1) Parallel the derivation in the text to find the metric on the 2-sphere in its usual form,

$$ds^2 = R^2(d\theta^2 + \sin^2\theta d\phi^2) \tag{2.83}$$

from the 3 dimensional Euclidean metric.

2) Show that on-shell, the graviton has degrees of freedom corresponding to a transverse ($d-2$ indices) symmetric traceless tensor.

3) Show that the metric $g_{\mu\nu}$ is covariantly constant ($D_\mu g_{\nu\rho} = 0$) by substituting the Christoffel symbols.

4) Prove that the general coordinate transformation on $g_{\mu\nu}$,

$$g'_{\mu\nu}(x') = g_{\rho\sigma}(x)\frac{\partial x^\rho}{\partial x'^\mu}\frac{\partial x^\sigma}{\partial x'^\nu} \tag{2.84}$$

reduces for infinitesimal tranformations to

$$\partial_\xi g_{\mu\nu}(x) = (\xi^\rho\partial_\rho)g_{\mu\nu} + (\partial_\mu\xi^\rho)g_{\rho\nu} + (\partial_\nu\xi^\rho)g_{\rho\mu}. \tag{2.85}$$

5) Prove that the commutator of two covariant derivatives when acting on a covariant vector gives the action of the Riemann tensor on it, eq. (2.25).

6) Parallel the calculation in 2 dimensions to show that the Penrose diagram of 3 dimensional Minkwoski space, with an angle ($0 \leq \phi \leq 2\pi$) supressed, is a triangle.

7) Substitute the coordinate transformation

$$X_0 = R\cosh\rho\cos\tau; \quad X_i = R\sinh\rho\Omega_i; \quad X_{d+1} = R\cosh\rho\sin\tau \tag{2.86}$$

to find the global metric of AdS space from the embedding $(2, d-1)$ signature flat space.

1) Write down the worldline reparametrization invariance for the first order action for the particle, both the finite and infinitesimal versions.

2) Calculate the equation of motion of the free particle in a gravitational field (the geodesic equation), from the action (7.3), with $\eta_{\mu\nu} \to g_{\mu\nu}$, and specialize to the Newtonian limit to recover motion in Newtonian gravity.

3) Calculate $L_m, \tilde{L}_m$ and $L_0 + \tilde{L}_0$ for the bosonic string.

4) Derive the worldsheet momentum P_σ.

5) Write down the states of the first massive closed string level.

6) Prove the supersymmetry of the Wess-Zumino term of the superstring.

7) Prove the kappa symmetry of the action $S_{kin} + S_{WZ}$ for the superstring.

8) Show that the coupling to $B_{\mu\nu}$ is of the type of p-brane sources, thus a string is a 1-brane source for the field $B_{\mu\nu}$.

9) Find a representation for the 11 dimensional Γ matrices in terms of the Pauli σ^i matrix tensor products.

10) Check that the reduction ansatz (7.128) takes (7.127) to (7.113).