Onset of the precision era for gluon saturation: two-particle correlations at NLO

Farid Salazar (UCLA/LBL)

$\Delta t \propto 1/\Delta E$

Physics Opportunities at an Electron-Ion Collider

May 5th, 2023

Based on

(1) 2108.06347 [JHEP 11 (2021) 222]
(2) 2208.13872 [JHEP 11 (2022) 169]
(3) 2304.03304 *[preprint]*

In collaboration with

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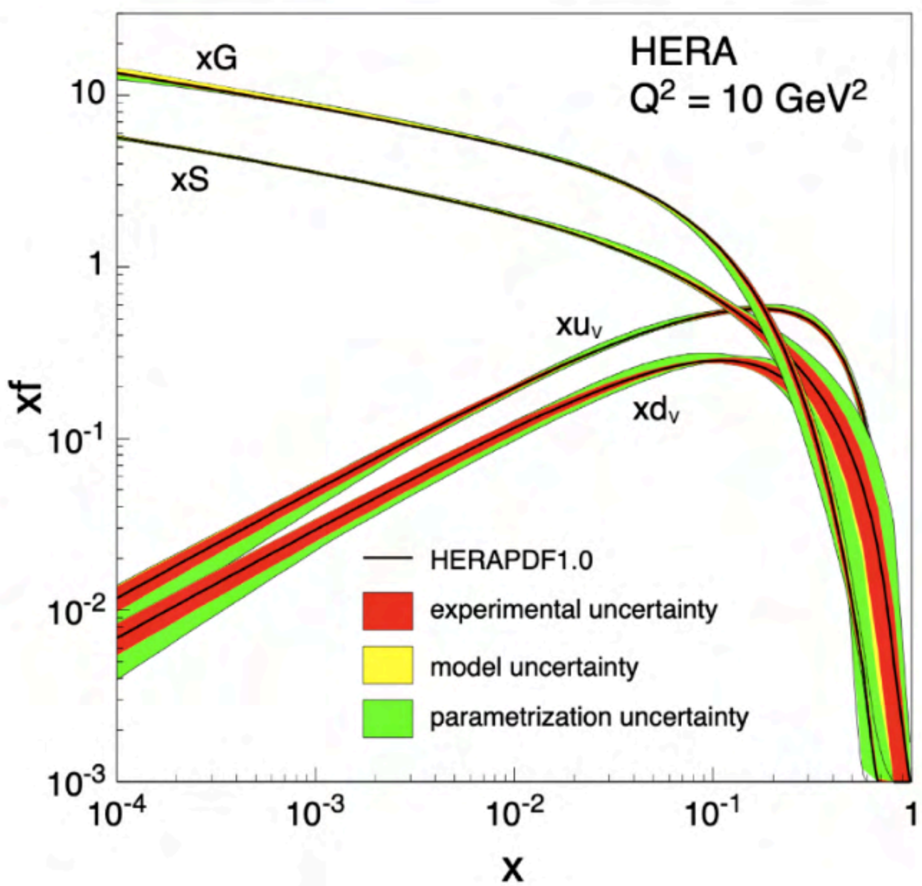
Tomasz Stebel (Jagiellonian)
Raju Venugopalan (BNL)

momentum

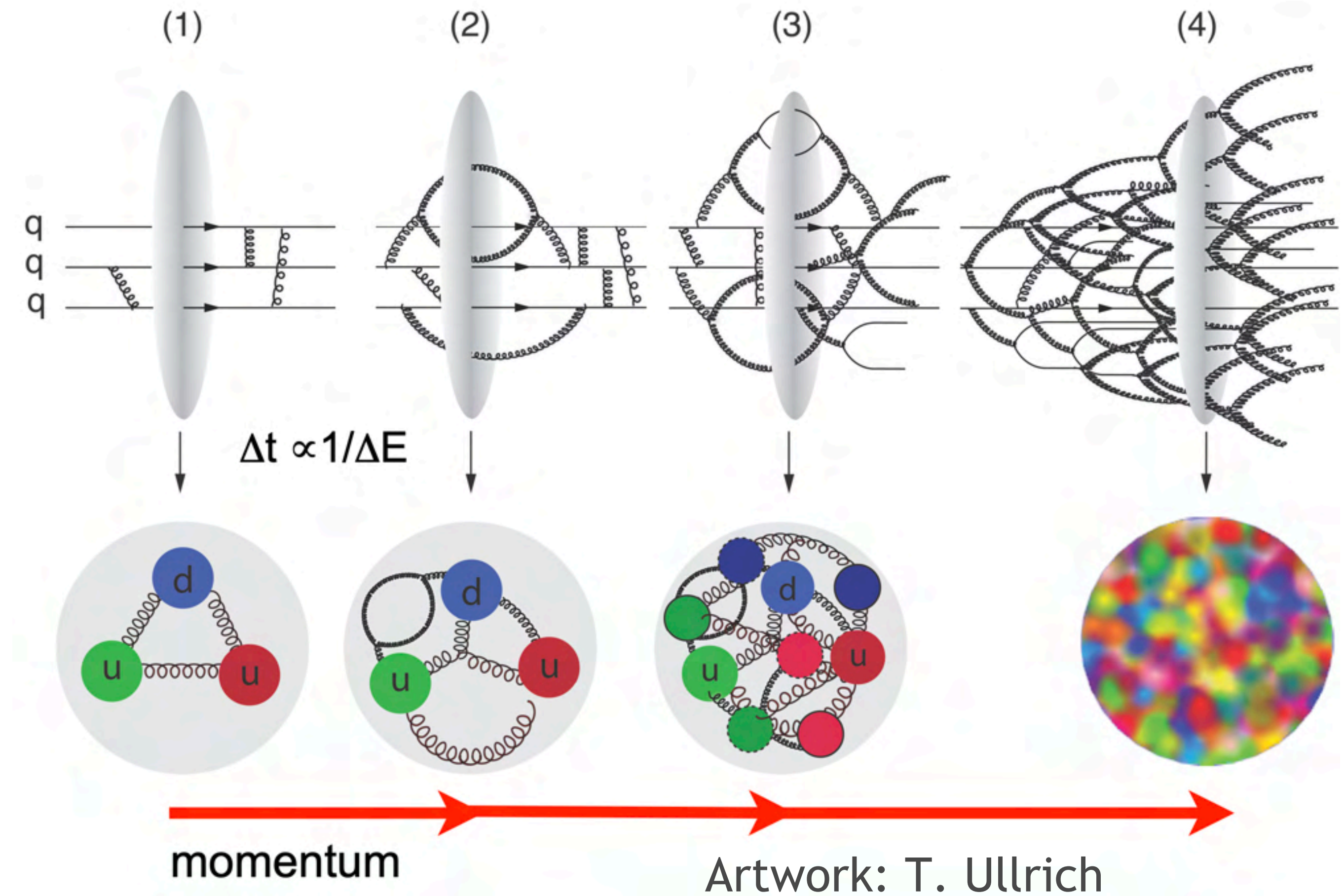
Gluon saturation on a nutshell review, status and challenges

Gluon saturation: review, status and challenges

Anatomy of QCD at high-energies

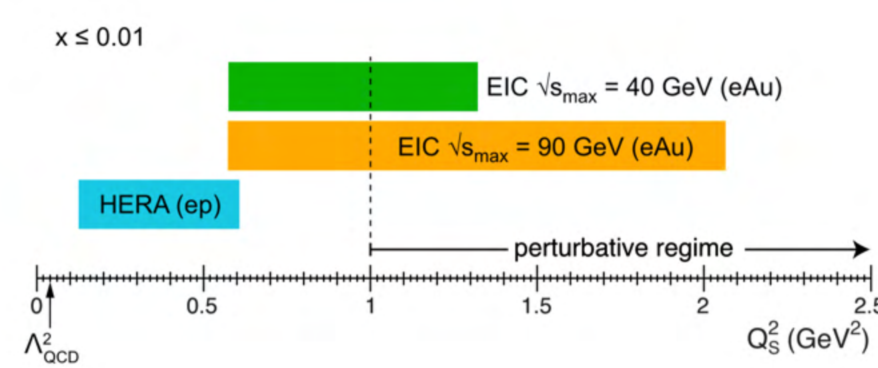


gluon density must saturate
at high-energies/small-x



Partonic picture superseded by **strong highly occupied fields**

Emergence of an energy and nuclear species dependent momentum scale $Q_s^2 \propto A^{1/3} s^{1/3}$



Multiple scattering (higher twist effects)

Non-linear evolution equations

For a review see Mining gluon saturation at colliders. FS, Morreale (Universe 2021)

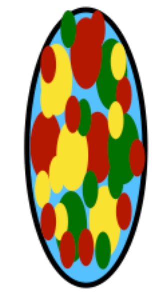
Gluon saturation: review, status and challenges

Color Glass Condensate in a nutshell

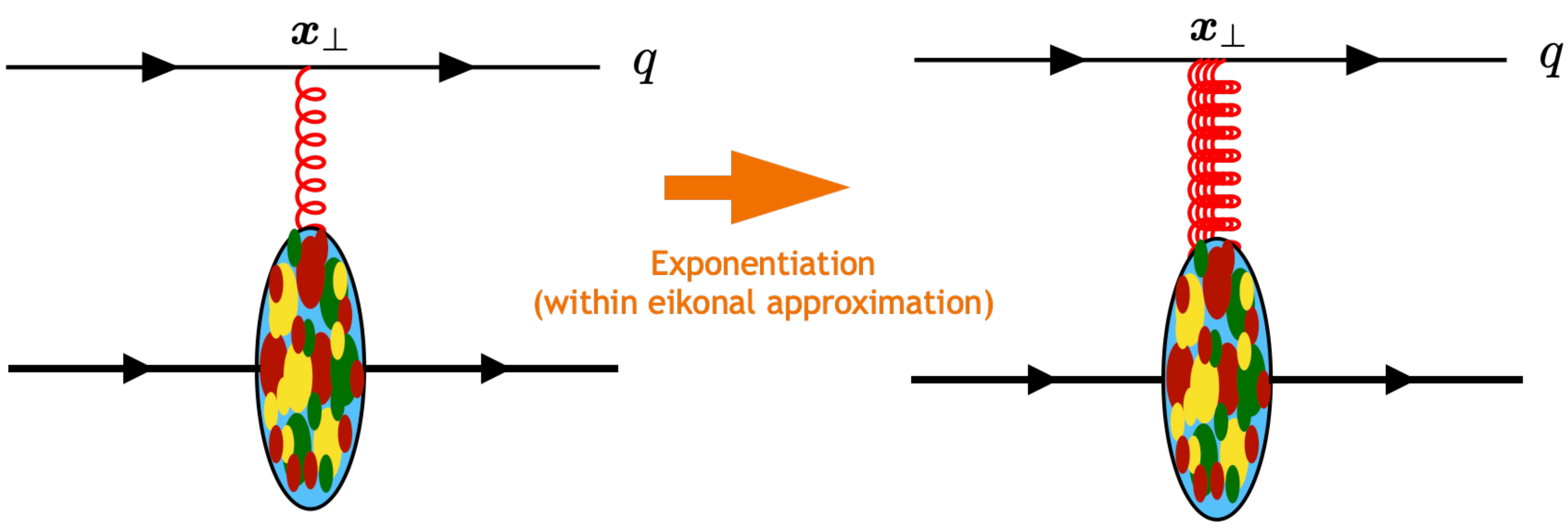
L. McLerran, R. Venugopalan (1993)

- **Effective field theory** for slow gluons sourced by fast partons

Color (QCD)
Glass (separation between slow and fast degrees of freedom)
Condensate (highly occupied system)



- **Multiple scattering** of probing partons with slow gluon field



Ayala, Jalilian-Marian, McLerran, Venugopalan (PRD 1995) Balistky (NPB 1996)

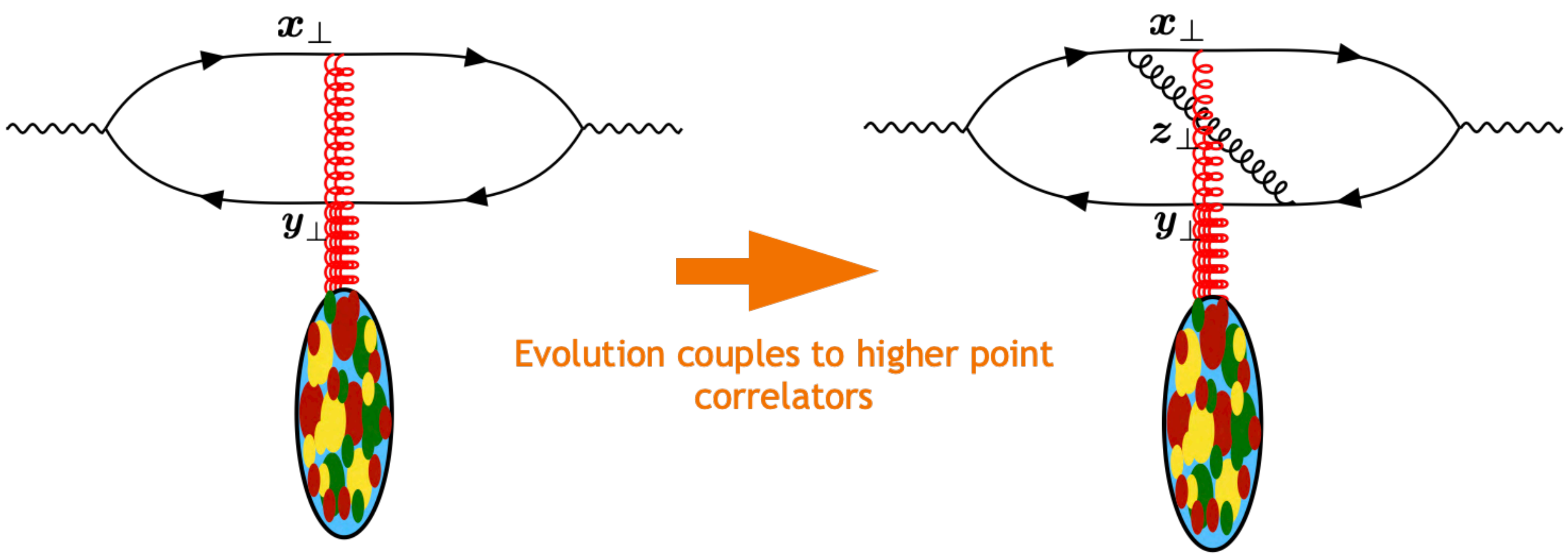
High-energy scattering dofs = light-like Wilson line:

$$V_{ij}(\boldsymbol{x}) = P \exp\left\{ ig \int dx^- A_{cl}^{+,a}(\boldsymbol{x}, x^-) t^a \right\}$$

Gluon saturation: review, status and challenges

Color Glass Condensate in a nutshell

- **Non-linear** renormalization group **evolution (BK-JIMWLK)**



$$\mathrm{Tr}\left[V(\boldsymbol{x}_\perp)V^\dagger(\boldsymbol{y}_\perp)\right]$$

$$\mathrm{Tr}\left[V(\boldsymbol{x}_\perp)t^a V^\dagger(\boldsymbol{y}_\perp)t^b\right] U_{ab}(\boldsymbol{z}_\perp)$$

I. Balistky (1995), Y. Kovchegov (1999)
J. Jalilian-Marian, E. Iancu, L. McLerran,
H. Weigert, A. Leonidov, A. Kovner (1996-2002)

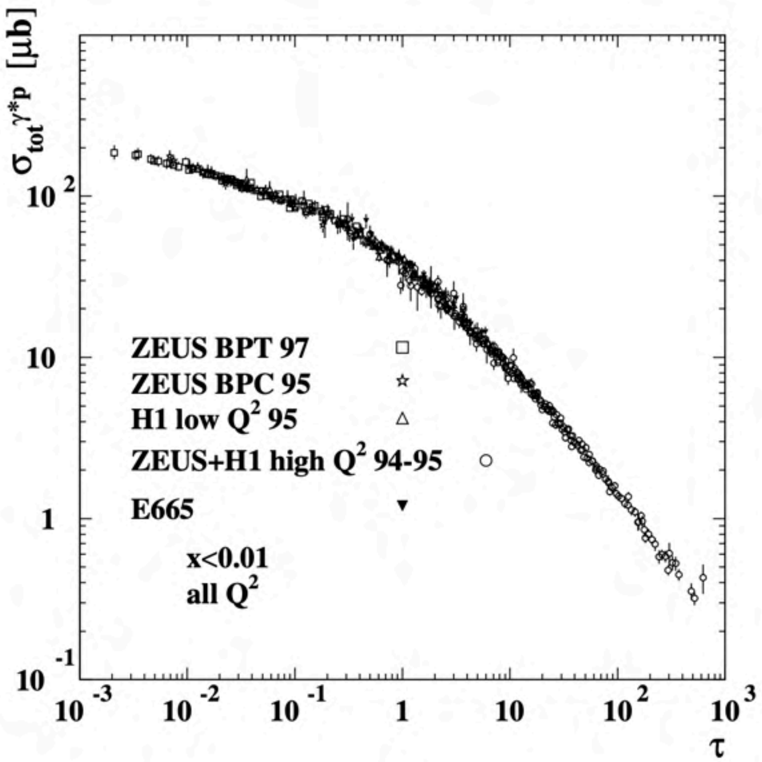
- Fast fields are non-perturbative, slow fields evolve perturbatively

- **Probing CGC with dilute projectile**
  = pQCD embedded in strong gluon (non-perturbative) background field

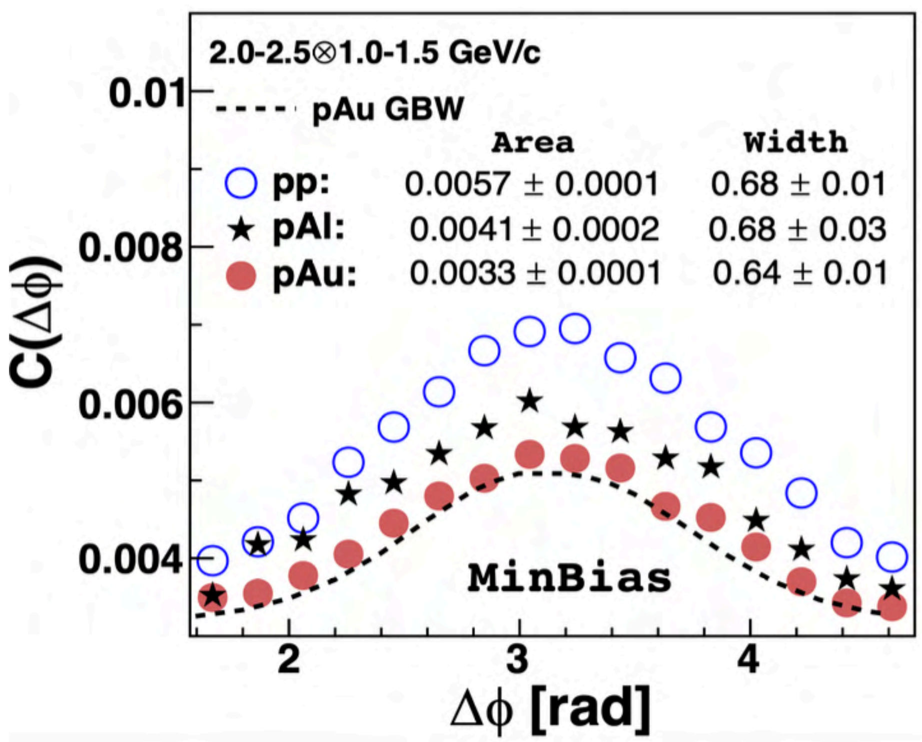
Gluon saturation: review, status and challenges

Experimental status

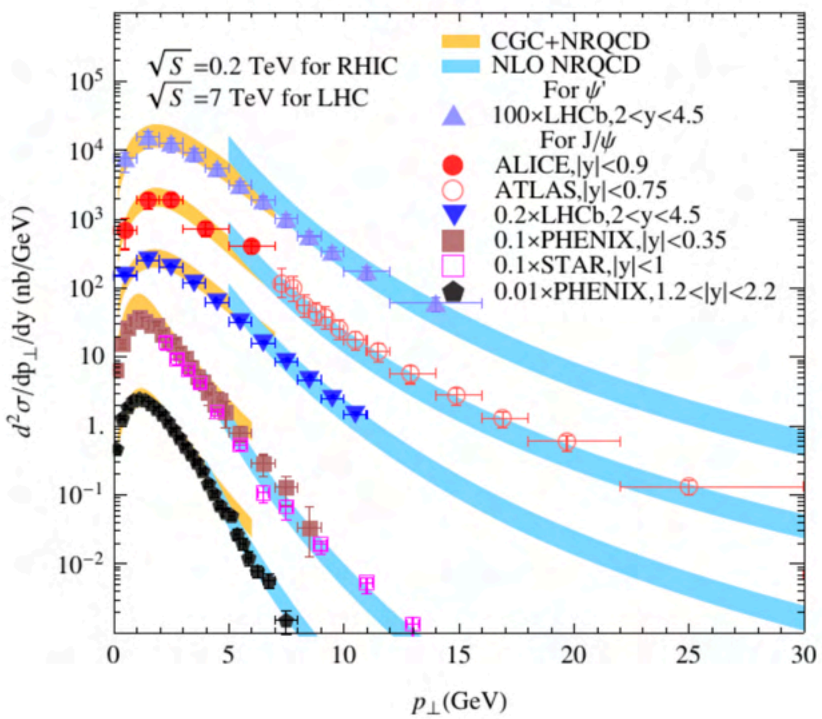
- Heavy-ion collisions, hadronic collisions, UPCs (e.g. RHIC and LHC)
- Deep-inelastic scattering (at HERA and future EIC)
- Inclusive, semi-inclusive, diffractive processes



Geometric scaling at RHIC



Dihadron suppression at RHIC



Quarkonium production at RHIC and LHC

Compelling but not definitive evidence yet!

For a review see Mining gluon saturation at colliders. FS, Morreale (Universe 2021)

Gluon saturation: review, status and challenges

Outstanding theoretical challenges

- **Higher-order calculations for precision**
- Identification of novel observables
- Modeling of initial conditions
- Spin Physics and saturation
- Event generators and global analysis
- Unification of dilute and dense QCD (beyond CGC)

recently funded SURGE Topical Collaboration supported by DOE

Discovery and characterization of gluon saturation principal goals of the future Electron-Ion Collider

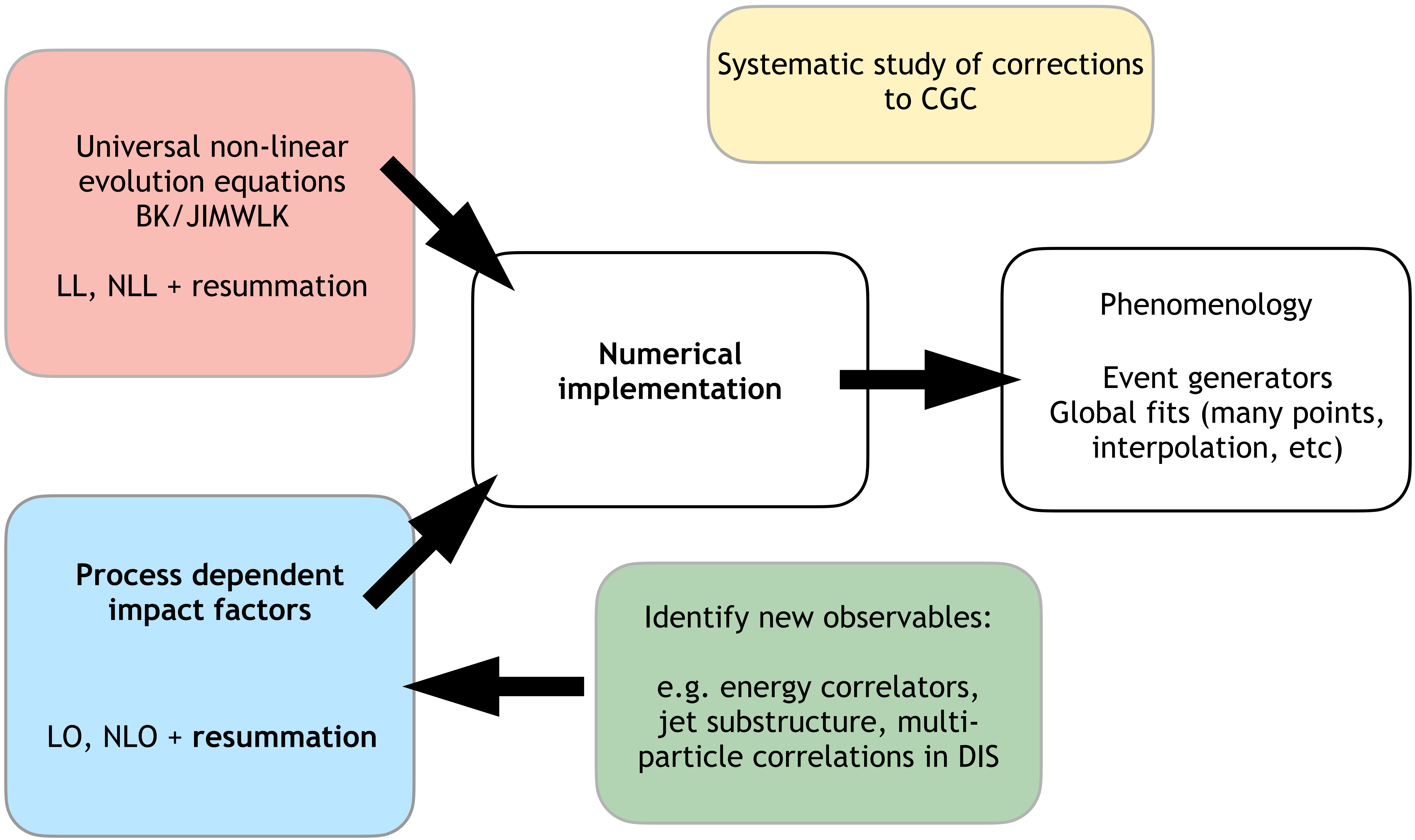
Other novel directions:

- Entanglement entropy and saturation, space-like and time-like correspondence, CGC-blackhole correspondence, color memory effect

For a short review see: *section 6 in Snowmass 2021 White Paper (arXiv: 2203.13199)*

Gluon saturation: review, status and challenges

Pipeline for NLO calculations



Gluon saturation: review, status and challenges

Lots of **recent progress** in understanding **saturation physics** in the **precision era** (NLO) with focus on fully inclusive process or one-particle production

The **more differential** the process (e.g. two-particle correlations) the **harder** the calculations (both **analytically** and **numerically**)

Our goal:

Promote **two-particle observables** in **saturation** to **NLO**

Observable:

Inclusive Dijet production in **DIS**

See Jamal's talk on Wednesday for dihadrons

Why?

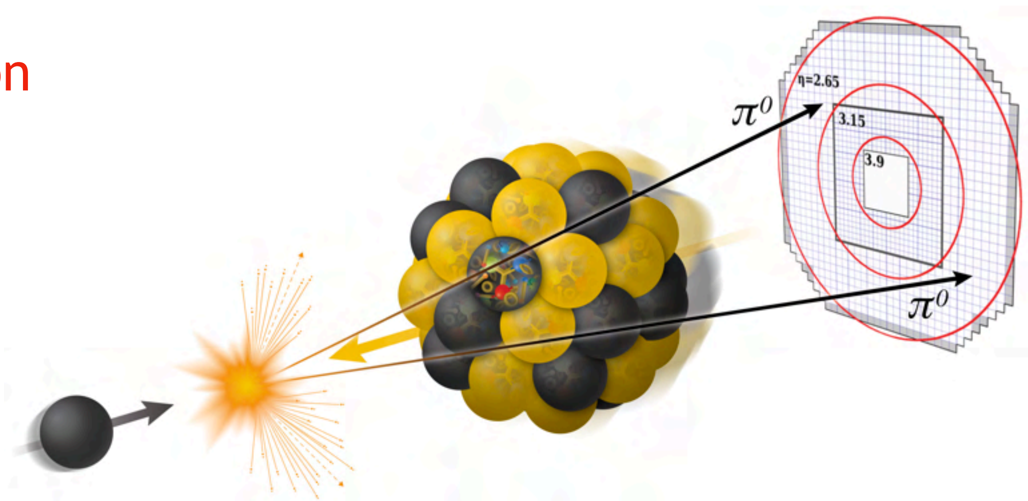
Will be **measured** at **EIC** and **theoretically clean**

See Elke's talk on Monday and Brian's talk on Thursday

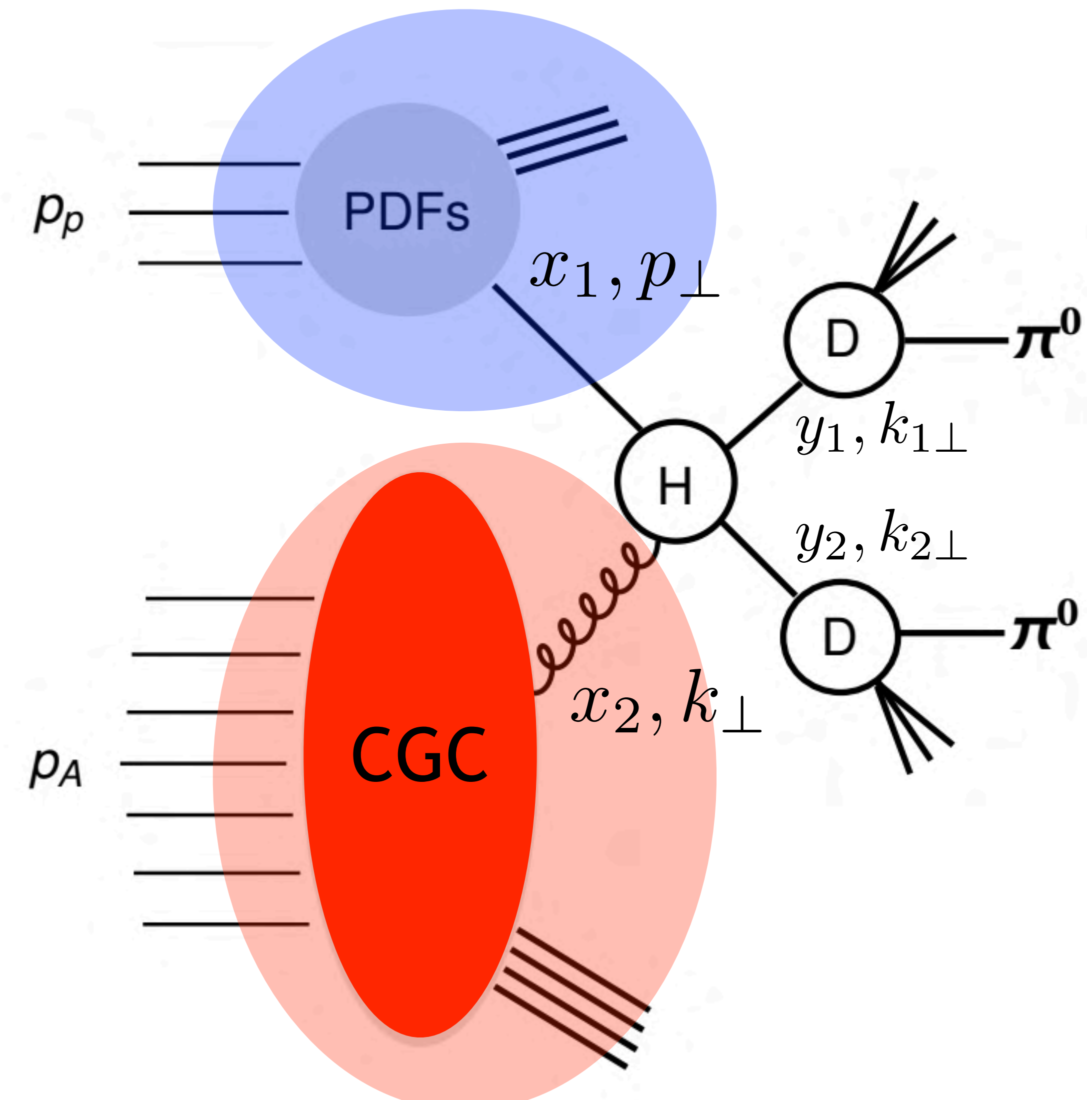
Two-particle correlations: a window to gluon saturation

Forward dihadrons in proton-nucleus collisions

Azimuthal correlations as a probe for gluon saturation
D. Kharzeev, E. Levin, L. McLerran *(2005)*



Hybrid dilute-dense formalism



$$x_1 = \frac{1}{\sqrt{s}}(k_{1\perp}e^{y_1} + k_{2\perp}e^{y_2}) \sim 1$$

$$x_2 = \frac{1}{\sqrt{s}}(k_{1\perp}e^{-y_1} + k_{2\perp}e^{-y_2}) \ll 1$$

$$p_\perp \sim \Lambda_{\mathrm{QCD}}$$

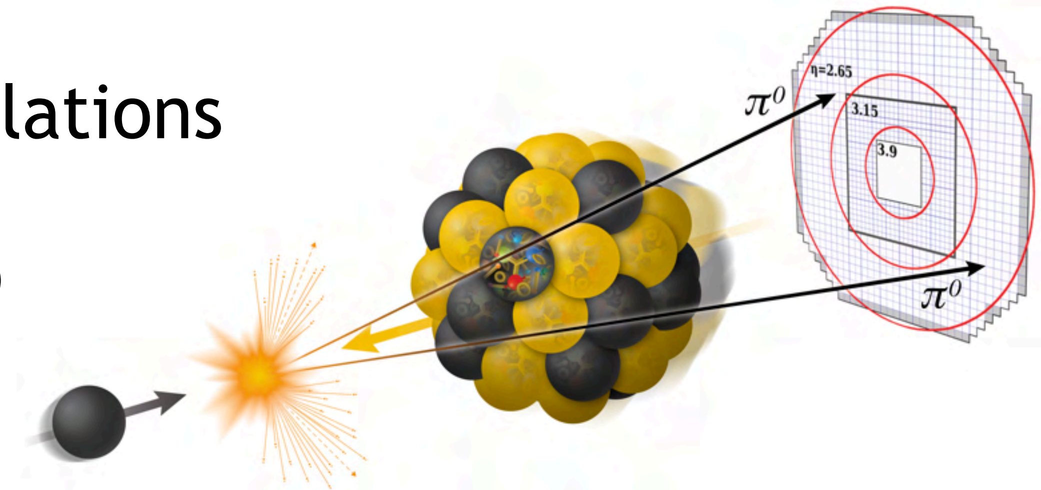
$$k_\perp \sim Q_s(x_2)$$

$$Q_s(x_2) \gg \Lambda_{\mathrm{QCD}}$$

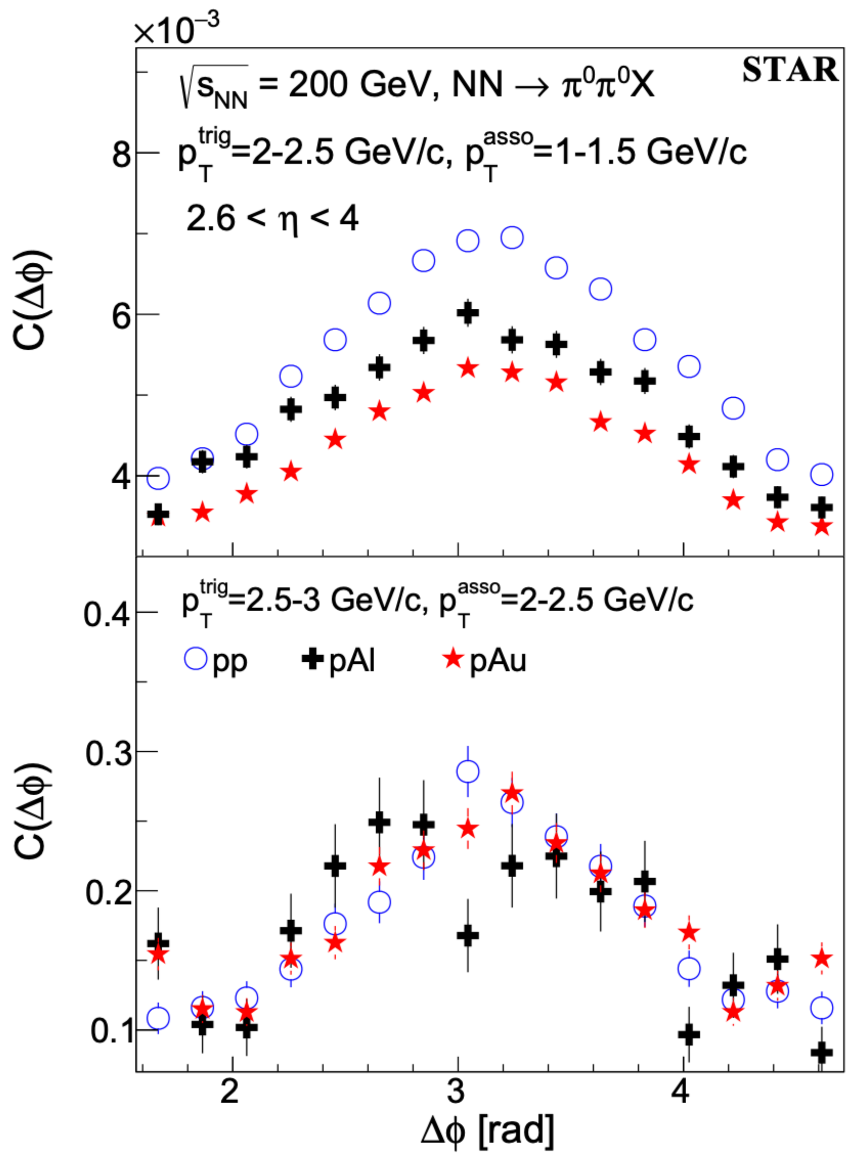
Dihadron momentum
imbalance $\sim Q_s$

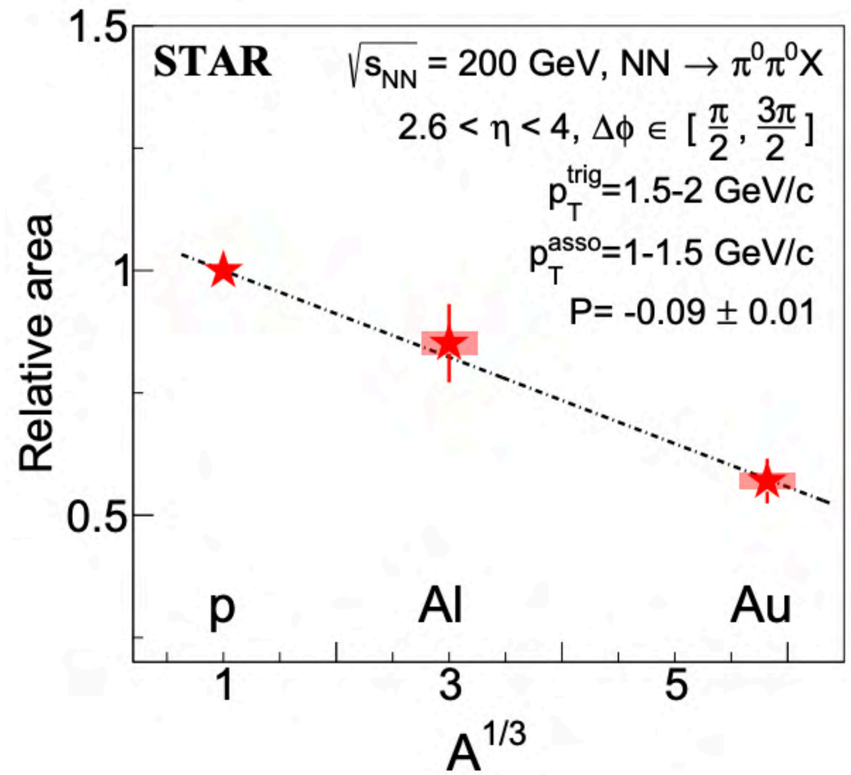
Forward dihadron at RHIC

Signatures of saturation in azimuthal correlations



STAR Collaboration. *Phys. Rev. Lett. 129, 092501 (2022)*





Suppression characteristic of saturation

$$Q_s^2 \propto A^{1/3}$$

Xiaoxuan Chu and Elke Aschenauer looking at the STAR detector

Outline

- Dijet production in DIS at small-x at NLO

  P. Caucal, FS, R. Venugopalan. *2108.06347* *[JHEP 11 (2021) 222]*

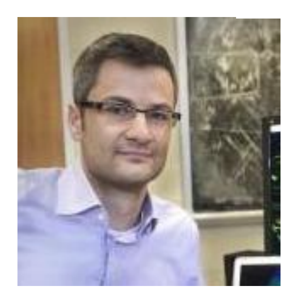
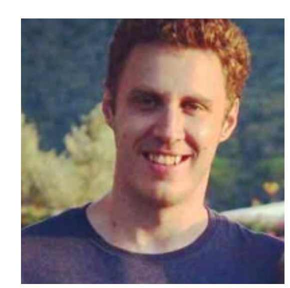
- The back-to-back limit: Sudakov and the small-x gluon TMD

  P. Caucal, FS, B. Schenke ,R. Venugopalan. *2208.13872* *[JHEP 11 (2022) 169]*

- Complete small-x TMD factorization at NLO

  P. Caucal, FS, B. Schenke, T. Stebel ,R. Venugopalan. *2304.03304* *[preprint]*

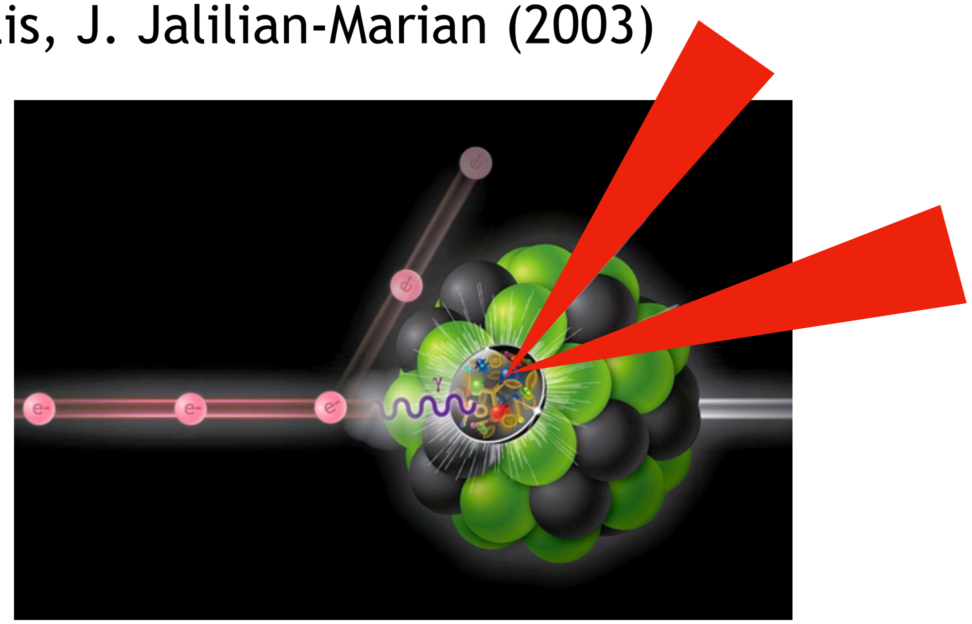
- Outlook

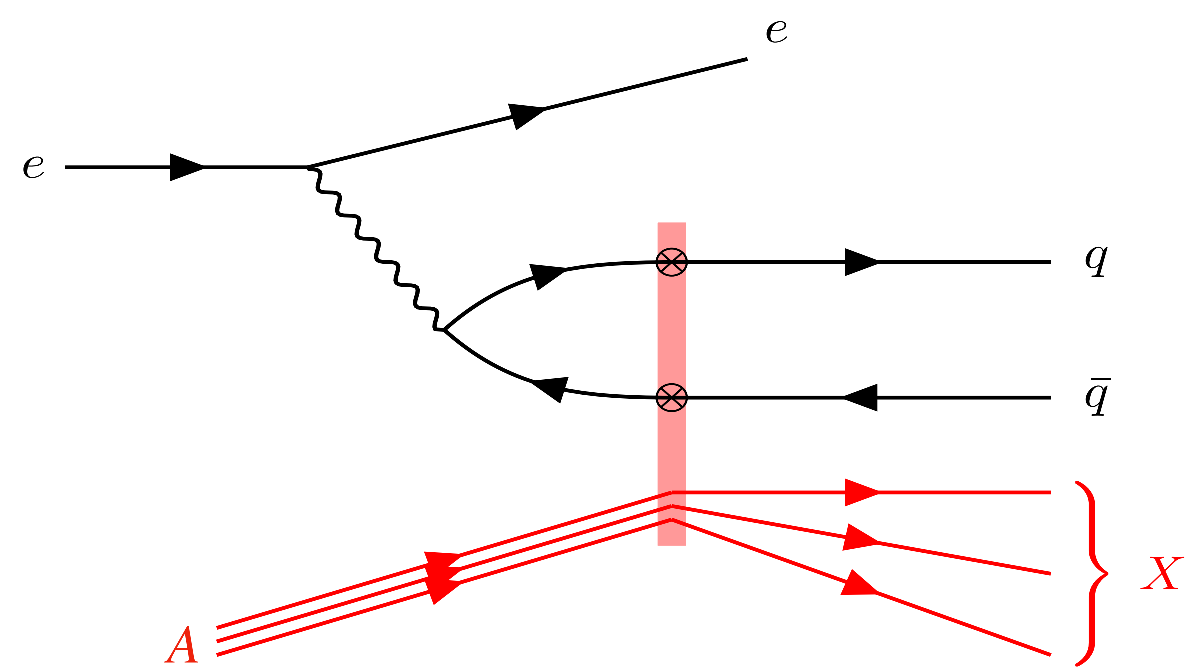

Paul Caucal Björn Schenke Tomasz Stebel Raju Venugopalan

Dijet production in DIS at LO

F. Gelis, J. Jalilian-Marian (2003)



No need for hybrid factorization. Pin down photon kinematics from electron. Only one channel.



Dijet differential cross-section:

$$\frac{\mathrm{d}\sigma^{\gamma^*_\lambda + A \to q\bar{q} + X}}{\mathrm{d}^2\boldsymbol{k_{1\perp}}\mathrm{d}^2\boldsymbol{k_{2\perp}}\mathrm{d}\eta_1\mathrm{d}\eta_2} \propto \int \mathrm{d}^8\boldsymbol{X}_\perp \; e^{-i\boldsymbol{k_{1\perp}}\cdot(\boldsymbol{x}_\perp - \boldsymbol{x}'_\perp)} e^{-i\boldsymbol{k_{2\perp}}\cdot(\boldsymbol{y}_\perp - \boldsymbol{y}'_\perp)}$$
$$\times \boxed{\langle \Xi_{\mathrm{LO}}(\boldsymbol{x}_\perp, \boldsymbol{y}_\perp; \boldsymbol{y}'_\perp \boldsymbol{x}'_\perp)\rangle_Y} \boxed{\mathcal{R}^\lambda(\boldsymbol{x}_\perp - \boldsymbol{y}_\perp, \boldsymbol{x}'_\perp - \boldsymbol{y}'_\perp)}$$

$$\Xi_{\mathrm{LO}}(\boldsymbol{x}_\perp, \boldsymbol{y}_\perp; \boldsymbol{y}'_\perp \boldsymbol{x}'_\perp) = 1 - S^{(2)}(\boldsymbol{x}_\perp, \boldsymbol{y}_\perp) - S^{(2)}(\boldsymbol{y}'_\perp, \boldsymbol{x}'_\perp) + S^{(4)}(\boldsymbol{x}_\perp, \boldsymbol{y}_\perp; \boldsymbol{y}'_\perp, \boldsymbol{x}'_\perp)$$

dipoles

quadrupole

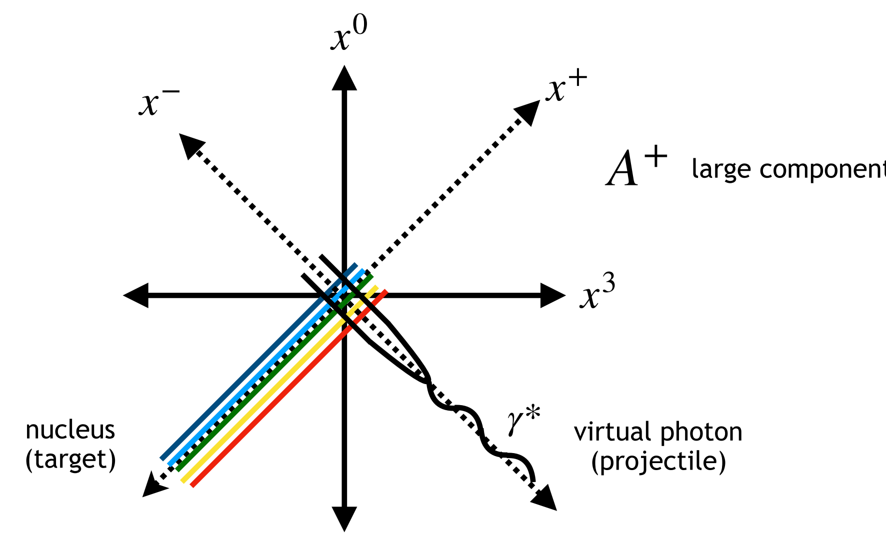
Implicitly contain saturation scale Q_s

First numerical evaluation
H. Mäntysaari, N. Mueller, FS, B. Schenke (PRL 2019)

Dijet production in DIS at NLO

Our calculation in a nutshell

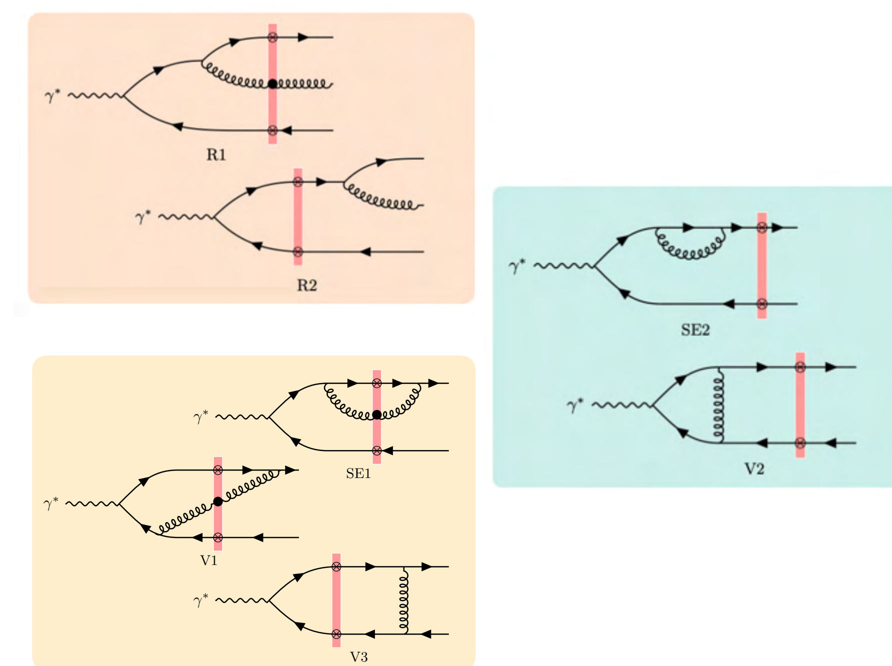
P. Caucal, FS, and R. Venugopalan. *JHEP 11 (2021) 222*



- Covariant perturbation theory Feynman rules in momentum space

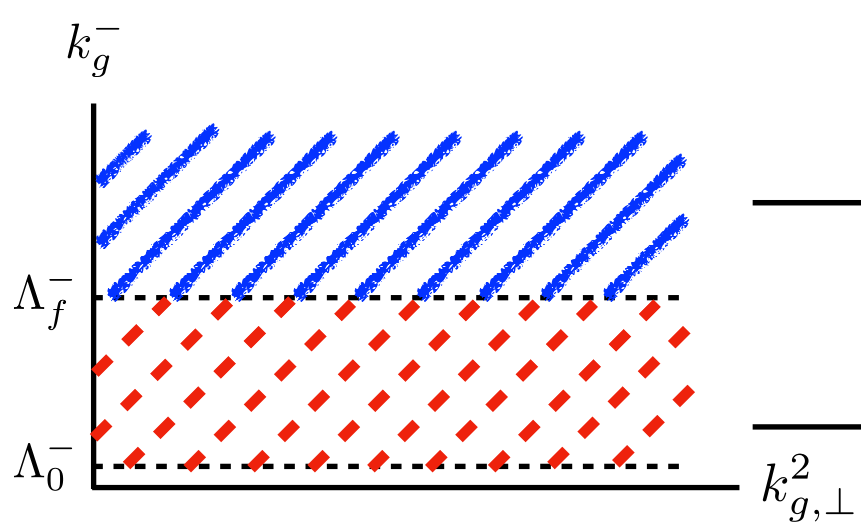
Dimensional regularization + longitudinal momentum cut-off + small-R cone algorithm

$$\int_{\Lambda_0^-} \frac{\mathrm{d}k_g^-}{k_g^-}\mu^\varepsilon \int \frac{\mathrm{d}^{2-\varepsilon}\boldsymbol{k_{g\perp}}}{(2\pi)^{2-\varepsilon}} f_{\Lambda^-}(k_g^-, \boldsymbol{k_{g\perp}})$$



- We showed cancellation of UV, soft and collinear divergences
- Absorbed large energy/rapidity logs into JIMWLK resummation
- Isolated genuine $\mathcal{O}(\alpha_s)$ contributions (aka NLO impact factor)

Gluon phase space (evolution vs impact factor)



- Impact factor

→ **Finite piece (free of large rapidity logs)**

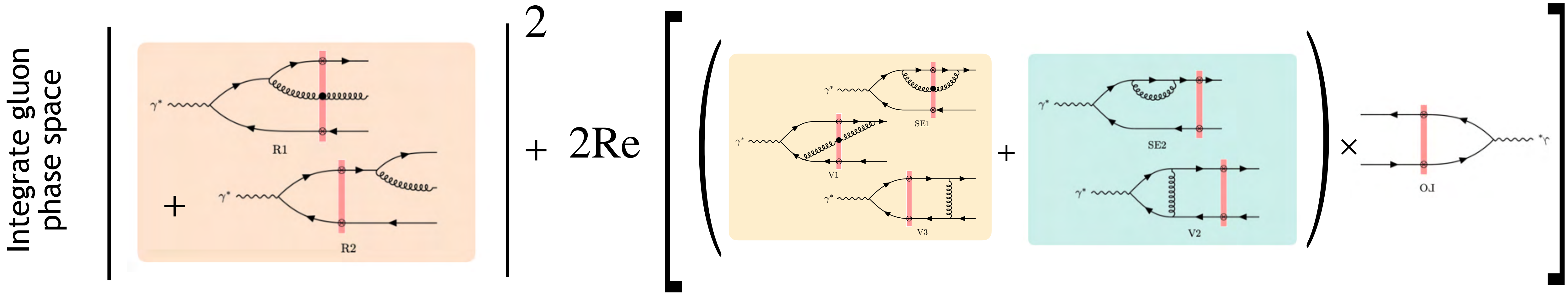
- Large rapidity (high-energy) logs

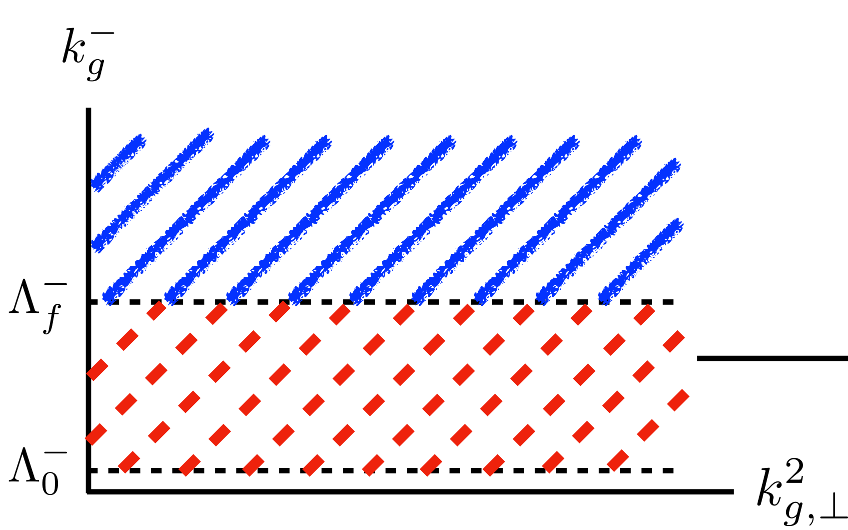
→ **Resummed via JIMWLK renormalization**

Dijet production in DIS at NLO

Small-x evolution: JIMWLK factorization

P. Caucal, FS, and R. Venugopalan. *JHEP 11 (2021) 222*





Small-x evolution of dipole and quadrupole!

$$\left.\frac{\mathrm{d}\sigma^{\lambda}_{\mathrm{NLO}}}{\mathrm{d}^2\boldsymbol{k_{1\perp}}\mathrm{d}\eta_1\mathrm{d}^2\boldsymbol{k_{2\perp}}\mathrm{d}\eta_2}\right|_{\mathrm{LLx}} = \frac{\alpha_{\mathrm{em}}e_f^2N_c}{(2\pi)^6}\delta(1-z_q-z_{\bar{q}})\int \mathrm{d}\boldsymbol{X}_\perp \mathcal{R}^{\lambda}_{\mathrm{LO}}(\boldsymbol{r}_{xy},\boldsymbol{r}_{x'y'})\ln\left(\Lambda_f^-/\Lambda^-\right)$$

$$\mathcal{H}_{\mathrm{LL}}\left\langle \Xi_{\mathrm{LO}}(\boldsymbol{x}_\perp,\boldsymbol{y}_\perp;\boldsymbol{x}'_\perp\boldsymbol{y}'_\perp)\right\rangle_Y$$

$$\begin{aligned}\times\frac{\alpha_sN_c}{4\pi^2}\Bigg\langle\int \mathrm{d}^2\boldsymbol{z}_\perp\Bigg\{&\frac{\boldsymbol{r}^2_{xy}}{\boldsymbol{r}^2_{zx}\boldsymbol{r}^2_{zy}}(2D_{xy}-2D_{xz}D_{zy}+D_{zy}Q_{y'x',xz}+D_{xz}Q_{y'x',zy}-Q_{xy,y'x'}-D_{xy}D_{y'x'})\\
&+\frac{\boldsymbol{r}^2_{x'y'}}{\boldsymbol{r}^2_{zx'}\boldsymbol{r}^2_{zy'}}(2D_{y'x'}-2D_{y'z}D_{zx'}+D_{zx'}Q_{xy,y'z}+D_{y'z}Q_{xy,zx'}-Q_{xy,y'x'}-D_{xy}D_{y'x'})\\
&+\frac{\boldsymbol{r}^2_{xx'}}{\boldsymbol{r}^2_{zx}\boldsymbol{r}^2_{zx'}}(D_{zx'}Q_{xy,y'z}+D_{xz}Q_{y'x',zy}-Q_{xy,y'x'}-D_{xx'}D_{y'y})\\
&+\frac{\boldsymbol{r}^2_{yy'}}{\boldsymbol{r}^2_{zy}\boldsymbol{r}^2_{zy'}}(D_{y'z}Q_{xy,zx'}+D_{zy}Q_{y'x',xz}-Q_{xy,y'x'}-D_{xx'}D_{y'y})\\
&+\frac{\boldsymbol{r}^2_{xy'}}{\boldsymbol{r}^2_{zx}\boldsymbol{r}^2_{zy'}}(D_{xx'}D_{y'y}+D_{xy}D_{y'x'}-D_{zx'}Q_{xy,y'z}-D_{zy}Q_{y'x',xz})\\
&+\frac{\boldsymbol{r}^2_{x'y}}{\boldsymbol{r}^2_{zx'}\boldsymbol{r}^2_{zy}}(D_{xx'}D_{y'y}+D_{xy}D_{y'x'}-D_{y'z}Q_{xy,zx'}-D_{xz}Q_{y'x',zy})\Bigg\}\Bigg\rangle_Y\end{aligned}$$

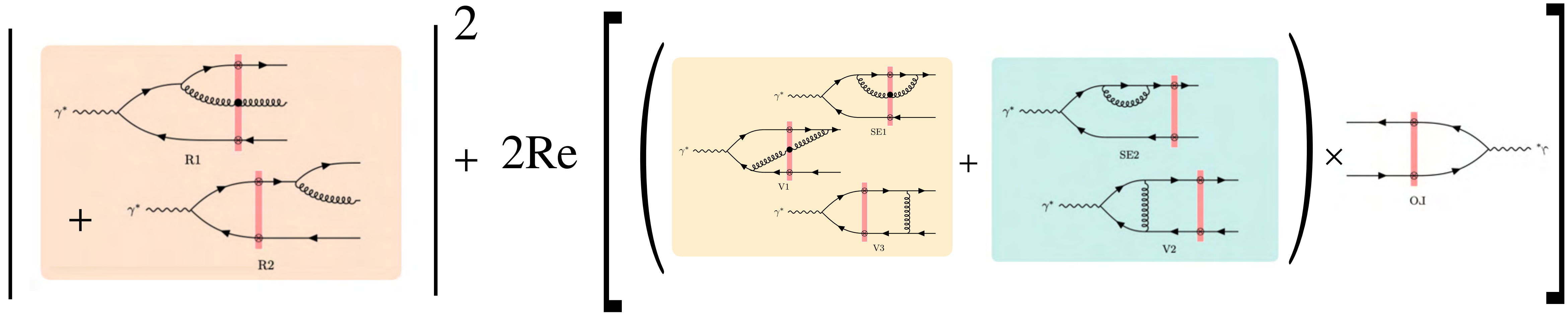
JIMWLK LL Hamiltonian acting on LO cross-section → Renormalization of Wilson line operators

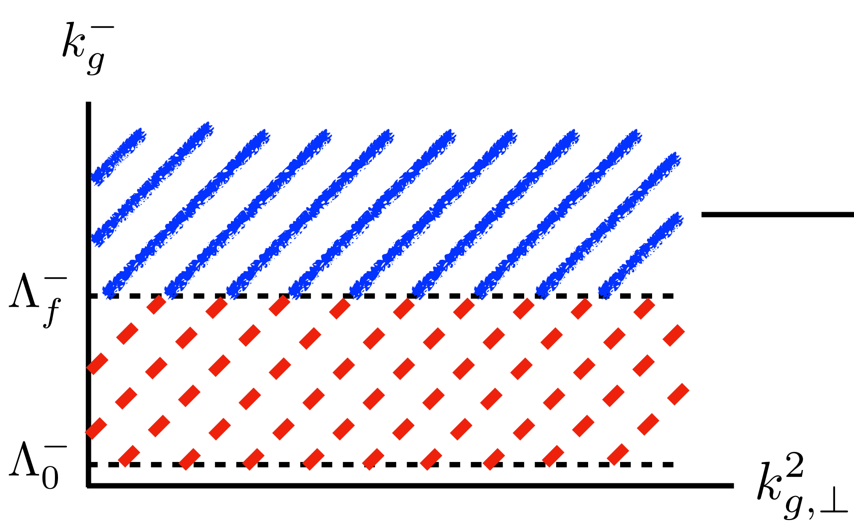
Dijet production in DIS at NLO

Impact factor

P. Caucal, FS, and R. Venugopalan. *JHEP 11 (2021) 222*

Integrate gluon phase space





$$\mathrm{d}\sigma_{\mathrm{R}_2\times\mathrm{R}_2,\mathrm{sud2}} = \frac{\alpha_{\mathrm{em}} e_f^2 N_c \delta_z^{(2)}}{(2\pi)^6}\int \mathrm{d}^8\boldsymbol{X}_\perp e^{-i\boldsymbol{k_{1\perp}}\cdot\boldsymbol{r}_{xx'}-i\boldsymbol{k_{2\perp}}\cdot\boldsymbol{r}_{yy'}}\mathcal{R}^\lambda_{\mathrm{LO}}(\boldsymbol{r}_{xy},\boldsymbol{r}_{x'y'}) \times C_F\Xi_{\mathrm{LO}}(\boldsymbol{x}_\perp,\boldsymbol{y}_\perp;\boldsymbol{x}'_\perp,\boldsymbol{y}'_\perp)\times\frac{\alpha_s}{\pi}\int_0^1\frac{\mathrm{d}\xi}{\xi}\left[1-e^{-i\xi\boldsymbol{k_{1\perp}}\cdot\boldsymbol{r}_{xx'}}\right]\ln\left(\frac{\boldsymbol{k_{1\perp}}^2\boldsymbol{r}_{xx'}^2R^2\xi^2}{c_0^2}\right)$$

$$\mathrm{d}\sigma_{\mathrm{R}_2\times\mathrm{R}'_2,\mathrm{sud2}} = \frac{\alpha_{\mathrm{em}} e_f^2 N_c \delta_z^{(2)}}{(2\pi)^6}\int \mathrm{d}^8\boldsymbol{X}_\perp e^{-i\boldsymbol{k_{1\perp}}\cdot\boldsymbol{r}_{xx'}-i\boldsymbol{k_{2\perp}}\cdot\boldsymbol{r}_{yy'}}\mathcal{R}^\lambda_{\mathrm{LO}}(\boldsymbol{r}_{xy},\boldsymbol{r}_{x'y'}) \times\Xi_{\mathrm{NLO},3}(\boldsymbol{x}_\perp,\boldsymbol{y}_\perp;\boldsymbol{x}'_\perp,\boldsymbol{y}'_\perp)\times\frac{(-\alpha_s)}{\pi}\int_0^1\frac{\mathrm{d}\xi}{\xi}\left[1-e^{-i\xi\boldsymbol{k_{1\perp}}\cdot\boldsymbol{r}_{xy'}}\right]\ln\left(\frac{\boldsymbol{P}_\perp^2\boldsymbol{r}_{xy'}^2\xi^2}{z_2^2c_0^2}\right)$$

$$\mathrm{d}\sigma^{\gamma_{\mathrm{L}}^\star+A\to q\bar{q}g+X}_{\mathrm{R,no-sud,LO}} = \frac{\alpha_{\mathrm{em}} e_f^2 N_c}{(2\pi)^8}\int \mathrm{d}^8\boldsymbol{X}_\perp e^{-i\boldsymbol{k_{1\perp}}\cdot\boldsymbol{r}_{xx'}-i\boldsymbol{k_{2\perp}}\cdot\boldsymbol{r}_{yy'}}(4\alpha_sC_F)\Xi_{\mathrm{LO}}(\boldsymbol{x}_\perp,\boldsymbol{y}_\perp;\boldsymbol{x}'_\perp,\boldsymbol{y}'_\perp) \times\frac{e^{-i\boldsymbol{k_{g\perp}}\cdot\boldsymbol{r}_{xx'}}}{(\boldsymbol{k_{g\perp}}-\frac{z_g}{z_1}\boldsymbol{k_{1\perp}})^2}\left\{8z_1z_2^3(1-z_2)^2Q^2\left(1+\frac{z_g}{z_1}+\frac{z_g^2}{2z_1^2}\right)K_0(\bar{Q}_{\mathrm{R2}}r_{xy})K_0(\bar{Q}_{\mathrm{R2}}r_{x'y'})\delta_z^{(3)} -\mathcal{R}^{\mathrm{L}}_{\mathrm{LO}}(\boldsymbol{r}_{xy},\boldsymbol{r}_{x'y'})\Theta(z_1-z_g)\delta_z^{(2)}\right\}+(1\leftrightarrow 2)$$

$$\mathrm{d}\sigma^{\gamma_{\mathrm{L}}^\star+A\to q\bar{q}g+X}_{\mathrm{R,no-sud,NLO_3}} = \frac{\alpha_{\mathrm{em}} e_f^2 N_c}{(2\pi)^8}\int \mathrm{d}^2\boldsymbol{X}_\perp e^{-i\boldsymbol{k_{1\perp}}\cdot\boldsymbol{r}_{xx'}-i\boldsymbol{k_{2\perp}}\cdot\boldsymbol{r}_{yy'}}(-4\alpha_s)\Xi_{\mathrm{NLO},3}(\boldsymbol{x}_\perp,\boldsymbol{y}_\perp;\boldsymbol{x}'_\perp,\boldsymbol{y}'_\perp) \times\frac{e^{-i\frac{z_g}{z_1}\boldsymbol{k_{1\perp}}\cdot\boldsymbol{r}_{xy'}}}{\boldsymbol{l}_\perp^2}\left\{8z_1^2z_2^2(1-z_2)(1-z_1)Q^2K_0(\bar{Q}_{\mathrm{R2}}r_{xy})K_0(\bar{Q}_{\mathrm{R2'}}r_{x'y'})\left[1+\frac{z_g}{2z_1}+\frac{z_g}{2z_2}\right] \times e^{-i\boldsymbol{l}_\perp\cdot\boldsymbol{r}_{xy'}}\frac{\boldsymbol{l}_\perp\cdot(\boldsymbol{l}_\perp+\boldsymbol{K}_\perp)}{(\boldsymbol{l}_\perp+\boldsymbol{K}_\perp)^2}\delta_z^{(3)}-\mathcal{R}^{\mathrm{L}}_{\mathrm{LO}}(\boldsymbol{r}_{xy},\boldsymbol{r}_{x'y'})\Theta\left(\frac{c_0^2}{\boldsymbol{r}_{xy'}^2}\geq\boldsymbol{l}_\perp^2\geq\boldsymbol{K}_\perp^2\right)\Theta(z_1-z_g)\delta_z^{(2)}\right\} +(1\leftrightarrow 2)$$

$$\mathrm{d}\sigma^{\gamma_{\mathrm{L}}^\star+A\to q\bar{q}g+X}_{\mathrm{R,no-sud,other}} = \frac{\alpha_{\mathrm{em}} e_f^2 N_c\delta_z^{(3)}}{(2\pi)^8}\int \mathrm{d}^8\boldsymbol{X}_\perp e^{-i\boldsymbol{k_{1\perp}}\cdot\boldsymbol{r}_{xx'}-i\boldsymbol{k_{2\perp}}\cdot\boldsymbol{r}_{yy'}}8z_1^3z_2^3Q^2\int\frac{\mathrm{d}^2\boldsymbol{z}_\perp}{\pi}\frac{\mathrm{d}^2\boldsymbol{z}'_\perp}{\pi}e^{-i\boldsymbol{k_{g\perp}}\cdot\boldsymbol{r}_{zz'}} \alpha_s\left\{-\frac{\boldsymbol{r}_{zx}\cdot\boldsymbol{r}_{z'x'}}{\boldsymbol{r}_{zx}^2\boldsymbol{r}_{z'x'}^2}K_0(QX_{\mathrm{R}})K_0(\bar{Q}_{\mathrm{R2}}r_{w'y'})\left(1+\frac{z_g}{z_1}+\frac{z_g^2}{2z_1^2}\right)\Xi_{\mathrm{NLO},1}(\boldsymbol{x}_\perp,\boldsymbol{y}_\perp,\boldsymbol{z}_\perp;\boldsymbol{w}'_\perp,\boldsymbol{y}'_\perp) +\frac{\boldsymbol{r}_{zy}\cdot\boldsymbol{r}_{z'x'}}{\boldsymbol{r}_{zy}^2\boldsymbol{r}_{z'x'}^2}K_0(QX_{\mathrm{R}})K_0(\bar{Q}_{\mathrm{R2'}}r_{w'y'})\left(1+\frac{z_g}{2z_1}+\frac{z_g}{2z_2}\right)\Xi_{\mathrm{NLO},1}(\boldsymbol{x}_\perp,\boldsymbol{y}_\perp,\boldsymbol{z}_\perp;\boldsymbol{w}'_\perp,\boldsymbol{y}'_\perp) +\frac{1}{2}\frac{\boldsymbol{r}_{zx}\cdot\boldsymbol{r}_{z'x'}}{\boldsymbol{r}_{zx}^2\boldsymbol{r}_{z'x'}^2}K_0(QX_{\mathrm{R}})K_0(QX'_{\mathrm{R}})\left(1+\frac{z_g}{z_1}+\frac{z_g^2}{2z_1^2}\right)\Xi_{\mathrm{NLO},4}(\boldsymbol{x}_\perp,\boldsymbol{y}_\perp,\boldsymbol{z}_\perp;\boldsymbol{x}'_\perp,\boldsymbol{y}'_\perp,\boldsymbol{z}'_\perp) -\frac{1}{2}\frac{\boldsymbol{r}_{zy}\cdot\boldsymbol{r}_{z'x'}}{\boldsymbol{r}_{zy}^2\boldsymbol{r}_{z'x'}^2}K_0(QX_{\mathrm{R}})K_0(QX'_{\mathrm{R}})\left(1+\frac{z_g}{2z_1}+\frac{z_g}{2z_2}\right)\Xi_{\mathrm{NLO},4}(\boldsymbol{x}_\perp,\boldsymbol{y}_\perp,\boldsymbol{z}_\perp;\boldsymbol{x}'_\perp,\boldsymbol{y}'_\perp,\boldsymbol{z}'_\perp) +(1\leftrightarrow 2)+c.c.\right\}-\frac{\alpha_{\mathrm{em}} e_f^2 N_c\delta_z^{(2)}}{(2\pi)^8}\alpha_s\Theta(z_f-z_g)\times\text{“slow”}$$

$$\mathrm{d}\sigma_{\mathrm{V,no-sud,LO}} = \frac{\alpha_{\mathrm{em}} e_f^2 N_c \delta_z^{(2)}}{(2\pi)^6}\int \mathrm{d}^8\boldsymbol{X}_\perp e^{-i\boldsymbol{k_{1\perp}}\cdot\boldsymbol{r}_{xx'}-i\boldsymbol{k_{2\perp}}\cdot\boldsymbol{r}_{yy'}}\mathcal{R}^\lambda_{\mathrm{LO}}(\boldsymbol{r}_{xy},\boldsymbol{r}_{x'y'})\Xi_{\mathrm{LO}}(\boldsymbol{x}_\perp,\boldsymbol{y}_\perp;\boldsymbol{x}'_\perp,\boldsymbol{y}'_\perp) \times\frac{\alpha_sC_F}{\pi}\left\{-\frac{3}{4}\ln\left(\frac{\boldsymbol{k_{1\perp}}^2\boldsymbol{k_{2\perp}}^2\boldsymbol{r}_{xy}^2\boldsymbol{r}_{x'y'}^2}{c_0^4}\right)-3\ln(R)+\frac{1}{2}\ln^2\left(\frac{z_1}{z_2}\right)+\frac{11}{2}+3\ln(2)-\frac{\pi^2}{2}\right\}$$

$$\mathrm{d}\sigma^{\lambda=\mathrm{L}}_{\mathrm{V,no-sud,NLO_3}} = \frac{\alpha_{\mathrm{em}} e_f^2 N_c \delta_z^{(2)}}{(2\pi)^6}\int \mathrm{d}^3\boldsymbol{X}_\perp e^{-i\boldsymbol{k_{1\perp}}\cdot\boldsymbol{r}_{xx'}-i\boldsymbol{k_{2\perp}}\boldsymbol{r}_{yy'}}8z_1^3z_2^3Q^2K_0(\bar{Q}r_{x'y'}) \times\frac{\alpha_s}{\pi}\int_0^{z_1}\frac{\mathrm{d}z_g}{z_g}\left\{K_0(\bar{Q}_{\mathrm{V3}}r_{xy})\left[\left(1-\frac{z_g}{z_1}\right)^2\left(1+\frac{z_g}{z_2}\right)(1+z_g)e^{i(\boldsymbol{P}_\perp+z_g\boldsymbol{q}_\perp)\cdot\boldsymbol{r}_{xy}}K_0(-i\Delta_{\mathrm{V3}}r_{xy}) -\left(1-\frac{z_g}{2z_1}+\frac{z_g}{2z_2}-\frac{z_g^2}{2z_1z_2}\right)e^{i\frac{z_g}{z_1}\boldsymbol{k_{1\perp}}\cdot\boldsymbol{r}_{xy}}\mathcal{J}_\odot\left(\boldsymbol{r}_{xy},\left(1-\frac{z_g}{z_1}\right)\boldsymbol{P}_\perp,\Delta_{\mathrm{V3}}\right)\right] +K_0(\bar{Q}r_{xy})\ln\left(\frac{z_gP_\perp r_{xy}}{c_0z_1z_2}\right)+(1\leftrightarrow 2)\right\}\Xi_{\mathrm{NLO},3}(\boldsymbol{x}_\perp,\boldsymbol{y}_\perp;\boldsymbol{x}'_\perp,\boldsymbol{y}'_\perp)+c.c.$$

$$\mathrm{d}\sigma^{\lambda=\mathrm{L}}_{\mathrm{V,no-sud,other}} = \frac{\alpha_{\mathrm{em}} e_f^2 N_c \delta_z^{(2)}}{(2\pi)^6}\int \mathrm{d}^8\boldsymbol{X}_\perp e^{-i\boldsymbol{k_{1\perp}}\cdot\boldsymbol{r}_{xx'}-i\boldsymbol{k_{2\perp}}\boldsymbol{r}_{yy'}}8z_1^3z_2^3Q^2K_0(\bar{Q}r_{x'y'})\int_0^{z_1}\frac{\mathrm{d}z_g}{z_g} \times\frac{\alpha_s}{\pi}\int\frac{\mathrm{d}^2\boldsymbol{z}_\perp}{\pi}\left\{\frac{1}{\boldsymbol{r}_{zx}^2}\left[\left(1-\frac{z_g}{z_1}+\frac{z_g^2}{2z_1^2}\right)e^{-i\frac{z_g}{z_1}\boldsymbol{k_{1\perp}}\cdot\boldsymbol{r}_{zx}}K_0(QX_V)-\Theta(z_f-z_g)K_0(\bar{Q}r_{xy})\right]\Xi_{\mathrm{NLO},1} -\frac{1}{\boldsymbol{r}_{zx}^2}\left[\left(1-\frac{z_g}{z_1}+\frac{z_g^2}{2z_1^2}\right)e^{-\frac{\boldsymbol{r}_{zx}^2}{\boldsymbol{r}_{xy}^2e^{\gamma_E}}}K_0(\bar{Q}r_{xy})-\Theta(z_f-z_g)e^{-\frac{\boldsymbol{r}_{zx}^2}{\boldsymbol{r}_{xy}^2e^{\gamma_E}}}K_0(\bar{Q}r_{xy})\right]C_F\Xi_{\mathrm{LO}} -\frac{\boldsymbol{r}_{zx}\cdot\boldsymbol{r}_{zy}}{\boldsymbol{r}_{zx}^2\boldsymbol{r}_{zy}^2}\left[\left(1-\frac{z_g}{z_1}\right)\left(1+\frac{z_g}{z_2}\right)\left(1-\frac{z_g}{2z_1}-\frac{z_g}{2(z_2+z_g)}\right)e^{-i\frac{z_g}{z_1}\boldsymbol{k_{1\perp}}\cdot\boldsymbol{r}_{zx}}K_0(QX_V) -\Theta(z_f-z_g)K_0(\bar{Q}r_{xy})\right]\Xi_{\mathrm{NLO},1}+(1\leftrightarrow 2)\right\}+c.c..$$

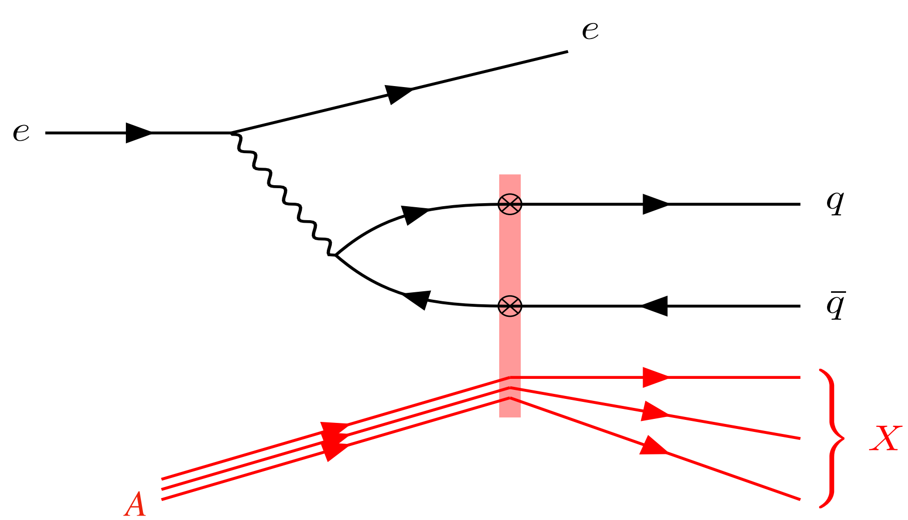
$$\mathrm{d}\sigma_{\mathrm{sud1}} = \frac{\alpha_{\mathrm{em}} e_f^2 N_c \delta_z^{(2)}}{(2\pi)^6}\int \mathrm{d}^8\boldsymbol{X}_\perp e^{-i\boldsymbol{k_{1\perp}}\cdot\boldsymbol{r}_{xx'}-i\boldsymbol{k_{2\perp}}\cdot\boldsymbol{r}_{yy'}}\mathcal{R}^\lambda_{\mathrm{LO}}(\boldsymbol{r}_{xy},\boldsymbol{r}_{x'y'})\times\frac{\alpha_s}{\pi} \times\left\{C_F\Xi_{\mathrm{LO}}(\boldsymbol{x}_\perp,\boldsymbol{y}_\perp;\boldsymbol{x}'_\perp,\boldsymbol{y}'_\perp)\left[\ln\left(\frac{z_f}{z_1}\right)\ln\left(\frac{\boldsymbol{r}_{xx'}^2}{|\boldsymbol{r}_{xy}||\boldsymbol{r}_{x'y'}|}\right)+\ln\left(\frac{z_f}{z_2}\right)\ln\left(\frac{\boldsymbol{r}_{yy'}^2}{|\boldsymbol{r}_{xy}||\boldsymbol{r}_{x'y'}|}\right)\right] +\Xi_{\mathrm{NLO},3}(\boldsymbol{x}_\perp,\boldsymbol{y}_\perp;\boldsymbol{x}'_\perp,\boldsymbol{y}'_\perp)\left[\ln\left(\frac{z_1}{z_f}\right)\ln\left(\frac{\boldsymbol{r}_{xy'}^2}{|\boldsymbol{r}_{xy}||\boldsymbol{r}_{x'y'}|}\right)+\ln\left(\frac{z_2}{z_f}\right)\ln\left(\frac{\boldsymbol{r}_{yx'}^2}{|\boldsymbol{r}_{xy}||\boldsymbol{r}_{x'y'}|}\right)\right]\right\}$$

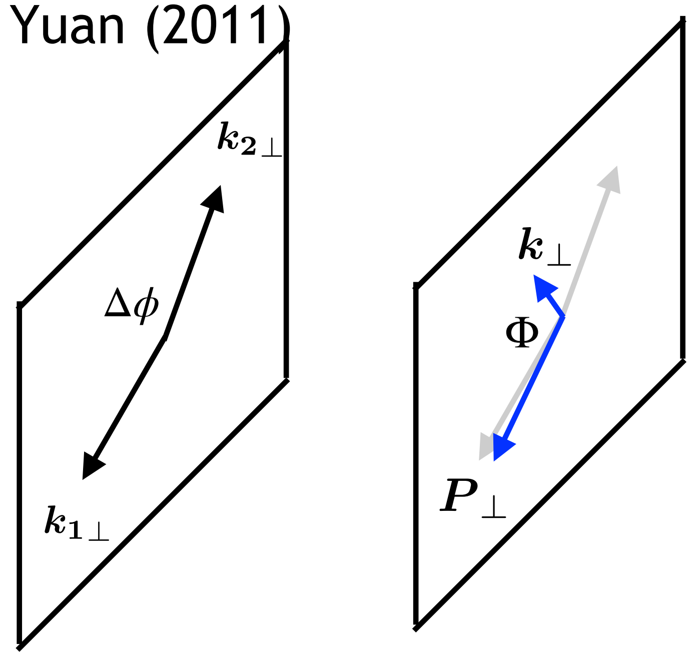
Only longitudinally polarized photon shown, lengthier expressions for transversely polarized photon

Back-to-back dijets at LO

Small-x Weizsäcker-Williams (WW) gluon distribution

F. Dominguez, C. Marquet, B-W. Xiao, F. Yuan (2011)





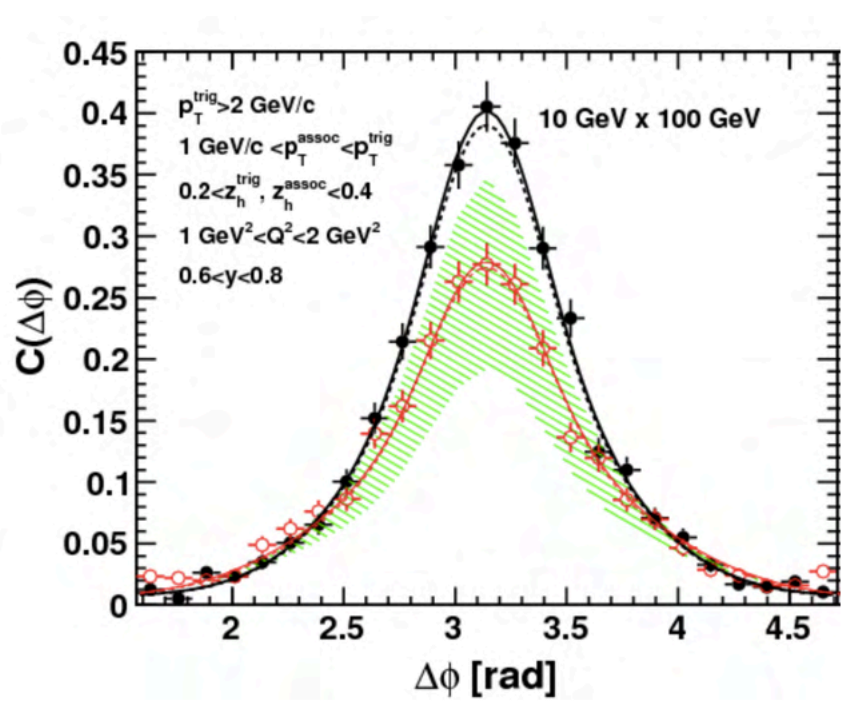
$$\boldsymbol{k}_\perp = \boldsymbol{k_1}_\perp + \boldsymbol{k_2}_\perp$$
$$\boldsymbol{P}_\perp = z_2 \boldsymbol{k_1}_\perp - z_1 \boldsymbol{k_2}_\perp$$

Perturbatively calculable

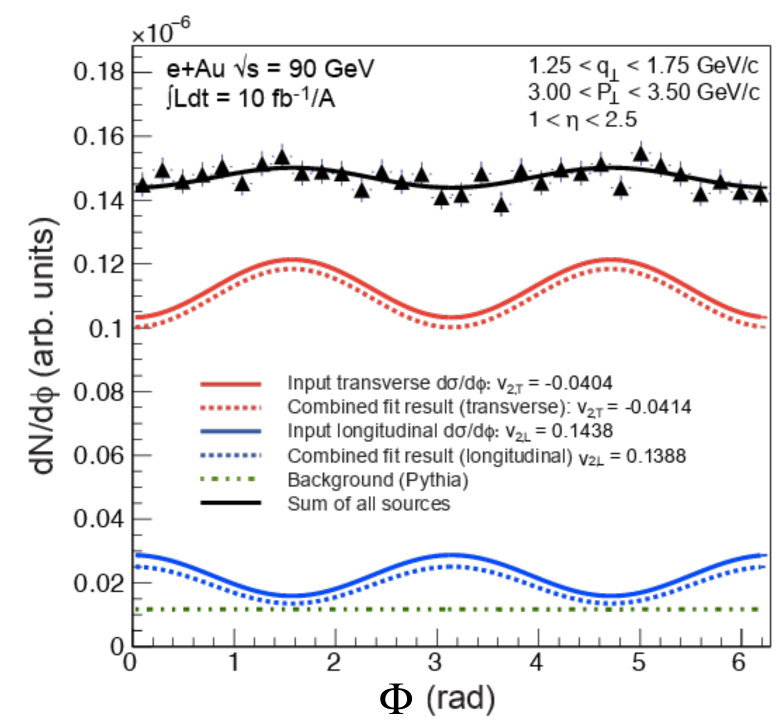
WW gluon TMD (all saturation contained here)

Dijets are produced back-to-back $k_\perp, Q_s \ll P_\perp$: $\mathrm{d}\sigma^{\gamma^*_\lambda + A \to q\bar{q} + X} \sim \mathcal{H}^{ij}(Q, \boldsymbol{P}_\perp)\ \alpha_s G_Y^{ij}(\boldsymbol{k}_\perp)$

CGC → small-x TMD-like factorization



Zheng, Aschenauer, Lee, Xiao (2014)



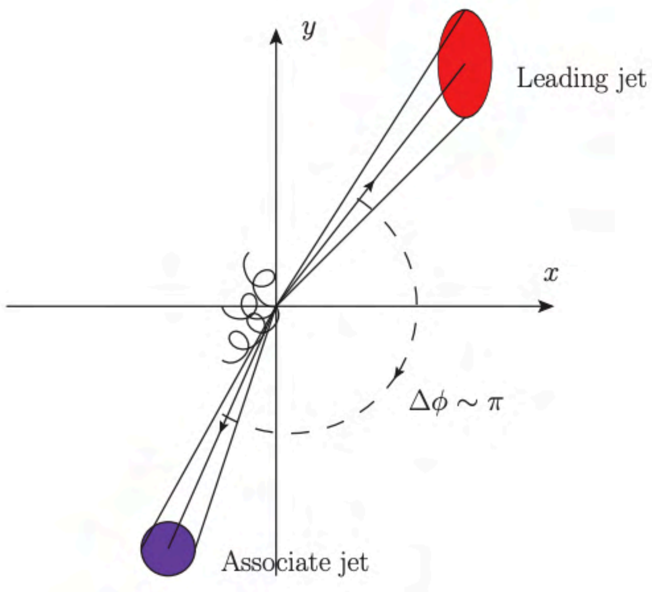
Dumitru, Skokov, Ullrich (2018)

Back-to-back dijets at NLO

Does CGG-TMD correspondence hold at NLO?

A.H. Mueller, B-W. Xiao, F. Yuan (2013)

$$q_\perp^2 \ll P_\perp^2 \ll s^2 \qquad \ln(s/P_\perp^2) \qquad \ln^2(P_\perp^2/q_\perp^2)$$



Conjecture: joint (soft) small-x + Sudakov resummation

$$\mathrm{d}\sigma^{\gamma_\lambda^* + A \to q\bar{q} + X} \propto \mathcal{H}(Q, \boldsymbol{P}_\perp) \int \frac{\mathrm{d}^2\boldsymbol{b}_\perp}{(2\pi)^2} e^{-i\boldsymbol{q}_\perp \cdot \boldsymbol{b}_\perp} \alpha_s \widetilde{G}_Y^0(\boldsymbol{b}_\perp) e^{-S_{\mathrm{Sud}}(\boldsymbol{b}_\perp, \boldsymbol{P}_\perp)}$$

Perturbative Sudakov factor:

$$S_{\mathrm{Sud}}(b_\perp^2, P_\perp^2) = \int_{c_0^2/b_\perp^2}^{P_\perp^2} \frac{\mathrm{d}\mu^2}{\mu^2} \left[A \ln\left(\frac{P_\perp^2}{\mu^2}\right) + B \right]$$

Soft gluon emissions $\longrightarrow$ **Change profile of azimuthal correlations**

See also Y. Hatta, B-W. Xiao, F. Yuan, J. Zhou (2021)

Can we derive these results from our NLO calculation? Can we obtain finite NLO pieces?

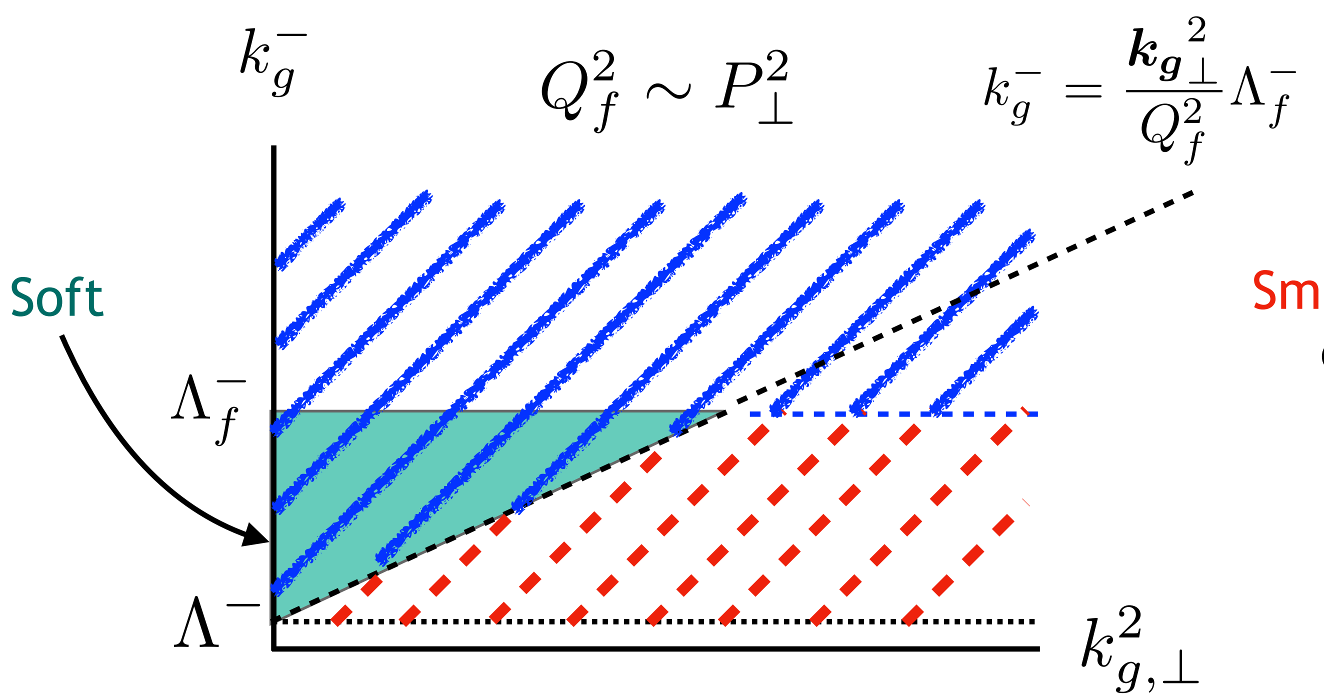
Back-to-back dijets at NLO

Small-x and Sudakov interplay: The need for kinematic constraint

P. Caucal, FS, B. Schenke, and R. Venugopalan *JHEP 11 (2022) 169*

The back-to-back limit of our NLO dijet led to Sudakov double log with opposite sign

Solution: impose a kinematic constraint which also enforces ordering in k_g^+ (target momenta)



Proposed by P. Taels, T. Altinoluk, C. Marquet, G. Beuf (2022) for dijet photo-production

Small-x evolution for WW follows well-known BK-JIMWLK equations **amended with a kinematic constrain** to separate small-x and soft gluons

Joint resummation of small-x

soft gluons at finite Nc and to single log accuracy

$$\mathrm{d}\sigma^{\gamma_\lambda^* + A \to q\bar{q} + X} = \mathcal{H}^{\lambda,ij}(Q, \boldsymbol{P}_\perp) \int \frac{\mathrm{d}^2\boldsymbol{b}_\perp}{(2\pi)^2} e^{-i\boldsymbol{q}_\perp \cdot \boldsymbol{b}_\perp} \alpha_s \widetilde{G}_Y^{ij}(\boldsymbol{b}_\perp) e^{-S_{\mathrm{Sud}}(\boldsymbol{b}_\perp, \boldsymbol{P}_\perp)} + \mathcal{O}(\alpha_s)$$

What about finite piece?

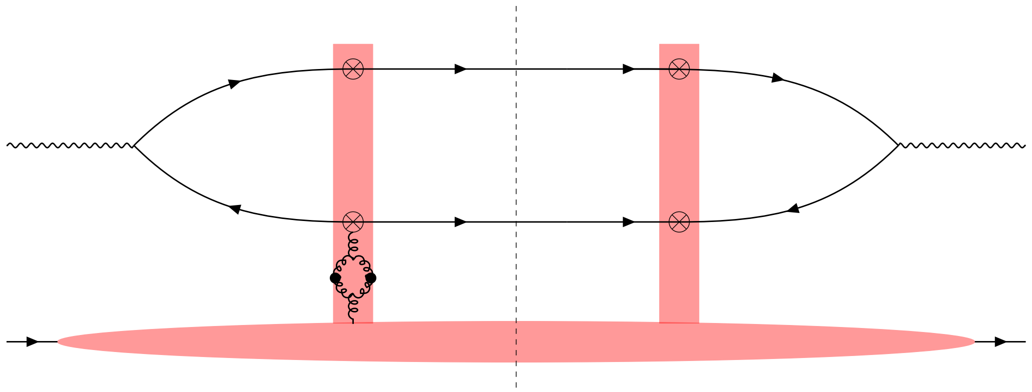
Back-to-back dijets at NLO

Caucal, FS, Schenke, Stebel, Venugopalan. 2304.03304

Sudakov single log from quantum correction to back-ground field

One-loop correction to the background field has two pieces: $\ln(1/x)$ piece (evolution) and **UV divergent contribution (running of the coupling)**

Ayala, Jalilian-Marian, McLerran, Venugopalan (1996)



$$\hat{G}_Y^{(1)}(\boldsymbol{r}_{bb'}) = -\alpha_s \beta_0 \left[\frac{1}{\varepsilon} + \text{finite}\right] \hat{G}_Y^{(0)}(\boldsymbol{r}_{bb'})$$

$$\beta_0 = (11N_c - 2n_f)/(12\pi)$$

Renormalize the WW distribution from the natural scale $c_0^2/r_{bb'}^2$ to hard scale μ_h^2

$$\hat{G}_{Y_f}(\boldsymbol{r}_{bb'}, \mu_h^2) = \boxed{\exp\left(\int_{c_0^2/\boldsymbol{r}_{bb'}^2}^{\mu_h^2} \frac{\mathrm{d}\mu^2}{\mu^2} \alpha_s \beta_0\right)} \hat{G}_{Y_f}(\boldsymbol{r}_{bb'}, c_0^2/\boldsymbol{r}_{bb'}^2)$$

Single Sudakov log

Final result for Sudakov form factor:

$$S_{\mathrm{Sud}}(b_\perp, \mu_h) = \int_{c_0^2/\boldsymbol{r}_{bb'}^2}^{\mu_h^2} \frac{\mathrm{d}\mu^2}{\mu^2} \frac{\alpha_s(\mu^2) N_c}{\pi} \left[\frac{1}{2}\ln\left(\frac{\mu_h^2}{\mu^2}\right) + \frac{C_F}{N_c} s_0 - \tilde{s}_f - \frac{\pi \beta_0}{N_c}\right]$$

$$s_0 = \ln\left(\frac{2(1+\cosh(\Delta\eta_{12}))}{R^2}\right) \qquad s_f = 1 - \ln\left(1 + \frac{Q^2}{M_{q\bar{q}}^2}\right)$$

Consistent with CSS formalism (see e.g. Hatta, Xiao, Yuan, Zhou 2021), except for an additional -1 factor

Back-to-back dijets at NLO

Caucal, FS, Schenke, Stebel, Venugopalan. 2304.03304

Finite pieces: correlators beyond the WW

$$\mathrm{d}\sigma^{\gamma_\lambda^*+A\to q\bar{q}+X} = \mathcal{H}^{ij}(Q,\boldsymbol{P}_\perp)\int \frac{\mathrm{d}^2\boldsymbol{b}_\perp}{(2\pi)^2} e^{-i\boldsymbol{q}_\perp\cdot\boldsymbol{b}_\perp}\alpha_s \widetilde{G}^{ij}_Y(\boldsymbol{b}_\perp)e^{-S_{\mathrm{Sud}}(\boldsymbol{b}_\perp,\boldsymbol{P}_\perp)} + \mathcal{O}(\alpha_s)$$

Pure $\mathcal{O}(\alpha_s)$

Subset involves complicated convolutions including operators beyond WW, but **needed for precision!**

$\Xi_{\mathrm{LO}}(\boldsymbol{x}_\perp,\boldsymbol{y}_\perp;\boldsymbol{x}'_\perp,\boldsymbol{y}'_\perp)$	$\langle 1 - D_{xy} - D_{y'x'} + Q_{xy,y'x'}\rangle$
$\Xi_{\mathrm{NLO},1}(\boldsymbol{x}_\perp,\boldsymbol{y}_\perp,\boldsymbol{z}_\perp;\boldsymbol{x}'_\perp,\boldsymbol{y}'_\perp)$	$\frac{N_c}{2}\langle 1 - D_{y'x'} + Q_{zy,y'x'}D_{xz} - D_{xz}D_{zy}\rangle - \frac{1}{2N_c}\Xi_{\mathrm{LO}}$
$\Xi_{\mathrm{NLO},2}(\boldsymbol{x}_\perp,\boldsymbol{y}_\perp,\boldsymbol{z}_\perp;\boldsymbol{x}'_\perp,\boldsymbol{y}'_\perp)$	$\frac{N_c}{2}\langle 1 - D_{y'x'} + Q_{xz,y'x'}D_{zy} - D_{xz}D_{zy}\rangle - \frac{1}{2N_c}\Xi_{\mathrm{LO}}$
$\Xi_{\mathrm{NLO},3}(\boldsymbol{x}_\perp,\boldsymbol{y}_\perp;\boldsymbol{x}'_\perp,\boldsymbol{y}'_\perp)$	$\frac{N_c}{2}\langle 1 - D_{xy} - D_{y'x'} + D_{xy}D_{y'x'}\rangle - \frac{1}{2N_c}\Xi_{\mathrm{LO}}$
$\Xi_{\mathrm{NLO},4}(\boldsymbol{x}_\perp,\boldsymbol{y}_\perp,\boldsymbol{z}_\perp;\boldsymbol{x}'_\perp,\boldsymbol{y}'_\perp,\boldsymbol{z}'_\perp)$	$\frac{N_c}{2}\langle 1 - D_{xz}D_{zy} - D_{y'z}D_{zx'} + Q_{xz,z'x'}Q_{y'z',zy}\rangle - \frac{1}{2N_c}\Xi_{\mathrm{LO}}$

- Blue correlators collapse to the WW gluon TMD, red correlators result in other TMDs. e.g.

$$\Xi_{\mathrm{LO}} \approx \boldsymbol{u}^i_\perp \boldsymbol{u}'^j_\perp \frac{1}{2N_c}\underbrace{(-2)\left\langle \mathrm{Tr}\left[\left(V(\boldsymbol{b}_\perp)\partial^i V^\dagger(\boldsymbol{b}_\perp)\right)\left(V(\boldsymbol{0}_\perp)\partial^j V^\dagger(\boldsymbol{0}_\perp)\right)\right]\right\rangle_Y}_{\alpha_s \widetilde{G}^{ij}_Y(\boldsymbol{b}_\perp)}$$

$$\Xi_{\mathrm{NLO},1} \approx -\boldsymbol{u}'^j_\perp \frac{1}{2N_c}\underbrace{(-2)\left\langle \mathrm{Tr}\left[V(\boldsymbol{b}_\perp)V^\dagger(\boldsymbol{z}_\perp)\right]\mathrm{Tr}\left[V(\boldsymbol{z}_\perp)V^\dagger(\boldsymbol{b}_\perp)\partial^j V(\boldsymbol{0}_\perp)V^\dagger(\boldsymbol{0}_\perp)\right]\right\rangle_Y}_{\alpha_s \widetilde{G}^j_{Y,\mathrm{NLO},1}(\boldsymbol{b}_\perp,\boldsymbol{z}_\perp)}$$

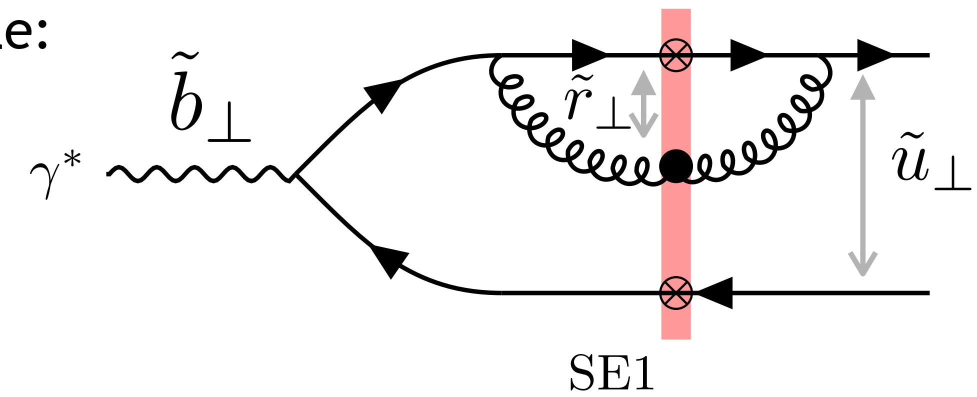
breaks TMD factorization?

Back-to-back dijets at NLO

Caucal, FS, Schenke, Stebel, Venugopalan.
2304.03304

Finite pieces: restoring TMD factorization

Example:



Correlation limit at NLO

$$q_\perp \ll P_\perp \longrightarrow \tilde{u}_\perp \ll \tilde{b}_\perp$$

Still involves operator beyond WW

Key observation: Perturbative factor constraint $\tilde{r}_\perp$:

$$K_0\left(\bar{Q}\sqrt{u_\perp^2 + \frac{z_g}{z_1 z_2} r_\perp^2}\right) \longrightarrow r_\perp^2 \lesssim \frac{z_1 z_2}{z_g} u_\perp^2 \sim \frac{1}{z_g P_\perp^2}$$

$z_g \sim \mathcal{O}(1)$ Expand around small $r_\perp$

Recover WW!

$$\widetilde{\Xi}_{\mathrm{NLO},1} = \left[C_F \tilde{\boldsymbol{u}}_\perp^i \boldsymbol{u}_\perp'^j + \left(\frac{N_c}{2} + \frac{1}{2N_c}\frac{z_g}{(z_1 - z_g)}\right)\tilde{\boldsymbol{r}}_\perp^i \boldsymbol{u}_\perp'^j\right]$$
$$\times \frac{1}{N_c}\left\langle \mathrm{Tr} V(\tilde{\boldsymbol{b}}_\perp)\partial^i V^\dagger(\tilde{\boldsymbol{b}}_\perp)\partial^j V(\boldsymbol{b}_\perp')V^\dagger(\boldsymbol{b}_\perp')\right\rangle_Y .$$

$z_g \ll 1$ Can't expand in $r_\perp$

Operator beyond WW non-linear evolution!

Operators can be found in the Evolution of WW by
F. Dominguez, A. Mueller, S. Munier, B-W. Xiao (2011)

$r_\perp \sim 1/k_{g\perp}$

Kinematic constraint interpolates between
non-linear (small $k_{g\perp}^2$ gluons) and linear (large $k_{g\perp}^2$ gluons) regimes

Back-to-back dijets at NLO

Caucal, FS, Schenke, Stebel, Venugopalan. 2304.03304

Complete small-x TMD factorization at NLO

$$\mathrm{d}\sigma^{\gamma^*_\lambda+A\to\mathrm{dijet}+X} = \mathcal{H}^{ij,\lambda}_{\mathrm{NLO}}(Q,\boldsymbol{P}_\perp;\mu_F;R)\int\frac{\mathrm{d}^2\boldsymbol{b}_\perp}{(2\pi)^2}e^{-i\boldsymbol{q}_\perp\cdot\boldsymbol{b}_\perp}\alpha_s\widetilde{G}^{ij}_Y(\boldsymbol{b}_\perp)e^{-S_{\mathrm{Sud}}(\boldsymbol{b}_\perp,\mu_F)} + \mathcal{O}(\alpha_s^2)$$

fully analytic result

$$\mathrm{d}\sigma^{(0),\lambda=\mathrm{L}} = \alpha_{\mathrm{em}}\alpha_s e_f^2\delta_z^{(2)}\mathcal{H}^{0,\lambda=\mathrm{L}}_{\mathrm{LO}}\left\{1+\frac{\alpha_s(\mu_h)}{\pi}\left[\frac{N_c}{2}\tilde{f}_1^{\lambda=\mathrm{L}}(\chi,z_f)+\frac{1}{2\pi N_c}\tilde{f}_2^{\lambda=\mathrm{L}}(\chi)\right]\right\}$$
$$\times\int\frac{\mathrm{d}^2\boldsymbol{r}_{bb'}}{(2\pi)^4}e^{-i\boldsymbol{q}_\perp\cdot\boldsymbol{r}_{bb'}}\hat{G}^0_{Y_f}(\boldsymbol{r}_{bb'})\,\widetilde{S}(\mu_h^2,\boldsymbol{r}_{bb'}^2)$$
$$+\alpha_{\mathrm{em}}\alpha_s e_f^2\delta_z^{(2)}\mathcal{H}^{0,\lambda=\mathrm{L}}_{\mathrm{LO}}\frac{\alpha_s(\mu_h)}{\pi}\left\{\frac{N_c}{2}\left[1+\ln(R^2)\right]+\frac{1}{2N_c}\left[-\ln(z_1z_2R^2)\right]\right\}$$
$$\times\int\frac{\mathrm{d}^2\boldsymbol{r}_{bb'}}{(2\pi)^4}e^{-i\boldsymbol{q}_\perp\cdot\boldsymbol{r}_{bb'}}\hat{h}^0_{Y_f}(\boldsymbol{r}_{bb'})\,\widetilde{S}(\mu_h^2,\boldsymbol{r}_{bb'}^2)$$

Analogous expression for elliptic anisotropy $\mathrm{d}\sigma^{(2),\lambda=L}$

$$f_1^{\lambda=\mathrm{L}}(\chi,z_f) = 7-\frac{3\pi^2}{2}-\frac{3}{2}\ln\left(\frac{z_1z_2R^2}{\chi^2}\right)-\ln(z_1)\ln(z_2)+2\ln\left(\frac{(1+\chi^2)z_f}{z_1z_2}\right)$$
$$-\ln(1+\chi^2)\ln\left(\frac{1+\chi^2}{z_1z_2}\right)+\left\{\mathrm{Li}_2\left(\frac{z_2-z_1\chi^2}{z_2(1+\chi^2)}\right)-\frac{1}{4(z_2-z_1\chi^2)}\right.$$
$$\left.+\frac{(1+\chi^2)(z_2(2z_2-z_1)+z_1(2z_1-z_2)\chi^2)}{4(z_2-z_1\chi^2)^2}\ln\left(\frac{z_2(1+\chi^2)}{\chi^2}\right)+(1\leftrightarrow 2)\right\}$$

$$\chi = \frac{\bar{Q}}{P_\perp}$$

Similar expression for f_2

The first proof of TMD factorization at NLO at small-x (modulo the non-linear evolution of the WW)

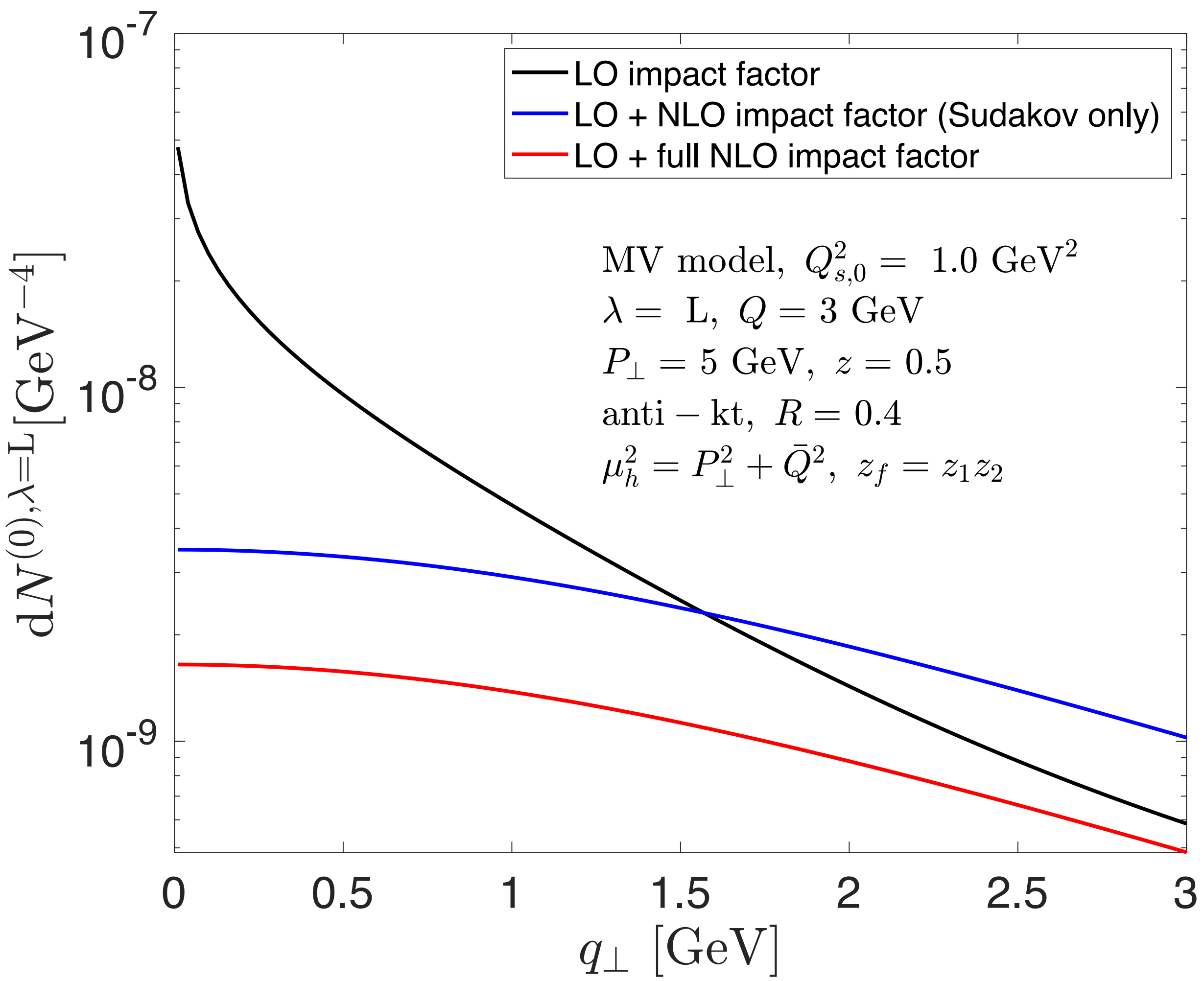
Back-to-back inclusive dijets in DIS at NLO

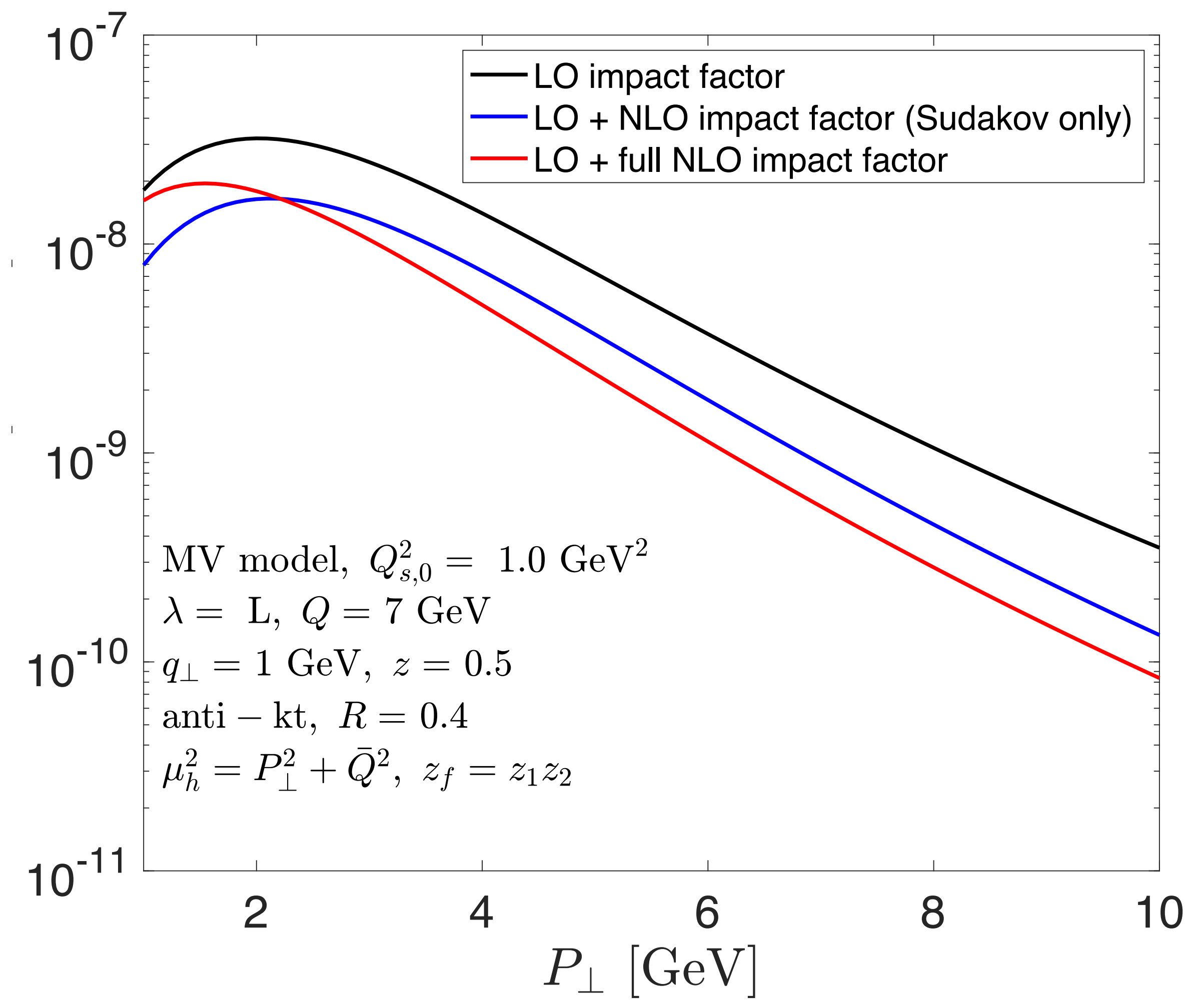
Numerical results (only $\gamma_L^* + A \to 2jet + X$) Caucal, FS, Schenke, Stebel, Venugopalan. 2304.03304

- WW from MV model (Gaussian approximation) without rapidity evolution
- Sudakov form factor with running of the coupling at two-loops + non-perturbative contribution

$$\frac{\mathrm{d}N^{\lambda=\mathrm{L}}}{\mathrm{d}^2\boldsymbol{P}_\perp \mathrm{d}^2\boldsymbol{q}_\perp \mathrm{d}\eta_1 \mathrm{d}\eta_2} = \mathrm{d}N^{(0),\lambda=\mathrm{L}}\left[1 + 2\sum_{n=1}^{\infty} v_{2n}^{\lambda=\mathrm{L}} \cos(2n\phi)\right]$$

ϕ angle between $\boldsymbol{P}_\perp$ and $\boldsymbol{q}_\perp$





Back-to-back inclusive dijets in DIS at NLO

Numerical results (only $\gamma_L^* + A \to 2jet + X$)

Caucal, FS, Schenke, Stebel, Venugopalan.
2304.03304

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$$\frac{\mathrm{d}N^{\lambda=\mathrm{L}}}{\mathrm{d}^2\boldsymbol{P}_\perp \mathrm{d}^2\boldsymbol{q}_\perp \mathrm{d}\eta_1 \mathrm{d}\eta_2} = \mathrm{d}N^{(0),\lambda=\mathrm{L}}\left[1 + 2\sum_{n=1}^{\infty} v_{2n}^{\lambda=\mathrm{L}}\cos(2n\phi)\right]$$

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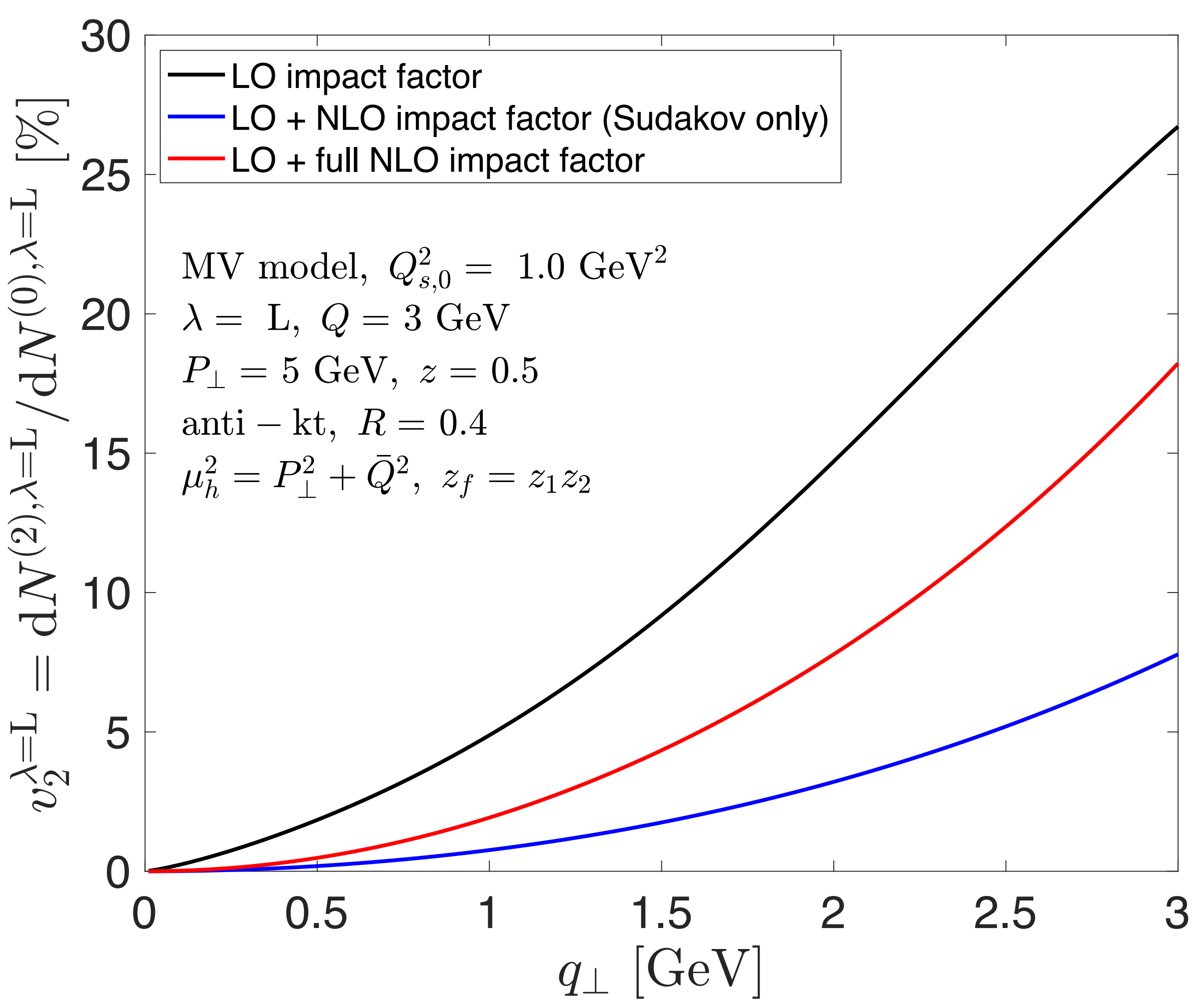
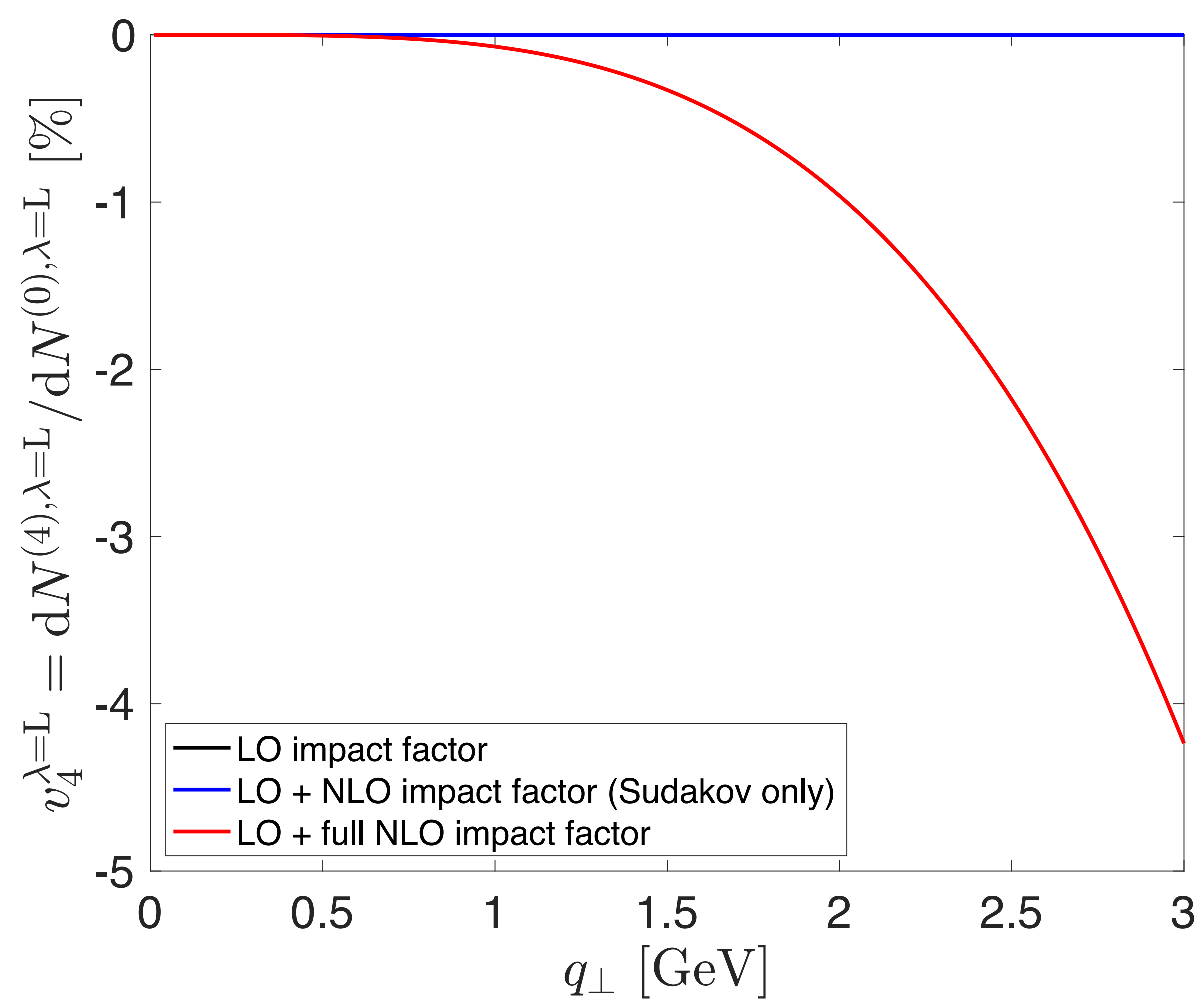




Summary

Motivation:

2-particle azimuthal correlations → powerful observables to search for saturation

Results:

full NLO calculation dijets in DIS $\xrightarrow{\text{back-to-back limit}}$

small-x and soft gluon resummation

NLO finite pieces

preliminary numerical results

Outlook: Include kinematically constrained small-x evolution in numerical result

Other observables: Dihadrons, UPCs, all two-particle observables

Could SCET-like techniques help us promote results beyond NLO/ more complex observables?

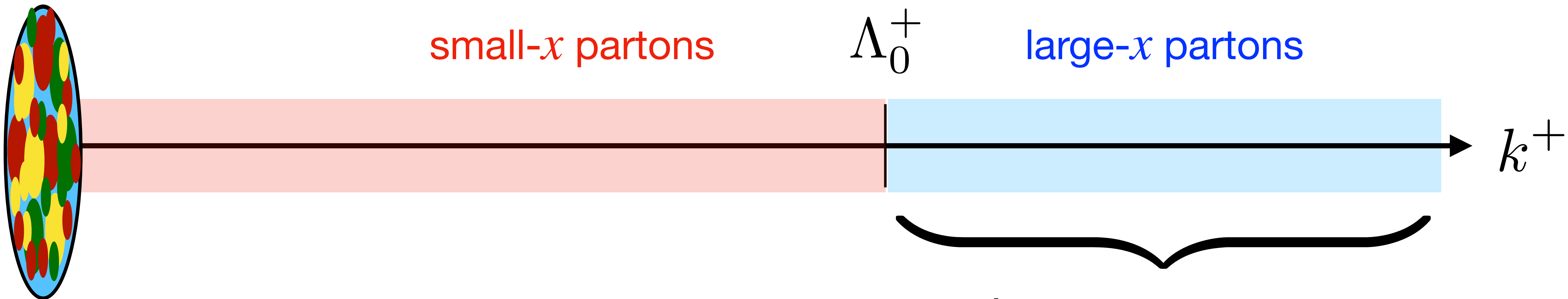
Back-up slides

The Color Glass Condensate

Sources and fields

L. McLerran, R. Venugopalan (1993)

Color (QCD)
Glass (separation slow vs fast modes)
Condensate (highly occupied system)



Back-ground field $\longleftarrow$ Integrate out current (along light-cone)

A double average:

$$\langle\langle \mathcal{O} \rangle\rangle = \int \underbrace{[\mathcal{D}\rho]\, W_{\Lambda_0}[\rho]}_{\text{CGC average for } \rho} \underbrace{\frac{\int^{\Lambda_0} [\mathcal{D}A]\, \mathcal{O} e^{i\mathcal{S}[A,\rho]}}{\int^{\Lambda_0} [\mathcal{D}A]\, e^{i\mathcal{S}[A,\rho]}}}_{\text{Path integral in the presence of } \rho}$$

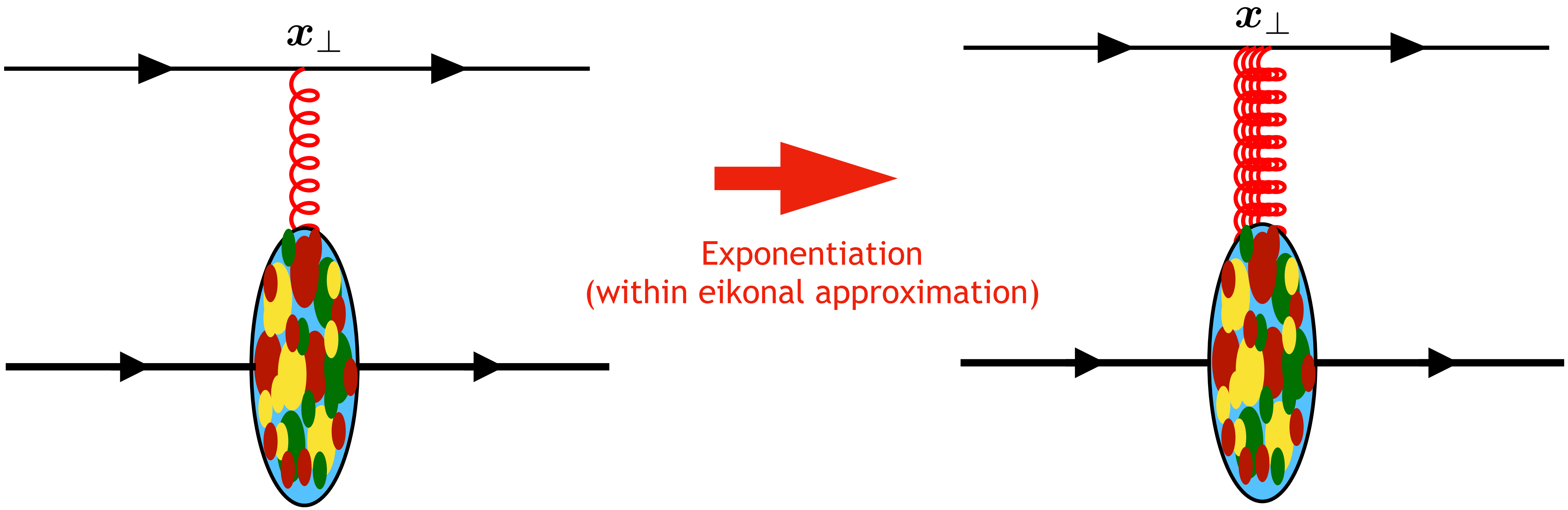
At leading order:

$$\langle\langle \mathcal{O} \rangle\rangle = \int [\mathcal{D}\rho]\, W_{\Lambda_0}[\rho] \mathcal{O}[A_{\mathrm{cl}}]$$

$\longleftarrow$ Classical solution in presence of ρ

The Color Glass Condensate

Multiple scattering and Wilson lines



Light-like Wilson line

$$V_{ij}(\boldsymbol{x}) = P \exp\left\{ ig \int dx^- A_{cl}^{+,a}(\boldsymbol{x}, x^-) t^a \right\}$$

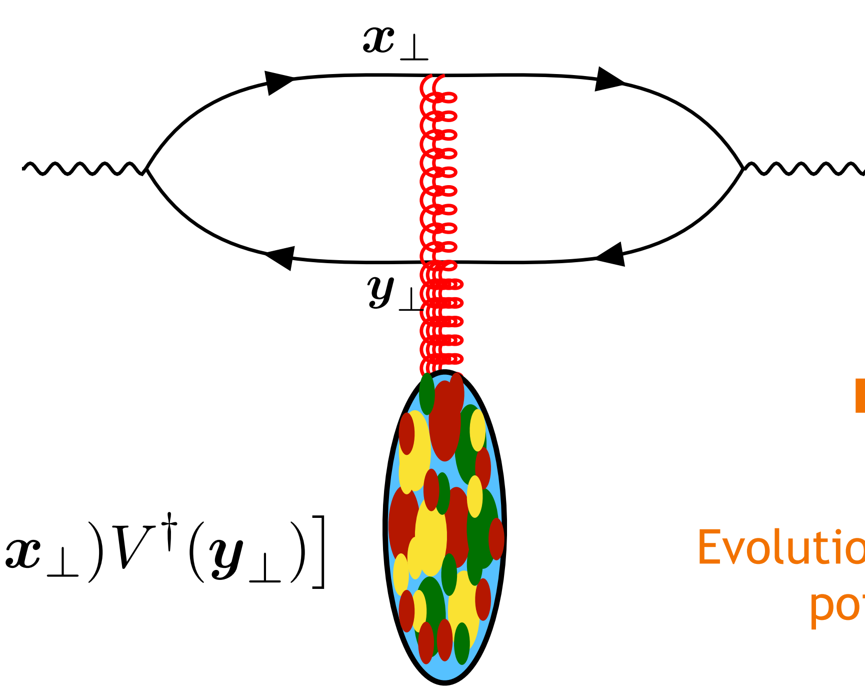
Observables built from Wilson lines, derivatives, etc… convoluted with perturbative factor (splitting functions)

$$\langle \mathcal{O} \rangle = \langle V V^\dagger \ldots \rangle$$

A. Ayala, J. Jalilian-Marian,
L. McLerran, R. Venugopalan (1995)
I. Balistky (1996)

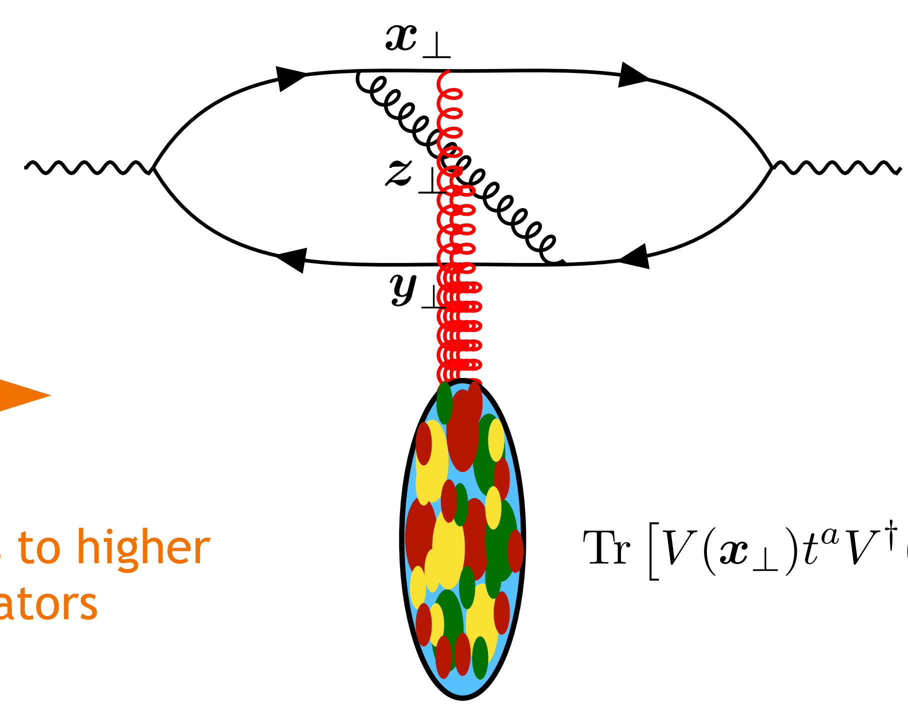
The Color Glass Condensate

Non-linear evolution of Wilson lines



$\mathrm{Tr}\left[V(\boldsymbol{x}_\perp)V^\dagger(\boldsymbol{y}_\perp)\right]$

Evolution couples to higher point correlators



$\mathrm{Tr}\left[V(\boldsymbol{x}_\perp)t^a V^\dagger(\boldsymbol{y}_\perp)t^b\right] U_{ab}(\boldsymbol{z}_\perp)$

Dipole: $$S_Y^{(2)}(\boldsymbol{x}_\perp - \boldsymbol{y}_\perp) = \frac{1}{N_c}\left\langle \mathrm{Tr}\left[V(\boldsymbol{x}_\perp)V^\dagger(\boldsymbol{y}_\perp)\right]\right\rangle_Y$$

Gluon emissions lead to non-linear evolution

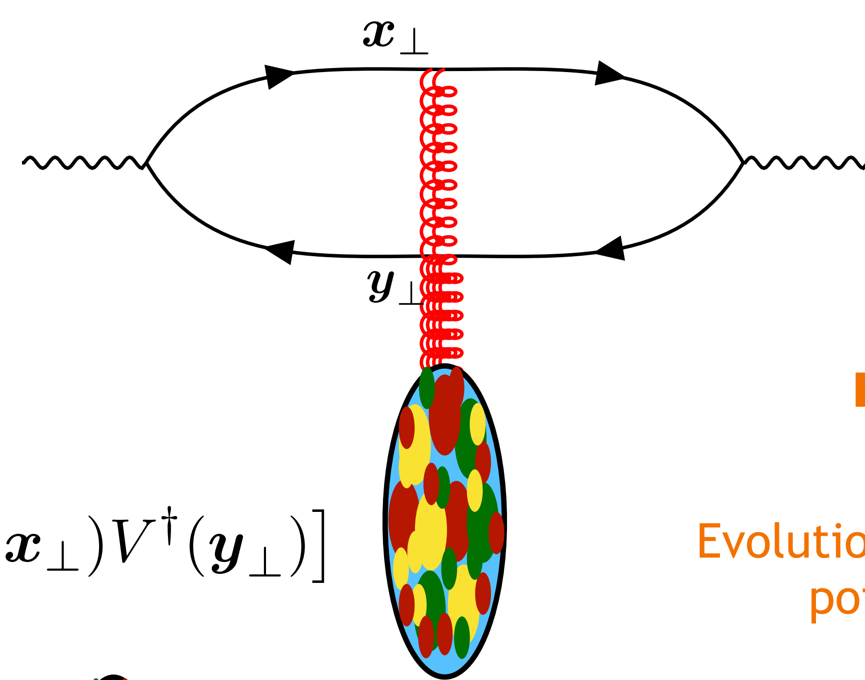
I. Balistky (1995), Y. Kovchegov (1999)

Balitsky-Kovchegov equation:

$$\frac{\mathrm{d}S_Y^{(2)}(\boldsymbol{r}_\perp)}{\mathrm{d}Y} = \frac{\alpha_s N_c}{2\pi^2}\int \mathrm{d}^2\boldsymbol{r}'_\perp \frac{\boldsymbol{r}_\perp^2}{\boldsymbol{r}'^2_\perp(\boldsymbol{r}_\perp - \boldsymbol{r}'_\perp)^2}\left[S_Y^{(2)}(\boldsymbol{r}'_\perp)S_Y^{(2)}(\boldsymbol{r}_\perp - \boldsymbol{r}'_\perp) - S_Y^{(2)}(\boldsymbol{r}_\perp)\right]$$

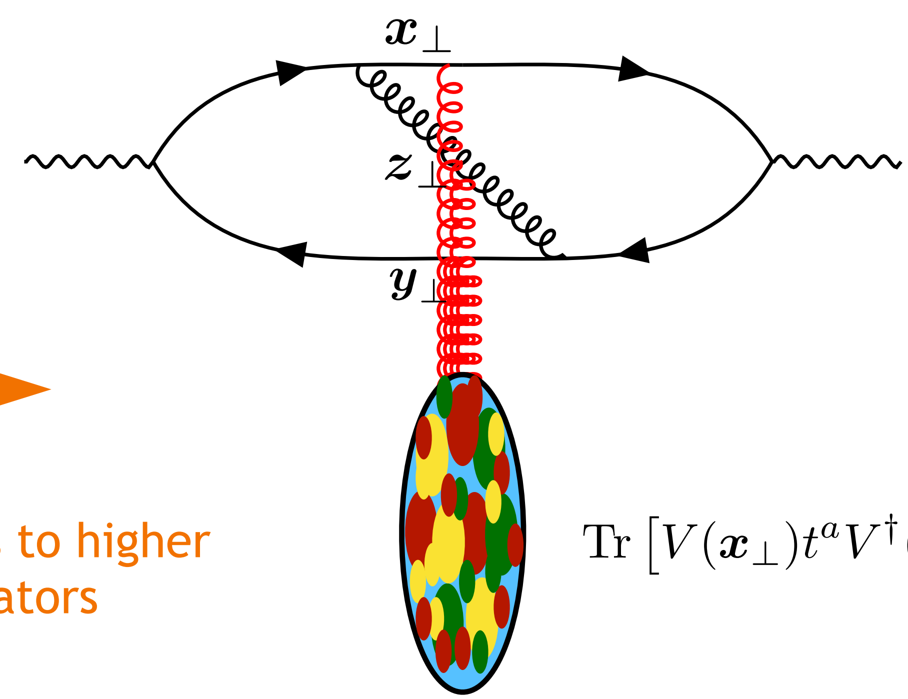
The Color Glass Condensate

Renormalization of sources

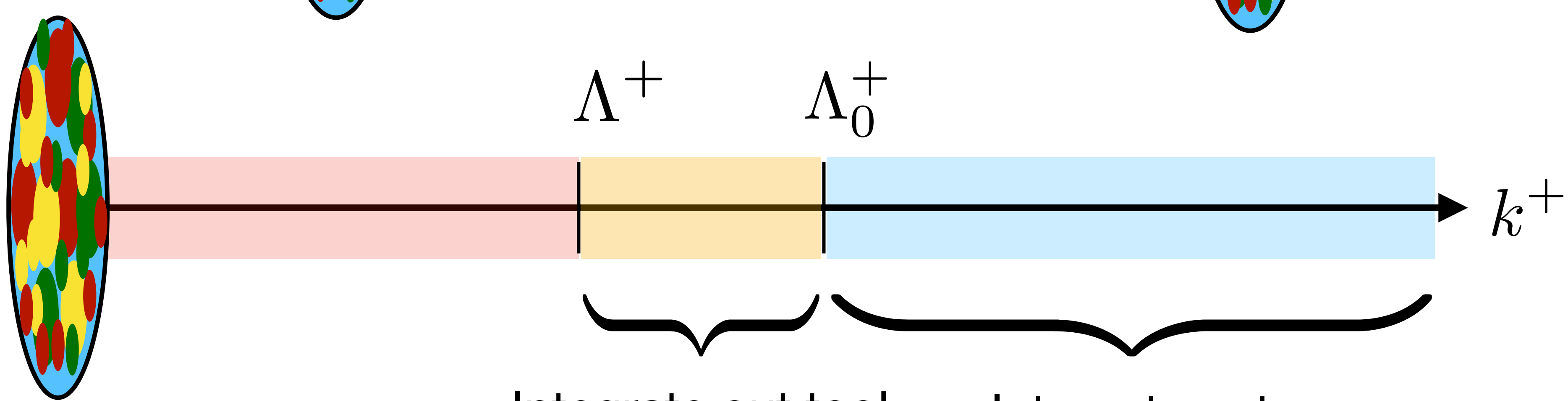


$\mathrm{Tr}\left[V(\boldsymbol{x}_\perp)V^\dagger(\boldsymbol{y}_\perp)\right]$

Evolution couples to higher point correlators



$\mathrm{Tr}\left[V(\boldsymbol{x}_\perp)t^aV^\dagger(\boldsymbol{y}_\perp)t^b\right]U_{ab}(\boldsymbol{z}_\perp)$

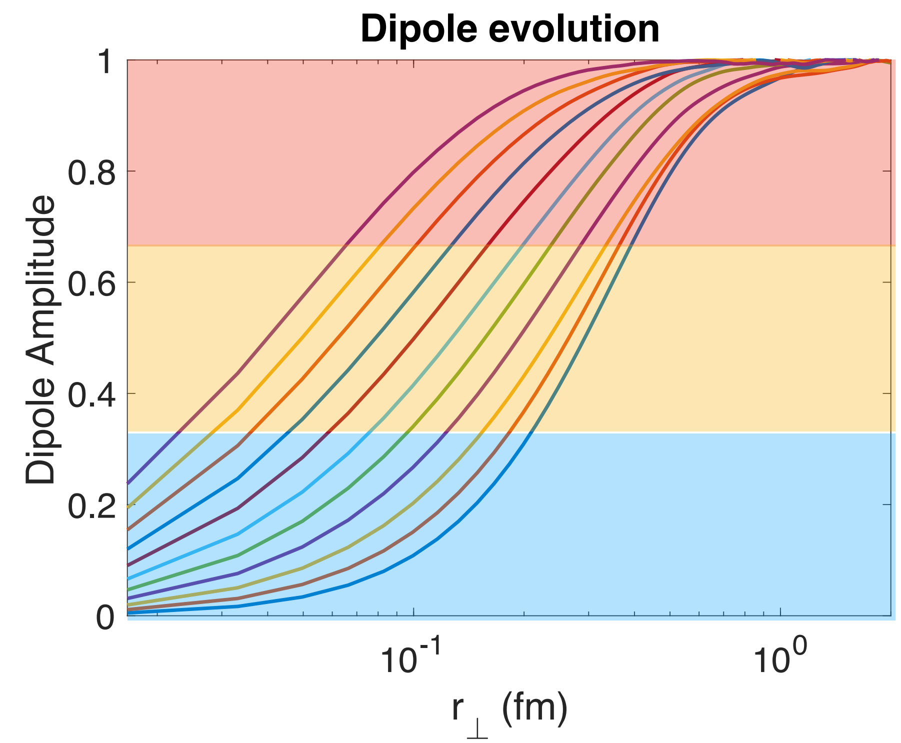


Small-x BK-JIMWLK resummation

$$W_{\Lambda_0}[\rho] \to W_\Lambda[\rho] \qquad \langle\langle\mathcal{O}\rangle\rangle = \int [\mathcal{D}\rho]\, W_\Lambda[\rho]\mathcal{O}[A_{\rm cl}]$$

J. Jalilian-Marian, E. Iancu, L. McLerran,
H. Weigert, A. Leonidov, A. Kovner (1996-2002)

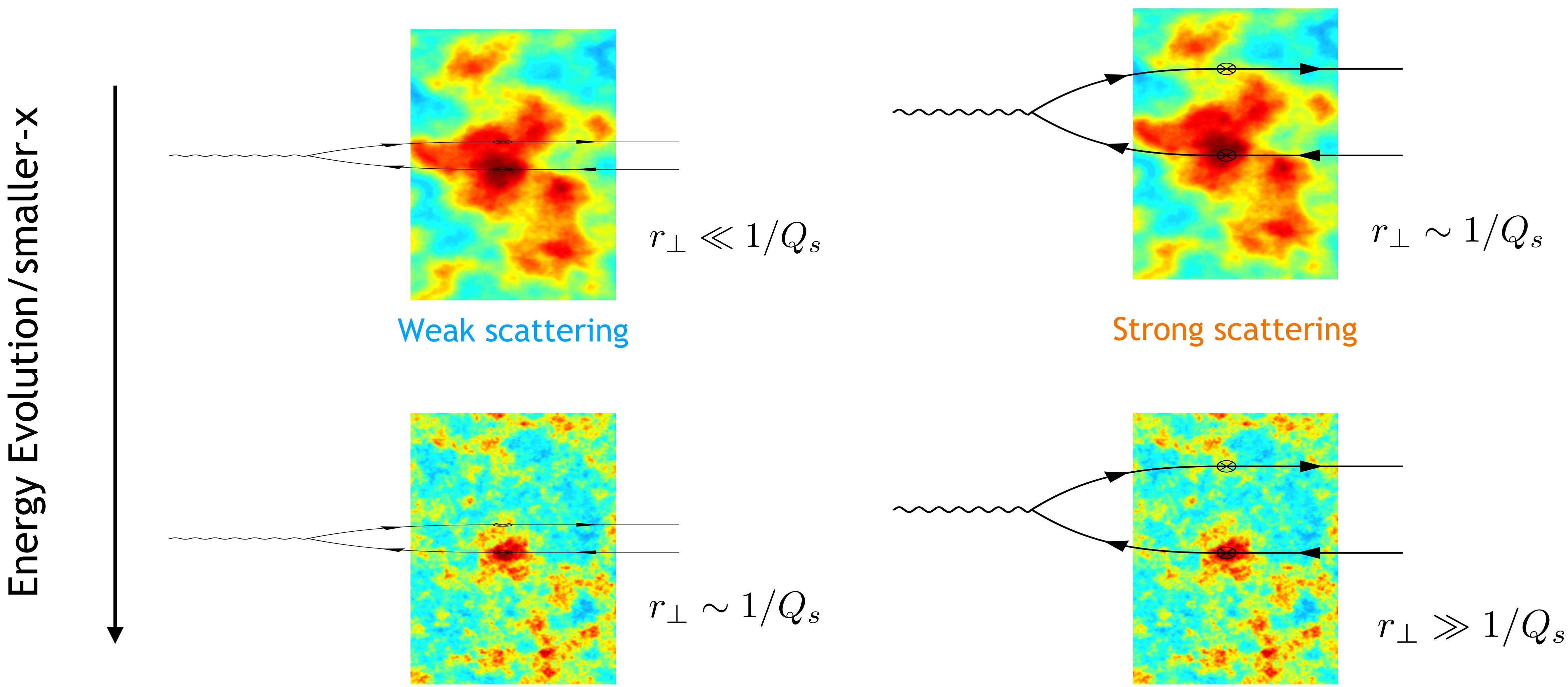
The Color Glass Condensate



Saturation scale

Dipole amplitude: $1 - \frac{1}{N_c}\langle \mathrm{Tr}\left[V(\boldsymbol{x}_\perp)V^\dagger(\boldsymbol{y}_\perp)\right]\rangle$

Q_s "correlation length" $Q_s^2 \propto A^{1/3}x^{-0.3}$



Weak scattering

Strong scattering

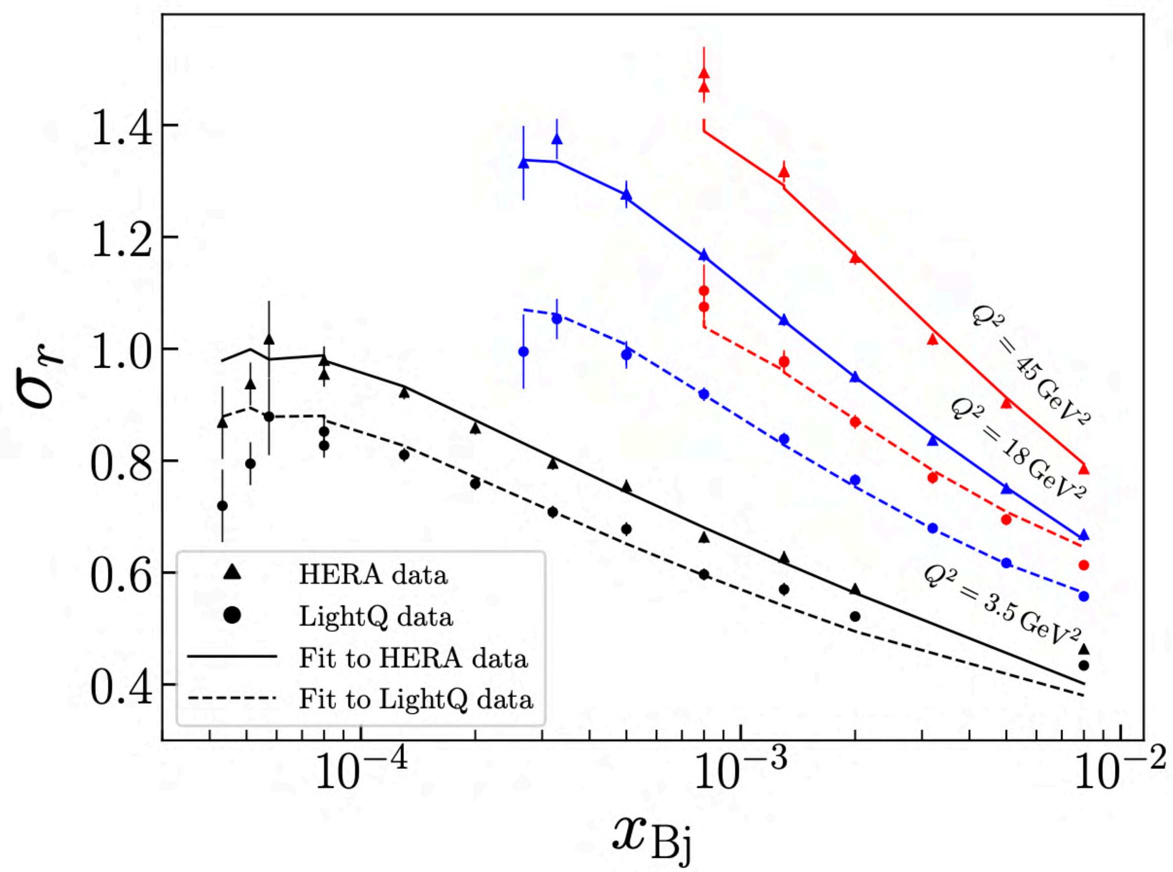
Strong scattering

Very strong scattering

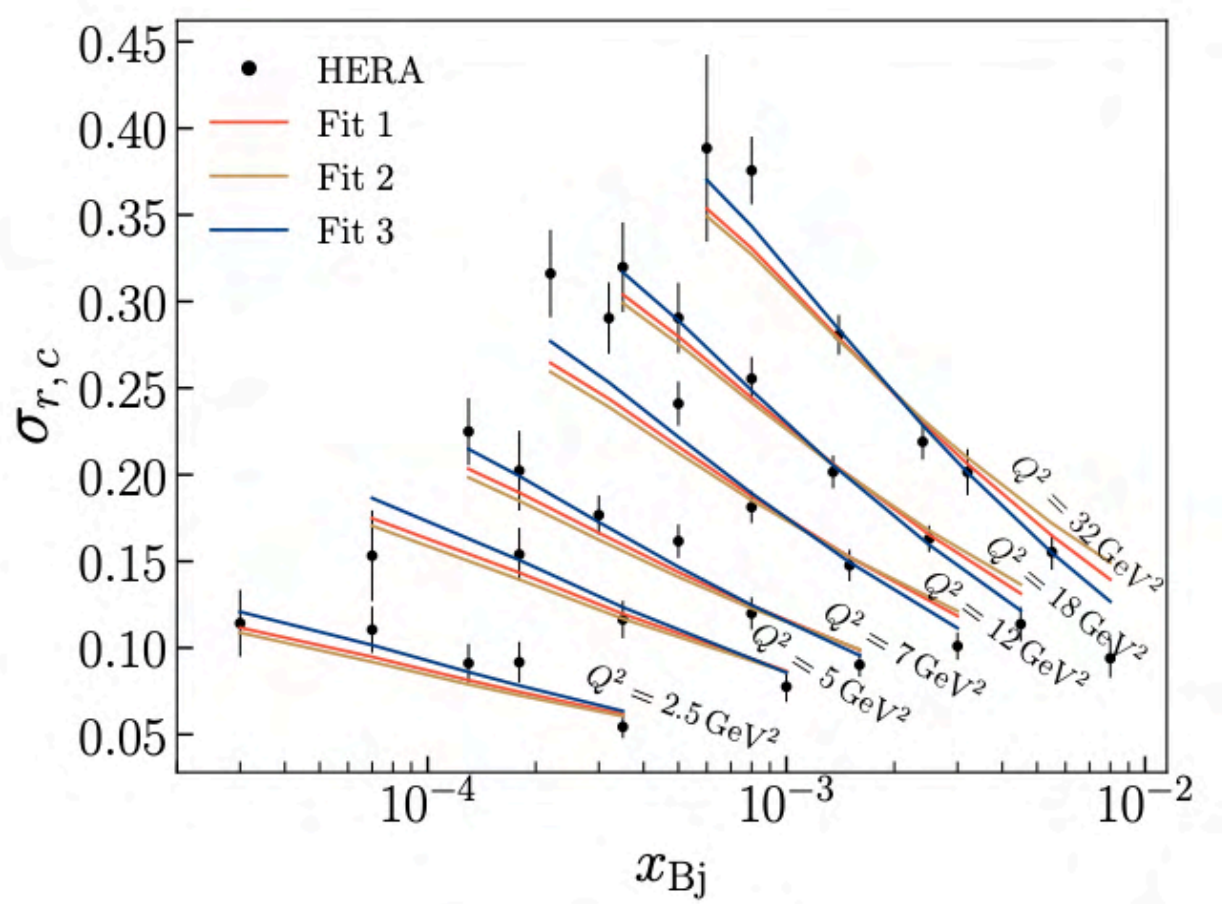
Precision Era for Gluon Saturation

Deep inelastic scattering structure functions

G. Beuf, T. Lappi, H. Hänninen, H. Mäntysaari (2020)



H. Hänninen, H. Mäntysaari, R Paatelainen, J. Penttala (2022)



Theory curves based on CGC with NLO impact factor and (most of) NLL BK equation

Consistent description of light and heavy quark structure functions from HERA

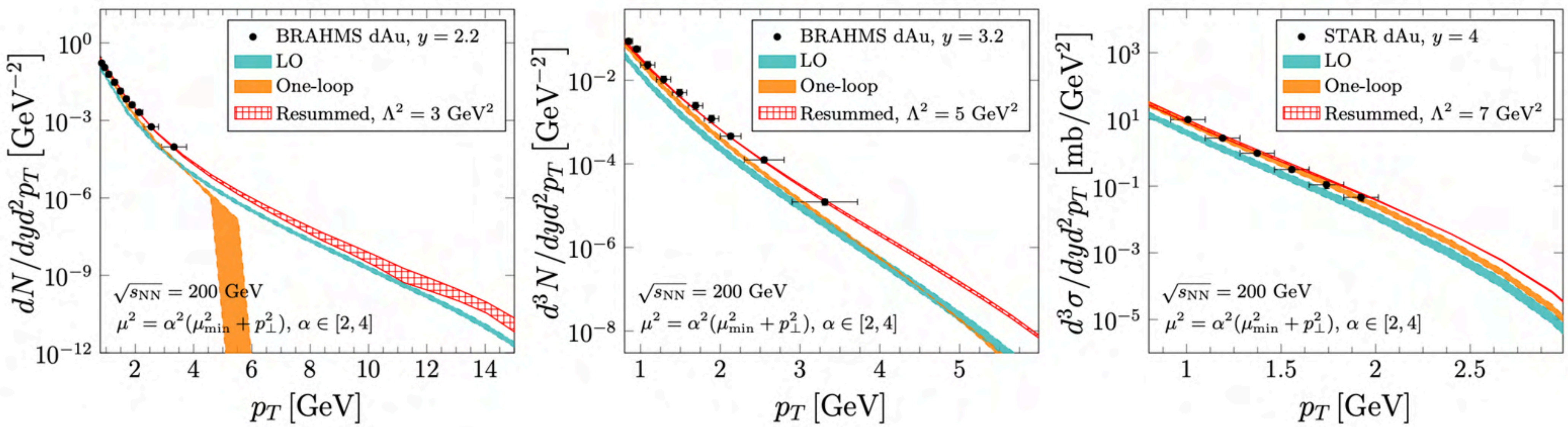
Precision Era for Gluon Saturation

Semi-inclusive single hadron production

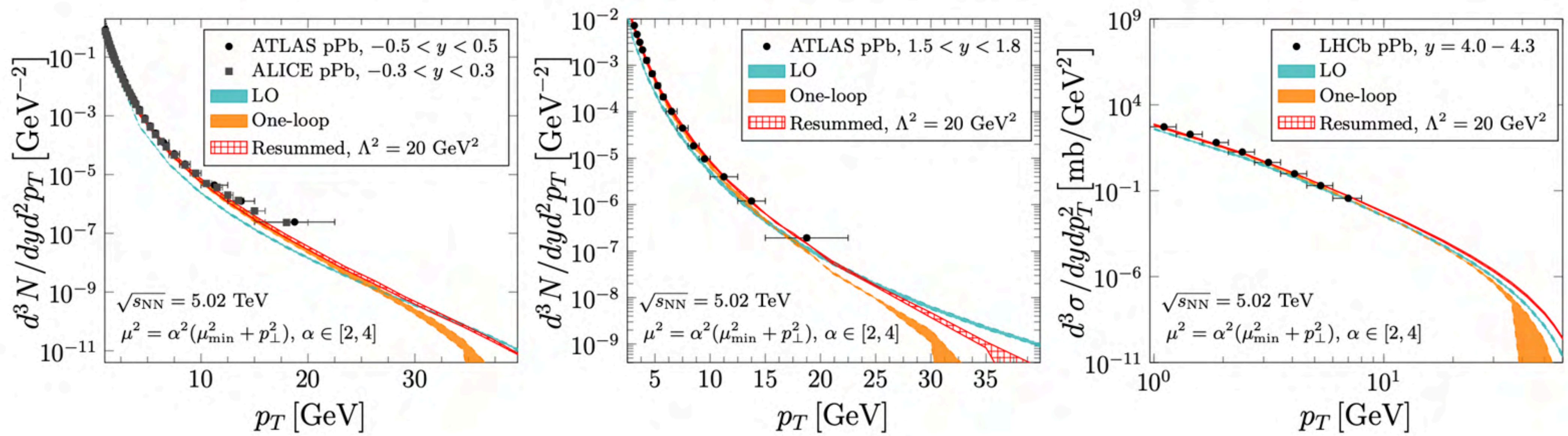
Theory curves based on CGC with NLO impact factor + rcBK evolution + threshold resummation

Comparison to RHIC data

Y. Shi, L. Wang, S. Wei, B. Xiao (2021)



Comparison to LHC data



See also H. Liu, X. Liu, Z. Kang (2020)