Light front time and the rest frame structure of hadrons

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in collaboration with Jerry Miller

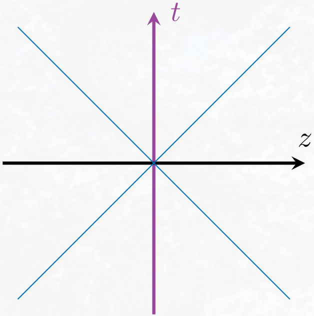
Introduction

- Form factors are extracted from experiment.
- Relationship to spatial densities is controversial.
  - Breit frame?
  - Light front?
  - Phase space formulation?
  - Localized wave packets?
  - Multipole moment densities?
- I won't solve controversy here.
- I propose a new conceptual framework for **light front densities**.
  - I will argue they provide **rest frame densities**.
- This talk is about **phenomenology**.
  - Use form factors from experiment (Kelly 2004, Riordan 2010).
  - I'm not arguing for light front quantization.

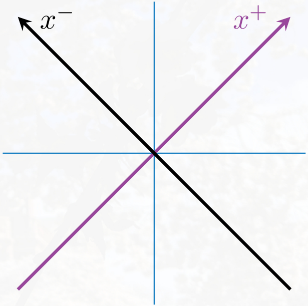
Light front coordinates

- **Light front coordinates** are a different foliation of spacetime.
- Entail a new **synchronization convention**.
- Entail a new spatial grid.

$$x^{\pm} = t \pm z \qquad \mathbf{x}_{\perp} = (x, y) \qquad x^{+} = t + z = \text{time}$$



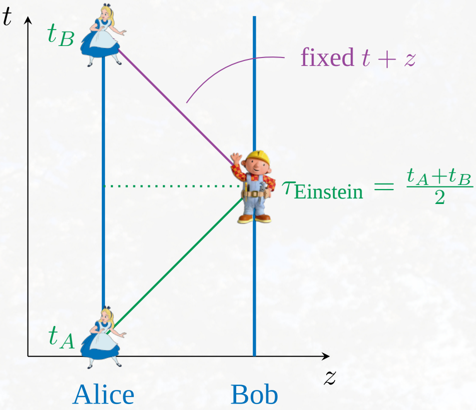
Minkowski coordinates



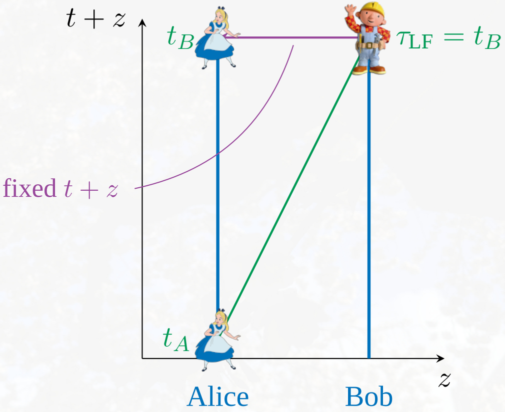
Light front coordinates

Synchronization conventions

Einstein synchronization



Light front synchronization



- **Einstein synchronization** defined to be isotropic.
- **Light front synchronization** defines hyperplanes with fixed $t + z$ to be "simultaneous."
  - Light travels instantaneously in $-z$ direction *by definition.*
  - We take what we see as literally happening now.

Equal-“time” surfaces are just a convention

- Relativity requires *round-trip* speed of light to be invariant.
- Convention that one-way speed of light be c is a *definition*, not an empirical fact.
  - Pointed out in Einstein’s original paper.
- Redefining “time” coordinate means changing this definition.
  - Light front coordinates do exactly this!

894 *A. Einstein.*

B durch einen in B befindlichen Beobachter möglich. Es ist aber ohne weitere Festsetzung nicht möglich, ein Ereignis in A mit einem Ereignis in B zeitlich zu vergleichen; wir haben bisher nur eine „A-Zeit“ und eine „B-Zeit“, aber keine für A und B gemeinsame „Zeit“ definiert. Die letztere Zeit kann nun definiert werden, indem man *durch Definition* festsetzt, daß die „Zeit“, welche das Licht braucht, um von A nach B zu gelangen, gleich ist der „Zeit“, welche es braucht, um von B nach A zu gelangen. Es gehe nämlich ein Lichtstrahl zur „A-Zeit“ t_A von A nach B ab, werde zur „B-Zeit“ t_B in B gegen A zu reflektiert und gelange zur „A-Zeit“ t'_A nach A zurück. Die beiden Uhren laufen definitionsgemäß synchron, wenn

$$t_B - t_A = t'_A - t_B.$$

Einstein, Ann. Phys. 322 (1905) 891

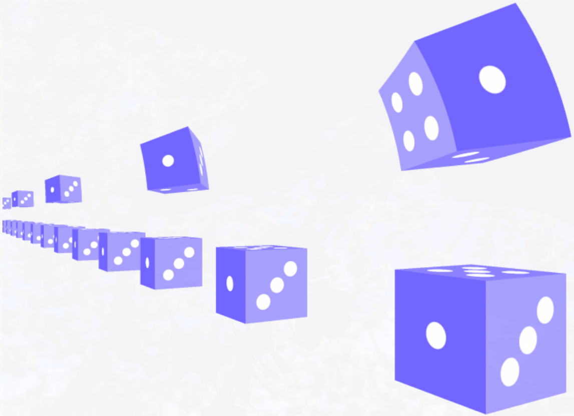
- **Technical review**: Anderson, Stedman & Vetharaniam, Phys. Rept. 295 (1998) 93
- **Didactic overview**: Veritasium, “Why No One Has Measured The Speed of Light” (YouTube)

Transverse boosts and Terrell rotations

- Lorentz-boosted objects *appear rotated*.
  - **Terrell rotation** (PR116, 1959)
  - Optical effect: contraction + delay
- **Light front transverse boost** *undoes* Terrell rotation:

$$B_x^{(\mathrm{LF})} = K_x - J_y$$

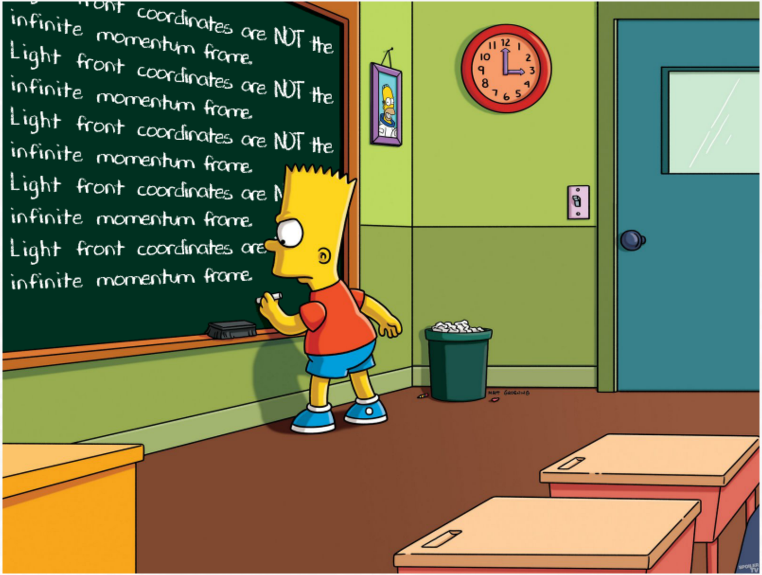
  - Standard boost + counter-rotation
  - Leaves x^+ (time) invariant
  - Part of the **Galilean subgroup**



Dice images by Ute Kraus,
`https://www.spacetimetravel.org/`

Not the IMF!

- All momenta can be finite.
- We didn't boost.
- LFCs are not the IMF.

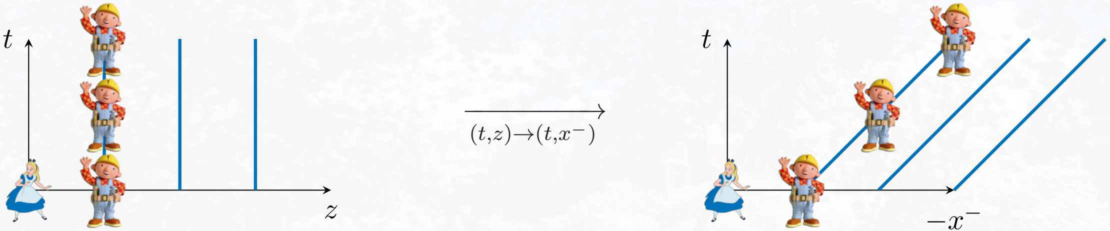


Not the rest frame!

- LFCs are not the IMF.
- They’re also not rest frames.
- They’re not even Cartesian.
- The reason is x^-.

What are reference frames?

- **Reference frame**: a hypothetical grid of reference points that define *spatial* coordinates.
  - Clocks are attached to grid points for time coordinate.
  - Synchronization scheme relates clock times.
  - Synchronization scheme not part of the "frame."
- A grid of (x, y, x^-) points is different than a grid of (x, y, z) points.
- $x^- = \text{fixed} = t - z$ means the LFC grid is *moving at the speed of light*.
- LFCs thus furnish a collection of **light-speed frames**.
  - The frames differ in x^- grid spacing (after longitudinal boost).



Why not use z?

- x^+ makes LFCs nice.
- x^- prevents us from getting rest frames.
- Why not use x^+ and z?

Tilted light front coordinates

Tilted coordinates

$$\tilde{\tau} = t + z$$
$$\tilde{x} = x$$
$$\tilde{y} = y$$
$$\tilde{z} = z$$

- Mind the strange metric…

$$\tilde{g}_{\mu\nu} = \begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ -1 & 0 & 0 & 0 \end{bmatrix}$$

- First defined by Blunden, Burkardt & Miller.
  - Phys. Rev. C61 (2000) 025206
- Use light front time.
  - Use light front synchronization!
  - Time invariant under **Galilean subgroup**.
- Use Cartesian spatial coordinates.
  - Can furnish a **rest frame**!

$$\mathrm{d}s^2 = \mathrm{d}\tilde{\tau}^2 - 2\,\mathrm{d}\tilde{\tau}\,\mathrm{d}\tilde{z} - \mathrm{d}\tilde{\boldsymbol{x}}_\perp^2$$
$$\partial^2 = -2\tilde{\partial}_z\tilde{\partial}_\tau - \tilde{\boldsymbol{\nabla}}^2$$

Momentum and velocity

- Energy & momentum are spacetime translation generators.

$$\mathrm{i}[\tilde{E}, \hat{M}] = \frac{\partial \hat{M}}{\partial \tilde{\tau}} \qquad -\mathrm{i}[\tilde{\boldsymbol{p}}, \hat{M}] = \tilde{\boldsymbol{\nabla}} \hat{M}$$

- On-shell dispersion relation:

$$\tilde{E} = \frac{m^2 + \tilde{\boldsymbol{p}}^2}{2\tilde{p}_z} = \frac{m^2 + \tilde{p}_z^2}{2\tilde{p}_z} + \frac{\tilde{\boldsymbol{p}}_\perp^2}{2\tilde{p}_z}$$

Energy-momentum

$$\tilde{E} = E$$
$$\tilde{p}_x = p_x$$
$$\tilde{p}_y = p_y$$
$$\tilde{p}_z = E + p_x = p^+$$

Velocity

$$\tilde{\boldsymbol{v}} = \boldsymbol{\nabla}_{\boldsymbol{p}} \tilde{E}$$
$$\tilde{v}_x = \tilde{p}_x / \tilde{p}_z$$
$$\tilde{v}_y = \tilde{p}_y / \tilde{p}_z$$
$$\tilde{v}_z = 1 - \tilde{E} / \tilde{p}_z$$

- **Rest** occurs when $\tilde{\boldsymbol{v}} = 0$.

Transverse boost

$$\begin{aligned}
\tilde{\tau}' &= \tilde{\tau} \\
\tilde{x}' &= \tilde{x} - v\tilde{\tau} \\
\tilde{y}' &= \tilde{y} \\
\tilde{z}' &= \tilde{z} + v\tilde{x} - \frac{v^2}{2}\tilde{\tau}
\end{aligned}$$

Longitudinal boost

$$\begin{aligned}
\tilde{\tau}' &= \mathrm{e}^{-\eta}\tilde{\tau} \\
\tilde{x}' &= \tilde{x} \\
\tilde{y}' &= \tilde{y} \\
\tilde{z}' &= \mathrm{e}^{\eta}\tilde{z} - \sinh(\eta)\tilde{\tau}
\end{aligned}$$

- Transverse boosts in **Galilean subgroup**.
- Longitudinal boosts induce redshift & blueshift.
  - **Redshift**: enlarged $\tilde{z}$ spacing, dilated time.
  - **Blueshift**: contracted $\tilde{z}$ spacing, quickened time.

Galilean subgroup

- Poincaré group has a $(2+1)$D **Galilean subgroup**.
  - $\tilde{\tau}$ is time and $\tilde{\boldsymbol{x}}_\perp$ is space under this subgroup.
  - $\tilde{p}_z = E_{\boldsymbol{p}} + p_z = p^+$ is the central charge.
  - $\tilde{\tau}$ and $\tilde{p}_z$ are invariant under this subgroup!

- Light front synchronization gives **fully relativistic** 2D picture that looks a lot like non-relativistic physics.
  - But with $\tilde{p}_z$ in place of m.

$$\frac{\mathrm{d}\tilde{\boldsymbol{p}}_\perp}{\mathrm{d}\tilde{\tau}} = \tilde{p}_z \frac{\mathrm{d}^2\tilde{\boldsymbol{x}}_\perp}{\mathrm{d}\tilde{\tau}^2}$$

$$\tilde{E} = \tilde{E}_{\text{rest}} + \frac{\tilde{\boldsymbol{p}}_\perp^2}{2\tilde{p}_z}$$

$$\tilde{\boldsymbol{v}}_\perp = \frac{\tilde{\boldsymbol{p}}_\perp}{\tilde{p}_z}$$

etc.

- Physical four-current density:

$$\int \mathrm{d}\tilde{z}\,\langle\Psi|\hat{j}^{\mu}(x)|\Psi\rangle = \int \mathrm{d}^3\tilde{\boldsymbol{R}}\,\mathscr{P}^{\mu}{}_{\nu}(\tilde{\boldsymbol{R}},\tilde{\tau},\Psi)\,\tilde{j}^{\nu}_{\text{internal}}(\tilde{\boldsymbol{x}}_{\perp}-\tilde{\boldsymbol{R}}_{\perp})$$

Internal density

Smearing function

invariant under LF boosts

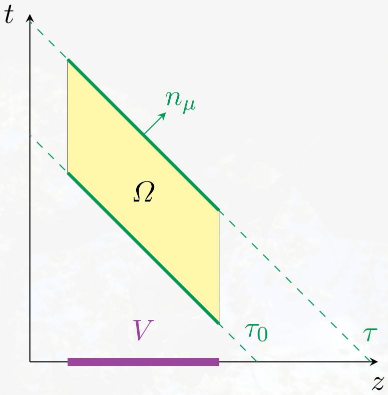
  - **Smearing function** contains all wave packet & velocity dependence.
  - Only **smearing function** modified by Lorentz boosts.
  - **Internal density** is boost-invariant. (due to Galilean subgroup)
  - **Internal density** is rest frame density!
- $\tilde{z}$ *still* must be integrated out for initial & final state to have same central charge.
  - That's why we're stuck with 2D densities.
  - But we made it clear we're dealing with ordinary space.

- Charge density *at fixed* $\tilde{\tau} = t + z$.
  - Since we're using light front synchronization.
- Charge density given by:

$$\tilde{j}^0 = j^0 + j^3 = j^+$$

- Temporal part of continuity equation:

$$\tilde{\partial}_\mu \tilde{j}^\mu = \frac{\partial \tilde{j}^0}{\partial \tilde{\tau}} + \tilde{\boldsymbol{\nabla}} \cdot \tilde{\boldsymbol{j}} = 0$$

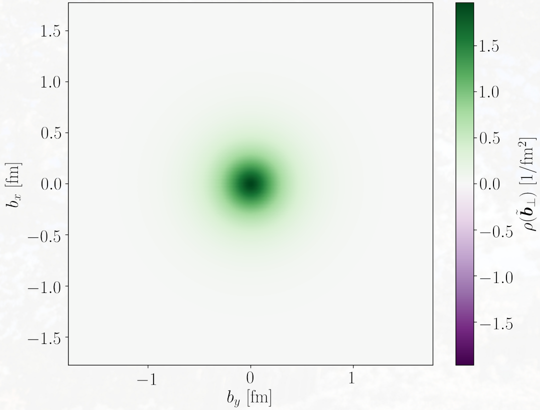


- Simple formula due to invariance under **Galilean subgroup**:

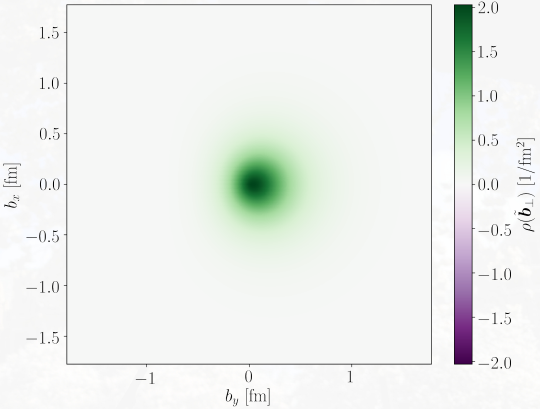
$$\tilde{j}^0_{\text{internal}}(\tilde{\boldsymbol{b}}_\perp, \hat{\boldsymbol{s}}) = \int \frac{\mathrm{d}^2 \tilde{\boldsymbol{\Delta}}_\perp}{(2\pi)^2} \frac{\langle p', \hat{\boldsymbol{s}} | \hat{j}^+(0) | p, \hat{\boldsymbol{s}} \rangle}{2p^+} \mathrm{e}^{-\mathrm{i}\tilde{\boldsymbol{\Delta}}_\perp \cdot \tilde{\boldsymbol{b}}_\perp}$$

$$\tilde{j}^0(\tilde{\boldsymbol{b}}_\perp, \hat{\boldsymbol{s}}) = \int \frac{\mathrm{d}^2\tilde{\boldsymbol{\Delta}}_\perp}{(2\pi)^2} \left(F_1(-\tilde{\boldsymbol{\Delta}}_\perp^2) + \frac{(\hat{\boldsymbol{s}} \times \mathrm{i}\tilde{\boldsymbol{\Delta}}_\perp)\cdot\hat{z}}{2m} F_2(-\tilde{\boldsymbol{\Delta}}_\perp^2) \right) \mathrm{e}^{-\,\mathrm{i}\tilde{\boldsymbol{\Delta}}_\perp\cdot\tilde{\boldsymbol{b}}_\perp} \,,$$

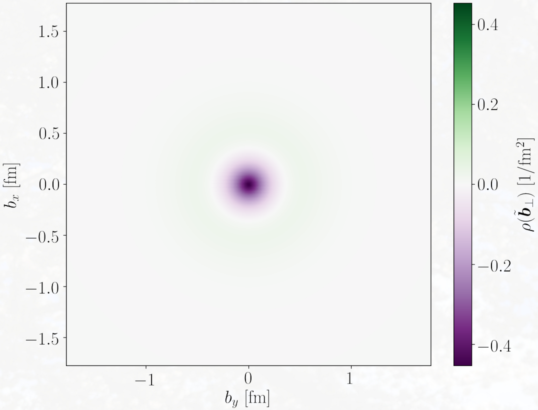
Longitudinal polarization



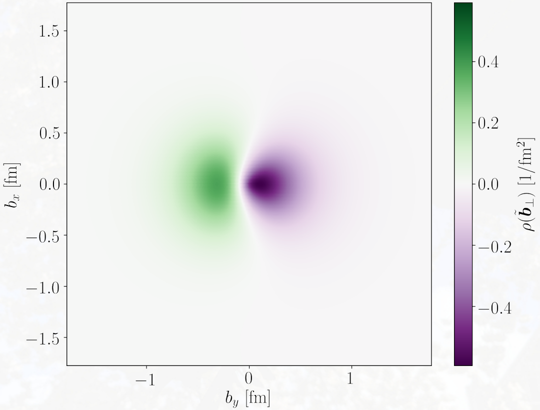
Transverse polarization



Longitudinal polarization



Transverse polarization

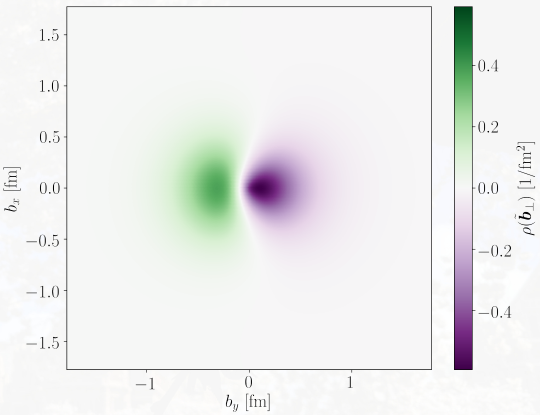


- Longitudinal polarization: negative core & diffuse positive cloud
  - Reproduces Miller, Phys. Rev. Lett. 99 (2007) 112001
- Transverse polarization: apparent electric dipole
  - Reproduces Carlson & Vanderhaegen, Phys. Rev. Lett. 100 (2008) 032004 (up to a sign)

So why modulations?

- Charge density of transpol. neutron.
  - Spin up ↑ along vertical axis.
- This is the charge density in every frame.
  - Includes the rest frame.
- Not an IMF artifact!
  - I never went to the IMF.
- Effect of **synchronization scheme**.
  - Effect of taking what we see literally.
  - This is a known effect; relativistic wheel.
  - Explained by George Gamow in 1938, *Mr Tompkins in Wonderland*

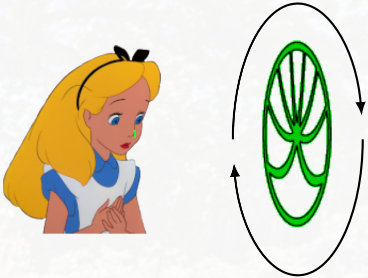
Trans. pol. neutron



The relativistic wheel

Static wheel

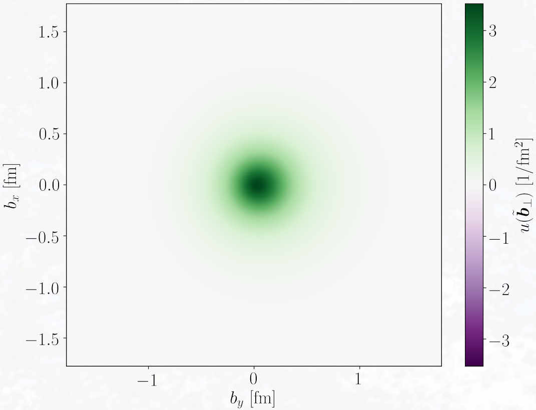
Spinning wheel



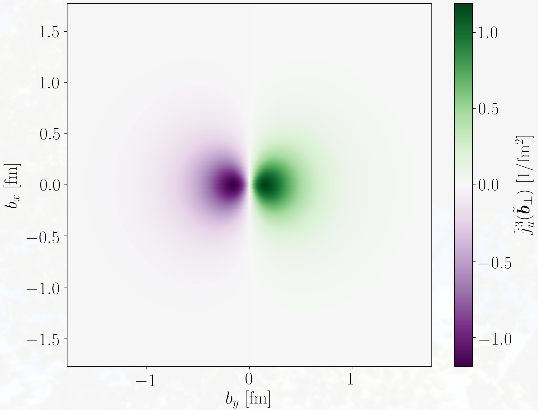
- **Static wheel** has regularly-placed spokes.
- Consider **spinning wheel**, axis oblique to observer.
  - *The wheel is considered at rest.*
- Spokes moving away are **redshifted**.
  - *Appear to* move slower.
  - Pile up; *appear to* become denser.
- Spokes moving towards are **blueshifted**.
  - *Appear to* move faster.
  - *Appear to* become rarer.
- These same distortions are present in nucleons!
  - **Light front densities bake in optical effects.**
- Also see videos at:
  `https://www.spacetimetravel.org/rad`
  (green wheel is relevant case)

Up quark density & current in the proton

Up quark density



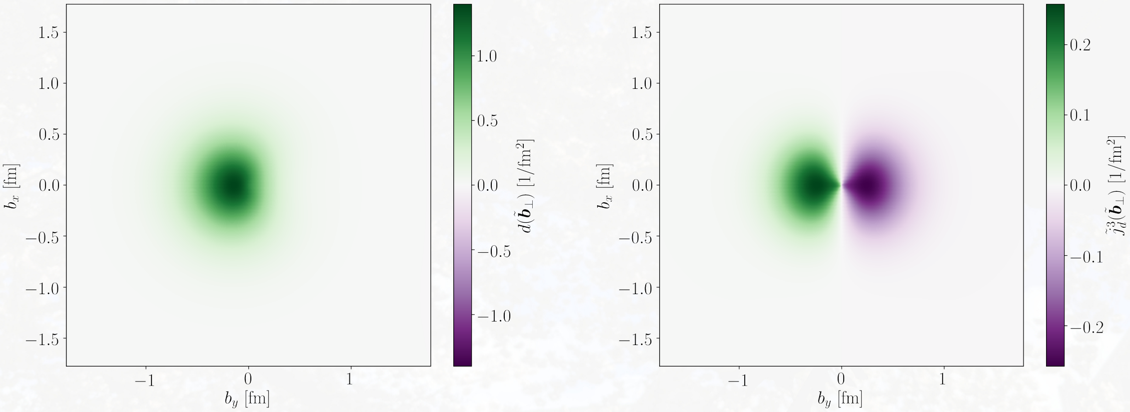
Up quark current ($\tilde{z}$ component)



- Convert proton & neutron $\rightarrow$ up & down (**flavor separation**).
- Small distortion for up quarks, but consistent with wheel picture.
- Purple means towards, green means away.

Down quark density & current in the proton

Down quark density | **Down quark current** ($\tilde{z}$ component)



- Bigger distortion in down quarks!
- Orbit & bunching in opposite direction from up quark.
- Purple means towards, green means away.

How the proton appears (rough estimates)

- Up quarks orbit along with proton spin.

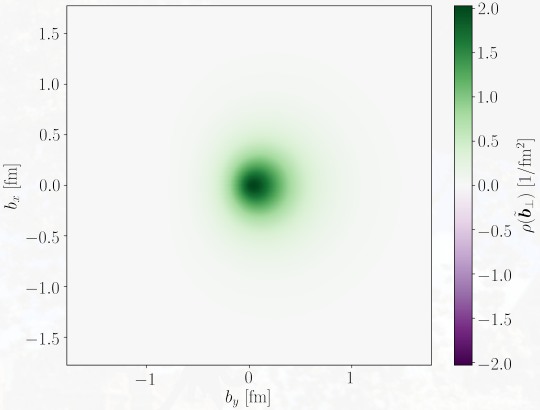
$$\omega_u \approx \quad 0.417\ c/\mathrm{fm} = \quad 125\ \mathrm{ZHz}$$

- Down quarks orbit (much faster) against proton spin.

$$\omega_d \approx -0.922\ c/\mathrm{fm} = -276\ \mathrm{ZHz}$$

- Constructively contribute to *apparent* dipole moment.
  - In transversely polarized states.
- Would be what a viewer really sees!
  - Known effect: the relativistic wheel.

Trans. pol. proton



Energy density

Momentum densities

$$\tilde{T}^{\mu}{}_{\nu}(x) = \begin{bmatrix} \tilde{T}^{0}{}_{0}(x) & \tilde{T}^{0}{}_{1}(x) & \tilde{T}^{0}{}_{2}(x) & \tilde{T}^{0}{}_{3}(x) \\ \tilde{T}^{1}{}_{0}(x) & \tilde{T}^{1}{}_{1}(x) & \tilde{T}^{1}{}_{2}(x) & \tilde{T}^{1}{}_{3}(x) \\ \tilde{T}^{2}{}_{0}(x) & T^{2}{}_{1}(x) & T^{2}{}_{2}(x) & T^{2}{}_{3}(x) \\ \tilde{T}^{3}{}_{0}(x) & \tilde{T}^{3}{}_{1}(x) & \tilde{T}^{3}{}_{2}(x) & \tilde{T}^{3}{}_{3}(x) \end{bmatrix}$$

Energy fluxes

Stress tensor

- All 16 components of EMT have clear meaning in tilted coordinates.
- The **energy density** integrates to the usual “instant form” energy.

$$\tilde{E} = E$$

  - *Relativistically exact* energy density.
  - Will give standard mass decomposition.
  - Can describe system at rest.
- Work in progress!

Smearing functions revisited

- Physical energy-momentum tensor:

$$\int \mathrm{d}\tilde{z}\, \langle \Psi | \hat{T}^{\mu}{}_{\nu}(x) | \Psi \rangle = \int \mathrm{d}^3 \tilde{\boldsymbol{R}}\, \mathscr{P}^{\mu}{}_{\nu\alpha}{}^{\beta}(\tilde{\boldsymbol{R}}, \tilde{\tau}, \Psi) [\tilde{T}^{\alpha}{}_{\beta}(\tilde{\boldsymbol{x}}_{\perp} - \tilde{\boldsymbol{R}}_{\perp})]_{\text{internal}}$$

Internal density

Smearing function

invariant under LF boosts

  - **Smearing function** contains all wave packet & velocity dependence.
  - Only **smearing function** modified by Lorentz boosts.
  - **Internal density** is boost-invariant. (due to Galilean subgroup)
  - **Internal density** is rest frame density!
- Separating **smearing function** and **internal density** is ambiguous.
- Need a fixed scheme for doing this separation.

A classical heuristic

- Classical system, at rest & at the origin, has an EMT:

$$\Theta^{\mu}{}_{\nu}(\tilde{\boldsymbol{x}}, \tilde{\tau}) = \Theta^{\mu}{}_{\nu}(\tilde{\boldsymbol{x}})$$

  - Stationary system: no explicit time dependence.
- Boost via Λ, then translate by $\tilde{\boldsymbol{r}}$:

$$T^{\mu}{}_{\nu}(\tilde{\boldsymbol{x}}; \Lambda, \tilde{\boldsymbol{r}}) = \Lambda^{\mu}{}_{\alpha} \Lambda_{\nu}{}^{\beta} \Theta^{\alpha}{}_{\beta}(\Lambda^{-1}[\tilde{\boldsymbol{x}} - \tilde{\boldsymbol{r}}])$$

- Unknown Λ & $\tilde{\boldsymbol{r}}$ with probability distribution ρ:

$$\langle T^{\mu}{}_{\nu}\rangle(\tilde{\boldsymbol{x}}) = \int \mathrm{d}\mu(\Lambda) \int \mathrm{d}^3\tilde{\boldsymbol{r}}\, \rho(\tilde{\boldsymbol{r}}, \Lambda) \Lambda^{\mu}{}_{\alpha} \Lambda_{\nu}{}^{\beta} \Theta^{\alpha}{}_{\beta}(\Lambda^{-1}[\tilde{\boldsymbol{x}} - \tilde{\boldsymbol{r}}])$$

- Can I just first-quantize this?

$$\langle T^{\mu}{}_{\nu}\rangle(\tilde{\boldsymbol{x}}) \xrightarrow[\text{quantize}]{} \mathrm{Tr}\left\{ \hat{\rho} \mathcal{Q}\left[\Lambda^{\mu}{}_{\alpha} \Lambda_{\nu}{}^{\beta} \Theta^{\alpha}{}_{\beta}(\Lambda^{-1}[\tilde{\boldsymbol{x}} - \tilde{\boldsymbol{r}}]) \right] \right\}$$

  - If so, what is the quantization map $\mathcal{Q}$?
  - AF, in preparation (no arxiv preprint yet)

Weyl quantization

- By **first quantization**, I mean promoting $\tilde{\boldsymbol{r}}$ & $\tilde{\boldsymbol{p}}$ to operators $\hat{\tilde{\boldsymbol{R}}}$ & $\hat{\tilde{\boldsymbol{P}}}$.
  - Lorentz boost Λ encodes hadron's momentum $\tilde{\boldsymbol{p}}$.
- Ambiguous in general; the following are all operators representing $\tilde{x}^4\tilde{p}_x^2$:
  - $\hat{\tilde{X}}^2\hat{\tilde{P}}_x^2\hat{\tilde{X}}^2$
  - $\frac{1}{2}\left(\hat{\tilde{X}}^4\hat{\tilde{P}}_z^2 + \hat{\tilde{P}}_z^2\hat{\tilde{X}}^4\right)$
  - $\hat{\tilde{P}}_z\hat{\tilde{X}}^4\hat{\tilde{P}}_z$
- **Weyl quantization** provides a fixed scheme for first-quantizing classical expressions.

$$\mathcal{Q}[f(\tilde{\boldsymbol{r}},\tilde{\boldsymbol{p}})] = \int \frac{\mathrm{d}^3\tilde{\boldsymbol{r}}}{(2\pi)^3} \int \frac{\mathrm{d}^3\tilde{\boldsymbol{p}}}{(2\pi)^3} \int \mathrm{d}^3\boldsymbol{a} \int \mathrm{d}^3\boldsymbol{b}\, f(\tilde{\boldsymbol{r}},\tilde{\boldsymbol{p}})\, \mathrm{e}^{\mathrm{i}\boldsymbol{a}\cdot(\hat{\tilde{\boldsymbol{R}}}-\tilde{\boldsymbol{r}})+\mathrm{i}\boldsymbol{b}\cdot(\hat{\tilde{\boldsymbol{P}}}-\tilde{\boldsymbol{p}})}$$

  - Weyl, Zeitschrift für Physik 46 (1927) 1

McCoy's formula

- Formula for Weyl quantization due to Neal McCoy:

$$\mathcal{Q}[\tilde{x}^r \tilde{p}_x^s] = \frac{1}{2^r} \sum_{k=0}^{r} \frac{r!}{k!(r-k)!} \hat{\tilde{P}}_x^k \hat{\tilde{X}}^s \hat{\tilde{P}}_x^{r-k}$$

  - McCoy, Proc. NAS 18 (1932) 674
  - Need canonical commutation relations to hold!
- Very helpful in **momentum representation**:

$$\begin{aligned}\mathrm{Tr}\left\{\hat{\rho}\mathcal{Q}[\tilde{x}^r \tilde{p}_x^s]\right\} &= \frac{1}{2^r} \sum_{k=0}^{r} \frac{r!}{k!(r-k)!} \int \frac{\mathrm{d}^3\tilde{\boldsymbol{p}}}{2\tilde{p}_z(2\pi)^3} \int \frac{\mathrm{d}^3\tilde{\boldsymbol{p}}'}{2\tilde{p}_z'(2\pi)^3} \langle\tilde{\boldsymbol{p}}|\hat{\rho}|\tilde{\boldsymbol{p}}'\rangle\langle\tilde{\boldsymbol{p}}'|\hat{\tilde{P}}_x^k \hat{\tilde{X}}^s \hat{\tilde{P}}_x^{r-k}|\tilde{\boldsymbol{p}}\rangle \\ &= \int \frac{\mathrm{d}^3\tilde{\boldsymbol{p}}}{2\tilde{p}_z(2\pi)^3} \int \frac{\mathrm{d}^3\tilde{\boldsymbol{p}}'}{2\tilde{p}_z'(2\pi)^3} \tilde{P}_x^r \langle\tilde{\boldsymbol{p}}|\hat{\rho}|\tilde{\boldsymbol{p}}'\rangle\langle\tilde{\boldsymbol{p}}'|\hat{\tilde{X}}^s|\tilde{\boldsymbol{p}}\rangle\end{aligned}$$

  - $\tilde{\boldsymbol{P}} = \frac{1}{2}\left(\tilde{\boldsymbol{p}} + \tilde{\boldsymbol{p}}'\right)$ is average between initial & final momentum.
  - This variable appears in form factor decompositions.
  - The boosted & translated EMT can be expanded as a formal series in position & momentum.

- Transverse position operators are local:

$$\langle\tilde{\boldsymbol{p}}|\hat{\tilde{X}}^i_\perp|\Psi\rangle = \mathrm{i}\frac{\partial}{\partial\tilde{p}_i}\Big[\langle\tilde{\boldsymbol{p}}|\Psi\rangle\Big]$$

- Longitudinal position non-local; Newton-Wigner-like:

$$\langle\tilde{\boldsymbol{p}}|\hat{\tilde{Z}}|\Psi\rangle = \mathrm{i}\left(\frac{\partial}{\partial\tilde{p}_z} - \frac{1}{2\tilde{p}_z}\right)\Big[\langle\tilde{\boldsymbol{p}}|\Psi\rangle\Big]$$

- Non-local $\hat{\tilde{Z}}$ leads to inconsistent $\tilde{\boldsymbol{P}}$ dependence between quantized heuristic and QFT.

$$\mathrm{Tr}\Big\{\hat{\rho}[\hat{T}^\mu{}_\nu(x)]_{\mathrm{QFT}}\Big\} \neq \mathrm{Tr}\Big\{\hat{\rho}\mathcal{Q}\Big[\Lambda^\mu{}_\alpha\Lambda_\nu{}^\beta\Theta^\alpha{}_\beta(\Lambda^{-1}[\tilde{\boldsymbol{x}}-\tilde{\boldsymbol{r}}])\Big]\Big\}$$

- Inconsistencies can be removed by integrating out $\tilde{z}$.

$$\int_{\mathbb{R}}\mathrm{d}\tilde{z}\,\mathrm{Tr}\Big\{\hat{\rho}[\hat{T}^\mu{}_\nu(x)]_{\mathrm{QFT}}\Big\} = \int_{\mathbb{R}}\mathrm{d}\tilde{z}\,\mathrm{Tr}\Big\{\hat{\rho}\mathcal{Q}\Big[\Lambda^\mu{}_\alpha\Lambda_\nu{}^\beta\Theta^\alpha{}_\beta(\tilde{\boldsymbol{x}}_\perp-\tilde{\boldsymbol{r}}_\perp)\Big]\Big\}$$

  - Λ^{-1} dropped due to Galilean subgroup.
  - Consistent $\tilde{\boldsymbol{P}}$ dependence requires defining $\Lambda(\tilde{\boldsymbol{P}})$ correctly.

Quantum Lorentz boosts

- *Classical* dispersion relation (tilted coordinates):

$$m^2 = 2\tilde{E}\tilde{p}_z - \tilde{\boldsymbol{p}}^2$$

  - Entails *classical* velocity formulas:

$$\tilde{\boldsymbol{v}}_\perp = \frac{\tilde{\boldsymbol{p}}_\perp}{\tilde{p}_z} \qquad \tilde{v}_z = \frac{1}{2}\left(1 - \frac{m^2 + \tilde{\boldsymbol{p}}_\perp^2}{\tilde{p}_z}\right)$$

- $\tilde{\boldsymbol{P}}$ is an *average* of two on-shell momenta.
  - $\tilde{\boldsymbol{p}}$ & $\tilde{\boldsymbol{p}}'$ obey classical (on-shell) dispersion relation.

$$m^2 - \frac{1}{4}\Delta^2 = 2\tilde{P}_0\tilde{P}_z - \tilde{\boldsymbol{P}}^2 \xrightarrow[\Delta_z=0]{} m^2 + \frac{1}{4}\tilde{\boldsymbol{\Delta}}_\perp^2 = 2\tilde{P}_0\tilde{P}_z - \tilde{\boldsymbol{P}}^2$$

  - $\Delta = p' - p$
  - Integrating out $\tilde{z}$ sets $\Delta_z = 0$.
  - Entails *average* velocity formulas (for $\tilde{\Delta}_z = 0$ *only*):

$$\tilde{\boldsymbol{V}}_\perp = \frac{\tilde{\boldsymbol{P}}_\perp}{\tilde{P}_z} \qquad \tilde{V}_z = \frac{1}{2}\left(1 - \frac{m^2 + \frac{1}{4}\tilde{\boldsymbol{\Delta}}_\perp^2 + \tilde{\boldsymbol{P}}_\perp^2}{\tilde{P}_z}\right)$$

  - It's $\tilde{\boldsymbol{V}}$, not $\tilde{\boldsymbol{v}}$, that must be used in quantized Lorentz boosts.

Average rest

- This changes the kinematic "rest" condition.
- Classically,

$$\tilde{\boldsymbol{p}}_{\text{rest}} = (0, 0, m)$$

- Quantum-mechanically,

$$\tilde{\boldsymbol{P}}_{\text{rest}} = \left(0, 0, \sqrt{m^2 + \frac{1}{4}\tilde{\boldsymbol{\Delta}}_\perp^2}\right)$$

- This makes *nearly all* tilted coordinate densities *different than* light front coordinate densites!
  - Charge density is an exception.

Smearing functions revisited (again)

- Physical energy-momentum tensor:

$$\int \mathrm{d}\tilde{z}\,\langle\Psi|\hat{T}^{\mu}{}_{\nu}(x)|\Psi\rangle = \int \mathrm{d}^3\tilde{\boldsymbol{R}}\,\mathscr{P}^{\mu}{}_{\nu\alpha}{}^{\beta}(\tilde{\boldsymbol{R}},\tilde{\tau},\Psi)[\tilde{T}^{\alpha}{}_{\beta}(\tilde{\boldsymbol{x}}_\perp-\tilde{\boldsymbol{R}}_\perp)]_{\text{internal}}$$

Internal density

Smearing function

invariant under LF boosts

- Quantized heuristic gives **smearing function**:

$$\mathscr{P}^{\mu}{}_{\nu\alpha}{}^{\beta}(\tilde{\boldsymbol{R}},\tilde{\tau},\Psi) = \int \frac{\mathrm{d}^3\tilde{\boldsymbol{P}}}{2\tilde{P}_z(2\pi)^3}\langle\tilde{\boldsymbol{p}}|\hat{\rho}|\tilde{\boldsymbol{p}}'\rangle\Lambda^{\mu}{}_{\alpha}(\tilde{\boldsymbol{V}})\Lambda_{\nu}{}^{\beta}(\tilde{\boldsymbol{V}})\,\mathrm{e}^{\mathrm{i}\tilde{\boldsymbol{P}}\cdot\tilde{\boldsymbol{R}}}$$

- Quantized heuristic gives **internal, rest frame density**:

$$[\tilde{T}^{\alpha}{}_{\beta}(\tilde{\boldsymbol{x}}_\perp-\tilde{\boldsymbol{R}}_\perp)]_{\text{internal}} = \int\frac{\mathrm{d}^2\tilde{\boldsymbol{\Delta}}_\perp}{(2\pi)^3}\frac{\langle\tilde{\boldsymbol{p}}'|\hat{T}^{\alpha}{}_{\beta}(0)|\tilde{\boldsymbol{p}}\rangle}{2m\sqrt{1+\tilde{\boldsymbol{\Delta}}_\perp^2/4m^2}}\,\mathrm{e}^{-\mathrm{i}\tilde{\boldsymbol{\Delta}}_\perp\cdot(\tilde{\boldsymbol{x}}_\perp-\tilde{\boldsymbol{R}}_\perp)}\Big|_{\tilde{\boldsymbol{P}}=\tilde{\boldsymbol{P}}_{\text{rest}}}$$

- **Gravitational form factors** defined via (spin-zero target):

$$\langle p'|\hat{T}^{\mu\nu}(0)|p\rangle = 2P^\mu P^\nu A(-\Delta^2) + \frac{1}{2}\Big(\Delta^\mu\Delta^\nu - g^{\mu\nu}\Delta^2\Big)D(-\Delta^2)$$

- **Traditional light front energy density** (spin-zero target):

$$\mathcal{E}_{\mathrm{LF}}(\boldsymbol{b}_\perp) = m\int\frac{\mathrm{d}^2\boldsymbol{\Delta}_\perp}{(2\pi)^2}\left[\left(1+\frac{\boldsymbol{\Delta}_\perp^2}{4m^2}\right)A(-\boldsymbol{\Delta}_\perp^2) + \frac{\boldsymbol{\Delta}_\perp^2}{2m^2}D(-\boldsymbol{\Delta}_\perp^2)\right]\mathrm{e}^{-\mathrm{i}\boldsymbol{\Delta}_\perp\cdot\boldsymbol{b}_\perp}$$

- **Tilted energy density** (spin-zero target):

$$\tilde{\mathcal{E}}(\tilde{\boldsymbol{b}}_\perp) = m\int\frac{\mathrm{d}^2\tilde{\boldsymbol{\Delta}}_\perp}{(2\pi)^2}\frac{1}{\sqrt{1+\frac{\tilde{\boldsymbol{\Delta}}_\perp^2}{4m^2}}}\left[\left(1+\frac{\tilde{\boldsymbol{\Delta}}_\perp^2}{4m^2}\right)A(-\tilde{\boldsymbol{\Delta}}_\perp^2) + \frac{\tilde{\boldsymbol{\Delta}}_\perp^2}{4m^2}D(-\tilde{\boldsymbol{\Delta}}_\perp^2)\right]\mathrm{e}^{-\mathrm{i}\tilde{\boldsymbol{\Delta}}_\perp\cdot\tilde{\boldsymbol{b}}_\perp}$$

  - Looks like Polyakov & Schweitzer's energy density with z integrated out.
  - (See Eq. (28a) of their Int.J.Mod.Phys.A 33 (2018) 1830025)

Pion energy density

- Phenomenological form factors:

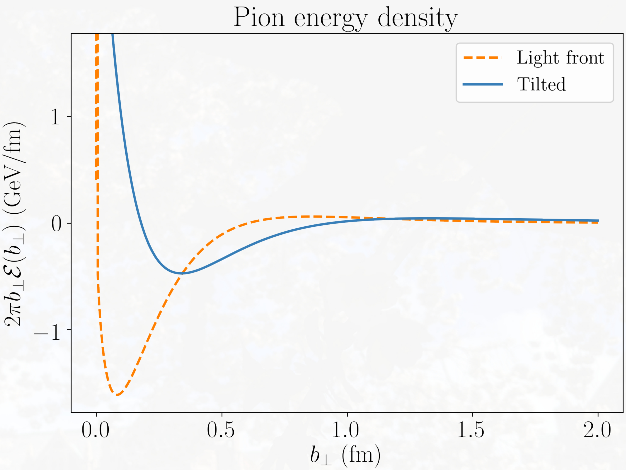
$$A(t) = \frac{1}{1 - t/m_{f_2}^2}$$

$$D(t) = \frac{-1}{(1 - t/m_{f_2}^2)(1 - t/m_\sigma^2)}$$

$$m_{f_2} = 1270 \text{ MeV}$$

$$m_\sigma = 630 \text{ MeV}$$

  - Forms inspired by Masjuan *et al* [PRD87 (2013) 014005]
  - Poles match Kumano's slopes [PRD97 (2018) 014020]
- AF, in preparation



- Densities differ tremendously.
  - Tilted density is **rest frame density**.
  - Light front has a delta; smeared for visibility.

The End

Thank you for your time!