat{x}, \hat{\xi}) = -\frac{\left(\frac{Q^2}{-\hat{l}^2}\right)^\epsilon}{\hat{\epsilon}^2(1 - \hat{x})} - \frac{3\left(\frac{Q^2}{-\hat{l}^2}\right)^\epsilon}{2\hat{\epsilon}(1 - \hat{x})} + \cdots - (\hat{x} \to -\hat{x}) \]
Summary

• Revisited QCD factorization for Compton scattering: Crucial topic for ongoing & future experiments including at EIC

• Importance to understand off-forward poles originating from chiral & trace anomalies

\[ T^{\mu\nu} \sim \frac{\langle F \tilde{F} \rangle}{l^2}, \quad \frac{\langle FF \rangle}{l^2} \] Unnoticed in literature
Summary

Perturbative calculations suggest that massless poles are induced in GPDs $\tilde{E}, H, E$

- Revisited QCD factorization for Compton scattering: Crucial topic for ongoing & future experiments including at EIC

However, we know there are no massless poles in axial and gravitational form factors (moments of GPDs)

- Importance to understand off-forward poles originating from chiral & trace anomalies

\[ T^{\mu\nu} \sim \frac{\langle F \tilde{F} \rangle}{l^2}, \quad \frac{\langle FF \rangle}{l^2} \quad \text{Unnoticed in literature} \]
Summary

Perturbative calculations suggest that massless poles are induced in GPDs $\tilde{E}, H, E$.

However, we know there are no massless poles in axial and gravitational form factors (moments of GPDs).

We proposed a possible scenario of pole cancellation.

This has to do with eta-meson & glueball mass generations.

The importance to understand off-forward poles originating from chiral & trace anomalies.

cf, the $\eta'$ mass problem:

Antisymmetric case:

$$\frac{1}{l^2 - m_{\eta'}^2}$$

Symmetric case:

$$\frac{A(l^2), B(l^2), D(l^2)}{l^2 - m_G^2}$$
Summary

• Revisited QCD factorization for Compton scattering: Crucial topic for ongoing & future experiments including at EIC

• Importance to understand off-forward poles originating from chiral & trace anomalies

\[ T^{\mu\nu} \sim \frac{\langle F \tilde{F} \rangle}{l^2}, \quad \frac{\langle FF \rangle}{l^2} \]  Unnoticed in literature

Off-forwardness is a physical factorization scheme that elucidates the physics of anomaly

Profound physical implication of poles; touches questions on mass generations, Chiral symmetry breaking, ...
Summary & outlook

- Revisited QCD factorization for Compton scattering: Crucial topic for ongoing & future experiments including at EIC
  - Unnoticed in literature

- Importance to understand off-forward poles originating from chiral & trace anomalies
  - Off-forwardness is a physical factorization scheme that elucidates the physics of anomaly
  - Profound physical implication of poles; touches questions on mass generations, Chiral symmetry breaking, ...

**Novel connections between DVCS & chiral/trace anomalies:**
This could be a new & potentially rich avenue for GPD research

**Imprint of anomaly on other physical processes:**
(Example: Deeply-virtual meson production)
Backup slides
Imprint of Anomalies in QCD Compton scattering

FIG. 1: Box diagrams for the Compton amplitude in off-forward kinematics.
Imprint of Anomalies in QCD Compton scattering

\[
\frac{g_P(l^2)}{2M} \approx \frac{-2M \Delta \Sigma}{l^2 - m_{\eta'}^2}, \quad i \frac{\langle P_2 | \frac{n_f \alpha_s}{4\pi} F \tilde{F} | P_1 \rangle}{\bar{u}(P_2) \gamma_5 u(P_1)} \approx -2M \Delta \Sigma \frac{m_{\eta'}^2}{l^2 - m_{\eta'}^2}.
\]
Example: Antisymmetric part of Compton amplitude

\begin{equation}
\bar{F}_1^{\text{off}}(x_B, l) \approx \frac{1}{2\pi} \sum_f \left(\frac{Q^2}{l^2} + C_{1g}^{\text{off}}\right) \otimes g(x_B) + \frac{1}{l^2} C^{\text{anom}} \otimes' F(x_B, \xi, l^2) \frac{u(P_2)u(P_1)}{2M}.
\end{equation}

\begin{equation}
\bar{F}_2^{\text{off}}(x_B, l) \approx x_B \frac{Q^2}{2\pi} \left(\sum_f \sum_f' \right) \left(\frac{Q^2}{l^2} + C_{2g}^{\text{off}}\right) \otimes g(x_B) + \frac{1}{l^2} C^{\text{anom}} \otimes' F(x_B, \xi, l^2) \frac{u(P_2)u(P_1)}{2M}.
\end{equation}

We recognize the expected structure of the one-loop corrections associated with the unpolarized gluon PDF \( g(x) \), with the splitting function \( P_{gg}(\hat{x}) = 2T_R((1 - \hat{x})^2 + \hat{x}^2) \). The coefficient functions are given by

\begin{align}
C_{1g}(\hat{x}) &= 2T_R((1 - \hat{x})^2 + \hat{x}^2) \left(\ln \frac{1}{\hat{x}(1 - \hat{x})} - 1\right), \\
C_{2g}(\hat{x}) &= 2T_R((1 - \hat{x})^2 + \hat{x}^2) \left(\ln \frac{1}{\hat{x}(1 - \hat{x})} - 1\right) + 8T_R \hat{x}(1 - \hat{x}).
\end{align}

In addition, we find a pole \( 1/l^2 \) in both \( \bar{F}_1^{\text{off}} \) and \( \bar{F}_2^{\text{off}} \) (but not in the difference \( \bar{F}_2^{\text{off}} - 2x_B\bar{F}_1^{\text{off}} \) relevant to the longitudinal structure function), with the following convolution formula

\begin{equation}
C^{\text{anom}} \otimes' F(x_B, \xi, l^2) = \int_{x_B}^1 \frac{dx}{x} K(\hat{x}, \hat{\xi}) F(x, \xi, l^2) - \frac{\theta(\xi - x_B)}{2} \int_{x_B}^1 \frac{dx}{x} L(\hat{x}, \hat{\xi}) F(x, \xi, l^2),
\end{equation}

where

\begin{align}
K(\hat{x}, \hat{\xi}) &= 2T_R \frac{\hat{x}(1 - \hat{x})}{1 - \hat{\xi}^2}, \\
L(\hat{x}, \hat{\xi}) &= 2T_R \frac{\hat{x}(\hat{\xi} - \hat{x})}{1 - \hat{\xi}^2}.
\end{align}
Imprint of Anomalies in QCD Compton scattering

Symmetric part of Compton amplitude \((\xi \neq 0)\):

\[
\int_0^1 dx_B x_B (H_f(x_B, \xi, l^2) - H_f(-x_B, \xi, l^2)) = \int_{-1}^1 dx_B x_B H_f(x_B, \xi, l^2) = A_f(l^2) + \xi^2 D_f(l^2),
\]

\[
\int_0^1 dx_B x_B (E_f(x_B, \xi, l^2) - E_f(-x_B, \xi, l^2)) = \int_{-1}^1 dx_B x_B E_f(x, \xi, l^2) = B_f(l^2) - \xi^2 D_f(l^2).
\]

Pole! (New result)

Polynomiality:

\[
\sum_f e_f^2 (A_f^{\text{bare}}(l^2) + \xi^2 D_f^{\text{bare}}(l^2)) \approx \frac{T_R \alpha_s}{12\pi l^2} \left( \sum_f e_f^2 \right) \left( \frac{\langle P | F^{\alpha\beta}(i\not\!D^+)^2 F_{\alpha\beta} | P \rangle}{(P^+)^2} + \xi^2 \langle P | F^2 | P \rangle \right),
\]

\[
\sum_f e_f^2 (B_f^{\text{bare}}(l^2) - \xi^2 D_f^{\text{bare}}(l^2)) \approx -\frac{T_R \alpha_s}{12\pi l^2} \left( \sum_f e_f^2 \right) \left( \frac{\langle P | F^{\alpha\beta}(i\not\!D^+)^2 F_{\alpha\beta} | P \rangle}{(P^+)^2} + \xi^2 \langle P | F^2 | P \rangle \right).
\]

(Non-local) trace anomaly manifests itself in high energy scattering amplitude & possibly breaks QCD factorization
Imprint of Anomalies in QCD Compton scattering

**Polynomiality:**
Symmetric part of Compton amplitude \((\xi \neq 0)\)

**Structure of convolution:**

\[
C_{\text{anom}} \otimes' \mathcal{F}(x_B, \xi, l^2) \equiv \int_{x_B}^{1} \frac{dx}{x} K(\hat{x}, \hat{\xi}) \mathcal{F}(x, \xi, l^2) - \frac{\theta(x - x_B)}{2} \int_{-1}^{1} \frac{dx}{x} L(\hat{x}, \hat{\xi}) \mathcal{F}(x, \xi, l^2)
\]

\[
K(\hat{x}, \hat{\xi}) = 2T_R \frac{\hat{x}(1 - \hat{x})}{1 - \hat{\xi}^2}
\]

\[
L(\hat{x}, \hat{\xi}) = 2T_R \frac{\hat{x}(\xi - \hat{x})}{1 - \hat{\xi}^2}
\]

**Twist-4 GPD:**

\[
\mathcal{F}(x, \xi, l^2) = -4xP^+M \int \frac{dz}{2\pi} e^{ix\cdot z} \langle P_2 | F_{\mu\nu}(-z^-/2) F_{\mu\nu}(z^-/2) | P_1 \rangle \frac{\bar{u}(P_2) u(P_1)}{\bar{u}(P_2) u(P_1)}
\]

**Non-local** trace anomaly manifests itself in high energy scattering amplitude & apparently breaks QCD factorization

Hatta, Zhao (2020); Radyushkin, Zhao (2021)

"Bare GPD" (tree level)

Perturbative pole (one loop)

Nonzero for nonzero skewness
Imprint of Anomalies in QCD Compton scattering

Polynomiality:

Symmetric part of Compton amplitude \((\xi \neq 0)\)

Structure of convolution:

\[
C_{\text{anom}} \otimes' \mathcal{F}(x_B, \xi, l^2) = \int_{x_B}^{1} \frac{dx}{x} K(\hat{x}, \hat{\xi}) \mathcal{F}(x, \xi, l^2) - \frac{\theta(\xi - x_B)}{2} \int_{-1}^{1} \frac{dx}{x} L(\hat{x}, \hat{\xi}) \mathcal{F}(x, \xi, l^2)
\]

- \(K(\hat{x}, \hat{\xi}) = 2T_R \frac{\hat{x}(1 - \hat{x})}{1 - \hat{\xi}^2}\)
- \(L(\hat{x}, \hat{\xi}) = 2T_R \frac{\hat{x}(\xi - \hat{x})}{1 - \hat{\xi}^2}\)

Example:

\[
\sum_f e_f^2 x_B (H_{f}^\text{bare}(x_B, \xi, l^2) - H_{f}^\text{bare}(-x_B, \xi, l^2)) \approx -\frac{\alpha_s}{2\pi} \left( \sum_f e_f^2 \right) \frac{x_B}{l^2} C_{\text{anom}} \otimes' \mathcal{F}(x_B, \xi, l^2 = 0)
\]

Twist-4 GPD:

Perturbative pole (one loop)

- Nonzero for nonzero skewness

Polynomiality

\[
\sum_f e_f^2 (A_f^\text{bare}(l^2) + \xi^2 D_f^\text{bare}(l^2)) \approx \frac{T_R \alpha_s}{12\pi l^2} \left( \sum_f e_f^2 \right) \left( \frac{\langle P| F^{\alpha\beta}(i \mathcal{D}^+)^2 F_{\alpha\beta}|P \rangle}{(P^+)^2} + \xi^2 \langle P| F^2 |P \rangle \right)
\]