- 4 dimensional on-shell supergravity is the first nontrivial case (with propagating degrees of freedom) and is composed of e_μ^a and ψ_μ.
- Supergravity (local supersymmetry) is of the type $\delta e_\mu^a = (k_N/2)\bar{\epsilon}\gamma^a\psi_\mu + ...$, $\delta\psi_\mu = (D_\mu\epsilon)/k_N + ...$
- The action for gravity in supergravity is the Einstein-Hilbert action in the vielbein-spin connection formulation.
- The action for the gravitino is the Rarita-Schwinger action.
- The most useful formulation is the 1.5 order formalism: second order formalism, but don't vary $\omega(e,\psi)$ by the chain rule.
- For each supersymmetry we have a gravitino. The maximal supersymmetry in $d = 4$ is $\mathcal{N} = 8$.
- Gauged supergravity is AdS supergravity, and is an extension by a gauge coupling parameter of the ungauged models.
- Supergravity theories in higher dimensions can contain antisymmetric tensor fields.
- The maximal dimension for a supergravity theory is d=11, with a unique model composed of $e_\mu^a, \psi_\mu, A_{\mu\nu\rho}$.

References and further reading

The vielbein and spin connection formalism for general relativity is harder to find in standard general relativity books, but one can find some information for instance in the supergravity review [14]. An introduction to supergravity, but one which might be hard to follow for the beginning student, is found in West [9] and Wess and Bagger [10]. A good supergravity course, that starts at an introductory level and reaches quite far, is [14]. In this chapter, I followed mostly [14] (you can find more details in sections 1.2-1.6 of the reference). A good and complete recent book is [15].

Exercises, Chapter 4

1) Check that

$$\omega_\mu^{ab}(e) = \frac{1}{2}e^{a\nu}(\partial_\mu e_\nu^b - \partial_\nu e_\mu^b) - \frac{1}{2}e^{b\nu}(\partial_\mu e_\nu^a - \partial_\nu e_\mu^a) - \frac{1}{2}e^{a\rho}e^{b\sigma}(\partial_\rho e_{c\sigma} - \partial_\sigma e_{c\rho})e_\mu^c \tag{4.47}$$

satisfies the no-torsion (vielbein) constraint, $T_{\mu\nu}^a = 2D_{[\mu}e_{\nu]}^a = 0$.

2) Find $\omega_\mu^{ab}(e,\psi) - \omega_\mu^{ab}(e)$ in the second order formalism for $\mathcal{N} = 1$ supergravity.

function rule" developed in [19]. To understand the meaning of extremal p-branes, one can look at the rule for making an extremal solution non-extremal, found in [20].

Exercises, Chapter 6.

1) Check the transformation from Schwarzschild coordinates to Kruskal coordinates.

2) Verify that the Penrose diagram for an astrophysical black hole (from a collapsing star) is the one in Fig.8b.

3) Consider the *ingoing Eddington-Finkelstein coordinates* v and r, with u defined in (6.16). Show that the metric becomes

$$ds^2 = -\left(1 - \frac{2MG}{r}\right) dv^2 + 2dvdr + r^2 d\Omega_2^2. \tag{6.88}$$

Similarly, consider the *outgoing Eddington-Finkelstein coordinates* u and r, with v defined in (6.16). Show that now the metric becomes

$$ds^2 = -\left(1 - \frac{2MG}{r}\right) du^2 - 2dudr + r^2 d\Omega_2^2. \tag{6.89}$$

4) Check that $H = 1 + a/r^{7-p}$ is a good harmonic function for a p-brane. Check that $r = 0$ is an event horizon (it traps light).

5) The electric current of a point charge is $j^\mu = Q\frac{dx^\mu}{d\tau}\delta^{d-1}(x^\mu(\tau))$. Write an expression for the $p+1-$form current of a p-brane, $j^{\mu_1\cdots\mu_{p+1}}$.

6) Prove that the change of coordinates

$$r^{D-3} = \bar{r}^{D-3} + r_H^{D-3} \tag{6.90}$$

takes the extremal black hole metric to

$$ds^2 = -f(\bar{r})^{-2}dt^2 + f(\bar{r})^{\frac{2}{D-3}}(d\bar{r}^2 + \bar{r}^2 d\Omega_{D-2}^2). \tag{6.91}$$

Maldacena in [30], but the paper is not so easy to read. It was then made more concrete first in [31] and then in the paper by Witten [32]. In particular, the state map and the "experimental evidence" was found in [32]. The comparison is done with the spectrum of 10d IIB supergravity on $AdS_5 \times S^5$, found in [33]. This dimensional reduction is only at the linear level. The full nonlinear reduction on S^5 is not yet done. For the other 2 cases of interest (discussed only in part III of this book) of AdS/CFT, $AdS_4 \times S^7$ and $AdS_7 \times S^4$, the nonlinear reduction was done in [34] (though it is not totally complete) for $AdS_4 \times S^7$ and in [35, 36] (completely) for the $AdS_7 \times S^4$ case.

Exercises, Chapter 10.

1) The metric for an "M2 brane" solution of $d = 11$ supergravity is given by

$$ds^2 = H^{-2/3}(d\vec{x}_3)^2 + H^{+1/3}(dr^2 + r^2 d\Omega_7^2); \quad H = 1 + \frac{2^5\pi^2 l_P^6}{r^6}. \tag{10.34}$$

Check that the same limit taken for D3 branes gives M theory on $AdS_4 \times S_7$ if $l_P \to 0$, $U \equiv r^2/l_P^3$ fixed.

2) Check that the $r \to 0$ limit of the D-p-brane metric gives $AdS_{p+2} \times S_{8-p}$ only for $p = 3$.

3) String corrections to the gravity action come about as g_s corrections to terms already present and α' corrections appear generally as $(\alpha'\mathcal{R})^n$, with $\mathcal{R}$ the Ricci scalar, or some particular contraction of Riemann tensors. What then do α' and g_s string corrections correspond to in SYM via AdS/CFT (in the $N \to \infty$, $\lambda = g_{YM}^2 N$ fixed and large limit)?

4) Show that the time it takes a light ray to travel from a finite point in AdS to the real boundary of space and back is finite, but the times it takes to reach the center of AdS ($x_0 = \infty$, or $r = 0$, or $\rho = 0$) is infinite. Try this in both Poincaré and global coordinates.

5) Consider a metric that interpolates in the radial coordinate r between AdS_4 with radius R and $AdS_2 \times S^2$ with radius $R/2$. Is a scalar that is marginally stable in AdS_4 (saturates the BF bound) also stable in $AdS_2 \times S^2$? How about if the $AdS_2 \times S^2$ radius is $R/3$?

6) Write down towers of chiral primary operators corresponding to massive vectors in AdS_5, based on $\mathcal{O}_n$ (by acting with Q's and $\bar{Q}$'s), and predict the vector masses $m_k^2 R^2$.